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COORDINATION OF A SINGLE-MANUFACTURER MULTI-RETAILER SUPPLY CHAIN WITH PRICE AND GREEN SENSITIVE DEMAND UNDER STOCHASTIC LEAD TIME

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Abstract: *When dealing with uncertainties in supply chain and ensuring customer satisfaction, efficient management of lead time plays a significant role. Likewise, besides managing inventory and pricing strategies adeptly in multi-retailer supply chain, it has become inevitable to the firms to embrace green and sustainable business practices. In this context, this paper considers a two-level supply chain consisting of a single manufacturer and multiple retailers in which the manufacturer produces a single product and delivers it to the retailers in some equal-sized batches. Each retailer faces a price and green sensitive market demand. The lead time is assumed to be a random variable which follows a normal distribution. Shortages for retailer inventory are allowed to occur and are completely backlogged. The centralized model and a decentralized model based on leader-follower Stackelberg gaming approach are developed. A price discount mechanism between the manufacturer and retailers is proposed. For the acceptance of this contract, the upper and lower limits of the price discount rate are established. Numerical outcomes exhibit that the price discount mechanism effectively coordinates the supply chain and enhances both environmental and economical performances. A sensitivity analysis with respect to some key parameters is performed, and certain managerial insights are emphasized.*

Key words: *Two-level supply chain, multiple retailers, stochastic lead time, price and green sensitive demand, price discount mechanism.*

1. Introduction

The growing importance of environmental protection and pollution reduction has been felt all over the world in recent years. Green supply chain management aims to prevent pollution while also producing environmentally friendly products. It involves many activities including green manufacturing, green packaging, green distribution,

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remanufacturing and waste management. Many industries (Walmart, Coca-Cola, Nike, Adidas, and others) are showing great interest in environmentally friendly supply chains. They are successfully influencing consumers' attitudes toward green products by emphasizing the benefits and necessity of a green supply chain. LG India has pioneered the creation of eco-friendly electronic gadgets. They have strictly used halogen or mercury, trying to reduce the use of dangerous substances in their products. TCS has already earned the title of Newsweek's Top Greenest Company in the world, with a global green score of 80.4% due to its worldwide recognized sustainability practices. Dell has promoted an efficient and effective safe disposal system by allowing their customers to return their product to the company for free. As consumer awareness grows, more people are willing to buy environmentally friendly products and are willing to pay more for those products. The government is also trying to make people aware of eco-friendly products through various guidelines and legislation. Researchers and practitioners are focusing on integrating environ-mental concerns into supply chain management.

Lead time plays a vital role in supply chain management. The assumption of deterministic lead time is not valid in most real world situations because of various reasons such as delays in production process, transit time, inspection, loading and unloading, and so on. Therefore, dealing with stochastic lead time is very fascinating and challenging. To avoid a planned shortage at the buyer's end and to efficiently manage the phenomena of early arrival, researchers are developing supply chain models with stochastic lead time (He et al., 2005; Lieckens and Vandaele, 2007; Barman et al., 2021b).

Price is another important factor that influences the customer demand. In this context, a good quality product with relatively lower price always attracts customers. In traditional supply chain management, the manufacturer determines the quality of the product and the retailers set their selling prices independently. Therefore, it has become an important managerial concern to implement an effective coordination between the manufacturer and the retailers for balancing the social and economical issues equitably. Suitable coordination schemes can improve the efficiency of the entire supply chain by creating incentives for all members to adopt it. Through such coordination mechanisms, the members of the supply chain develop a collaborative relationship between themselves. Researchers have performed a significant amount of work to coordinate the supply chain with an appropriate contract such as revenue sharing contract (Zhang and Feng, 2014; Mondal and Giri, 2021), cost sharing contract (Saha and Goyal, 2015; Zhu et al., 2018), delay in payments (Ebrahimi et al., 2019; Duary et al., 2022), etc.

In today's competitive market, manufacturers do not rely on a single retailer to sell their produced goods; instead, they deal with multiple retailers. In this study, we consider a two-level supply chain which is comprised of a single manufacturer and multiple retailers trading for a single product. The manufacturer delivers the retailers' order quantities in equal-sized batches and invests in green technologies to produce eco-friendly products. The product's greening level and selling price influence customer demand at each retailer. Replenishment lead time is assumed to be random. Both the centralized and decentralized models are considered. We demonstrate cooperation between the manufacturer and the retailers by a price discount mechanism. Our primary goal is to fulfill the research gap and find answers to the following research questions:

- What will be the optimal strategies of the manufacturer and retailers when the market demand is price and green sensitive?

- What is the impact of a price discount contract on the optimal decisions of the supply chain?
- Is the price discount mechanism capable of coordinating the supply chain?
- What is the effect of greening investment on the profitability of the supply chain?

The contributions of this study are as follows: Firstly, we incorporate a price discount mechanism with green initiatives in a single-manufacturer multi-retailer supply chain model under stochastic lead time. Secondly, we examine whether the proposed price discount contract is able to coordinate the supply chain or not. Finally, we look at the influence of the price discount contract on supply chain members' profitability and determine the conditions under which they accept the price discount contract.

The rest of this paper is structured as follows: Section 2 contains a brief review of the existing literature relevant to this work. Section 3 introduces notations and assumptions that are used throughout the paper. The problem description is given in Section 4. In section 5, mathematical models are formulated. Section 6 is devoted to numerical analysis. A sensitivity analysis of some key parameters is performed in Section 7. Section 8 discusses some managerial implications of this study. Finally, Section 9 concludes the paper with some limitations and future research directions.

2. Literature Review

In this section, we review some of the existing literatures which are related to our current work across four research streams: price- and green-sensitive demand, stochastic lead time, single-manufacturer multi-retailer supply chain model and price discount contract.

2.1 Price- and green-sensitive demand

Price is one of the important factors that influence market demand. A preliminary work focusing on price dependent demand was carried out by Whitin and Thomson (1955). Later, many researchers and practitioners (Ho et al., 2008; Yang et al., 2009; Lin and Ho, 2011; Atamer et al., 2013; Rad et al., 2014; Jaggi et al.,2015; Alfares and Ghaithan, 2016) have done numerous works on price dependent demand. Researchers and practitioners are currently focused on issues including the reduction of harmful effects of production on the environment. Swami and Shah (2013) studied a vertical supply chain consisting of a single manufacturer and a single retailer where the members put an effort for greening their operations, and the customer demand at the retailer's end is price and green sensitive. Zanoni et al. (2014) investigated a two-level joint economic lot size model with customer demand sensitive to price and environmental quality, and concluded that investing in improving a product's environmental performance is more beneficial, and implementing an integrated policy can increase both environmental and economic performance. Ghosh and Shah (2015) explored the positive impact of a cost sharing contract on the optimal decisions of a green supply chain to enhance the profit level and produce items with higher greening quality. Li et al. (2016) initiated e-commerce in green supply chain management and proposed a coordination mechanism for decentralized dual channel green supply chain. Basiri and Heydari (2017) investigated coordination issues in a green supply chain with a non-green traditional product and a substitutable green product under price, greening level and sales effort dependent demand. Giri et al. (2018) analyzed a two-level closed-loop supply chain model where the customer demand is affected by selling price, warranty period and greening level of the product. They proposed a revenue sharing contract in order to develop both social and economic performances. Heydari et al. (2019) developed a three-tier dual channel supply chain model with price and green sensitive demand that is not only economically beneficial but also reduces the selling price in both channels. Heydari et al. (2021) proposed a hybrid coordination scheme of cost sharing contract and revenue sharing contract in a two-level green supply chain with price and green sensitive demand. In a two-level supply chain model with imperfect production system, price, advertisement, and green sensitive customer demand, Giri and Dash (2022) established a cost-sharing contract between the manufacturer and the retailer. Sepehri and Gholamian (2022) investigated the impacts of shortages in a sustainable inventory model with price and emission sensitive demand considering quality improvement and inspection process concurrently.

2.2 Stochastic lead time

To address the shortcomings of deterministic lead time, researchers devised supply chain models that take into account the stochastic nature of lead time. Sajadieh et al. (2009) developed a single vendor single buyer supply chain model with stochastic lead time following exponential distribution and deterministic demand, and exhibited a significant cost reduction in integrated system than decentralized ones. Hoque (2013) presented an integrated inventory model with stochastic lead time following normal distribution under combined equal and unequal batch shipment policy. Lin (2016) considered an integrated vendor-buyer model with stochastic lead time, and demonstrated that further investment can reduce lead time variability and achieve enough savings for the entire system. Giri and Masanta (2019) derived optimal production and shipment policy for a closed-loop supply chain model with stochastic lead time, and observed that learning in production and remanufacturing leads to a significant cost reduction for the supply chain. Giri and Masanta (2020) developed a closed-loop supply chain model with learning in production, price and quality sensitive demand under stochastic lead time, and elaborated the positive impact of learning in production process on the optimal decisions. Sarkar et al. (2020a) investigated an integrated vendor-buyer model considering time value of money with partially backlogged shortage under stochastic lead time where the lead time is variable but dependent on the order size of the buyer and production rate at the vendor. Safarnezhad et al. (2021) derived optimal ordering, pricing and inspection policies in a vendor-buyer supply chain model with price dependent demand and stochastic lead time. Hoque (2021) developed a single-manufacturer multi-retailer supply chain model under stochastic lead time where the manufacturer delivers the lots to the retailers either only with equal batch sizes or only with unequal batch shipments.

2.3 Single-manufacturer multi-retailer supply chain model

To come closer to the reality, focusing on multi-retailer models has become a great topic of interest for the researchers. Recently, Giri and Roy (2016) considered a supply chain model consisting of a single manufacturer and multiple retailers with price sensitive customer demand. They found that lead time reduction by paying extra crashing cost does not affect the retail price significantly but enhances the entire system profit. Chen and Sarker (2017) investigated a single-manufacturer multi-retailer production-inventory model for deteriorating items with price sensitive demand under just-in-time delivery environment. They solved the model using particle swarm optimization (PSO) and quantum-behaved PSO (QBPSO) techniques. Majumder et al. (2018) studied a single-vendor multi-buyer supply chain model with variable production rate and controllable lead time reduction where the production cost at the vendor is a function of the production rate. Chan et al. (2018) proposed a coordination mechanism in a single-vendor multi-buyer supply chain model with stochastic demand, and synchronized the manufacturer's production cycle and retailers' ordering cycle. Ben-Daya et al. (2019) developed a single manufacturer multi-retailer closed-loop supply chain model with an environment-friendly approach of remanufacturing the used products under consignment stock policy. Giri et al. (2020b) developed a single-manufacturer multi-retailer inventory model with stochastic lead time and price sensitive demand. Esmaeili and Nasrabadi (2021) presented a single-vendor multi-retailer supply chain model for deteriorating items with trade credit and inflationary conditions, where the demand is price sensitive. Najafnejhad et al. (2021) used an imperialist competitive algorithm to solve a single-vendor multi-retailer inventory model under vendor managed inventory policy considering upper limits of inventories as decision variables. Nandra et al. (2021b) studied a single-vendor multi-buyer model that took into account variable production cost, imperfect items and environmental factors. Malleeswaran and Uthayakumar (2022) introduced a discrete investment for ordering cost reduction in a single-manufacturer multi-retailer EPQ model with green and environmental sensitive consumer demand and reworking system under carbon emissions policies.

