

## SOLVING FUZZY DYNAMIC SHIP ROUTING AND SCHEDULING PROBLEM THROUGH MODIFIED GENETIC ALGORITHM

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**Abstract:** *Main motto of ship routing and scheduling is to reduce the total transportation cost of each ship or vessel without interrupting the demand and supply. In this study, we have proposed a ship routing and scheduling model for commercial ships where, to ensure unhindered demand and supply of products at various ports in a fixed time frame, the dynamic demand and supply of each port were considered under a fuzzy environment. Additionally, simultaneous loading and unloading and a fixed load factor is used to minimize port time and reduce risks, and this aspect of our work makes it realistically inclined. We also show, in our work, speed optimization to reduce fuel consumption and carbon emission. In practice, cost parameters cannot be always determined, it fluctuates at a certain range from time to time. We have treated the imprecise cost parameters as triangular fuzzy numbers. With a view to working with the developed model, a modified genetic algorithm (MGA) with a new selection technique, namely an in-vitro-fertilization-based crossover, and a generation-dependent mutation is proposed. The proposed sustainable ship routing algorithm with dynamic demand and supply in an uncertain environment gives a novelty in the literature. Another novelty is incurred through the proposed MGA in the heuristic search algorithms. This algorithm has produced numerical results superior to those of other heuristic algorithms. We have also established the efficiency of the proposed algorithm through statistical experiments.*

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## 1. Introduction

Shipping industries play an important role in the growth of the economy of a country, and forms a core part of its trade. In the 20<sup>th</sup> century, the maritime transportation grew exponentially. Bulk cargo is generally moved either by sea, or by road, or, by air. Seaborne trade is the cheapest form, and it has spread to 89.6% of total global trade in terms of volume and 70.1% in terms of value in the recent years. In ship routing, each ship starts its journey from initial ports at the beginning of the planning horizon and visits from one port to another port with its containers. A ship visits a set of ports within any planning horizon, and at each of these ports some loading/unloading operations are carried out. To transport the cargos, a heterogeneous fleet of ships are used. There are a fixed capacity and sailing speed for each individual ship. This model aims to design appropriate shipping routes and schedules to minimizing the costs associated with transportation. The present investigation considered heterogeneous ships with a variety of capacities. Determining the navigation of ships or vessels in the maritime environment is a ship routing problem that is similar to the vehicle routing problem on land. Some of the challenges in ship routing and scheduling are as follows:

1. Because a trip may last for numerous days, each ship has a fixed 'time window' to operate at the port. A fixed time is allotted to each ship to complete the operation (loading/unloading) at the port.
2. A linear ship route is not always possible because of several factors.
3. Sometimes two captains may not agree on the same route for the same container for several reasons.
4. Because of variable weather conditions, maritime transport is highly uncertain. Calculation of the traveling time and cost is difficult because the travel time is affected by the wind speed, direction, and current.

Commercial ship operations are categorized as follows: 1) linear, 2) tramp and 3) industrial. Similar to bus transport, a linear operation in sea involves visiting all ports in its route, and ending at the destination port. Tramp ships are analogous to taxis, and their goal is to maximize profit while ensuring unhindered demand and supply of products. Industrial shipment minimizes the cost of shipping products by using the best route. Cargo is allotted to ships at the source port, which is then transported to the destination. Industrial shipping is of two types, namely cargo routing and inventory routing. The cargo routing problem is specified by the time frame for loading and unloading operations, and the demand and supply at specific ports may not be fixed. By contrast, in the inventory routing problem, the product requirement at a particular port is fixed. In this research, we have focused on the cargo routing problem.

In practice, the demand and supply of products at the port are not fixed, it varies from time to time. To satisfy this condition, the loading/unloading operation of some products is determined spontaneously at the port according to the demand/supply. Because of uncertain market conditions, dynamic demand/supply is typical. Loading and unloading operations are performed simultaneously to optimize the port time as well as port congestion. In this study, we considered those operations simultaneously where there is spontaneous and simultaneous loading/unloading. The traveling costs depend on various factors, such as the geographical areas, the weather conditions

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during the sailing period, and the product being transported, and hence are considered as vague or fuzzy in this model. We have thus worked to minimize the shipping cost and maintain sustainability keeping uncertainty in mind, at the same time ensuring that all cargoes are lifted from the loading port and discharged at the unloading port. This model used container-based cargo to transport goods. Risk factors such as, thunderstorms, tsunamis, icebergs, and piracy, which depends on the time period of sailing and choosing the routes, are also considered as fuzzy numbers. The total risk is considered to be less than the maximum risk which is assigned for the entire trip of that ship. It is this fuzzy approach renders the problem realistic.

### **1.1. Motivation**

In ship routing and scheduling problems, a ship with a fixed capacity starts its voyage and visits a set of ports in a fixed time frame to satisfy the demand and supply of those ports. A port can be visited by only one ship at a particular time. De et al. (2017) considered numerous sub-time window concepts in ship routing and scheduling problems. The travelling time of a ship is the time to travel from one port to another port and its operation time is the time needed mainly for loading and unloading. However, within a fixed time frame, due to lack of proper scheduling of ships, there is a delay in cargo transportation from the origin port to the destination port. A penalty cost is imposed for early/late arrival or loading/unloading delay. If any ship exceeds the fixed time frame, it had to pay a penalty calculated according to the additional hours it operates outside the fixed time frame. In this model, loading and unloading operations of various goods is performed simultaneously but unloading operation from a ship is performed before the loading operation starts. Here the main research motive is total cost minimization and optimum path determination for an industrial ship routing and scheduling in a fixed time frame, in which risk is minimum in a sustainable environment but in uncertain market conditions. Because of COVID-19, social behavior as well as demand and supply fluctuate randomly, affecting the shipping industry. Imran et al. (2020) designed a ship routing and scheduling model for static demand and supply, but the model is not robust in uncertain or dynamic situations. Our second research motive is the facilitating the transport of cargo whose demand and supply are determined dynamically. Yang et al. (2021) designed a tramp ship routing model to address port congestion because of static demand and supply. The result revealed that dynamic demand and supply reduce port congestion and render the system robust. Next, we focused on container-based cargo shipping to minimize the cost and path of travel. Container-based shipping provides an end-to-end approach to customers and shipping companies.

Finally, we focused on increasing the efficiency and effectiveness of the solution method. In this study, we develop a novel selection technique and generation-dependent mutation to achieve a better solution. A nature-based heuristic algorithm requires less time and computational cost. Considering fuzzy numbers, we introduced uncertainty or dynamic attributes to the model and adopted the possibility and necessity approach by using the graded mean integration value defuzzification method.

Thus, we can once again the novelties of the proposed study are as follows:

We developed a dynamic cargo ship routing and scheduling algorithm under uncertain environments.

To maintain sustainability conditions, a novel mathematical function was incorporated to study the fuel consumption corresponding to the total carbon emission and the limitations of the journey considered.

We developed an optimal route design with a maximum allowable risk for that trip.

A novel GA (new selection, crossover, and mutation strategies introduced) was developed to address the proposed model.

We considered traveling cost and risk factors as fuzzy values for uncertain environment.

The rest of this paper is organized as follows: Section 2 describes studies relevant to the present investigation. Section 3 describes mathematical preliminaries. Section 4 discusses the proposed dynamic ship routing model. The defuzzification of the proposed model is presented in Section 5. In section 6, a modified GA (MGA) is presented. Section 7 illustrates the outcome of numerical experiments. Statistical test significance is discussed in Section 8. Section 9 provides results and discussions. Managerial insights are provided in Section 10. Finally, conclusions and future scope are presented in Section 11.

## 2. Literature Study

The ship routing and scheduling problem (SRSP) is similar to the traveling salesman problem (TSP) for cargo on land. Bausch et al. (1998) was the first to develop a short-term ship routing and scheduling model. Subsequently, numerous studies have focused on ship routing and scheduling problems. Psaraftis (2019) described a few conceptual hypotheses for the SRSP. However, the study did not provide any practical implementations derived from these. Alfandri et al. (2019) proposed a weekly demand service between a pair of ports for barge containers using a liner service to maximize the profit. However, transport delays may occur in liner shipping for certain routes because of uncertain weather conditions. Rabbani et al. (2019) described a model to determine the best route to minimize the shipping costs and carbon emissions and maximize job creation in ships and ports. In this study, variable speed was considered. Cost minimization and job creation maximization contrast each other. Noshokaty (2021) used information technology in tramp ship routing and scheduling problems to address commodity forecasting. In this model, the method of solution increases the time complexity, when all constraints are considered in commodities forecasting. Zhao et al. (2019) described how operation measures and objectives depend on fuel prices and vessel loads. To generate profits, the speed of a vessel is decreased for carrying a higher vessel load. This scenario sometimes results in missing the time window because of the low-speed voyage. Homsy et al. (2020) described the generation of elementary routes for tackling routing problems related to industrial and tramp ships. However, load-dependent fuel consumption, sea condition, and emission-related vital points were not considered there. Liqan et al. (2020) considered ocean currents for speed optimization to minimize the total fuel consumption. However, this optimization model had been tested for only one ship, and hence the results cannot be exactly generalized. Numerous algorithms have been proposed to optimize ship routing problems, and each algorithm has some pros and cons. Laura et al. (2016) used algorithms, such as the Dijkstra's algorithm, dynamic programming, and iterative methods, to solve ship route optimization problems. These solution methods are problem-dependent, and each method has advantages and disadvantages. These methods are computationally expensive for NP-hard problems. De et al. (2017) described a hybrid particle swarm optimization (PSO) algorithm to solve time-window-based ship routing problems with multidimensional features. In this study, PSO marginally increases the computational time. Wang et al. (2018) proposed a

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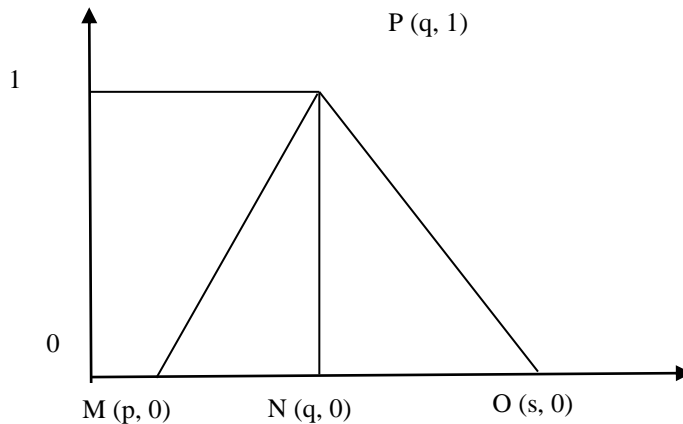
### 3. Mathematical Preliminaries

In this study, we presented preliminary fuzzy number concepts and their membership values and the defuzzification technique required for the proposed model.

