Decision Making: Applications in Management and Engineering
Vol. 2, Issue 2, 2019, pp. 54-64.
ISSN: 2560-6018
eISSN:2620-0104
DOI: https://doi.org/10.31181/dmame1902049s

AN APPROACH TO RANK PICTURE FUZZY NUMBERS FOR DECISION MAKING PROBLEMS

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Received: 25 January 2019;
Accepted: 27 April 2019;
Available online: 29 April 2019.

Original scientific paper

Abstract: *Comparison of picture fuzzy numbers (PFNs) are performed using score and accuracy values. But when both of the score and accuracy values are equal, those PFNs are said to be identical. This article presents a novel method to compare the PFNs even when the score and accuracy values of those PFNs are equal. The proposed ranking method is based on positive ideal solution, positive and negative goal differences, and score and accuracy degrees of the picture fuzzy numbers. A new score function is proposed to calculate the actual score value which depends on the positive and negative goal differences and the neutral degree. Finally, a real-life example has been used to illustrate the efficiency of the proposed method.*

Key words*: Picture fuzzy set, picture fuzzy number, positive ideal solution, positive goal difference, negative goal difference.*

1. Introduction

Picture fuzzy set (PFS) (Cong, 2014) is an extension of fuzzy set (FS) (Zadeh, 1965) and intuitionistic fuzzy set (IFS) (Li, 2008, Das et al., 2018, Das et al., 2017), and it can easily manage the uncertain nature of human thoughts by introducing the neutral and refusal membership degrees. In PFS, the authors have divided the hesitation margin of IFS into two parts which are neutral membership degree and refusal membership degree. When both of the neutral and refusal membership degrees are zero, i.e., hesitation margin becomes zero, then the PFS returns to IFS. Sometimes, FS and IFS find it difficult to express the situations when human thoughts involve more options like 'yes', 'abstain', 'no' and 'refusal'. PFS is preferable to handle

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this type of situations using the positive, neutral, negative, and refusal membership degrees. The general election of a country is a good example of such situation where the voters can give their opinions like 'vote for', 'abstain', 'vote against' and the 'refusal' of the election (Cuong and Kreinovich, 2013, 2014). Suppose one candidate and 1000 voters are participating in an election process. Among them, 400 voters vote for the candidate, 100 voters are not interested in casting their vote, i.e., they remain to abstain from the voting process, 300 voters are giving their vote against the candidate, and 200 voters refuse to cast their vote for the candidate, i.e., they vote for NOTA. This kind of situation may happen in reality and it is outside the scope of IFS and FS, since FS and IFS don't support neutrality. As another example, suppose an expert takes the opinion of a person regarding some object. Now the person may say that 0.4 is the possibility that the object is good, 0.3 is the possibility that the object is not good, 0.2 is the possibility that the object is good and as well as not good, and 0.1 is the possibility that (s)he does not know about the object. This issue is also not handled by the FSs or IFSs. Due to having the capability of accepting more opinions, PFS has become an important tool to deal with imprecise and ambiguous information and been applied in many real-life problems by some researchers (Zhang and Xu, 2012, Cuong and Kreinovich, 2013, Cuong, 2013).

Son (2016) investigated the application of PFS in clustering algorithms to exploit the hidden knowledge from a mass of data sets by proposing a hierarchical picture clustering (HPC) method. Motivated by the application of PFS in decision making, Garg (2017) proposed a series of aggregation operators in the context of PFS and presented a decision-making approach using the proposed aggregation operators. Wang et al. (2017) proposed picture fuzzy set based geometric aggregation operator and compared two picture fuzzy numbers (PFNs) using score and accuracy functions. Guiwu (2016) used PFS in decision-making problem and proposed cross entropy of PFSs. Using the idea of cross entropy, the author introduced a new ranking method in PFS environment. Singh (2015) defined correlation coefficient of PFS and applied it to clustering analysis problem. Many authors have contributed to rank the corresponding numbers in the framework of fuzzy sets (Atanassov and Georgiev, 1993, Das et al., 2015, 2017, 2018), intuitionistic fuzzy sets (Atanassov, 1989,Bhatiaand Kumar, 2013, Kumar and Kaur, 2012) and intuitionistic multi fuzzy sets (Li, 2005, Li, 2008, Liu, 2007, Si and Das, 2017). Most of these ranking methods are based on the comparative analysis of a pair of sets, measurement of the distance between the sets, and measurement of the distance of a set from a central point. In the comparative and distance measurement methods, the membership, non-membership and neutral degrees are considered to have similar importance. Another important concern is that the neutral membership degree is considered just like a positive or negative membership degree. No focus is given even when the neutral degree increases or decreases. But in our real life, some situations are totally different and the membership, non-membership and neutral degrees play different roles and have various types of functionality in the decision making process or ranking among them.