2.4 Price discount contract

Coordination between manufacturers and retailers has received a lot of attention as a means of improving inventory control, and researchers have done a lot of work to coordinate the supply chain with the appropriate contract. As we consider a price discount coordination scheme in our study, we cover some literatures which address similar issues. Viswanathan and Piplani (2001) analyzed a single-vendor multi-buyer model with a coordination mechanism in which the vendor specifies the replenishment period and all the buyers agree to order at the same time in exchange for a price discount. Li et al. (2011) investigated the impact of a price discount mechanism in a single-vendor single-buyer supply chain model with service level constraint and controllable lead time. Aljazzar et al. (2017) dealt with a three-level supply chain with two types of trade credit mechanism, and concluded that implementing both delay in payment and price discount coordination mechanisms at a time lead more profit for the entire supply chain rather adopting these contracts individually. Nouri et al. (2018) proposed a compensation-based wholesale price contract between the manufacturer and the retailer where the customer demand is stochastic and dependent on innovation and promotional efforts. Furthermore, they devised a profit-sharing strategy on the basis of bargaining power of the members. Xu et al. (2018) investigated the role of a price discount contract in coordinating a dual-channel supply chain under carbon emission capacity regulation, with consumer demand in both online and offline channels influenced by the product's selling price. They provided the necessary conditions for which the price discount contract coordinates the dual-supply chain in both online and offline modes. Sarkar et al. (2020b) suggested a price discount coordination mechanism in a two-level supply chain with price sensitive customer demand to encourage the supply chain players to take part in joint decision-making strategy. Yang et al. (2021) explored the optimal cooperation strategy between an upstream supplier and two competing manufacturers considering a wholesale price contract and manufacturers' technology investment. In order to reduce products' carbon emissions. Zu et al. (2021) analyzed a single-manufacturer single-retailer supply chain model under two different mechanisms viz. wholesale price contract and consignment contract. Zhang et al. (2022) performed a comparative analysis between wholesale price contract and cost-sharing contract in a two-level green supply chain model. They looked at which contract is more effective in improving the product's greenness and promoting demand, taking into account the consumer reference pricing effect.

2.5 Research gaps in the existing literature

Table 1 summarizes the research gaps in the existing literature as follows:

- Although there are numerous research papers available that explore stochastic lead time and single-manufacturer multi-retailer supply chain models, no attempt has been made to maximize individual profits of supply chain stakeholders. The majority of these research focused on maximizing (or minimizing) overall supply chain profit (or cost).
- Most of these studies considered deterministic customer demand. They overlooked some crucial factors such as the selling price, greening level, promotional effort, advertising and product quality, all of which have an impact on market demand.
- No one has incorporated environmental awareness into a single-manufacturer multi-retailer supply chain model with stochastic lead time, and none of these studies looked at the influence of greening investment on both the supply chain's economic and environmental performance.
- Almost no study has ever suggested a channel coordination mechanism.

The above literature review reveals a significant research gap and indicates that no attempt has been made in implementing price discount coordination mechanism in a single-manufacturer multi-retailer supply chain model with price and green sensitive demand under stochastic lead time. It would be interesting and contributory to consider all the genuine issues like the stochastic nature of lead time, the impact of retail price and environmental awareness on market demand, single-manufacturer multi-retailer business situations and so on under one umbrella. Although, Hoque (2021) extended the model of Hoque (2013) in multi-retailer scenario, but he considered the demand of each retailer as deterministic and minimizes the total cost of the supply chain. In this paper, our aim is to fulfill this research gap and implement an appropriate coordination scheme which efficiently improves each supply chain member's profitability as well as environmental performance. A comparison of the present work with the relevant existing literature is presented in Table 1.

Table 1. A comparison of the present model with some existing literature

Authors	Retailer	Batch shipment	Demand	Lead time	Coordination
Sajadieh et al. (2009)	Single	Equal	Deterministic	Stochastic	No
Li et al. (2011)	Single	Equal	Deterministic	Controllable	Price discount
Hoque (2013)	Single	Equal & unequal	Deterministic	Stochastic	No
Sarkar et al. (2017)	Single	Equal	Deterministic	No	No
Giri et al. (2018)	Single	No	Price, green and warranty period sensitive	No	Revenue sharing
Sarkar et al. (2018)	Multiple	Equal	Deterministic	Variable	No
Giri and Masanta (2019)	Single	Equal	Deterministic	Stochastic	No
Giri et al. (2020a)	Single	Equal & unequal	Price and green dependent	No	Cost sharing
Sarkar et al. (2020b)	Single	Equal	Price dependent	No	Price discount
Agrawal and Yadav (2020)	Multiple	Equal	Price dependent	constant	Profit sharing
Esmaeili and Nasrabadi (2021).	Multiple	No	Price dependent	No	No
Nandra et al. (2021a)	Multiple	Equal	Deterministic	Controllable	No
Sarkar et al. (2021)	Single	Equal	Online & offline price dependent	Distribution free approach & normal	No
Safarnezhad et al. (2021)	Single	No	Price dependent	Stochastic	No
Hoque (2021)	Multiple	Equal or unequal	Deterministic	Stochastic	No
This paper	Multiple	Equal	Price and green dependent	Stochastic	Price discount

3. Notations and Assumptions

The following notations are used for developing the proposed model:

Parameters:

R	production rate (units/ year)
A_v	set-up cost per set-up ($/set-up)
h_v	manufacturer's holding cost per item per unit time ($/unit /year)
F	transportation cost per batch shipment($/shipment)
w	unit wholesale price($/unit)
I	greening investment parameter ($)
N	number of retailers (positive integer)

Q	total order quantity $[=\sum_{i=1}^{N} Q_i]$(units)
D	total market demand $[=\sum_{i=1}^{N} D_i]$(units /year)
L	lead time, a random variable with p.d.f. $f_L(.)$

i-th retailer:

A_i	ordering cost per order ($/order)
h_i	holding cost per item per unit time ($/unit /year)
D_i	demand rate $[R > \sum_{i=1}^{N} D_i]$(units /year)
a_i	basic market demand (units /year)
α_i	consumer sensitivity coefficient to greening level
β_i	consumer sensitivity coefficient to retail price
Q_i	order quantity (units)
c_i	shortage cost per item per unit time ($/unit /year)
r_i	reorder point(units)
σ_i	standard deviation of the lead time

Decision variables:

n	number of batches delivered to each retailer (positive integer)
θ	greening improvement level
q_i	batch size of the i-th retailer (units)
p_i	unit retail price of the i-th retailer ($/unit)
ϕ	price discount ratio, $\phi \in [0, 1]$
$(.)\text{^}d$	decision variable in decentralized policy
$(.)\text{^}c$	decision variable in centralized policy
$(.)\text{^}co$	decision variable in coordinated mechanism

Profit functions:

AEP_m	average expected profit of the manufacturer($/year)
AEP_i	average expected profit of the i-th retailer($/year)
AEP_s	average expected profit of the supply chain ($/year)

The basic assumptions for developing the proposed model are as follows:

1. A single manufacturer produces a single item and meets the demand of multiple retailers (Sarkar et al., 2018).
2. The manufacturer transfers the products to the retailers in a number of equal sized batches (Sarkar et al., 2020b).
3. The retailers face a consumer demand dependent on the selling price and greenness of the product (Ghosh and Shah, 2015). We assume that the demand rate of the i-th retailer is a linear function of retail price and greening level of the product given by $D_i(p_i, \theta) = a_i - \beta_i p_i + \alpha_i \theta$, where a_i is the basic market demand, β_i and α_i are positive integers such that $a_i + \alpha_i \theta > \beta_i p_i$ for all $i = 1,2,\ldots,N$.
4. The manufacturer produces the product at a constant production rate R in one set-up and the production rate is greater than the sum of demands of all retailers i.e., $R > \sum_{i=1}^{N} D_i$ (Hoque, 2021).
5. Shortages are allowed and are assumed to be completely backlogged (Sarkar et al., 2018).

6. The i-th retailer places his next order when his inventory stock level reaches to a certain reorder level r_i (Hoque, 2013).
7. The lead time to meet the retailer's demand is a random variable which follows a normal distribution and the lead time for each shipment is independent of the others (Hoque, 2013).
8. Annual greening investment for the product is taken as $I\theta^2$, which is increasing and convex in the greening improvement level θ (Ghosh and Shah, 2015).

4. Problem Definition

This study develops a green supply chain model where the single manufacturer deals with multiple retailers for a single product. Figure 1 exhibits the schematic diagram of the proposed model.

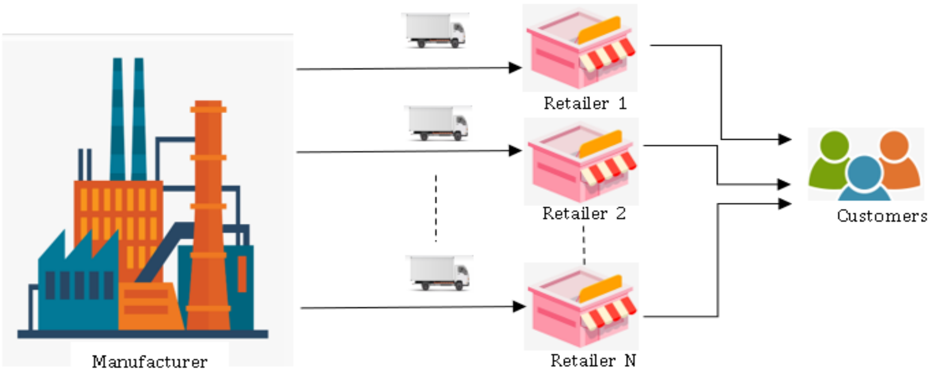

Figure 1. Logistics diagram of the proposed single-manufacturer multi-retailer green supply chain model

The manufacturer produces the items at a fixed production rate in a single set-up and delivers the order quantities of the retailers with an equal sized batch shipment policy. Due to various unavoidable circumstances such as late start in production, varying transportation time, loading, unloading, etc., the batches may arrive early or late at the retailers. To deal with this type of delivery uncertainty, lead time is treated as a stochastic random variable which follows a normal distribution. Customer demand is assumed to be affected by the retail price and environmental performance of the product. The manufacturer adopts a green investment strategy to maintain his environmental responsibility as well as stimulate the customer demand in an eco-conscious market. In both decentralized and centralized settings, the manufacturer's and all retailers' optimal pricing and inventory strategies are derived. Following that, a wholesale price discount contract is implemented between the manufacturer and the retailers to coordinate the supply chain.

5. Model Formulation

We suppose that the manufacturer sells the produced items to N retailers. The manufacturer transfers the ordering quantity Q_i of the i-th retailer in n equal batches of size q_i. Total order quantity of N retailers is Q. Therefore, $Q_i = nq_i$ and $Q = \sum_{i=1}^{N} Q_i$.

The i-th retailer places the next order when the inventory stock reaches to a level r_i. The shipment is expected to arrive to the retailer's end at or before the time of selling this r_i quantity. The mean lead time is $\frac{r_i}{D_i}$. Due to various reasons, the batches may reach early or late. We assume that the lead time follows a normal distribution. Depending on the length of the lead time, three cases may arise:

Case (i) *When the batch q_i reaches to the retailer earlier i.e.,* $0 < l_i < \frac{r_i}{D_i}$.