#### 3.1. Triangular Fuzzy Numbers

Here,  $\tilde{A} = (p, q, r)$  is a normalized triangular fuzzy number (TFN), which is a subset of  $S$  (real numbers). It's membership function is given as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < p \\ \frac{x-p}{q-p}, & p \leq x \leq q \\ \frac{s-x}{s-q}, & q \leq x \leq s \\ 0, & x > s \end{cases} \quad (1)$$



**Figure 1.** Graphical representation of Triangular Fuzzy Number

**3.2. Fuzzy Possibility and Necessity Approach**

Assuming  $\tilde{p}$  and  $\tilde{q}$  are two fuzzy numbers with membership functions  $\mu_{\tilde{p}}(x)$  and  $\mu_{\tilde{q}}(y)$ , respectively  $x, y \in S$

$$\text{pos}(\tilde{p} * \tilde{q}) = \sup \left\{ \min \left( \mu_{\tilde{p}}(x), \mu_{\tilde{q}}(y) \right), x, y \in S \right\} \quad (2)$$

where \* any one of the relations  $<, >, =, \leq, \geq$  and  $S$  represents real numbers.

$$\text{nes}(\tilde{p} * \tilde{q}) = \overline{\text{pos}(\tilde{p} * \tilde{q})} \quad (3)$$

where nes represents necessity.

If  $\tilde{p}, \tilde{q}, \tilde{r} \subseteq S$  and  $\tilde{p} = f(\tilde{q}, \tilde{r})$  where  $f : S \times S \rightarrow S$  is a binary operation,

then the membership function  $\mu_{\tilde{p}}(x)$  of  $\tilde{a}$  is defined as follows:

$$\mu_{\tilde{p}}(z) = \sup \left\{ \min \left( \mu_{\tilde{p}}(x), \mu_{\tilde{q}}(y) \right), x, y \in S \text{ and } z = f(x, y) \right\} \quad (4)$$

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For a TFN  $\tilde{B} = (b_1, b_2, b_3)$  has three parameters  $b_1 < b_2 < b_3$ . By using (1), the membership function  $\mu_{\tilde{B}}(x)$  is expressed as follows:

$$\mu_{\tilde{B}}(x) = \begin{cases} \frac{x-b_1}{b_2-b_1}, & \text{for } b_1 \leq x < b_2 \\ \frac{b_3-x}{b_3-b_2}, & \text{for } b_2 \leq x \leq b_3 \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

From the aforementioned definitions, the following lemmas can easily be derived:

Lemma 1. If  $\tilde{B} = (b_1, b_2, b_3)$  is a TFN with  $0 < b_1$  and  $d$  is a crisp number, then

$$\text{pos}(\tilde{p} < d) \geq \alpha_1, \text{ if } \frac{d-b_1}{b_2-b_1} \geq \alpha_1.$$

Lemma 2. If  $\tilde{B} = (b_1, b_2, b_3)$  is a TFN with  $0 < b_1$  and  $d$  is a crisp number, then

$$\text{nes}(\tilde{p} < d) \geq \alpha_1 \text{ if } \frac{b_3-d}{b_3-b_2} \leq 1-\alpha_1.$$

Lemma 3. If  $\tilde{B} = (b_1, b_2, b_3)$  and  $\tilde{E} = (e_1, e_2, e_3)$  is TFN with  $0 < b_1$  and  $0 < e_1$

$$\text{then } \text{pos}(\tilde{B} < \tilde{E}) \geq \alpha_1 \text{ if } \frac{e_3-b_1}{e_3-e_2+b_2-b_1} \geq \alpha_1.$$

Lemma 4. If  $\tilde{B} = (b_1, b_2, b_3)$  and  $\tilde{E} = (e_1, e_2, e_3)$  is TFN with  $0 < b_1$  and  $0 < e_1$ ,

$$\text{then } \text{nes}(\tilde{B} < \tilde{E}) \geq \alpha_1 \text{ if } \frac{b_3-e_1}{e_2-e_1+b_3-b_2} \geq 1-\alpha_1.$$

where  $\alpha_1$  is a predefined possibility.

### 3.3. Defuzzification of the fuzzy number by using graded mean integration

The fuzzy number graded mean integration method with the integral value of graded mean  $\alpha_1$ -level of the generalized fuzzy number was used for defuzzification.

Assuming  $\tilde{B}$  is a generalized fuzzy number, then the graded mean integration value (GMIV) of  $\tilde{B}$  is denoted by the following equation:

$$\begin{aligned} P(\tilde{B}) &= \int_0^1 x\{(1-u)L^{-1}(x) + uR^{-1}(x)\}dx / x dx . \\ &= 2 \int_0^1 x\{(1-u)L^{-1}(x) + uR^{-1}(x)\}dx . \end{aligned} \quad (6)$$

Where  $u \in [0,1]$

if,  $u = 1$  , an optimistic value.

if,  $u = 0$  , a pessimistic value.

and if,  $u = 0.5$  , moderately optimistic value.

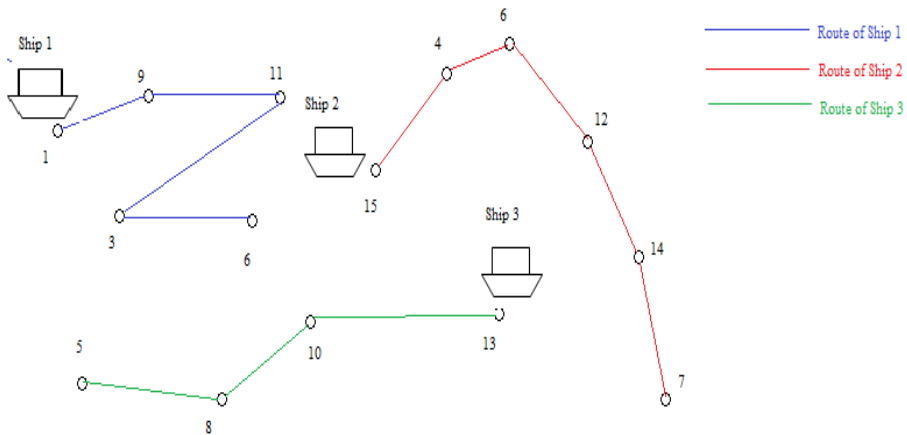
Using the GMIV of TFN  $\tilde{B} = (b_1, b_2, b_3)$  Equation (6) becomes a crisp value  $[(1-u)b_1 + 2b_2 + ub_3] / 3$ .

When  $u = 0.5$  , the expression becomes  $\frac{1}{6}[b_1 + 4b_2 + b_3]$ .

#### 4. Proposed Dynamic Ship Routing Model

To develop the proposed model, a set of ships, a group of ports, and a set of products were considered. Typically, a ship visits some of the ports to ensure unhindered demand and supply of those ports within a time frame. In this model, a ship starts its journey from a port, visits several ports, and ends its journey at the destination port, which is neither the starting port nor a visited port. For a ship, exceeding the given time window results in a penalty cost. Loading and unloading are performed at the port, but the unloading operation is performed before loading. In practice, the cost may not be deterministic, is imprecise, and considered as fuzzy parameters.

The model was developed by using various dependent and independent variables and several constraints. The assumption of this model are as follows.



**Figure 2.** Graphical representation of the proposed model

##### 4.1. Assumptions of the model

To construct the model mathematically, we assumed the following points:

1. A ship visits a set of ports in its voyage in a fixed time frame.



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2. We consider heterogeneous ships for this model. Each ship has a different capacity.
3. A container contains only one product at a time. It has a particular demand port and supply port. In between these two ports, containers cannot be unloaded from the ship.
4. A ship sails at a certain speed in which fuel consumption is minimum and it does not violate the time frame. The total carbon emission does not exceed the maximum carbon emission index.
5. Costs and risks are not fixed, but they vary in a certain range. However, the total risks do not exceed the maximum allowable risk for that route.
6. Ship bunkering, cleaning, and maintenance are performed at the operation time at the port if required.
7. On a weekly basis, each ship visits the port. Demand and supply vary dynamically at ports.

#### 4.2. Mathematical Model

Mathematical indices and parameters are stated in **Table 1**. Mathematical formulations are given below. The indices obtained from De et al. (2017) work partially.

**Table 1.** Symbol and indices

Indices		Sets	
Symbols	Descriptions	Symbols	Descriptions
$l, m$	Time period	$L$	Time periods set
$s$	Ships	$S$	Ships set
$p, q$	Ports	$P$	Ports set
$r$	Products	$R$	Products set
Parameters			
Symbols	Descriptions		
$D_{pq}$	Distance between port $p$ and port $q$ .		
$T_{pl}^A$	The beginning of the time window at port $p$ in the time period $l$ .		
$T_{pl}^B$	The ending of the time window at port $p$ in the time period $l$ .		
$C_{pr}$	Operational cost (loading/unloading) for a single unit of product $r$ at port $p$ .		
$T_{pr}$	The loading/unloading time required for single unit of product $r$ at port $p$ .		
$W_{plr}$	The demand of product $r$ at port $p$ in time period $l$ .		
$C_{pqs}$	The transportation cost from port $p$ to port $q$ for ship $s$ .		
$E_{sl}$	Storage capacity of ship $s$ in time period $l$ .		
$Y_{pr}$	Total storage capacity associated with the depot at port $p$ for the product $r$ .		
$U_{pr}$	The setup time required for operation (loading/unloading) at port $p$ for product $r$ .		

$C_{pl}$	Penalty cost per hour, associated with operation delay in time $l$ at port $p$ .
$H_s$	Carbon emission per hour for ship $s$ .
$g$	Constant (Relationship constant of speed and fuel consumption).
$f_s$	Fuel consumption rate per hour for ship $s$ .
$e_{\max}$	Maximum carbon emission for that trip.
$B_{\max}$	Maximum bunker consumption for that trip in crushing speed.
$J_{pr}$	1, if product $r$ has a supply at port $p$ . 2, if a demand for product $r$ at port $p$ . 0, if no demand/supply for product $r$ at port $p$ .