In this article, we propose a new method to calculate the score to rank the PFNs using positive ideal solution, negative ideal solution and average neutral value of the alternatives. Neutral degree of PFS has an active role in our proposal. We consider the average value of the neutral degree as a pivot point concerning the all other neutral degrees. Then, we provide a practical example to analyze the proposed method for ranking to take the decision.

Remaining of the article organized is as follows. Some relevant ideas of picture fuzzy sets are recalled in Section 2. We propose the new ranking method in Section 3 followed by a real-life example in Section 4. A comparative study is given in Section 5. Finally, we conclude in Section 6.

2. Preliminaries

This section briefly presents the relevant ideas of picture fuzzy set and some of its operations.

2.1 Picture fuzzy set

A picture fuzzy set (PFS) A on the universe X is an object in the form of

$$A=\left\{\left(x,\mu_A(x),\eta_A(x),\nu_A(x)\right)\middle| x\in X\right\} \tag{1}$$

where $\mu_A(x)\in[0,1]$ be the degree of positive membership of x in A, similarly $\eta_A(x)\in[0,1]$ and $\nu_A(x)\in[0,1]$ are respectively called the degrees of neutral and negative membership of x in A. These three parameters $\left(\mu_A(x),\eta_A(x)\ and\ \nu_A(x)\right)$ of the picture fuzzy set A satisfy the following condition $\forall x\in X, 0\le\mu_A(x)+\eta_A(x)+\nu_A(x)\le 1$.

Then, the degree of refusal membership $\rho_A(x)$ of x in A can be estimated accordingly,

$$\forall x\in X, \rho_A(x)=1-\left(\mu_A(x)+\eta_A(x)+\nu_A(x)\right) \tag{2}$$

The neutral membership $\left(\eta_A(x)\right)$ of x in A can be consideredasdegree of positive membership as well as degree of negative membership whereas refusal membership $\left(\rho_A(x)\right)$ can be explained as not to take care of the system. When, $\forall x\in X, \eta_A(x)=0$, then the PFS reduces into IFS.

For a fixed $x\in A$, $\left(\mu_A(x),\eta_A(x),\nu_A(x),\rho_A(x)\right)$ is called picture fuzzy number (PFN),where $\mu_A(x)\in[0,1]$, $\eta_A(x)\in[0,1]$, $\nu_A(x)\in[0,1]$, $\rho_A(x)\in[0,1]$ and

$$\mu_A(x)+\eta_A(x)+\nu_A(x)+\rho_A(x)=1 \tag{3}$$

Simply, PFN is representedas $\left(\mu_A(x),\eta_A(x),\nu_A(x)\right)$.

2.2 Operations on PFS

For two PFSs $A=(\mu_a,\eta_a,\nu_a)$ and $B=(\mu_b,\eta_b,\nu_b)$, Cong (2014) defined some operations as given below.

$$A\cup B=\left\{\left(x,\max\left(\mu_A(x),\mu_B(x)\right),\min\left(\eta_A(x),\eta_B(x)\right),\min\left(\nu_A(x),\nu_B(x)\right)\right)\middle| x\in X\right\} \tag{4}$$

$$A\cap B=\left\{\left(x,\min\left(\mu_A(x),\mu_B(x)\right),\min\left(\eta_A(x),\eta_B(x)\right),\max\left(\nu_A(x),\nu_B(x)\right)\right)\middle| x\in X\right\} \tag{5}$$

$$\overline{A}=\left\{\left(x,\nu_A(x),\eta_A(x),\mu_A(x)\right)\middle| x\in X\right\} \tag{6}$$

Cuong and Kreinovich (2013) and Cuong (2013) defined some properties on PFSs as given below.

$$A \subseteq B \text{ If } \left(\forall x \in X, \mu_A(x) \le \mu_B(x), \eta_A(x) \le \eta_B(x), \nu_A(x) \ge \nu_B(x)\right) \tag{7}$$

$$A = B \text{ If } \left(A \subseteq B \text{ and } B \subseteq A\right) \tag{8}$$

If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$ (9)

$$\overline{\overline{A}} = A \tag{10}$$

2.3 Distance between picture fuzzy sets

Distances between the two PFSs are defined in (Cuong and Son, 2015, Son, 2016). The distance between two PFSs $A = (\mu_a, \eta_a, \nu_a)$ and $B = (\mu_b, \eta_b, \nu_b)$ in $X = \{x_1, x_2, ..., x_n\}$ is calculated as follows.