In this case, similar to Hoque (2013), the inventory holding area of the i-th retailer can be determined from Figure 2(a) as

$$= Area\ (\triangle ABCD + \triangle EFG + \square GHDE)$$

$$= \frac{1}{2}(r_i - D_i l_i + r_i) l_i + \frac{1}{2}(q_i - D_i l_i)\frac{(q_i - D_i l_i)}{D_i} + \frac{r_i(q_i - D_i l_i)}{D_i}$$

$$= \frac{1}{2}\left[\frac{q_i^2}{D_i} + 2q_i\left(\frac{r_i}{D_i} - l_i\right)\right] \text{ where } r_i = \frac{q_i D_i}{R}$$

Then the order quantity Q_i of the i-th retailer is given by

$$\frac{n}{2}\left[\frac{q_i^2}{D_i} + 2q_i\left(\frac{r_i}{D_i} - l_i\right)\right]$$

The holding cost refers to the investment in storing the unsold products. The expected inventory holding cost for the order quantity Q_i of the i-th retailer is

$$h_i \int_0^{\frac{r_i}{D_i}} \frac{n}{2}\left[\frac{q_i^2}{D_i} + 2q_i\left(\frac{r_i}{D_i} - l_i\right)\right] f_L(l_i) dl_i$$

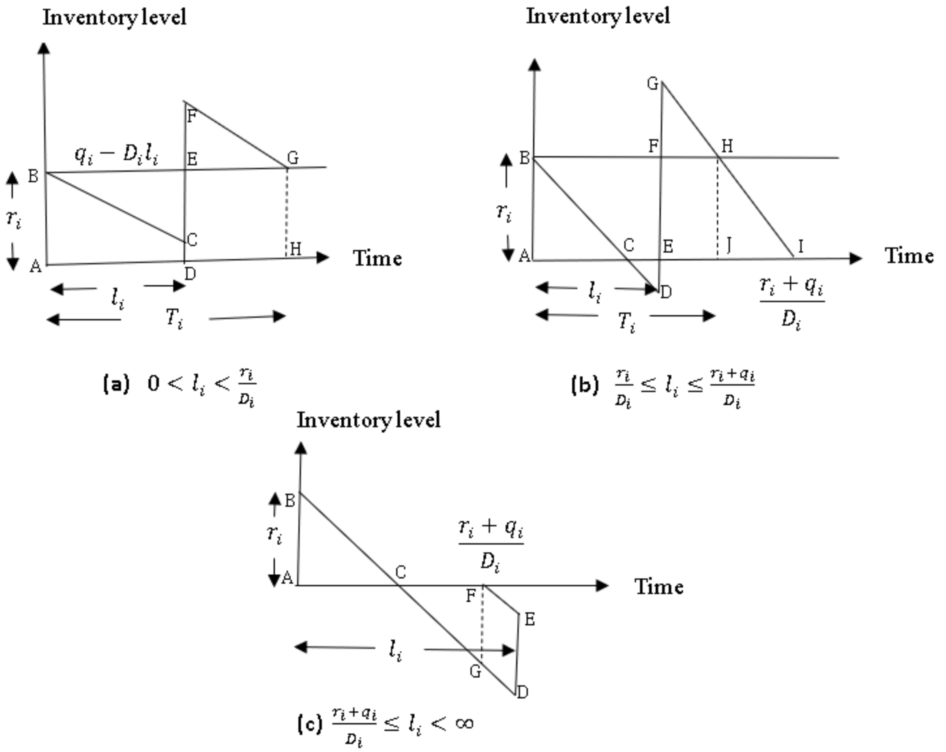

Figure 2. Inventory of i- th retailer under stochastic lead time

Case (ii) When the batch q_i reaches late to the i-th retailer and the lead time l_i lies in the range $\frac{r_i}{D_i} \le l_i \le \frac{r_i+q_i}{D_i}$.

In this case, shortages occur at the retailer's end. From Figure 2(b), the shortage area at the i-th retailer is obtained as

$$= Area\ (\triangle CDE)$$
$$= \frac{1}{2D_i}(D_i l_i - r_i)^2.$$

So, the expected shortage cost of the i-th retailer for n batches is given by

$$\frac{nc_i}{2}\int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(D_i l_i - r_i)^2}{D_i} f_L(l_i)dl_i$$

Inventory holding area of the i-th retailer for the batch q_i is

$$= Area\ (\triangle ABC + \triangle FGH + \square EFHJ)$$
$$= \frac{r_i^2}{2D_i} + \frac{(q_i - D_i l_i)^2}{2D_i} + \frac{r_i(q_i - D_i l_i)}{D_i}$$
$$= \frac{(q_i - D_i l_i + r_i)^2}{2D_i}$$

Hence the expected inventory holding cost of the i-th retailer for n shipments is obtained as

$$nh_i \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(q_i - D_i l_i + r_i)^2}{2D_i} f_L(l_i)dl_i$$

It is assumed that, during this delay period, the batches remain in the manufacturer's stockhouse. So, it causes an extra holding cost to the manufacturer. The extra inventory for this delayed delivery is $\sum_{i=1}^{N} \frac{nq_i(D_i l_i - r_i)}{D_i}$. So, in this case, the additional inventory holding cost for the manufacturer is

$$h_v \sum_{i=1}^{N} \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{nq_i(D_i l_i - r_i)}{D_i} f_L(l_i)dl_i.$$

Case(iii) When the batch q_i arrives late to the retailer with lead time in the range $\frac{r_i+q_i}{D_i} \le l_i < \infty$.

In this case, shortages occur at the retailer's end and from Figure 2(c), the shortage area for the batch q_i is obtained as

$$= Area\ (\square CDEF)$$
$$= \frac{q_i^2}{2D_i} + Area(\square\ DEFG)$$

So, the expected shortage cost of the i-th retailer for all batch shipments is

$$nc_i \int_{\frac{r_i+q_i}{D_i}}^{\infty} \left[\frac{q_i^2}{2D_i} + q_i\left(\frac{D_i l_i - q_i - r_i}{D_i}\right)\right] f_L(l_i)dl_i$$

Similar to case(ii), the additional expected inventory holding cost for the manufacturer is

$$h_v \sum_{i=1}^{N} \int_{\frac{r_i+q_i}{D_i}}^{\infty} \frac{nq_i(D_i l_i - r_i)}{D_i} f_L(l_i)dl_i.$$

Combining all three cases, the expected holding cost of the i-th retailer for all batches is given by

$$nh_i\int_0^{\frac{r_i}{D_i}}\frac{1}{2}\left[\frac{q_i^2}{D_i}+2q_i\left(\frac{r_i}{D_i}-l_i\right)\right]f_L(l_i)dl_i+nh_i\int_{\frac{r_i}{D_i}}^{\frac{r_{i+}q_i}{D_i}}\frac{(q_i-D_il_i+r_i)^2}{2D_i}f_L(l_i)dl_i$$

and the expected shortage cost for all batch shipments is

$$nc_i\int_{\frac{r_i}{D_i}}^{\frac{r_{i+}q_i}{D_i}}\frac{(D_il_i-r_i)^2}{2D_i}f_L(l_i)dl_i+nc_i\int_{\frac{r_{i+}q_i}{D_i}}^{\infty}\left[\frac{q_i^2}{2D_i}+q_i\left(\frac{D_il_i-q_i-r_i}{D_i}\right)\right]f_L(l_i)dl_i$$

5.1. Decentralized Model (DM)

In the decentralized model, the manufacturer and the retailers independently take their decisions in order to maximize their own profits. Here we consider the retailers to be the Stackelberg leader and the manufacturer as the follower. The manufacturer sets the number of shipments and greening level of the products. Then taking these responses into consideration, the retailers decide their optimal retail price and batch sizes.

Average expected profit of the manufacturer

The manufacturer's total extra holding cost from cases(ii) and (iii) is

$$h_v\sum_{i=1}^{N}\int_{\frac{r_i}{D_i}}^{\infty}\frac{nq_i(D_il_i-r_i)}{D_i}f_L(l_i)dl_i$$

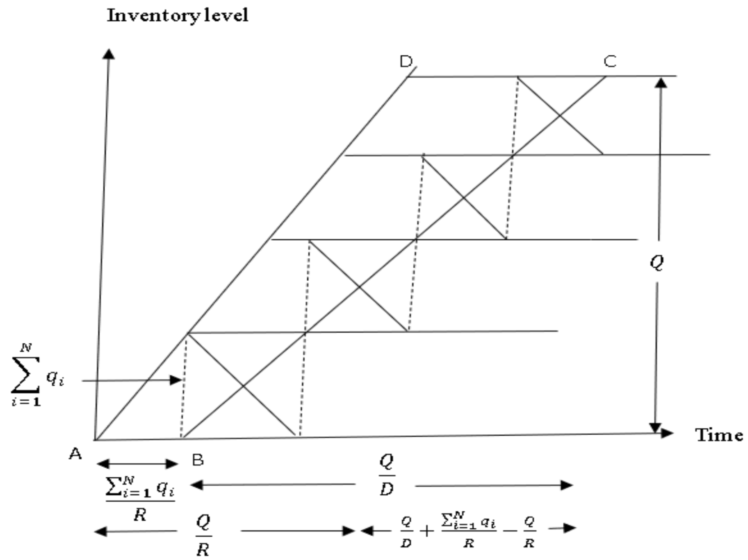

Figure 3. Joint inventory of the manufacturer and the retailers

In Figure 3, the trapezium ABCD represents the joint inventory of the manufacturer and retailers. The average inventory of the manufacturer-retailer system is

$$= Area\ (\triangle\ ABCD)\times\frac{D}{Q}$$

$$= \frac{1}{2} \times (AB + CD) \times Q \times \frac{D}{Q}$$

$$= \frac{1}{2}\left[\frac{\sum_{i=1}^{N} q_i}{R} + \left(\frac{Q}{D} + \frac{\sum_{i=1}^{N} q_i}{R} - \frac{Q}{R}\right)\right] D = \frac{D\sum_{i=1}^{N} q_i}{R} + \frac{Q}{2}\left(1 - \frac{D}{R}\right)$$

Average inventory holding of N retailers is

$$\sum_{i=1}^{N}\left(\frac{{q_i}^2}{2D_i}\right)\left(\frac{D_i}{Q_i}\right) = \sum_{i=1}^{N}\frac{{q_i}^2}{2Q_i}$$

Therefore, the average inventory holding of the manufacturer is

$$\frac{D\sum_{i=1}^{N} q_i}{R} + \frac{Q}{2}\left(1 - \frac{D}{R}\right) - \sum_{i=1}^{N}\frac{{q_i}^2}{2Q_i}$$

The set-up cost incorporates the costs of materials and labours to get ready the machinery system for processing the new production lot of goods. It plays an important role in start-up of a new business and smooth running of it. The i-th retailer places an order of quantity Q_i. The manufacturer produces the total order quantity $Q = \sum_{i=1}^{N} Q_i$. The cycle length of the manufacturer is $\frac{Q}{D}$. Therefore, the average set up cost is $\frac{A_v D}{Q}$.