The following binary variables were assumed:

$$X_{plqms} = \begin{cases} 1, & \text{if ship } s \text{ began its operation at port } p \text{ in} \\ & \text{time } l \text{ and then travel from port } p \text{ to} \\ & \text{port } q \text{ and initiate its operation at port } q \\ & \text{in time } m. \\ 0, & \text{otherwise; } \quad l, m \in L; \quad p, q \in P. \end{cases}$$

$$O_{plsr} = \begin{cases} 1, & \text{if product } r \text{ is loaded/unloaded in ship } s \text{ at port } p \\ & \text{in time period } l. \\ 0, & \text{otherwise; } \quad r \in R; \quad l \in L; \quad s \in S; \quad p \in P. \end{cases}$$

$$Z_{pls} = \begin{cases} 1, & \text{if finally ends its travel at port } p \text{ after an operation} \\ & \text{which is started at time } l \text{ by ship } s. \\ 0, & \text{otherwise; } \quad l \in L; \quad s \in S; \quad p \in P. \end{cases}$$

The following continuous variables were assumed:

$t_{pl}^1$ : The starting time of operation at port  $p$  in time period  $l$ ;  $l \in L, p \in P$ .

$t_{pl}^E$ : The operation ending time at port  $p$  in period  $l$ ;  $l \in L, p \in P$ .

$N_{pl}$ : Total operating time outside the time frame in time period  $l$  at port  $p$ ;  $l \in L, p \in P$ .

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$Q_{plsr}^1$  : Total quantity of product  $r$  loaded/unloaded at port  $p$  from ship  $s$  in time  $l$ ;

$$l \in L, p \in P, r \in R, s \in S; Q_{plsr}^1 = 0 \text{ if } J_{pr} = 0.$$

$cnt_{ik}$  :  $i$  number of containers of  $k$ th type, where  $i=1,2,3,4,5,6,7$ ;  $k=1$ .

$I_{plsr}^1$  : Total amount of product  $r$  available on ship  $s$  after an operation that

started in time  $l$  while departing from port  $p$ ;  $l \in L, p \in P, r \in R, s \in S$ .

$S_{plr}^1$  : The stock level at port  $p$  of product  $r$  in time period  $l$ ;  $l \in L, p \in P, r \in R$ .

$V_{pqs}$  : Velocity of ship  $s$  while traveling from port  $p$  to port  $q$ .  $p, q \in P; s \in S$ .

$$V_{pqs} = 0 \text{ if } X_{plqms} = 0 \text{ and } p = q.$$

$Rf_{pq}$  : Risk factor between port  $p$  and port  $q$ ;  $p, q \in P$ .

$r_{\max}$  : Maximum risk on that trip.

Objective function:

$$\text{minimize } \sum_{p,q \in P} \sum_{l \in L} \sum_{s \in S} C_{pqs} X_{plqms} + \sum_{p \in P} \sum_{l \in L} \sum_{s \in S} \sum_{r \in R} C_{pr} O_{plsr} Q_{plsr}^1 + \sum_{p \in P} \sum_{l \in L} C_{pl} N_{pl} \quad (7)$$

Equation (7) minimizes the traveling cost, operation cost, and penalty cost.

The constraints are as follows:

$$\sum_{p \in P} \sum_{l \in L} \sum_{s \in S} X_{plqms} \geq 1 \quad \forall q \in P, \forall m \in L \quad (8)$$

Constraint (8) represents that at least a single ship can operate in port  $p$  at the given time  $l$ .

$$\sum_{p \in P} \sum_{l \in L} Z_{pls} = 1 \quad \forall s \in S \quad (9)$$

Constraint (9) represents that ship  $s$  at some port  $p$  ended its route at the time  $l$ .

$$T_{pl}^A \leq t_{pl}^1 \leq T_{pl}^B \quad \forall p \in P, \forall l \in L \quad (10)$$

Constraint (10) represents the time frame range.

$$\left( t_{pl}^1 - t_{pm}^E - \frac{D_{pq}}{V_{pqs}} \right) X_{plqms} \geq 0 \quad \forall p, q \in P; \forall l, m \in L; \forall s \in S \quad (11)$$

Constraint (11) represents that after finishing the operation at port  $p$ , ship  $s$  travels some distance between port  $p$  and port  $q$  with a fixed velocity.

If  $\left( t_{pl}^1 - t_{pm}^E - \frac{D_{pq}}{V_{pqs}} \right) X_{plqms} = 0$ , then the ship is not traveling in the sea, that is,

$$X_{plqms} = 0.$$

$$t_{pl}^E - t_{pl}^1 - \sum_{s \in S} \sum_{r \in R} U_{pr} - \sum_{s \in S} \sum_{r \in R} T_{pr} Q_{plsr}^1 = 0 \quad \forall p \in P, \forall l \in L, \forall s \in S \quad (12)$$

Constraint (12) represents the sum of the setup time, starting time of operation, and that the total loading/unloading time is equal to the ending of each operation.

$$t_{pl}^E - T_{pl}^B = N_{pl} \quad (13)$$

Constraint (13) represents penalty time at port  $p$ .

$$E_{sl} O_{plsr} - \sum_{p \in P} \sum_{r \in R} Q_{plsr}^1 cnt_{ik} \leq E_{s(l+1)} \quad \forall p \in P, \forall l \in L, \forall s \in S, \forall r \in R \quad (14)$$

Constraint (14) represents that after unloading the product from the ship, the empty space in the ship should be greater than the previous period. Products are unloaded container wise.

$$I_{p(l-1)sr}^1 \pm \sum_{p \in P} \sum_{r \in R} J_{pr} Q_{plsr}^1 cnt_{ik} \leq E_{sl} \quad \forall p \in P, \forall l \in L, \forall s \in S, \forall r \in R \quad (15)$$

Constraint (15) represents that the quantity product  $r$  in ship  $s$  at port  $p$  in time period  $(l-1)$  is onboard and addition or removal of the quantity of product  $r$  loaded/unloaded at port  $p$  in time period  $l$  should not exceed the empty capacity of ship  $s$  in time period  $l$ .

$$\sum_{q \in P} \sum_{l \in L} E_{sl} X_{plqms} - \sum_{r \in R} I_{plsr}^1 \geq 0 \quad \forall p, q \in P; \forall l, m \in L; \forall s \in S, \forall r \in R \quad (16)$$

Constraint (16) represents that an upper bound of ship  $s$  of product  $r$  while sailing does not exceed the ship capacity.

$$S_{p(m-1)r}^1 + \sum_{r \in R} Q_{pmrs}^1 - W_{pmr} - S_{pmr}^1 = 0 \quad (17)$$

Constraint (17) represents that the demand for each product  $r$  is satisfied at time  $m$  at port  $p$ .

$$g \sum_{p,q \in P} D_{pq} \log(V_{pqs}) + \sum_{p \in P} \sum_{l \in L} (t_{pl}^E - t_{pl}^1) f_{ps} \leq B_{\max} \quad \forall p \in P, \forall l \in L, \forall s \in S \quad (18)$$

Constraint (18) represents that the total fuel consumption for ship  $s$  for a trip is less than maximum bunker consumption. Here, fuel consumption is considered as a logarithmic function of velocity. When ships travel from port  $p$  to port  $q$ , the fuel consumption is  $g \log(V_{pqs})$ .

$$\sum_{p \in P} \sum_{l \in L} (t_{pl}^E - t_{pl}^1) f_{ps} \geq 0$$

A certain fuel consumption occurs at the port when ship  $s$  is in port  $p$ .

$$S_{plr}^1 - Y_{pr} \leq 0 \quad \forall p \in P, \forall l \in L, \forall r \in R \quad (19)$$

Constraint (19) represents the storage capacity of port  $p$  for product  $r$ .

$$\sum_{p,q \in P} Rf_{pq} \leq r_{\max} \quad (20)$$

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Constraint (20) represents that the total risk achieved by the port is less than equal to the maximum risk on that trip.

$$H_s * (g * \sum_{p,q \in P} D_{pq} \log(V_{pqs}) + \sum_{p \in P} \sum_{l \in L} (t_{pl}^E - t_{pl}^1) f_{ps}) \leq e_{\max} \quad p \neq q, \forall s \in S \quad (21)$$

Constraint (21) represents the total carbon emission at port and sea, which is always less than the maximum carbon emission index for that trip. In the proposed model, we assumed speed to be a logarithmic function, which is appropriate for considering traveling time and carbon emission.

$$X_{plqms} \in \{0,1\} \quad \forall p,q \in P; \forall l,m \in L; \forall s \in S, p \neq q \quad (22)$$

$$Z_{pls} \in \{0,1\} \quad \forall p \in P, \forall l \in L, \forall s \in S \quad (23)$$

$$O_{plsr} \in \{0,1\} \quad \forall p \in P, \forall l \in L, \forall s \in S, \forall r \in R \quad (24)$$

Constraints (22) – (24) represent the binary variables.

$$V_{pqs} \geq 0 \quad \forall p,q \in P; \forall s \in S; P \neq q \quad (25)$$

$$S_{plr}^1 \geq 0 \quad \forall p \in P, \forall l \in L, \forall r \in R \quad (26)$$

$$Q_{plsr}^1, I_{plsr}^1 \quad \forall p \in P, \forall l \in L, \forall r \in R, \forall s \in S \quad (27)$$

$$t_{pl}^1, t_{pl}^E, N_{pl} \quad \forall p \in P, \forall l \in L \quad (28)$$

Constraints (25) – (28) represent the nonnegative constraints.

## 5. Defuzzification technique for the proposed model

The traveling costs and the risk factors are fuzzy numbers, which are denoted as  $\tilde{C}(i, j)$  and  $\tilde{r}(i, j)$  respectively, where  $\tilde{r}_{\max}$  is the maximum risk level of a particular path. In the proposed model, the total traveling cost and risk are expressed as follows:

$$\left. \begin{array}{l} \text{to minimize } z = \sum_{i,j=1}^M \tilde{C}(x_i, x_j) \\ \text{subject to } \sum_{i,j}^M \tilde{r}(x_i, x_j) \leq \tilde{r}_{\max} \end{array} \right\} \quad (29)$$

Where  $x_i \neq x_j, i, j = 1, 2, \dots, M$ .

$M$  = total number of nodes on that route.

### 5.1. Possibility Approaches (optimistic)

By using Equation (2), we obtain the following objective and constants:

$$\begin{aligned}
 & \text{minimize } F \\
 & \text{Subject to } \left. \begin{aligned}
 & \text{pos} \left( \sum_{i,j=1}^M \tilde{C}(x_i, x_j) < F \right) \geq \alpha_2 \\
 & \text{pos} \left( \sum_{i,j}^M \tilde{r}(x_i, x_j) \leq \tilde{r}_{\max} \right) \geq \beta_2
 \end{aligned} \right\} \tag{30}
 \end{aligned}$$

Where  $x_i \neq x_j, i, j= 1, 2, \dots, M$ .

$M$  = total number of nodes on that route.

where  $\alpha_2, \beta_2$  are levels of possibility, respectively (predefined).

Traveling fuzzy cost for a path is  $\tilde{C}(i, j) = (C(i, j)_1, C(i, j)_2, C(i, j)_3)$ , and

corresponding fuzzy risk is  $\tilde{r}(i, j) = (r(i, j)_1, r(i, j)_2, r(i, j)_3)$ , and

maximum risk  $\tilde{r}_{\max} = (r_1, r_2, r_3)$ .

The aforementioned problem is reduced by using Lemma 1 and Lemma 3 as follows:

$$\begin{aligned}
 & \text{to minimize } F \\
 & \text{Subject to } \left. \begin{aligned}
 & \frac{F - F_1}{F_2 - F_1} \geq \alpha_2 \\
 & \frac{r_3 - R_1}{r_3 - r_2 + R_2 - R_1} \geq \beta_2
 \end{aligned} \right\} \tag{31}
 \end{aligned}$$

Where  $F_k = \sum_{i,j}^M C(x_i, x_j)_k, k=1, 2, 3$ .

and  $R_k = \sum_{i,j}^M r(x_i, x_j)_k, k=1, 2, 3$ .

Where  $x_i \neq x_j, i, j= 1, 2, \dots, M$ .

$M$  = total number of nodes on that route.

The objective function in Equation (31) becomes:

Minimize  $F_1 + \alpha_2(F_2 - F_1)$

Subject to  $\frac{r_3 - R_1}{r_3 - r_2 + R_2 - R_1} \geq \beta_2$

Here,  $\alpha_2, \beta_2$  are possibility levels (predefined).

### 5.2. Necessity Approaches (pessimistic)

By using Equation (3), we obtain the following objective and constants:

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Minimize  $F$

$$\left. \begin{array}{l} \text{Subject to nes} \left( \sum_{i,j=1}^M \tilde{C}(x_i, x_j) < F \right) \geq \alpha_3 \\ \text{nes} \left( \sum_{i,j}^M \tilde{r}(x_i, x_j) \leq \tilde{r}_{\max} \right) \geq \beta_3 \end{array} \right\} \quad (32)$$

Where  $x_i \neq x_j, i, j = 1, 2, \dots, M$ .

$M$  = total number of nodes on that route.

Traveling fuzzy cost for a path is  $\tilde{C}(i, j) = (C(i, j)_1, C(i, j)_2, C(i, j)_3)$ , and

corresponding fuzzy risk is  $\tilde{r}(i, j) = (r(i, j)_1, r(i, j)_2, r(i, j)_3)$ , and

$$\text{maximum risk } \tilde{r}_{\max} = (r_1, r_2, r_3) .$$

The aforementioned problem is reduced by using Lemma 2 and Lemma 4:

To minimize  $F$

$$\left. \begin{array}{l} \text{Subject to } \frac{F_3 - F}{F_3 - F_2} \leq 1 - \alpha_3 \\ \frac{R_3 - r_1}{r_2 - r_1 + R_3 - R_2} \leq 1 - \beta_3 \end{array} \right\} \quad (33)$$

Where  $F_k = \sum_{i,j}^M C(x_i, x_j)_k, k=1, 2, 3$ .

and  $R_k = \sum_{i,j}^M r(x_i, x_j)_k, k=1, 2, 3$ .

Where  $x_i \neq x_j, i, j = 1, 2, \dots, M$ .

$M$  = total number of nodes on that route.

The objective function in Equation (33) becomes:

$$\text{Minimize } F_3 - (1 - \alpha_3)(F_3 - F_2)$$

$$\text{Subject to } \frac{R_3 - r_1}{r_2 - r_1 + R_3 - R_2} \leq \beta_3$$

Here, necessity levels are  $\alpha_3, \beta_3$  (predefined).

### 5.3. GMIV Approach

By applying GMIV for the defuzzification on SRSP using Equation (29), we obtain the following equation:

To minimize  $z = \frac{1}{6}[F_1 + 4F_2 + F_3]$

Subject to  $\frac{1}{6}[R_1 + 4R_2 + R_3] \leq \frac{1}{6}[r_1 + 4r_2 + r_3]$

Here, we consider  $w = 0.5$ , for an optimal solution.

## 6. MGA

In 1975, Holland was the first to propose natural and artificial system GA, which was subsequently used in natural evolution and optimization problems. Adaptive heuristic search algorithms evolved from GA. This algorithm used a random search technique to obtain an optimized result. Although random search is used, it is not random, and the previous information is used to direct the search in an appropriate search space to obtain optimum results. GA provides superior optimization when high dimensional search spaces and numerous variables are present.

### 6.1. Proposed Bird's nest building material selection technique

To solve the ship routing and scheduling model through MGA, we proposed a novel Bird's nest building material selection technique (BBMST) for selection. Not all birds excel at making nests. Hamerkop (*Scopus umbretta*), ruby-throated hummingbird (*Archilochus colubers*), sociable weaver (*Philetairus socius*), baya weaver (*Ploceus philippinus*), etc make good nests. Among these birds, "sociable weaver" birds select the best material for nest building, and they check the building material with their beak. We selected the best solution from all possible solutions set to obtain the optimum result in ship routing.

In the BBMST algorithm, first, the fitness values of all chromosomes are calculated, and the minimum cost path, which is the main objective of this model, is implemented. In each iteration, we selected the minimum cost of the chromosomes, and based on the cost of other chromosomes, we calculated the threshold value. The probability of selection was randomly generated, which is between 0 and 1. If the threshold value is less than the probability of selection, then the current chromosome is selected, otherwise the minimum threshold valued chromosome is considered as fittest for the IVF crossover process.

Let  $C[i][j]$  represent the traveling cost between  $i^{\text{th}}$  port to  $j^{\text{th}}$  port,  $N$  is the set of ports, and  $pv$  is the total number of chromosomes.

---

#### Algorithm1: BBMST selection process

---

**Require:** Set of given port  $N$  and the size of the population is  $pv$ .

**Ensure:** A set of fittest chromosomes.

1. **Begin**
2.     for ( $i = 1$  to  $pv$ )
3.         judge fitness;
4.     end for;
5.      $b = \text{chromosome}_{(\text{minimum-fitness})}$  ;
6.      $b_1 = \text{Avg}_{\text{fitness}}$  of population ;
7.     for( $i= 1$  to  $pv$ )



**Algorithm1:** BBMST selection process

8. generate a temporary population, where fitness <  $b_1$ ;
9. end for;
10. for( $i= 1$  to  $pv$ )
11. randomly select three edges  $S_i, j \in (1, 2, 3)$ ;
12. generate  $r_i = (\text{cost}(S_i)/ \text{fitness}_i) // \text{cost}(S_i)$  represents the traveling cost between two end vertices/nodes of  $S_j$ .
13.  $R = \text{random}[0, 1]$ ;
14. if (any two  $r_i \leq R$ )
15. select the chromosome  $i$ ;
16. else
17. select the chromosome  $b$ ;
18. end for
19. **End**



**Figure 3.** IVF Crossover

**6.2. IVF crossover**

In the proposed MGA, IVF is used for the crossover process. In the IVF process, three parents, namely one father (Pr1), one mother (Pr2), and a surrogate mother (Sr) are selected randomly, depending on their crossover probability ( $P_c$ ) and bring them in mating pool, and two children were created, child1 and child2. First randomly generate a node (for example:  $a_i$ ) and place it at the first position at child1 and update the parents with  $a_i$ . Next, the least cost valued node is selected from three parents in mating pool. Repeat the previous step until all the nodes are not selected and always check whether the same node is selected previously or not. Similarly, the same steps are performed for child2. In the mating pool of GA, two new offspring are created with superior information, which is inherited from parents. The IVF is a parallel flow process, which receives the input portion of various parents from the GA population. The individuals are supplied by the IVF process. Figure 3 displays the IVF process in a pictorial form. The algorithm steps of IVF crossover are given below:

---

**Algorithm2:** IVF crossover

---

1. **Begin**
  2. Choose three parents (Pr1, Pr2 and Sr) randomly from mating pool depending on crossover probability  $P_c$ .
  3. Nodes in three parents are Pr1:  $(a_1, a_2, \dots, a_N)$ , Pr2:  $(r_1, r_2, \dots, r_N)$  and Sr  $(s_1, s_2, \dots, s_N)$ .
  4. Randomly choose a node  $a_i$ .
  5. Place that  $a_i$  node at the beginning of child1 and update the three parents.
  6. In child1 place  $a_i$  at the first position.
  7. Find the minimum cost between  $(a_i, a_1)$ ,  $(a_i, r_1)$  and  $(a_i, s_1)$ .
  8. Choose the minimum cost node and place it to the next position of child1 and update parents.
  9. Repeat steps 7 and 8 until all nodes are not selected without repetition of any node.
  10. Repeat the same steps for child2.
  11. **End**
- 

**6.3. Generation-dependent mutation**

After the crossover process, the algorithm goes through the mutation process. In this algorithm, we used generation-dependent mutation. The mutation prevents the solution to be trapped in a local minimum and premature convergence. The mutation maintains genetic diversity in the population. In a population, a random change in some of genes result in one chromosome. This phenomenon produces a novel offspring with a new genetic structure. Mutation helps escape from local minima, find the global minima, and maintain the diversity of the population. The probability of generation-dependent mutation ( $p_m$ ) is expressed as follows:

$$p_m = \frac{k}{\sqrt{\text{Number of current generation}}}, \quad k \in [0, 1]$$

When the number of generations increases,  $p_m$  decreases normally.