Investment for greening supports the environmentally-conscious business practices. In this case, the manufacturer's average greening investment is $I\theta^2$.
The average expected profit of the manufacturer is

$$AEP_m(n,\theta) = wD - h_v\left[\frac{D\sum_{i=1}^{N} q_i}{R} + \frac{Q}{2}\left(1 - \frac{D}{R}\right) - \sum_{i=1}^{N}\frac{q_i^2}{2Q_i}\right] - h_v\sum_{i=1}^{N}\int_{\frac{r_i}{D_i}}^{\infty}(D_i l_i - r_i) f_L(l_i) dl_i$$
$$- \frac{A_v D}{Q} - I\theta^2$$
$$= wD - \frac{A_v D}{Q} - h_v\left[\frac{D\sum_{i=1}^{N} q_i}{R} + \frac{Q}{2}\left(1 - \frac{D}{R}\right) - \sum_{i=1}^{N}\frac{q_i^2}{2Q_i}\right] - \sum_{i=1}^{N}\frac{h_v D_i \sigma_i}{\sqrt{2\pi}} - I\theta^2 \tag{1}$$

From (1), we have

$$\frac{\partial AEP_m}{\partial n} = \frac{A_v D}{n^2 s} - \frac{h_v s}{2} + \frac{h_v s D}{2R} - \frac{h_v s}{2n^2} \tag{2}$$

$$\frac{\partial AEP_m}{\partial \theta} = \left[w - \frac{A_v}{Q} - \frac{h_v s}{R} + \frac{h_v Q}{2R} - \left(\sum_{i=1}^{N}\frac{h_v \sigma_i}{\sqrt{2\pi}}\right)\right] u - 2I\theta \tag{3}$$

$$\frac{\partial^2 AEP_m}{\partial n^2} = -\frac{2A_v D}{n^3 s} + \frac{h_v s}{n^3} \tag{4}$$

$$\frac{\partial^2 AEP_m}{\partial \theta^2} = -2I \tag{5}$$

$$\frac{\partial^2 AEP_m}{\partial n \partial \theta} = \frac{A_v u}{n^2 s} + \frac{h_v s u}{2R} \tag{6}$$

$$\frac{\partial^2 AEP_m}{\partial \theta \partial n} = \frac{A_v u}{n^2 s} + \frac{h_v s u}{2R}, \text{ where } s = \sum_{i=1}^{N} q_i \text{ and } u = \sum_{i=1}^{N} \alpha_i \tag{7}$$

Proposition 1. The average expected profit function of the manufacturer is jointly concave in n and θ if $8IR^2 ns(2A_v Ds - h_v s^2) > (2A_v Ru + h_v un^2 s^2)^2$.

Proof. Considering n as real, the Hessian matrix is

$$H = \begin{pmatrix} \frac{\partial^2 AEP_m}{\partial \theta^2} & \frac{\partial^2 AEP_m}{\partial \theta \partial n} \\ \frac{\partial^2 AEP_m}{\partial n \partial \theta} & \frac{\partial^2 AEP_m}{\partial n^2} \end{pmatrix} = \begin{pmatrix} -2I & \frac{A_v u}{n^2 s} + \frac{h_v s u}{2R} \\ \frac{A_v u}{n^2 s} + \frac{h_v s u}{2R} & -\frac{2A_v D}{n^3 s} + \frac{h_v s}{n^3} \end{pmatrix} \tag{8}$$

Here, $\frac{\partial^2 AEP_m}{\partial \theta^2} = -2I < 0$. So, the expected average profit function of the manufacturer will be concave in θ and n if $|H| > 0$. Substituting the values of the partial derivatives from the above and using the condition $|H| > 0$, we get after simplification, $8IR^2ns(2A_vDs - h_vs^2) > (2A_vRu + h_vun^2s^2)^2$.

Proposition 2. At the equilibrium, the optimal number of shipments to each retailer, and the optimal greening level of the product are as follows:

$$n^* = \sqrt{\frac{R(2A_vD - h_vs^2)}{h_vs^2(R-D)}} \tag{9}$$

$$\theta^* = \frac{\left[w - \frac{A_v}{Q} - \frac{h_vs}{R} + \frac{h_vQ}{2R} - \left(\sum_{i=1}^{N} \frac{h_v\sigma_i}{\sqrt{2\pi}}\right)\right]u}{2I} \tag{10}$$

Proof. At the equilibrium, we have

$$\frac{\partial AEP_m}{\partial n} = \frac{A_vD}{n^2s} - \frac{h_vs}{2} + \frac{h_vsD}{2R} - \frac{h_vs}{2n^2} = 0 \tag{11}$$

and $$\frac{\partial AEP_m}{\partial \theta} = \left[w - \frac{A_v}{Q} - \frac{h_vs}{R} + \frac{h_vQ}{2R} - \left(\sum_{i=1}^{N} \frac{h_v\sigma_i}{\sqrt{2\pi}}\right)\right]u - 2I\theta = 0 \tag{12}$$

Solving equations (11) and (12), we get the optimal values of n and θ as given in equations (9) and (10) above.

For integer optimal value of n,

$$n^{opt} = \begin{cases} \lfloor n^* \rfloor, & if\ AEP_m(\lfloor n^* \rfloor, \theta) \geq AEP_m(\lceil n^* \rceil, \theta) \\ \lceil n^* \rceil, & if\ AEP_m(\lfloor n^* \rfloor, \theta) \leq AEP_m(\lceil n^* \rceil, \theta) \end{cases}$$

Taking these response functions of the manufacturer, the retailers then set their batch sizes and retail prices.

Average expected profit of the i-th retailer

Since the expected cycle length for the i-th retailer is $\frac{\mathrm{Q_i}}{\mathrm{D_i}}$, therefore, the average ordering cost of the i-th retailer is given by $\frac{A_iD_i}{Q_i}$.

From manufacturing to delivery to the end customer and even returns, transportation is essential to the entire production process. It is practically impossible for a logistics firm to conduct business efficiently without transportation. As the number of shipments increases, the transportation cost increases. Since the manufacturer delivers order quantity to the i-th retailer in n shipments and the expected cycle length for the i-th retailer is $\frac{Q_i}{D_i}$, therefore, the average variable transportation cost is $\frac{nFD_i}{Q_i}$.

The expected total profit of the i-th retailer is

$$p_iQ_i - wQ_i - A_i - nF - \frac{nh_i}{2}\left[\int_0^{\frac{r_i}{D_i}}\left[\frac{q_i^2}{D_i} + 2q_i\left(\frac{r_i}{D_i} - l_i\right)\right]f_L(l_i)dl_i\right.$$
$$\left.+ \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(q_i - D_il_i + r_i)^2}{D_i} f_L(l_i)dl_i\right] - \frac{nc_i}{2}\left[\int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(D_il_i - r_i)^2}{D_i} f_L(l_i)dl_i\right.$$
$$\left.+ \int_{\frac{r_i+q_i}{D_i}}^{\infty}\left[\frac{q_i^2}{D_i} + 2q_i\left(\frac{D_il_i - q_i - r_i}{D_i}\right)\right]f_L(l_i)dl_i\right]$$

Therefore, the average expected profit of the i-th retailer is obtained as

$$AEP_i(q_i,p_i) = p_iD_i - wD_i - \frac{(A_i+nF)D_i}{Q_i} - \frac{h_i}{2}\left[\int_0^{\frac{r_i}{D_i}} [q_i + 2(r_i - D_il_i)]f_L(l_i)dl_i + \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(q_i-D_il_i+r_i)^2}{q_i} f_L(l_i)dl_i\right] - \frac{c_i}{2}\left[\int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(D_il_i-r_i)^2}{q_i} f_L(l_i)dl_i + \int_{\frac{r_i+q_i}{D_i}}^{\infty} [q_i + 2(D_il_i - q_i - r_i)]f_L(l_i)dl_i\right] \quad (13)$$

Proposition 3. The average expected profit of the i-th retailer is concave in q_i for given p_i if $\frac{2(A_i+nF)D_i}{n} + (h_i + c_i)\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} D_i^{\,2}l_i^{\,2}f_L(l_i)dl_i > 0$.

Proof. Differentiating (13) twice with respect to q_i, we obtain

$$\frac{\partial AEP_i}{\partial q_i} = \frac{(A_i+nF)D_i}{nq_i^2} - \frac{h_i}{2}\int_0^{\frac{q_i}{R}} \left(\frac{R+2D_i}{R}\right) f_L(l_i)dl_i - \frac{h_i}{2}\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \left(\frac{q_i^2(R+D_i)^2 - R^2D_i^2l_i^2}{q_i^2R^2}\right) f_L(l_i)dl_i + \frac{c_i}{2}\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \left(\frac{R^2D_i^2l_i^2 - q_i^2D_i^2}{q_i^2R^2}\right) f_L(l_i)dl_i + \frac{c_i}{2}\int_{\frac{q_i(R+D_i)}{RD_i}}^{\infty} \left(\frac{R+2D_i}{R}\right) f_L(l_i)dl_i \quad (14)$$

$$\frac{\partial^2 AEP_i}{\partial q_i^2} = -\frac{2(A_i+nF)D_i}{nq_i^3} - h_i\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{D_i^2l_i^2}{q_i^3} f_L(l_i)dl_i - c_i\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{D_i^2l_i^2}{q_i^3} f_L(l_i)dl_i \quad (15)$$

For given p_i, the average expected profit function of the i-th retailer is concave in q_i if $\frac{\partial^2 AEP_i}{\partial q_i^2}$ is negative.

This implies the condition $\frac{2(A_i+nF)D_i}{n} + (h_i + c_i)\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} D_i^{\,2}l_i^{\,2}f_L(l_i)dl_i > 0$.

Proposition 4. *The average expected profit of the i-th retailer is concave in p_i for given q_i if* $2\beta_i + (h_i + c_i)\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i^2(q_i-Rl_i)^2}{q_iR^2} f_L(l_i)dl_i > 0$.

Proof. Differentiating (13) with respect to p_i, we obtain

$$\frac{\partial AEP_i}{\partial p_i} = D_i - \beta_ip_i + h_i\int_0^{\frac{q_i}{R}} \frac{\beta_i(q_i - Rl_i)}{R} f_L(l_i)dl_i + h_i\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i(q_i - Rl_i)[q_iR + D_i(q_i - Rl_i)]}{q_iR^2} f_L(l_i)dl_i + w\beta_i + \frac{(A_i+nF)\beta_i}{Q_i} + c_i\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_iD_i(q_i - Rl_i)^2}{q_iR^2} f_L(l_i)dl_i - c_i\int_{\frac{q_i(R+D_i)}{RD_i}}^{\infty} \frac{\beta_i(q_i - Rl_i)}{R} f_L(l_i)dl_i \quad (16)$$

$$\frac{\partial^2 AEP_i}{\partial p_i^2} = -2\beta_i - (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i^2 (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i$$

(17)

Since β_i, h_i and c_i all are positive, therefore, it implies that

$$2\beta_i + (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i^2 (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i > 0.$$

Therefore, the average expected profit function AEP_i is concave in p_i for given q_i if the above condition satisfies.

Solution Algorithm

Taking the best response from the manufacturer, the average expected profit of the i-th retailer can be optimized using the following solution algorithm. To optimize the expected average profit of the i-th retailer, we consider initial guess values to the decision variables of the remaining $(N-1)$ retailers.

Step 1: Set $k = 1$.

Step 2: Set $i = 1$ and $q_j = q_j^{(k-1)}, p_j = p_j^{(k-1)}$ for all $j = i+1, i+2, \ldots.., N$.

Step 3: Optimize AEP_i taking n and θ from the response functions of the manufacturer and $q_j = q_j^{(k-1)}, p_j = p_j^{(k-1)}$ for all $j = i+1, i+2, \ldots.., N$. Set the optimal results as $q_i = q_i^{(k)}$ and $p_i = p_i^{(k)}$.

Step 4: Set $i = i + 1$.

Step 5: Optimize AEP_i taking n and θ from the manufacturer's response functions and $q_j = q_j^{(k)}, p_j = p_j^{(k)}$ for $j = 1,2, \ldots, i-1$ and $q_j = q_j^{(k-1)}, p_j = p_j^{(k-1)}$ for $j = i+1, i+2, \ldots., N$. Set the optimal results as $q_i = q_i^{(k)}$ and $p_i = p_i^{(k)}$.

Step 6: Repeat steps 4 and 5 until $i = N$.

Step 7: Stop if $q_j^{(k)} = q_j^{(k-1)}$ and $p_j^{(k)} = p_j^{(k-1)}$ for all $j = 2,3, \ldots.., N$ and consider $q_j^{(*)} = q_j^{(k)}$ and $p_j^{(*)} = p_j^{(k)}$ for all $j = 1,2,3, \ldots.., N$. Otherwise, set $k = k + 1$ and repeat steps 2 to 6.

Step 8: Evaluate the optimal values of n^* and θ^* taking q_j^* and p_j^* for all $j = 1,2,3, \ldots.., N$.

Step 9: Using these results, calculate optimal values of AEP_m and AEP_s.

5.2. Centralized Model (CM)

In this scenario, the manufacturer and all the retailers of the supply chain act jointly as a single decision maker. They determine the optimal selling prices of the product, greening improvement level, number of shipments and batch sizes in order to maximize the entire system profit rather than focusing on their individual profits. The average expected profit of the supply chain is

$$AEP_s(n, \theta, q_i, p_i) = \sum_{i=1}^N p_i D_i - \frac{A_v D}{Q} - h_v \left[\frac{D \sum_{i=1}^N q_i}{R} + \frac{Q}{2}\left(1 - \frac{D}{R}\right) - \sum_{i=1}^N \frac{q_i^2}{2Q_i} \right]$$

$$- \sum_{i=1}^N \frac{h_v D_i \sigma_i}{\sqrt{2\pi}} - I\theta^2 - \sum_{i=1}^N \left[\frac{(A_i + nF) D_i}{Q_i} + \frac{h_i}{2} \left[\int_0^{\frac{r_i}{D_i}} [q_i + 2(r_i - \right.\right.$$

$$D_i l_i)] f_L(l_i) dl_i + \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(q_i - D_i l_i + r_i)^2}{q_i} f_L(l_i) dl_i \Bigg] + \frac{c_i}{2} \Bigg[\int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(D_i l_i - r_i)^2}{q_i} f_L(l_i) dl_i + \int_{\frac{r_i+q_i}{D_i}}^{\infty} [q_i + 2(D_i l_i - q_i - r_i)] f_L(l_i) dl_i \Bigg] \Bigg] \quad (18)$$

Proposition 5. The average expected profit of the system is concave in n for given θ, q_i and p_i if $h_v s^2 < 2(A_v D + sg)$, where $g = \sum_{i=1}^{N} \frac{A_i D_i}{q_i}$ and the optimal number of shipments is given by

$$n^* = \sqrt{\frac{R(2A_V D - h_v s^2 + 2sg)}{h_v s^2 (R - D)}} \quad (19)$$

Proof. Considering n as real, from equation (18), we derive the following partial derivatives:

$$\frac{\partial AEP_S}{\partial n} = \frac{A_v D}{n^2 s} - \frac{h_v s}{2} + \frac{h_v s D}{2R} - \frac{h_v s}{2n^2} + \frac{g}{n^2} \quad (20)$$

$$\frac{\partial^2 AEP_S}{\partial n^2} = -\frac{2A_v D}{n^3 s} + \frac{h_v s}{n^3} - \frac{2g}{n^3} \quad \text{where } s = \sum_{i=1}^{N} q_i \quad (21)$$

The average expected profit of the system will be concave in n, for given θ, q_i and p_i, if $\frac{\partial^2 \mathrm{AEP_s}}{\partial \mathrm{n}^2} < 0$, which implies that $h_v s^2 < 2(A_v D + sg)$.

If the above condition holds then the system profit function attains the maximum value with respect to n, and the optimal value of n can be obtained by using the first order optimality condition i.e., $\frac{\partial AEP_S}{\partial n} = 0$. Solving it for n, one can get the optimal number of shipments as $n^* = \sqrt{\frac{R(2A_V D - h_v s^2 + 2sg)}{h_v s^2 (R-D)}}$.

For integer optimal value of n,

$$n^{opt} = \begin{cases} \lfloor n^* \rfloor, & if\ AEP_s(\lfloor n^* \rfloor, \theta, q_i, p_i) \geq AEP_s(\lceil n^* \rceil, \theta, q_i, p_i) \\ \lceil n^* \rceil, & if\ AEP_s(\lfloor n^* \rfloor, \theta, q_i, p_i) \leq AEP_s(\lceil n^* \rceil, \theta, q_i, p_i) \end{cases}$$

Proposition 6. For given values of n, q_i and p_i, the average expected profit function of the supply chain is concave in θ if $2I + \sum_{i=1}^{N} (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\alpha_i^2 (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i > 0$.

Proof. From equation (18), we get

$$\frac{\partial AEP_s}{\partial \theta} = -\left[\frac{A_v}{Q} + \frac{h_v s}{R} - \frac{h_v Q}{2R} + \left(\sum_{i=1}^{N} \frac{h_v \sigma_i}{\sqrt{2\pi}} \right) \right] u - 2I\theta - \sum_{i=1}^{N} \Bigg[-p_i \alpha_i + \frac{(A_i + nF)\alpha_i}{Q_i}$$
$$+ h_i \int_0^{\frac{q_i}{R}} \frac{\alpha_i (q_i - Rl_i)}{R} f_L(l_i) dl_i$$
$$+ h_i \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\alpha_i (q_i - Rl_i)[q_i R + D_i (q_i - Rl_i)]}{q_i R^2} f_L(l_i) dl_i$$
$$+ c_i \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\alpha_i D_i (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i - c_i \int_{\frac{q_i(R+D_i)}{RD_i}}^{\infty} \frac{\alpha_i (q_i - Rl_i)}{R} f_L(l_i) dl_i \Bigg] \quad (22)$$

$$\frac{\partial^2 AEP_S}{\partial\theta^2} = -2I - \sum_{i=1}^{N}(h_i + c_i)\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\alpha_i^2(q_i - Rl_i)^2}{q_i R^2} f_L(l_i)dl_i \tag{23}$$

It can be easily seen that $\frac{\partial^2 AEP_S}{\partial\theta^2} < 0$ if

$2I + \sum_{i=1}^{N} (h_i + c_i)\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\alpha_i^2(q_i - Rl_i)^2}{q_i R^2} f_L(l_i)dl_i > 0$. Therefore, we can conclude that the average expected profit function of the supply chain system is concave in θ for given n, q_i and p_i, if this condition holds.

Proposition 7. The average expected profit function of the supply chain system is concave in q_i for given n, θ and p_i if

$$\frac{2A_v D}{s^3} + \frac{2(A_i + nF)D_i}{q_i^3} + n(h_i + c_i)\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{D_i^2 l_i^2}{q_i^3} f_L(l_i)dl_i > 0$$

Proof. Differentiating (18) with respect to q_i, we get

$$\frac{\partial AEP_S}{\partial q_i} = \frac{A_v D}{ns^2} - \frac{h_v D}{R} - \frac{nh_v}{2}\left(1 - \frac{D}{R}\right) + \frac{h_v}{2n} + \frac{(A_i + nF)D_i}{nq_i^2} - \frac{h_i}{2}\int_0^{\frac{q_i}{R}}\left(\frac{R+2D_i}{R}\right) f_L(l_i)dl_i$$
$$-\frac{h_i}{2}\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}}\left(\frac{q_i^2(R+D_i)^2 - R^2 D_i^2 l_i^2}{q_i^2 R^2}\right) f_L(l_i)dl_i$$
$$+\frac{c_i}{2}\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}}\left(\frac{R^2 D_i^2 l_i^2 - q_i^2 D_i^2}{q_i^2 R^2}\right) f_L(l_i)dl_i$$
$$+\frac{c_i}{2}\int_{\frac{q_i(R+D_i)}{RD_i}}^{\infty}\left(\frac{R+2D_i}{R}\right) f_L(l_i)dl_i \tag{24}$$

$$\frac{\partial^2 AEP_S}{\partial q_i^2} = -\frac{2A_v D}{ns^3} - \frac{2(A_i + nF)D_i}{nq_i^3} - h_i\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{D_i^2 l_i^2}{q_i^3} f_L(l_i)dl_i$$
$$-c_i\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{D_i^2 l_i^2}{q_i^3} f_L(l_i)dl_i \tag{25}$$

Now, the average expected profit of the entire supply chain is concave in q_i for given n, θ and p_i, if $\frac{\partial^2 AEP_S}{\partial q_i^2} < 0$, which gives

$$\frac{2A_v D}{s^3} + \frac{2(A_i + nF)D_i}{q_i^3} + n(h_i + c_i)\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{D_i^2 l_i^2}{q_i^3} f_L(l_i)dl_i > 0.$$

Proposition 8. The average expected profit function of the supply chain system is concave in p_i for given n, θ and q_i if $2\beta_i + (h_i + c_i)\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i^2(q_i - Rl_i)^2}{q_i R^2} f_L(l_i)dl_i > 0$.

Proof. Differentiating (18) with respect to p_i, we get

$$\frac{\partial AEP_s}{\partial p_i} = D_i - \beta_i p_i + \frac{A_v\beta_i}{Q} + \frac{h_v\beta_i s}{R} - \frac{h_v Q\beta_i}{2R} + \frac{h_v\beta_i\sigma_i}{\sqrt{2\pi}} + \frac{(A_i + nF)\beta_i}{Q_i} + h_i\int_0^{\frac{q_i}{R}} \frac{\beta_i(q_i - Rl_i)}{R} f_L(l_i)dl_i + h_i\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i(q_i - Rl_i)[q_iR + D_i(q_i - Rl_i)]}{q_iR^2} f_L(l_i)dl_i + c_i\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i D_i(q_i - Rl_i)^2}{q_iR^2} f_L(l_i)dl_i - c_i\int_{\frac{q_i(R+D_i)}{RD_i}}^{\infty} \frac{\beta_i(q_i - Rl_i)}{R} f_L(l_i)dl_i \quad (26)$$

$$\frac{\partial^2 AEP_s}{\partial p_i^2} = -2\beta_i - (h_i + c_i)\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i^2(q_i - Rl_i)^2}{q_iR^2} f_L(l_i)dl_i \quad (27)$$

The average expected profit function of the entire supply chain is concave in p_i for given n, θ and q_i, if $\frac{\partial^2 AEP_s}{\partial p_i^2} < 0$ i.e., if $2\beta_i + (h_i + c_i)\int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i^2(q_i - Rl_i)^2}{q_iR^2} f_L(l_i)dl_i > 0$.

5.3. Coordinated Model (COM)

Supply chain players agree to accept the joint-decision making policy only if it provides a better profit than the decentralized model scenario. To motivate supply chain members to make integrated decisions, an incentive strategy is required. In this section, we propose a coordination mechanism between the manufacturer and the retailers, which motivates the members to accept the integrated decision-making policy. In this coordination mechanism, the manufacturer requests the retailers to decide their optimal batch sizes (q_i) and retail prices (p_i) according to the centralized policy and, in return, the manufacturer decreases his wholesale price (w).