**6.3.1. Mutation Process**

The proposed ship routing and scheduling model is a node-dependent problem. To mutate the chromosome  $X = (x_1, x_2, \dots, x_N), (y_1, y_2, \dots, y_N)$ , to find the mutated node  $T = p_m * N$ , where  $N$  = total number of nodes in a chromosome. If  $r2 < p_m$ ,  $r2 \in \text{random} [0, 1]$ , then the corresponding chromosome is to be selected for mutation.

First, we generate two distinct numbers  $x_i, x_j$  randomly between  $[1, N]$ . To obtain mutated solutions,  $x_i$  and  $x_j$  are interchanged. The same process repeated  $T$  times to obtain the best offspring.

---

**Algorithm3:** Generation-dependent mutation

---

1. **Begin**
  2. Set  $g$ = number of the current generation.
  3.  $p_m = \frac{k}{\text{sqrt}(g)}$ ,  $k \in [0,1]$
  4. Calculate  $T = p_m * N$  ; // the total number of mutated nodes in the  
// chromosome.
  5. for ( $i=0$  to  $\text{pop-size}$ ) //pop-size = population size.
  6.  $r2 \in = \text{rand}(0, 1)$ ;
  7. if( $r2 < p_m$ )
  8. Select current chromosome;
  9.  $c = \text{rand}[1, N]$ ;
  10.  $d = \text{rand}[1, N]$ ;
  11. if ( $c == d$ )
  12. Goto step 9;
  13. end for
  14. for ( $j = 1$  to  $N$ ) //  $N$ =total number of nodes of a route .
  15. if ( $x[j] == c$ )
  16.  $s = j$ ;
  17. if( $x[j] == d$ )
  18.  $t = j$ ;
  19.  $x[s] = d$  ; //replace  $c$  by  $d$ .
  20.  $x[t] = c$  ; //replace  $d$  by  $c$ .
  21. end for
  22. Repeat steps 8 to 20 up to  $T$  times.
  23. end if
  24. end for
  25. **End**
- 

Because the process is an NP-hard problem, the exact approach may cause high computational time. Meta-heuristic approaches provide an optimal result in a reasonable time. In this study, we proposed a modified GA-based BBMST, in vitro fertilization, and generation-dependent mutation.

#### 6.4. Complexity Analysis

We considered  $P$  as the initial population,  $N$  as the number of nodes,  $g$  as the number of generations,  $p_c$  as the probability of crossover,  $p_m$  as the probability of mutation, and  $p_c > p_m$ . Therefore, the time complexity of three operations, selection, crossover, and mutation are  $O(PN)$ ,  $O(P_c N^2)$ , and  $O(P_m N^2)$ , respectively. For the proposed algorithm, complexity is as follows:

6.4.1. Time complexity

Because the proposed MGA consists of BBMST selection, IVF crossover, and generation-dependent mutation, the time complexity of this algorithm is the maximum time complexity of these three processes.  $O(PN)$ ,  $O(PN^2)$ , and  $O(PN^2)$  are the time complexity of three processes. Therefore,  $O(PN^2)$  is the time complexity of the proposed algorithm.

6.4.2. Space complexity

In the proposed MGA, the total space is considered as the population multiplied by the number of nodes, that is,  $PN$ . As  $p > N$ , the space complexity of the proposed algorithm is  $O(PN)$ .

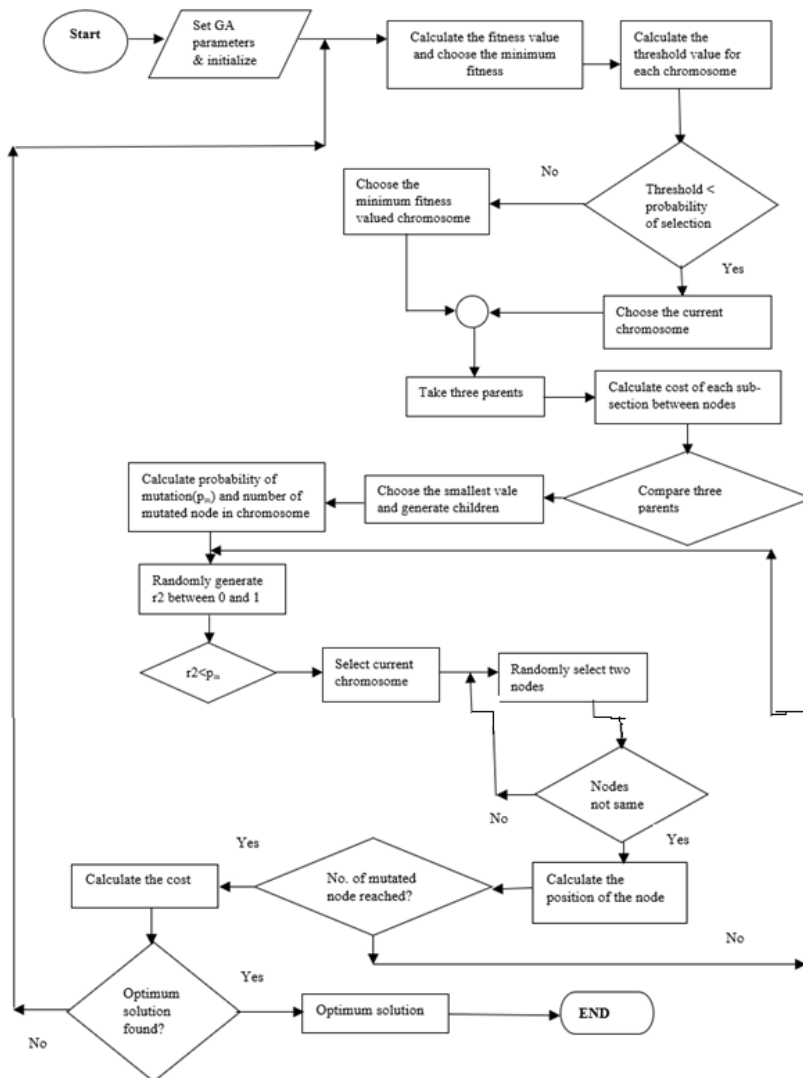


Figure 4. Flowchart of MGA

### 7. Numerical Experiments

The optimum value is obtained by running this algorithm up to 500 generations considering the crossover probability as 0.61 and considering generation-dependent mutation. We used a 2.40 GHz i7 processor with 12GB of RAM with Windows 10 for our numerical calculations. The use of fuzzy theory renders the model robust.

In this model, we considered 3 ships and 15 ports. We collected data from the Haldia Port and traveling cost in 10k of each cell value in matrix, which is expressed as follows:

**Table 2.** Input Data: Deterministic Traveling Cost Matrix

i/j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	∞	25	28	28	30	26	15	37	40	30	41	23	31	4	35
2	17	∞	20	28	25	14	30	14	28	4	37	25	32	50	11
3	14	8	∞	10	25	15	9	32	40	30	31	47	22	33	45
4	28	30	7	∞	20	25	30	35	22	37	11	33	40	21	32
5	37	22	35	30	∞	20	25	30	9	28	33	44	15	27	19
6	25	30	25	8	28	∞	32	40	32	30	15	34	27	41	11
7	28	25	30	22	37	40	∞	10	32	20	36	45	8	25	13
8	20	5	32	40	35	25	40	∞	22	37	10	37	29	15	50
9	30	40	35	25	20	22	37	32	∞	28	42	31	30	7	33
10	28	30	28	20	11	32	37	40	30	∞	36	22	32	23	16
11	12	24	37	29	52	19	37	6	42	31	∞	25	14	36	39
12	35	21	13	46	34	29	37	28	19	30	17	∞	16	34	29
13	42	36	31	26	25	12	30	24	19	27	36	23	∞	7	24
14	38	15	24	42	18	29	46	27	33	19	19	45	25	∞	31
15	41	29	11	28	41	27	34	29	9	28	16	45	29	34	∞

The loading and unloading operation cost for each product and penalty cost for the outside time from timeframe for each port and cost in 1 k of each cell of the matrix is presented in Table 3.

**Table 3.** Operation Cost and Penalty Cost Matrix

Operation Cost for Each Product															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	15	8	11	13	12	7	14	9	10	8	7	14	9	12	12
2	8	8	12	7	9	8	9	9	12	9	9	9	8	9	9
3	7	9	8	8	7	7	8	7	9	10	6	9	13	9	9
4	10	8	15	5	6	10	9	7	10	12	8	7	13	8	7
Penalty Cost of Ports															
	15	12	10	16	18	12	8	10	9	14	7	8	10	9	11

By using the deterministic traveling cost presented in Table 2, operation cost and penalty cost presented in Table 3, we obtained the results for the MGA and GA to obtain the optimum cost of the proposed algorithm. The final path for the four ships and their optimum cost for both the algorithms are presented in the given table:

**Table 4.** Result for Deterministic Model

SI NO	GA		MGA		Operation Cost	Penalty Cost
	Final Path	Traveling Cost	Final Path	Traveling Cost		
1	2-12-6-4-1-10	146	2-6-4-1-12-10	131	5720	22
	11-8-7-5-9	109	7-8-11-5-9	97	4080	40
	14-15-3-13	83	14-3-13-15	79	2843	41
2	11-1-8-2-10-12	156	11-8-1-2-10-12	112	5769	22
	13-4-5-15-7	144	4-5-13-15-7	138	3269	21
	3-6-9-14	105	14-3-6-9	108	3605	60
3	3-2-10-5-8-4	121	10-5-2-8-3-4	117	6293	44
	9-14-11-12-6	84	12-9-14-11-6	72	3602	38
	13-15-1-7	120	1-7-13-15	74	2748	21
4	4-2-15-13-7	128	2-15-13-4-7	124	4173	9
	8-12-11-9-5-14	178	12-11-8-5-9-14	112	4751	50
	6-1-10-3	115	10-3-6-1	108	3719	44
5	2-13-5-15-3	101	13-5-2-15-3	83	4695	53
	12-9-1-8-11-6	140	12-9-8-11-1-6	124	4856	50
	7-14-10-4	91	7-14-10-4	92	3092	0
6	6-1-8-2-10	140	8-2-6-1-10	102	5272	12
	12-14-11-3-5-4	170	12-3-5-14-11-4	141	4667	54
	7-13-15-9	83	7-13-15-9	83	2704	37
7	11-2-1-9-4	134	9-11-1-2-4	135	4678	40
	12-7-8-14-6	116	12-7-8-14-6	116	4208	10
	13-15-3-10-5	113	15-3-13-10-5	100	3757	53
8	2-4-11-12-10	106	4-12-11-2-10	95	4421	10
	6-15-3-8-9	106	8-3-6-15-9	97	3890	60
	13-14-5-1-7	105	1-14-5-13-7	95	4332	33
9	2-10-9-5-7	107	2-10-5-9-7	89	5223	40
	12-6-15-4-14	127	12-15-6-4-14	123	2933	10
	1-3-8-11-13	126	8-11-1-3-13	114	4487	53