Suppose that the manufacturer offers the price discount scheme to the i-th retailer as follows:

$$w = \begin{cases} w, & if\ q_i < q_i^{c^*} \\ w(1 - \phi_i), & if\ q_i \geq q_i^{c^*} \end{cases} \quad (28)$$

For this price discount scheme, the average expected profit of the i-th retailer becomes

$$AEP_i^{co}(q_i^c, p_i^c, \phi_i) = p_i^c D_i^c - (1 - \phi_i)wD_i^c - \frac{(A_i + n^c F)D_i^c}{Q_i^c} - \left[\frac{h_i}{2}\int_0^{\frac{r_i}{D_i^c}}[q_i^c + 2(r_i - D_i^c l_i)]f_L(l_i)dl_i + h_i\int_{\frac{r_i}{D_i^c}}^{\frac{r_i+q_i^c}{D_i^c}} \frac{(q_i^c - D_i^c l_i + r_i)^2}{2q_i^c} f_L(l_i)dl_i\right] - \left[\frac{c_i}{2}\int_{\frac{r_i}{D_i^c}}^{\frac{r_i+q_i^c}{D_i^c}} \frac{(D_i^c l_i - r_i)^2}{q_i^c} f_L(l_i)dl_i + \frac{c_i}{2}\int_{\frac{r_i+q_i^c}{D_i^c}}^{\infty} [q_i^c + 2(D_i^c l_i - q_i^c - r_i)]\, f_L(l_i)dl_i\right] \quad (29)$$

and the average expected profit of the manufacturer becomes

$$AEP_m^{co}(n^c,\theta^c,\phi_i) = \sum_{i=1}^{N}(1-\phi_i)wD_i^c - \frac{A_v D^c}{Q^c} - h_v[\frac{D^c \sum_{i=1}^{N} q_i^c}{R} + \frac{Q^c}{2}\left(1-\frac{D^c}{R}\right) - \sum_{i=1}^{N}\frac{q_i^c}{2n^c}] - \sum_{i=1}^{N}\frac{h_v D_i^c \sigma_i}{\sqrt{2\pi}} - I\theta^2 \tag{30}$$

Proposition 9. The minimum value of ϕ_i for which the i-th retailer accepts the coordination mechanism is

$$\phi_i^{min} = \frac{\left(p_i^d D_i^d - p_i^c D_i^c\right) - w\left(D_i^d - D_i^c\right) - \Delta^d + \Delta^c}{wD_i^c}$$

where,

$$\Delta = \frac{(A_i + nF)D_i}{Q_i} + \frac{h_i}{2}\left[\int_0^{\frac{r_i}{D_i}}[q_i + 2(r_i - D_i l_i)]\, f_L(l_i)dl_i + \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(q_i - D_i l_i + r_i)^2}{q_i} f_L(l_i)dl_i\right] + \left[\frac{c_i}{2}\int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(D_i l_i - r_i)^2}{q_i} f_L(l_i)dl_i + \frac{c_i}{2}\int_{\frac{r_i+q_i}{D_i}}^{\infty}[q_i + 2(D_i l_i - q_i - r_i)] f_L\,(l_i)dl_i\right]$$

Proof. The retailer's goal in engaging in the coordination is to find the minimum discount level so that his profit is more or equal to the profit in the decentralized situation. So,

$$AEP_i^{co}(q_i^c, p_i^c, \phi_i) \geq AEP_i^d(q_i^d, p_i^d) \tag{31}$$

Solving the inequality (31), we get

$$\phi_i \geq \frac{\left(p_i^d D_i^d - p_i^c D_i^c\right) - w\left(D_i^d - D_i^c\right) - \Delta^d + \Delta^c}{wD_i^c} \tag{32}$$

Therefore, if the wholesale price discount offered by the manufacturer does not satisfy the above condition, the i-th retailer will not accept the contract. So, to motivate the i-th retailer, the manufacturer should give at least ϕ_i discount level given by

$$\phi_i^{min} = \frac{\left(p_i^d D_i^d - p_i^c D_i^c\right) - w\left(D_i^d - D_i^c\right) - \Delta^d + \Delta^c}{wD_i^c} \tag{33}$$

Proposition 10. The maximum discount level offered by the manufacturer to the i-th retailer is given by

$$\phi_i^{max} = \frac{w\left(D_i^c - D^d\right) - \nabla^c + \nabla^d + \sum_{\substack{j=1 \\ j \neq i}}^{N}(1-\phi_j)wD_j^c}{wD_i^c} \tag{34}$$

where, $\nabla = \frac{A_v D}{Q} + h_v \left[\frac{D \sum_{i=1}^{N} q_i}{R} + \frac{Q}{2}\left(1 - \frac{D}{R}\right) - \sum_{i=1}^{N} \frac{q_i}{2n}\right] + \sum_{i=1}^{N} \frac{h_v D_i \sigma_i}{\sqrt{2\pi}} + I\theta^2$

Proof. The manufacturer will offer the price discount scheme if he/she gains more profit after giving price discount to all the retailers in this coordination than the decentralized scenario. So, if the manufacturer provides a ϕ_i discount level to the i-th retailer, then

$$AEP_m^{co}(n^c, \theta^c, \phi_i) \geq AEP_m^d(n^d, \theta^d) \tag{35}$$

Simplifying (35), we get,

$$\phi_i \leq \frac{w(D_i^c - D^d) - \nabla^c + \nabla^d + \sum_{\substack{j=1 \\ j \neq i}}^{N} (1-\phi_j) w D_j^c}{w\, D_i^c} \tag{36}$$

So, if the manufacturer gives ϕ_i% price discount to the i-th retailer, then the maximum allowable discount level for the manufacturer will be

$$\phi_i^{max} = \frac{w(D_i^c - D^d) - \nabla^c + \nabla^d + \sum_{\substack{j=1 \\ j \neq i}}^{N} (1-\phi_j) w D_j^c}{w\, D_i^c}$$

From Propositions (9) and (10), it can be observed that, the i-th retailer will accept the discount offer for all $\phi \geq {\phi_i}^{min}$. Therefore, all the N retailers will accept the discount scheme (28) if $\phi \geq \phi_{min}$, where

$\phi_{min} = max\{{\phi_1}^{min}, {\phi_2}^{min}, {\phi_3}^{min}, \ldots\ldots\ldots\ldots, {\phi_N}^{min}\}$ and the manufacturer will provide this price discount only if

$$\phi \leq \phi_{max} = \min\{\phi_1^{max}, \phi_2^{max}, \phi_3^{max}, \ldots\ldots, \phi_N^{max}\}$$

Hence, for all ϕ in $[\phi_{min}, \phi_{max}]$ the coordination through the price discount scheme (28) will result better profit level for both the manufacturer and the retailers than the decentralized scenario. Since the manufacturer sells the product to all the retailers at the same wholesale price, therefore, we assume that he/she offers the same price discount ratio ϕ to each retailer.

6. Numerical Analysis

In this section, we consider three numerical examples to analyze the behaviour of our proposed model and its applicability. Here we focus on the scenario where one manufacturer is trading with two retailers.

Example 1: The following set of parameter-values presented in Table 2 are considered to demonstrate the proposed model numerically. As it is difficult to get access to the actual industrial data, some of the parameter-values are taken from Hoque (2013) and the rest are hypothetical.

The p.d.f. of lead time (l_i) of the i-th retailer is $f_L(l_i) = \frac{1}{\sqrt{2\Pi}\sigma_i} e^{-\frac{1}{2\sigma_i^2}\left(l_i - \frac{r_i}{D_i}\right)^2}$.

Table 2. Set of parameter-values for Example 1

Parameter	Value	Parameter	Value
R	3000 units/ year	a_1	1500 units / year
A_v	\$400 /set up	a_2	1500 units / year
A_1	\$40 /order	β_1	4
A_2	\$45 / order	β_2	4.5
w	\$100 / unit	α_1	2
F	\$10 /shipment	α_2	1.5
h_v	\$3.5/ unit / year	σ_1	0.12
h_1	\$5.8 / unit / year	σ_2	0.13
h_2	\$5 / unit / year	I	\$40
c_1	\$7 / unit / year	N	2
c_2	\$7 / unit / year		

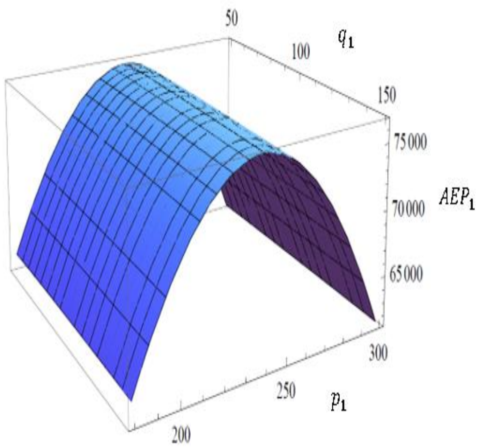

Figure 4. Concavity of average expected profit function of the first retailer

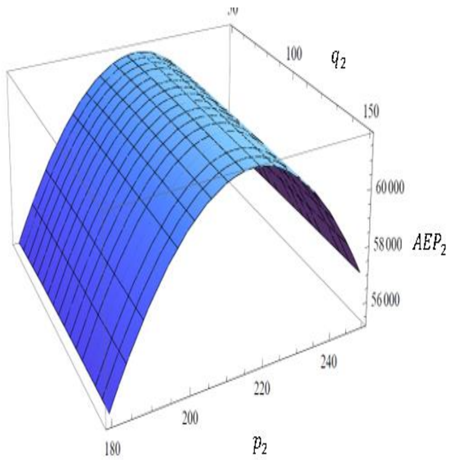

Figure 5. Concavity of average expected profit function of the second retailer

As shown in Figures 4 and 5, for given parameter-values, the average expected profit functions of both the retailers are found to be concave with respect to the batch sizes and retail prices of the product. The optimal results are obtained using the computational software Mathematica 10.0 with the command FindMaximum.

From the numerical results given in Table 3, we observe that the optimal order quantity, retail price, greening level of the product and number of shipments decided in the centralized scenario gives more system profit than that obtained in the decentralized scenario. In the centralized scenario, both the retailers can sell the product to the end customers at a cheaper price than the decentralized case. Since the customer demand is assumed to be price sensitive, the lower priced product attracts more customers.

Table 3. Optimal results for different models

Models	n^*	θ^*	q_1^* (unit)	q_2^* (unit)	p_1^* (\$/unit)	p_2^* (\$/unit)	$\phi_{\min}$	ϕ_{max}	ϕ	AEP_1^* (\$/year)	AEP_2^* (\$/year)	AEP_m^* (\$/year)	AEP_s^* (\$/year)
DM	4	4.36	91.98	111.79	238.74	217.53	-	-	-	76577	61799	105584	243960
COM	6	7.87	71.77	79.96	189.82	168.34	0.14	0.27	0.205	83132	66707	115684	265523
CM	6	7.87	71.77	79.96	189.82	168.34	-	-	-	-	-	-	265523

Also, the manufacturer can produce more greener product by making the optimal decisions jointly. As the product's greening level has a positive impact on customer demand, customer demand in the centralized case is considerably higher than in the decentralized case, and all the retailers increase their order quantities. As a result, joint-decision making generates a higher system profit than the separate profit optimization.

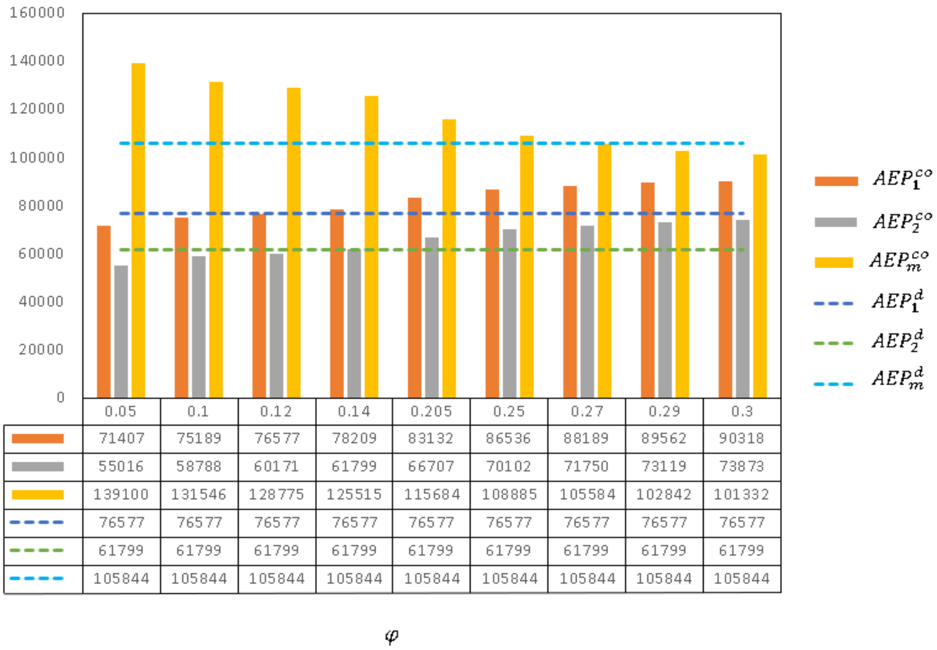

Figure 6. Price discount rate vs. average expected profit

It is also observed from Table 3 that the channel coordination can be achieved through the price discount mechanism between the manufacturer and the retailers. The optimal results of the models reflect that embracing the price discount coordination mechanism boosts not only the total system profit but also the profits of individual supply chain members. For the first retailer, the minimum discount ratio to undertake the coordination mechanism is obtained as 0.12 and for the second retailer, it is 0.14. It is clear that if the manufacturer offers 12% discount, the first retailer will accept the offer but the second retailer will not, as it will cause a loss to him. Therefore, to motivate both the retailers for participating in the coordination, the manufacturer has to give at least 14% price discount. Again, the maximum discount ratio for which the manufacturer does not face any loss is obtained as 0.27. So, the manufacturer can offer each retailer a maximum discount of 27%. Therefore, the win-win situation which occurs in the interval $[\phi_{min}, \phi_{max}]$ is appeared as [0.14,0.27]. For any value of ϕ in this interval, the price discount mechanism becomes profitable for the manufacturer and both the retailers than the decentralized scenario. For ϕ lying in the interval [0.14,0.27], the average expected profits of the first and second retailers vary within the intervals [\$78215, \$88049] and [\$61799, \$71610], respectively and the average expected profit of the manufacturer lies within the interval [\$105584, \$125503]. In all cases, the average expected system profit remains \$265523 i.e., our suggested coordination method effectively achieves channel coordination and results in the supply chain members sharing extra profit that occurs in the centralized scenario. Naturally, whenever the value of ϕ increases from ϕ_{min}, the retailers profitability increases gradually and attains their maximum profits at ϕ_{max} while the manufacturer's profit decreases, and at ϕ_{max}, the manufacturer attains the same profitability as that of the decentralized case. This fact is plotted in Figure 6. The supply chain members can fix the value of ϕ through bargaining. Here we take the value of ϕ as the mean of the feasible interval [0.14,0.27] i.e., 0.205.

Example 2: We consider the set of parameter values given in Table 4 to demonstrate the model, and the optimal results thus obtained are provided in Table 5.

Table 4. Set of parameter-values for Example 2

Parameter	Value	Parameter	Value
R	4000 units/ year	a_1	2000 units / year
A_v	\$500 /set up	a_2	1800units / year
A_1	\$50 /order	β_1	4.2
A_2	\$50/ order	β_2	5
w	\$90/ unit	α_1	3
F	\$15/shipment	α_2	2.5
h_v	\$3/ unit / year	σ_1	0.12
h_1	\$6/ unit / year	σ_2	0.13
h_2	\$5.5/ unit / year	I	\$30
c_1	\$7.4 / unit / year	N	2
c_2	\$7.4 / unit / year		

Table 5 shows that, when compared to a decentralized system, integrated decision making provides higher supply chain profit. Both the order quantity of each retailer as well as the product's greening improvement level increase in the centralized scenario compared to the decentralized scenario. In addition, the product's retail price falls at both the retailers. As a consequence, customers are enticed by a greener product at a lesser cost, which significantly increases market demand. In the coordinated model, the minimum wholesale price discount ratios for the two retailers are obtained as 2% and 7%, respectively, while the maximum allowable price discount ratio for the manufacturer is 17%. Therefore, for any price discount lying in the interval [7%, 17%], a win-win situation arises, i.e., the wholesale price contract benefits every member of the supply chain. The value of ϕ is taken as the mean of this feasible interval [7%, 17%] i.e., 12%.

Table 5. Optimal results for different models

Models	n^*	θ^*	q_1^* (unit)	q_2^* (unit)	p_1^* ($/unit)	p_2^* ($/unit)	$\phi_{\min}$	ϕ_{max}	ϕ	AEP_1^* ($/year)	AEP_2^* ($/year)	AEP_m^* ($/year)	AEP_s^* ($/year)
DM	4	8.22	132.42	161.82	286.18	227.19	-	-	-	161024	93638	131443	386105
COM	5	19.94	112.12	124.16	245.58	185.35	0.07	0.17	0.12	170783	97731	140097	408611
CM	5	19.94	112.12	124.16	245.58	185.35	-	-	-	-	-	-	408611

From Table 5, it can be noticed that, by accepting the wholesale price discount contract, the profits of the two retailers are increased by 6% and 4%, respectively. Furthermore, the manufacturer earns about 7% more profit from this contract.

6.1. A comparative study with existing literature

In this section, we attempt to compare the findings of our study to some previous research. Sarkar et al. (2020b) developed a single-vendor single-buyer model with equal-sized batch shipment policy and price-dependent demand in this direction. They did, however, take into account variable backorder and the inspection process, that are not considered in this study. Furthermore, their model didn't take into account for stochastic lead time and greening investment. To compare the proposed model to Sarkar et al. (2020b), common parameter values from Sarkar et al. (2020b) are used, while the remaining parameter values are chosen at random. The proposed model is compared to Sarkar et al.'s (2020b) model in two different situations: without greening investment and with greening investment. The parameter values considered are given in Table 6.

Table 6. Set of parameter-values for comparative study

Parameter	Value	Parameter	Value
a_1	11,000 units / year	β_1	320
A_v	$200/set up	α_1	3
A_1	$20/order	σ_1	0.02
w	$10/ unit	I	$80
F	$5/shipment	N	1
h_v	$2/ unit / year	$\frac{D}{R}$	0.4
h_1	$5/ unit / year		
c_1	$7.5/ unit / year		

For the case of without green sensitivity of the customer demand, we set $\alpha_1 = 0$, $I = 0$. Figure 7 shows a comparative graphical representation of the average expected supply chain profit. Their centralized model obtains optimal batch size as 279 units, optimal number of shipments as 5, optimal retail price as $17 and optimal profit of the entire supply chain as $92021. Whereas our proposed model without green investment results the optimal batch size as 155.2 units, optimal number of shipments as 9, optimal retail price as $17.30, and the average expected supply chain profit as $92287. Furthermore, the proposed model with stochastic lead time and greening investment provides the optimal batch size as 223.26 units, optimal number of shipments as 8, optimal retail price as $28.24, optimal green level as 35.07 and the average expected supply chain profit as $152186. As a conclusion of the above numerical results, it is apparent that adding the stochastic lead time and greening investment strategy makes the supply chain significantly more profitable.

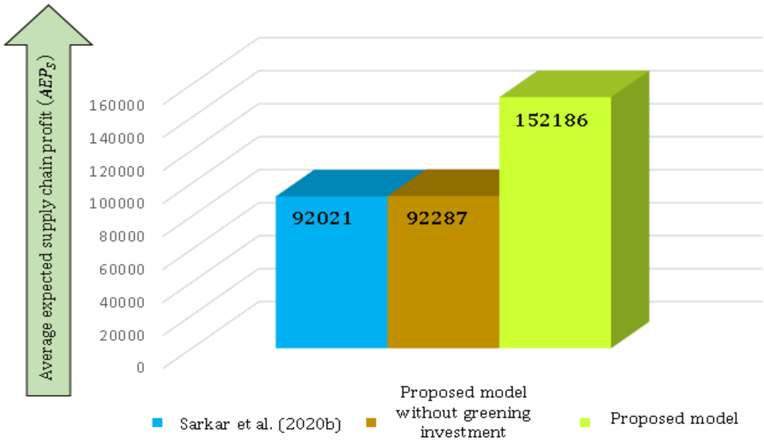

Figure 7. Comparison with existing literature

7. Sensitivity analysis

In order to explore the impact of model parameters on the optimal decisions as well as the average expected system profit, in this section we vary one parameter-value at a time while keeping other parameter-values unchanged in Example 1. The results are shown in Table 7 from which the following conclusions can be drawn:

From Table 7 and Figure 8, a significant change in overall profit of the system under the price discount coordination mechanism is observed for higher basic market demand. The first retailer can charge a higher price for the product whenever the customer demand increases at his side. This is because the first retailer compensates the effect of higher price by the higher market demand. He places order for more quantity from the manufacturer. Consequently, the profit of the first retailer as well as the manufacturer increases significantly. The changes in the order quantity and retail price of the product for the second retailer are almost negligible. As a result, the overall profit of the system increases. Similar scenario occurs whenever the market demand increases at the second retailer's side.

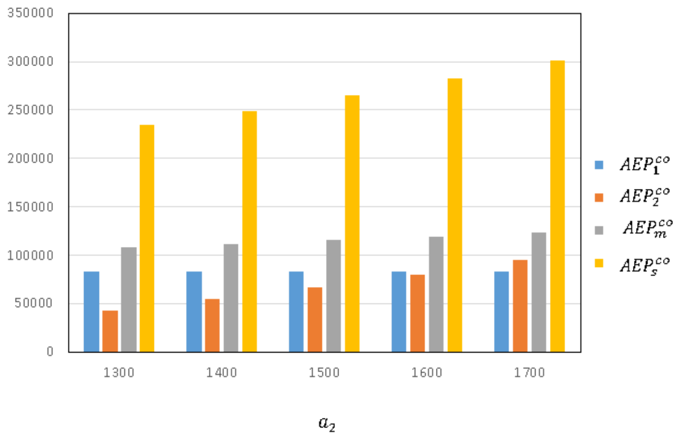

Figure 8. Average expected profit vs. a_2

Table 7. Sensitivity analysis of the parameters a_1, β_1, α_1 and F

Parameters	Values	n*	θ*	q_1^* (unit)	q_2^* (unit)	p_1^* ($/unit)	p_2^* ($/unit)	AEP_1^{co} ($/year)	AEP_2^{co} ($/year)	AEP_m^{co} ($/year)	AEP_s^{co} ($/year)
a_1	1300	6	7.24	65.20	78.88	164.687	168.26	55569	66592	108057	230219
	1400	6	7.55	68.50	79.41	177.253	168.301	68719	66649	111879	247242
	1500	6	7.87	71.77	79.96	189.82	168.34	83132	66707	115684	265523
	1600	6	8.19	75.10	80.55	202.387	168.383	98812	66765	119486	285065
	1700	6	8.50	78.20	81.17	214.955	168.424	115761	66823	123285	305865
β_1	3	6	9.47	71.90	80.07	253.508	168.607	131649	67014	114825	313488
	3.5	6	8.55	71.83	80.01	217.082	168.455	103873	66838	115343	286055
	4	6	7.87	71.77	79.96	189.82	168.34	83132	66707	115684	265523
	4.5	6	7.34	71.58	79.57	168.652	168.253	67049	66605	115917	249571
	5	6	6.92	71.48	79.90	151.736	168.183	54229	66524	116088	236841
α_1	1.5	6	6.67	71.58	79.87	189.104	168.143	82276	66478	116083	264837
	2	6	7.87	71.77	79.96	189.82	168.34	83132	66707	115684	265523
	2.5	6	9.08	72.01	80.07	190.69	168.543	84177	66940	115210	266327
	3	6	10.31	72.27	80.18	191.718	168.747	85419	67177	114654	267250
	3.5	6	11.56	72.58	80.30	192.908	168.954	86868	67418	114012	268298
F	0	11	7.87	38.99	43.67	189.752	168.282	83199	66751	115816	265766
	10	6	7.87	71.77	79.96	189.82	168.34	83132	66707	115684	265523
	20	4	7.87	97.95	111.89	189.89	168.41	83084	66680	115588	265352
	30	4	7.87	103.46	116.70	189.911	168.421	83022	66626	115563	265211
	40	3	7.87	124.43	144.46	189.98	168.48	82983	66605	115496	265084