To simulate practical applications, we obtained the fuzzy traveling cost matrix. Here, the triangular cost fuzzy matrix is used for the proposed model as follows:

**Table 5.** Input Data: Fuzzy Cost Matrix

i/j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	$\infty$	(22, 24, 27)	(25, 27, 29)	(24, 28, 30)	(31, 33, 35)	(24, 27, 29)	(13, 16, 18)	(32, 34, 36)	(39, 41, 42)	(29, 31, 33)	(40, 43, 45)	(20, 22, 24)	(30, 32, 34)	(15, 16, 18)	(31, 34, 38)
2	(16, 18, 20)	$\infty$	(20, 22, 24)	(24, 26, 29)	(23, 25, 27)	(13, 15, 17)	(29, 31, 33)	(11, 14, 16)	(26, 29, 31)	(14, 16, 17)	(36, 39, 40)	(23, 26, 28)	(31, 33, 35)	(47, 50, 52)	(11, 13, 15)
3	(13, 15, 17)	(16, 17, 20)	$\infty$	(9, 12, 14)	(23, 26, 28)	(14, 16, 18)	(17, 19, 21)	(29, 31, 33)	(36, 39, 42)	(27, 31, 34)	(28, 31, 35)	(42, 45, 48)	(45, 48, 50)	(18, 21, 23)	(40, 43, 45)
4	(24, 27, 29)	(25, 28, 31)	(16, 18, 19)	$\infty$	(19, 21, 23)	(24, 26, 27)	(29, 31, 33)	(30, 33, 35)	(18, 21, 23)	(34, 36, 37)	(9, 12, 14)	(29, 31, 34)	(38, 41, 43)	(21, 23, 25)	(30, 33, 35)
5	(33, 35, 37)	(21, 23, 25)	(32, 35, 37)	(28, 30, 33)	$\infty$	(20, 22, 24)	(20, 23, 26)	(31, 33, 35)	(8, 10, 12)	(27, 31, 36)	(31, 34, 37)	(42, 45, 47)	(13, 16, 18)	(26, 29, 31)	(18, 20, 22)
6	(24, 26, 28)	(29, 31, 33)	(24, 27, 29)	(16, 19, 21)	(27, 29, 31)	$\infty$	(31, 33, 35)	(39, 41, 43)	(30, 31, 35)	(25, 28, 31)	(14, 16, 18)	(33, 37, 39)	(25, 29, 32)	(38, 42, 44)	(9, 12, 14)
7	(27, 29, 31)	(24, 26, 28)	(29, 31, 33)	(21, 23, 24)	(32, 36, 39)	(40, 42, 43)	$\infty$	(9, 11, 12)	(28, 31, 33)	(19, 21, 23)	(33, 35, 38)	(42, 44, 48)	(6, 9, 11)	(20, 23, 25)	(11, 13, 15)
8	(20, 21, 23)	(15, 19, 21)	(31, 35, 37)	(37, 41, 43)	(33, 36, 38)	(23, 25, 27)	(38, 40, 43)	$\infty$	(21, 23, 25)	(35, 38, 40)	(8, 11, 13)	(32, 36, 38)	(23, 28, 31)	(13, 18, 21)	(47, 51, 55)
9	(31, 33, 34)	(36, 38, 40)	(34, 36, 37)	(24, 26, 27)	(21, 23, 25)	(20, 23, 25)	(36, 38, 39)	(29, 31, 33)	$\infty$	(26, 30, 34)	(40, 42, 44)	(31, 33, 35)	(31, 34, 38)	(6, 8, 10)	(29, 31, 33)
10	(27, 29, 31)	(30, 32, 34)	(25, 27, 29)	(19, 21, 23)	(10, 12, 13)	(31, 33, 35)	(35, 38, 43)	(39, 41, 43)	(31, 32, 35)	$\infty$	(36, 39, 44)	(19, 21, 23)	(29, 31, 35)	(22, 24, 26)	(16, 18, 20)
11	(10, 13, 15)	(23, 25, 27)	(38, 41, 43)	(25, 32, 35)	(51, 53, 55)	(19, 21, 23)	(34, 38, 41)	(17, 19, 21)	(42, 45, 48)	(29, 30, 32)	$\infty$	(24, 28, 32)	(13, 17, 21)	(32, 37, 40)	(37, 40, 44)
12	(34, 36, 37)	(22, 23, 24)	(13, 16, 18)	(44, 48, 48)	(35, 38, 38)	(27, 31, 31)	(35, 39, 39)	(27, 30, 30)	(19, 21, 23)	(28, 31, 31)	(17, 20, 20)	(14, 18, 18)	(32, 35, 35)	(27, 31, 31)	(27, 31, 31)
13	(37, 39, 41)	(36, 40, 40)	(32, 35, 35)	(23, 27, 27)	(24, 28, 28)	(11, 15, 15)	(29, 33, 33)	(20, 25, 24)	(19, 24, 24)	(25, 32, 32)	(32, 40, 40)	(20, 24, 24)	(7, 10, 10)	(23, 27, 27)	(23, 27, 27)
14	(35, 37, 39)	(13, 16, 18)	(23, 27, 27)	(41, 44, 44)	(17, 21, 21)	(28, 31, 31)	(42, 47, 47)	(25, 29, 29)	(29, 34, 34)	(17, 21, 21)	(15, 20, 20)	(41, 46, 46)	(24, 27, 27)	(32, 35, 35)	(32, 35, 35)
15	(39, 41, 43)	(27, 29, 31)	(10, 12, 14)	(28, 29, 30)	(38, 41, 43)	(27, 29, 30)	(31, 33, 35)	(29, 31, 33)	(7, 10, 11)	(27, 30, 30)	(15, 19, 19)	(42, 47, 47)	(25, 29, 29)	(33, 35, 37)	$\infty$

**Table 6.** Result for Fuzzy Model

SI NO	GA		MGA	
	Final Path	Traveling Cost	Final Path	Traveling Cost
1	2-4-12-6-1-10	512.83	2-6-4-1-12-10	444.83
	5-7-8-11-9	305.83	7-8-11-5-9	305.17
	15-3-14-13	184.67	15-3-14-13	184.67
2	11-2-8-1-10-12	502.67	11-1-8-2-10-12	447.17
	13-4-5-15-7	380.00	4-5-15-13-7	373.67
	3-6-9-14	222.67	3-6-9-14	222.67
3	3-2-10-5-8-4	447.16	3-2-8-10-5-4	434.33
	9-14-11-12-6	266.17	12-9-14-11-6	251.33
	13-15-1-7	296.50	13-15-1-7	296.50
4	4-2-15-13-7	394.50	2-15-4-13-7	364.00
	8-12-11-9-5-14	631.17	12-11-8-5-9-14	436.34
	6-1-10-3	282.00	3-6-10-1	239.00
5	5-2-13-15-3	361.00	5-2-15-13-3	329.33
	8-12-11-8-1-6	514.00	12-9-8-11-1-6	442.50
	7-14-10-4	222.17	14-10-4-7	236.17
6	8-1-2-6-10	392.67	1-2-8-6-10	336.17
	3-14-11-12-5-4	535.50	4-11-12-5-3-4	536.67
	7-13-15-9	137.67	7-15-9-13	181.50
7	11-1-2-9-4	398.00	11-2-1-9-4	325.83
	7-8-14-12-6	362.83	12-7-8-14-6	327.83
	13-15-3-10-5	324.17	13-15-3-10-5	324.17
8	2-4-11-12-10	344.50	2-12-4-11-10	313.83
	6-15-3-8-9	279.00	6-15-3-8-9	279.00
	13-14-1-5-7	366.17	13-14-5-1-7	285.83
9	10-5-2-9-7	352.33	2-5-9-10-7	342.33
	12-6-15-4-14	362.33	12-6-15-4-14	362.33
	8-11-3-1-13	350.83	3-1-8-11-13	312.50

By using the fuzzy triangular cost matrix presented in Table 5, we calculated the final path for three ships and their optimum cost for our MGA and GA. The aforementioned table reveals the results for the fuzzy data.

Table 7 presents the deterministic risk matrix between every two ports. We considered a value between 0 and 1 as the deterministic risk factor.



**Table 7.** Input Data: Deterministic Risk Matrix

i/j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	∞	0.5	0.8	0.7	0.82	0.59	0.51	0.61	0.38	0.71	0.57	0.61	0.63	0.58	0.42
2	0.78	∞	0.6	0.52	0.9	0.32	0.47	0.51	0.48	0.6	0.63	0.58	0.71	0.61	0.5
3	0.56	0.7	∞	0.5	0.47	0.49	0.55	0.63	0.7	0.8	0.67	0.6	0.53	0.63	0.39
4	0.61	0.77	0.73	∞	0.51	0.54	0.64	0.74	0.48	0.5	0.61	0.62	0.45	0.64	0.8
5	0.48	0.67	0.55	0.68	∞	0.55	0.51	0.62	0.7	0.8	0.71	0.72	0.5	0.54	0.49
6	0.39	0.57	0.66	0.68	0.88	∞	0.91	0.75	0.76	0.88	0.58	0.71	0.71	0.55	0.72
7	0.73	0.52	0.7	0.53	0.71	0.56	∞	0.59	0.67	0.63	0.56	0.66	0.67	0.59	0.6
8	0.78	0.54	0.67	0.69	0.58	0.73	0.55	∞	0.77	0.68	0.67	0.77	0.69	0.47	0.54
9	0.61	0.56	0.47	0.66	0.64	0.83	0.66	0.88	∞	0.57	0.73	0.74	0.72	0.6	0.69
10	0.49	0.63	0.56	0.59	0.63	0.81	0.72	0.48	0.49	∞	0.68	0.61	0.56	0.63	0.63
11	0.53	0.68	0.57	0.58	0.7	0.49	0.55	0.57	0.58	0.72	∞	0.63	0.57	0.43	0.7
12	0.5	0.76	0.62	0.62	0.63	0.66	0.71	0.44	0.57	0.61	0.6	∞	0.59	0.39	0.61
13	0.49	0.48	0.67	0.63	0.59	0.51	0.55	0.47	0.49	0.5	0.7	0.78	∞	0.51	0.53
14	0.47	0.54	0.64	0.72	0.76	0.66	0.56	0.73	0.61	0.57	0.45	0.48	0.47	∞	0.51
15	0.63	0.7	0.73	0.51	0.69	0.82	0.48	0.6	0.63	0.58	0.49	0.7	0.46	0.51	∞

Because risk factors are dependent on various weather conditions and other factors, it is appropriate to take risk factors as a fuzzy value instead of a crisp value. The table displays the fuzzy triangular risk factor between ports. In our problem, we obtained the TFN for the risk matrix.