Figure 9 depicts how the price sensitivity of the consumer demand affects the decision variables and the system profit. The figure shows that when the price sensitivity of consumer demand increases, the greater price of the product influences the customers' choice of alternatives. As a result, if customer demand becomes more price sensitive, the corresponding retailers lower their product prices to meet market demand, reducing the product's greenness. Figure 9 and Table 7 show that, under the coordination scheme, the average expected profit of both retailers and the total system profit decrease at a decreasing rate as price elasticity increases.

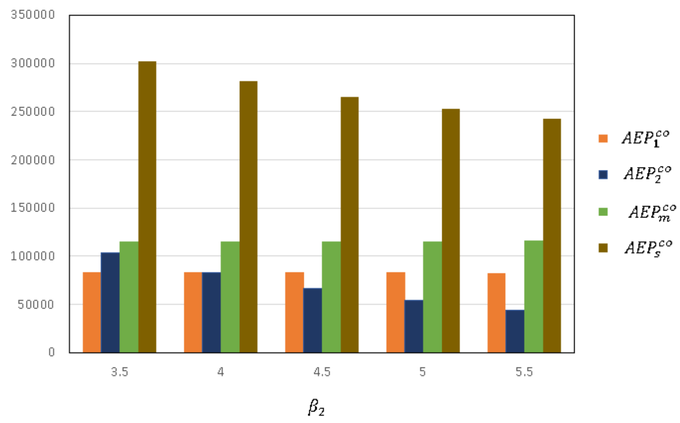

Figure 9. Average expected profit vs. β_2

Table 7 shows the effect of customers' environmental awareness on optimal decisions and supply chain members' profitability. When the values of α_1 and α_2 increase, customers are more concerned about the environmental performance of the product and they are willing to spend more for environmentally friendly products. In such a scenario, to satisfy the customers requirement, the manufacturer increases the greening level of the product. This fact is presented in Figure 10.

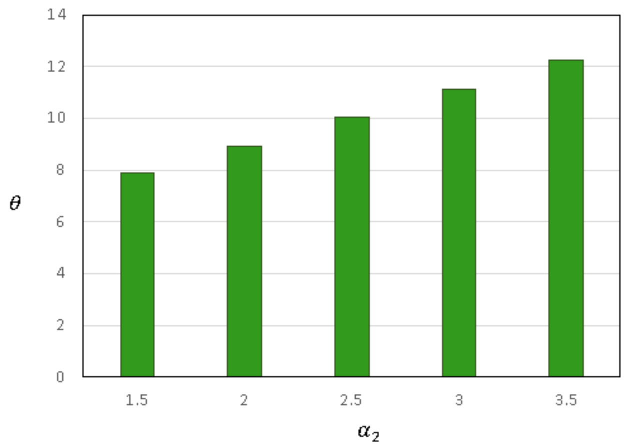

Figure 10. Product's greening level vs. α_2

However, the higher greening level increases the expense of the manufacturer. So, the average expected profit of the manufacturer gradually decreases. On the other hand, the retailers can enhance the retail price of the product and achieve higher profitability with higher greener product. It is further observed that the average expected system profit increases for greater values of α_1 and α_2.

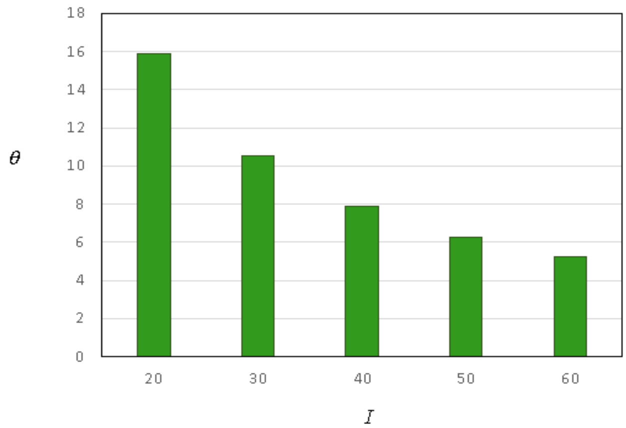

Figure 11. Product’s greening level vs. I

Table 7 illustrates the effect of the greening investment on the coordinated profit of the supply chain members and the profit of the entire supply chain. Figure 11 shows that the product's greening level falls rapidly while I increases. When I increases, the manufacturer produces lower greener product in order to curb his expenditure but it makes a negative impact on customer demand. So, for higher I, the profitability of the retailers decreases and the average expected profit of the entire supply chain also decreases gradually.

The effect of the transportation cost is found to be negligible on the supply chain’s profitability. If we ignore the transportation cost then the optimal number of shipments is obtained as 11. As the transportation cost increases, the optimal number of shipments declines from 11. Figure 12 reflects that the system profit decreases at a diminishing rate as F increases.

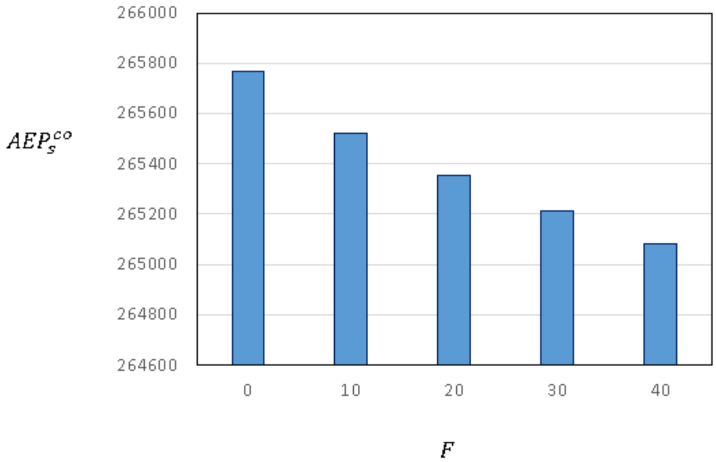

Figure 12. Average expected profit vs. F

8. Managerial Insights

From the numerical study and sensitivity analysis of our proposed model, the following key managerial insights are derived:

(1) Business managers can improve the sales volume and economic efficiency by adopting green manufacturing technologies and suitable coordination scheme. Though, from the point of view of social welfare, it is always desirable to produce green products, the firms should estimate the profitable growth before adopting green manufacturing. From the outcomes of sensitivity analysis, it is evident that environmental awareness of the consumers and greening investment play a crucial role in the profitability of the supply chain members. By participating in the proposed price discount coordination mechanism, the business managers can improve the greening quality of the product to a remarkable higher level. It not only increases their profits but also maintains their social responsibility and increases their reputation in the business market for adopting such green initiative.

(2) The proposed price discount scheme is capable of coordinating the supply chain. Under this mechanism, the manufacturer reduces his wholesale price and increases greening level of the product and encourages the retailers to set their prices and ordering quantities according to the centralized model. This improves the economic level of all members. Moreover, by participating in such coordination, the end customers get more eco-friendly product at a cheaper price than if they used an individual optimization strategy. Retailers should also remember that when consumers' price sensitivity is too high, they should lower their sales price to retain profitability.

(3) The business managers may not always agree to adopt joint decision-making process even though it yields higher profit for the entire supply chain but it may not be profitable for all the chain members. To convince the members to make coordination, such price discount scheme is very effective as in this scheme increment of each member’s profitability is guaranteed. All the members could enjoy the coordination agreement as it is beneficial both socially and economically.

(4) The delivery of the order quantity may not reach to the retailer’s end in time due to various reasons such as variation in transportation time, inspection time, loading and unloading times, etc. Therefore, the business managers should understand the stochastic nature of the lead time and account for all possibilities of early arrival, on time arrival, and late arrival to conduct the business efficiently.

9. Concluding Remarks

In this study, we have designed a two-level supply chain model consisting of a single manufacturer and multiple retailers. To develop a realistic model, the lead time between placing an order and receiving its delivery is taken to be stochastic in nature. The retailers face a price sensitive demand from the end customers. The customer demand is also affected by the greening improvement level of the product as determined by the manufacturer. We have studied the decentralized model where supply chain members optimize their own profits without worrying about the profit of others. Stackelberg gaming approach is used where the retailers are assumed to act as the leader and the manufacturer as the follower. A solution algorithm is suggested to find the optimal solution of the proposed model. The performance of the whole supply chain is also investigated under integrated decision-making model. Though the entire supply chain experiences a better economical and environmental performances in the centralized scenario but it may not be beneficial for all the members individually. Since the retail price of the product is decided by the retailers and the manufacturer determines the greening level of the product, and both these factors influence the customers demand, it is therefore essential to make these decisions in an efficient and coordinated manner which enriches the profit levels of each members. A price discount mechanism has been proposed to convince the supply chain members to make decisions in a coordinated manner. The maximum and minimum satisfactory discount rates are found so that all the members become interested for participating in this price discount coordination. This coordination mechanism is effective in both cases whether the market demand is high or low.

There are some limitations of this study and the present model can be extended in many directions to further enhance the scope of our study. It is widely adopted in the literature but the policy of equal sized batch shipment is very limited in nature, and it may not be always possible to supply the order quantities of all the retailers in some integer number of equal sized batches. So, it would be more realistic to consider a combined equal and unequal sized batch shipment policy (Hoque, 2013). Another limitation of this study is that it is based on a single product being traded between the manufacturer and the retailers. To simulate a real-world scenario, it can be expanded to include many items (Barman et al., 2021a) and multiple manufacturers. Another shortcoming of our study is the consideration of complete backlogging strategy. It is desirable to consider partially backlogging of shortages for a more realistic approach (Duary et al., 2022). In our study, we have considered constant production rate, perfect production system at the manufacturer. One can enrich the study by taking into account variable production rate (Sarkar et al., 2018) and/or imperfect production system (Sepehri and Gholamian, 2022). The competition between the retailers will be another interesting research idea (Mondal and Giri, 2020). Our developed model can be modified by considering bargaining between manufacturer and retailers to share the profits among all the members (Nouri et al., 2018). In our study, we have proposed a price discount coordination scheme. It would be interesting to employ other contracts such as greening cost sharing contract between the manufacturer and the retailers (Giri and Dash, 2022). Consideration of set up cost reduction investment (Sarkar et al., 2017), and promotional effort (Ebrahimi et al., 2019) would also be fruitful extensions of this model.

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