**Table 8.** Fuzzy Risk Matrix

i/j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	$\infty$	0.48, 0.51, 0.61	0.75, 0.81, 0.85	0.67, 0.7, 0.78	0.79, 0.83, 0.87	0.53, 0.57, 0.63	0.47, 0.5, 0.56	0.58, 0.6, 0.65	0.35, 0.37, 0.4	0.69, 0.7, 0.75	0.53, 0.56, 0.61	0.58, 0.6, 0.66	0.6, 0.64, 0.69	0.53, 0.57, 0.61	0.39, 0.41, 0.44
2	0.75, 0.59, 0.08	$\infty$	0.58, 0.59, 0.63	0.49, 0.51, 0.54	0.87, 0.91, 0.92	0.3, 0.33, 0.35	0.45, 0.48, 0.5	0.49, 0.52, 0.55	0.46, 0.49, 0.51	0.58, 0.61, 0.62	0.61, 0.63, 0.65	0.55, 0.57, 0.6	0.69, 0.72, 0.73	0.59, 0.62, 0.63	0.49, 0.51, 0.53
3	0.53, 0.55, 0.59	0.68, 0.69, 0.74	$\infty$	0.48, 0.5, 0.53	0.45, 0.49, 0.49	0.47, 0.54, 0.57	0.52, 0.64, 0.65	0.6, 0.71, 0.73	0.68, 0.81, 0.83	0.78, 0.81, 0.83	0.65, 0.66, 0.68	0.58, 0.61, 0.63	0.5, 0.54, 0.55	0.61, 0.64, 0.66	0.79, 0.81, 0.84
4	0.59, 0.62, 0.64	0.73, 0.76, 0.79	0.7, 0.74, 0.77	$\infty$	0.48, 0.52, 0.55	0.52, 0.55, 0.57	0.62, 0.65, 0.67	0.72, 0.75, 0.76	0.46, 0.48, 0.5	0.48, 0.51, 0.53	0.59, 0.62, 0.64	0.58, 0.61, 0.63	0.43, 0.44, 0.46	0.6, 0.65, 0.67	0.47, 0.48, 0.51
5	0.45, 0.47, 0.5	0.63, 0.66, 0.7	0.53, 0.55, 0.57	0.64, 0.67, 0.7	$\infty$	0.53, 0.56, 0.58	0.49, 0.52, 0.53	0.6, 0.63, 0.65	0.68, 0.71, 0.73	0.79, 0.82, 0.83	0.7, 0.72, 0.74	0.69, 0.71, 0.73	0.48, 0.51, 0.53	0.52, 0.55, 0.57	0.46, 0.48, 0.5
6	0.35, 0.38, 0.41	0.55, 0.57, 0.61	0.62, 0.65, 0.68	0.65, 0.68, 0.71	0.86, 0.88, 0.9	$\infty$	0.89, 0.91, 0.93	0.73, 0.76, 0.78	0.73, 0.87, 0.89	0.85, 0.87, 0.9	0.56, 0.59, 0.6	0.69, 0.72, 0.74	0.69, 0.73, 0.74	0.53, 0.56, 0.58	0.69, 0.72, 0.74
7	0.7, 0.74, 0.78	0.49, 0.53, 0.55	0.68, 0.71, 0.74	0.5, 0.54, 0.56	0.68, 0.72, 0.74	0.53, 0.55, 0.58	$\infty$	0.56, 0.6, 0.69	0.65, 0.66, 0.69	0.6, 0.64, 0.65	0.53, 0.55, 0.58	0.63, 0.65, 0.67	0.65, 0.68, 0.69	0.57, 0.61, 0.63	0.58, 0.61, 0.63
8	0.75, 0.77, 0.8	0.51, 0.53, 0.56	0.65, 0.68, 0.7	0.65, 0.68, 0.71	0.56, 0.59, 0.61	0.7, 0.72, 0.75	0.52, 0.56, 0.58	$\infty$	0.75, 0.78, 0.79	0.66, 0.69, 0.71	0.65, 0.68, 0.7	0.74, 0.76, 0.79	0.46, 0.48, 0.51	0.52, 0.55, 0.57	0.44, 0.46, 0.49
9	0.61, 0.64, 0.45	0.55, 0.6, 0.53	0.48, 0.5, 0.55	0.65, 0.68, 0.55	0.65, 0.6, 0.77	0.82, 0.85, 0.69	0.65, 0.89, 0.45	0.87, 0.89, 0.47	$\infty$	0.56, 0.58, 0.66	0.74, 0.76, 0.66	0.73, 0.75, 0.59	0.73, 0.75, 0.53	0.61, 0.6, 0.6	0.68, 0.7, 0.6
10	0.48, 0.52, 0.51	0.64, 0.67, 0.63	0.55, 0.58, 0.54	0.57, 0.61, 0.56	0.62, 0.65, 0.68	0.79, 0.84, 0.51	0.73, 0.75, 0.55	0.47, 0.5, 0.55	0.5, 0.53, 0.55	$\infty$	0.67, 0.7, 0.7	0.62, 0.64, 0.6	0.55, 0.58, 0.55	0.63, 0.65, 0.61	0.64, 0.66, 0.68
11	0.54, 0.59, 0.47	0.66, 0.71, 0.71	0.56, 0.59, 0.59	0.59, 0.61, 0.73	0.71, 0.73, 0.5	0.48, 0.5, 0.57	0.54, 0.57, 0.59	0.57, 0.6, 0.75	0.73, 0.75, 0.6	$\infty$	0.63, 0.65, 0.65	0.56, 0.59, 0.59	0.42, 0.55, 0.58	0.57, 0.57, 0.57	0.71, 0.73, 0.73
12	0.51, 0.55, 0.45	0.75, 0.78, 0.43	0.63, 0.65, 0.64	0.6, 0.63, 0.6	0.62, 0.65, 0.55	0.72, 0.73, 0.49	0.45, 0.47, 0.53	0.58, 0.6, 0.47	0.62, 0.63, 0.47	0.59, 0.63, 0.68	$\infty$	0.58, 0.62, 0.75	0.38, 0.4, 0.47	0.61, 0.63, 0.52	0.61, 0.63, 0.52
13	0.48, 0.51, 0.44	0.47, 0.5, 0.51	0.67, 0.69, 0.61	0.64, 0.66, 0.6	0.58, 0.6, 0.54	0.52, 0.58, 0.51	0.55, 0.5, 0.5	0.48, 0.5, 0.53	0.48, 0.5, 0.73	0.49, 0.71, 0.81	0.79, 0.81, 0.81	$\infty$	0.5, 0.53, 0.53	0.4, 0.47, 0.47	0.4, 0.56, 0.56
14	0.46, 0.5, 0.6	0.54, 0.57, 0.68	0.63, 0.67, 0.7	0.73, 0.75, 0.77	0.75, 0.68, 0.65	0.65, 0.57, 0.8	0.54, 0.57, 0.44	0.74, 0.76, 0.56	0.62, 0.63, 0.61	0.58, 0.6, 0.57	0.46, 0.49, 0.47	0.47, 0.49, 0.68	$\infty$	0.47, 0.49, 0.48	0.51, 0.54, 0.48
15	0.64, 0.66	0.71, 0.73	0.72, 0.75	0.52, 0.54	0.68, 0.7	0.83, 0.84	0.47, 0.5	0.59, 0.63	0.64, 0.66	0.6, 0.61	0.49, 0.51	0.71, 0.73	0.77, 0.73	0.52, 0.48	$\infty$ , 0.53

The table below displays the risk achieved by the deterministic risk factor and fuzzy risk factor from Tables 7 and 8, respectively, at a particular path for both the algorithms.

**Table 9.** Result for Risk Factor

SI NO	Risk Value Achieved for Deterministic Model			Risk Value Achieved for Fuzzy Model		
	GA	MGA	Maximum Risk ( $R_{max}$ )	GA	MGA	Maximum Risk ( $R_{max}$ )
1	2.78	2.65	7.23	2.30	2.23	6.44
	3.14	3.14	9.01	2.09	1.93	5.23
	2.09	1.77	5.25	3.01	3.01	8.41
2	2.46	2.30	6.01	2.11	2.12	6.44
	2.70	2.62	9.21	1.81	1.74	5.23
	2.05	1.89	7.44	3.26	3.26	8.41
3	2.86	2.55	8.88	2.48	2.39	6.44
	2.98	2.99	9.00	1.69	1.55	5.23
	1.81	1.71	6.68	2.70	2.70	8.41
4	2.46	2.23	8.34	2.34	2.27	6.32
	3.11	3.05	7.28	2.99	2.98	8.56
	2.22	1.44	6.98	1.47	1.42	5.20
5	2.64	2.49	8.95	2.50	2.47	6.32
	3.25	3.21	8.90	3.46	3.23	8.56
	1.68	1.75	8.32	2.15	1.75	5.20
6	2.53	2.50	8.59	2.07	1.95	6.32
	3.05	3.09	9.17	2.91	2.70	8.56
	1.71	1.71	7.64	2.11	1.96	5.20
7	2.41	2.35	8.29	2.93	2.83	8.76
	2.85	2.85	8.61	2.04	1.96	6.73
	1.93	1.83	6.25	2.15	2.15	7.68
8	2.51	2.50	7.48	3.54	3.53	8.76
	3.53	3.49	10.28	2.31	2.29	6.73
	1.70	1.64	8.25	2.45	2.23	7.68
9	1.81	1.72	7.32	1.90	1.85	5.20
	1.93	1.95	8.17	2.31	2.26	7.66
	2.50	2.46	8.54	3.44	3.42	8.32

## 8. Statistical Test

We compared the statistical test quantitative decision of the proposed algorithm with other those of other conventional algorithms. The analysis of variance (ANOVA) was performed to indicate the statistical significance of the proposed algorithm with the conventional algorithms (RWGA and PBGA).

### 8.1. ANOVA for the efficiency test

The performance of MGA, RW selection-based GA (RWGA), and probabilistic selection-based GA (PBGA) for solving the standard ship routing problem was compared. We obtained various parametric values, such as the different number of vessels, V3, V4, and V5; different number of containers, C7, C10, C15, C20, and C23, and four instances, namely instance 1 (I1), instance 2 (I2), instance 3 (I3), and instance 4 (I4). Various instances obtained various source ports and destination ports for the container, which changed the path of ships. In this model, nine types of data sets were considered for the ANOVA test. These nine data sets were categorized into two, short sea and deep-sea categories. Short sea data sets are represented as three ships and seven containers (V3C7), three ships and ten containers (V3C10), three ships and fifteen containers (V3C15), four ships and fifteen containers (V4C15), four ship and 20 containers (V4C20), five ship and 20 containers (V5C20). Deep sea data sets are represented as three ships and 15 containers (V3dC15), three ships, and 20 containers (V3dC20), four ships, and 23 containers (V4dC23). We obtained the result

for three types of the algorithm by using the standard ship routing library given by Hemmati et al., (2014). The results for benchmark data are expressed as follows:

**Table 10.** Comparative Results for Benchmark Data

		Short Sea					Deep-Sea			
		V3C7	V3C10	V3C15	V4C15	V4C20	V5C20	V3dC15	V3dC20	V4dC23
MGA	I1	876483	1313111	664424	454848	655617	642901	17263074	24286620	46565448
	I2	876580	1162171	593090	458165	642039	669206	15369754	24621576	48265056
	I3	899058	1320132	650774	418006	667173	548434	15906567	26041704	44681920
	I4	897202	1392051	683387	424909	663714	627702	15743047	24509110	48519056
RWGA	I1	1323232	1813450	861581	461202	800196	798402	22402576	33192620	65667660
	I2	1331933	1624214	895387	498499	738716	747174	2143396	32698632	65663988
	I3	1185146	1846947	801951	491465	725799	736120	21872220	32146192	60964440
	I4	1335549	1779040	861432	437312	727999	788223	21539136	34659412	61995028
PBGA	I1	1245731	1553214	842612	482627	800231	788321	20982165	33101152	50501933
	I2	1013155	1411325	791313	471737	712913	792319	19036152	31987645	52901340
	I3	1256324	1402389	782451	500321	773211	699939	18862371	29136691	57631210
	I4	1200135	1428362	802314	492001	690023	700291	19116692	29900267	7010005

For all three algorithms, we obtained 500 maximum generation (Max-gen) and 100 as the population size (Pop-size). The three algorithms were tested by using ship routing benchmark data. The results were obtained 100 win runs for all three algorithms. We compared more than two algorithms for efficiency tests. Therefore, the ANOVA is the best choice. We considered 100 runs and used the number of successful runs for three algorithms, namely MGA, RWGA, and PBGA, which are given in the following table:

**Table 11.** Number of wins run for different Algorithms

		Short Sea					Deep-Sea			
		V3C7	V3C10	V3C15	V4C15	V4C20	V5C20	V3dC15	V3dC20	V4dC23
MGA	I1	82	97	80	84	78	87	81	78	87
	I2	86	89	87	88	95	91	82	84	88
	I3	91	92	82	93	90	94	79	90	81
	I4	90	96	88	94	86	91	89	93	86
RWGA	I1	72	69	64	62	61	62	67	68	70
	I2	75	70	71	69	64	63	66	68	69
	I3	72	76	68	75	71	74	64	70	72
	I4	74	68	68	71	72	70	69	70	68
PBGA	I1	60	61	54	56	60	59	54	62	55
	I2	61	62	58	61	55	54	63	55	60
	I3	59	55	61	54	56	61	60	58	56
	I4	58	55	63	59	61	55	61	59	58

The ANOVA test presents a comparison of flexibility between the groups and flexibility within the groups. The sum of the square provides a total variation between the groups and within groups. The degree of freedom is calculated between groups as (number of samples of the individual group - 1) and within groups as (sum of all the samples - number of the sample of individual groups). The mean of the square is the sum of the square divided by the degree of freedom (mean of square = sum of square/degree of freedom). F is calculated as the comparison of the mean of the square between groups and the mean of the square within groups. The given table presents the ANOVA test result for the proposed model and two other algorithms:

**Table 12.** ANOVA Test Result

Instance No.	Source of Variation	Sum of Square	Degree of freedom	Mean of Square	F
Instance 1	Between Groups	3149.85	2	1574.93	76.62
	Within Groups	493.33	24	20.55	
	Total	3643.18	26		
Instance 2	Between Groups	3931.18	2	1965.60	149.71
	Within Groups	315.11	24	13.13	
	Total	4246.30	26		
Instance 3	Between Groups	4124.74	2	2062.37	116.31
	Within Groups	425.56	24	17.73	
	Total	4550.30	26		
Instance 4	Between Groups	4605.41	2	2302.71	291.98
	Within Groups	189.56	24	7.90	
	Total	4794.96	26		

The total sample size is nine for each algorithm, and the number of the algorithm is three. The critical value of  $V \approx 3.40$  and  $p$  is extremely smaller than  $\alpha = 0.05$  for all the cases. Because the  $F$  critical value is smaller than  $F$ , we rejected the null hypothesis, which provides statistical significance to the result. A significant difference was observed between the algorithms. Therefore, the proposed modified algorithm is more efficient than the other two algorithms.

## 9. Result discussion

We considered various sets of input values for dynamic ship routing and scheduling problems to achieve appropriate numerical outcomes. Table 2 provides the crisp data input for the traveling cost. Table 3 provides the operation cost of each product and the penalty cost for violating the time frame in case of each port. Table 4 displays the path and cost as the output from our ship routing model with a crisp cost matrix obtained by using classical GA and MGA. Table 5 provides the triangular fuzzy input data for traveling cost. Table 6 presents the output of our ship routing model considering fuzzy data obtained using classical GA and MGA. In Tables 4 and 6, we can see that the proposed algorithm provided the optimum result for crisp data input as well as the fuzzy data input. The use of crisp risk factors is presented in Table 7, whereas the use of fuzzy risk factors is exhibited in Table 8. Table 9 displays the risk achieved for the deterministic as well as the fuzzy models for both GA and our MGA. Here,  $R_{\max}$  for a route represents the maximum risk value on that route. Table 9 reveals that the proposed MG exhibited the best result for both the crisp and fuzzy data sets. The comparative results for the ship routing benchmark instances for the MGA, RWGA, and PBGA are displayed in Table 10, which reveals that the proposed MGA provides the best results for benchmark instances. In Table 11, we have compared the winning runs of our algorithm and those of two other algorithms. Table 12 displays the ANOVA test results for the three algorithms working on four instances of the data input for the standard ship routing problem. From Table 12, it seems we can conclude that the proposed modified algorithm works more efficiently and provides better results than the other two algorithms.

## 10. Managerial Insights

The substitution of the new model gives more space to the manager for making decisions regarding which approach to adopt for incurring maximum more profit for the organization. With the implementation of our model, a manager has the freedom to design the shipping route under an uncertain environment, a situation which can be easily applied to the shipping industry in real life. Our proposed model can handle dynamic demand and supply, and also generate optimum routes and minimize the overall costs of the transportation by ship, which is the principal motto of any ship routing and scheduling algorithm. Risk factors are considered in calculating optimum routes, which provides a realistic approach to the shipping industry. The use of cost minimization and time window reduces overall costs and maintain shipping discipline. That is why the proposed model can generate more profit than other previous models. The demand and supply of products at the port may vary because of market conditions or social issues and use of this model leads to less congestion at the port. Considering the cost and risks parameter as the fuzzy number makes the model realistic because costs and risks factor are changed time to time. This dynamic environment makes ship routing and scheduling problems more suitable for practical use. Therefore, management should adopt this realistic model, which is cost effective and manages the risks well.

## 11. Conclusions and Future Scope

In our model, we have assumed that ships start their sailing from starting ports, visit other ports to ensure the unhindered demand and supply of products at those ports, and end their routing in their destination ports. We obtained the optimum paths and traveling costs for various ships, and hence the optimal cost is calculated by adding the cost of traveling, loading–unloading operations, and penalties (levied to ensure port discipline). Cost efficiency and environmental friendliness are prioritized by considering a few aspects such as the dynamic demand and supply models followed in ports, container-based shipping, and techniques for saving fuel. Fuzzy cost and fuzzy risk factor are introduced to represent the impreciseness of the parameters. Therefore, in our model, a real-time ship routing approach is proposed in an uncertain environment. The advantages of this model are its ability to the determined optimal path and cost when various real-life parameters are uncertain. This model maintains the port time as well as the traffic, and handles the container congestion very smoothly. We have used BBMST selection and IVF crossover techniques in our work. A generation-dependent mutation was used in the proposed algorithm in which, when the number of generations increased, the algorithm provided a better result. The advantages of the proposed methodology are that the novel BBMST selects the better solution for further steps, which gives optimal solution in less computational time. The IVF-based crossover method has been used on very few instances of work before. The proposed MGA clearly revealed itself as being superior other RWGA and PBGA in our numerical experiments. The ANOVA test revealed that the proposed algorithm works efficiently.

However, the model has some limitations. The maximum risk ( $R_{max}$ ) is selected randomly to ensure the total risk does not exceed the maximum risk. Dynamic weather routing may define various risks of a route for a ship for a particular time, which is not considered. We did not consider the container type, so our models are based only on standard containers. There also are some limitations in the solving

Solving fuzzy dynamic ship routing and scheduling problem through modified genetic... method: as the approach is heuristic, a high computational complexity exists. When numerous constraints are considered, the MGA falls short in solving the problem.

The futures cope of this model is actually quite wide. Various types of containers (foldable container and smart container) can be introduced in the future. We can predict the dynamic risks of each route using Artificial Intelligence and modern technology in the future. Various ranking-based fuzzy approaches, including the type-2 fuzzy approach, can be used to solve complex uncertain routing problems. We may use rough numbers, fuzzy systems, or even neutrosophic numbers in the future.

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