Normalized Hamming distance

$$d_H(A,B) = \frac{1}{n}\sum_{i=1}^{n}\left(\left|\mu_a(x_i) - \mu_b(x_i)\right| + \left|\eta_a(x_i) - \eta_b(x_i)\right| + \left|\nu_a(x_i) - \nu_b(x_i)\right|\right) \tag{11}$$

Normalized Euclidean distance

$$d_E(A,B) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\left(\mu_a(x_i) - \mu_b(x_i)\right)^2 + \left(\eta_a(x_i) - \eta_b(x_i)\right)^2 + \left(\nu_a(x_i) - \nu_b(x_i)\right)^2\right)} \tag{12}$$

Example 1: Let A ={(0.7,0.2,0.1),(0.8,0.1,0.1),(0.7,0.1,0.2)} and B={(0.6,0.2,0.2),(0.8,0.2,0.0), (0.9,0.0,0.1)} are two picture fuzzy sets of dimensions 3. Then

$$d_H(A,B) = \frac{1}{3}\begin{pmatrix} \left(|0.7-0.6| + |0.2-0.2| + |0.1-0.2|\right) \\ +\left(|0.8-0.8| + |0.1-0.2| + |0.1-0.0|\right) \\ +\left(|0.7-0.9| + |0.1-0.0| + |0.2-0.1|\right) \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} (0.1+0.0+0.1) \\ +(0.0+0.1+0.1) \\ +(0.2+0.1+0.1) \end{pmatrix} = \frac{1}{3}(0.2+0.2+0.4) = \frac{0.8}{3} = 0.26$$

Wang et al. (2017) defined some special operations of picture fuzzy set. They proposed the following operations on PFNs $A = (\mu_a, \eta_a, \nu_a)$ and $B = (\mu_b, \eta_b, \nu_b)$.

$$A.B = (\mu_a + \eta_a)(\mu_b + \eta_b) - \eta_a\eta_b,\ \eta_a\eta_b,\ 1-(1-\nu_a)(1-\nu_b) \tag{13}$$

$$A^\lambda = (\mu_a + \eta_a)^\lambda - \eta_a{}^\lambda,\ \eta_a{}^\lambda,\ 1-(1-\nu_a)^\lambda,\ \lambda > 0 \tag{14}$$

Example 2: Let A =(0.7,0.2,0.1) and B=(0.6,0.2,0.2) are two picture fuzzy sets and λ=5.

A.B=(0.7+0.2)*(0.6+0.2)-0.2*0.2, 0.2*0.2, 1-(1-0.1)*(1-0.1)=(0.68,0.04,0.19).

$A^\lambda = A^5 = (0.7+0.2)^5 - (0.2)^5, (0.2)^5, 1-(1-0.1)^5 =$ (0.16807-0.00032), 0.00032, 1-0.59

=(0.16,0.00032,0.41)

2.4 Comparison of Picture fuzzy sets

Wang et al. (2017) used the score function and accuracy function to compare the PFSs. Let $C=(\mu_c, \eta_c, \nu_c, \rho_c)$ be a picture fuzzy number, then a score function *S(C)* is being defined as $S(C)=\mu_c-\nu_c$ and the accuracy function *H(C)* is given by $H(C)=\mu_c+\nu_c+\eta_c$ where $S(C)\in[-1,1]$ and $H(C)\in[0,1]$. Then, for two picture fuzzy numbers *C* and *D*

I. If $S(C)>S(D)$, then *C* is higher than *D*, denoted by *C>D*;

II. If $S(C)=S(D)$, then

  a. $H(C)=H(D)$, implies that *C* is equivalent to *D*, denoted by *C=D*;

  b. $H(C)>H(D)$, implies that *C* is higher than *D*, denoted by *C>D*.

Example 3: Let C = (0.7,0.2,0.1) and D=(0.6,0.2,0.2) are two picture fuzzy sets. Now, *S(C)*=0.7-0.1=0.6, *S(D)*=0.6-0.2=0.4, *H(C)*=0.7+0.2+0.1=0.9, *H(D)*=0.6+0.2+0.2=1. Since *S(C)>S(D)*, therefore *C>D*.

3. Proposed method for ranking PFNs

It is known that the ranking of the fuzzy sets depends on the membership value of the elements. Fuzzy numbers with higher membership value are ranked first. In an intuitionistic fuzzy set (IFS), the rank of Intuitionistic fuzzy numbers (IFNs) depend on membership values as well as non-membership values. The IFS which has the highest membership value and smaller non-membership value will have the first rank (Zhang and Xu, 2012). Below, some situations are given, which appears during the ranking of IFNs.

Let $A=(\mu_a, \nu_a)$ and $B=(\mu_b, \nu_b)$ be two IFNs, then

I. If $\mu_a \succ \mu_b$ and $\nu_a \prec \nu_b$ then $A \succ B$

II. If $\mu_a \prec \mu_b$ and $\nu_a \succ \nu_b$ then $A \prec B$

III. If $\mu_a = \mu_b$ and $\nu_a = \nu_b$ then $A = B$

Here, situation 1 and 2 clearly defines the rank between the IFSs *A* and *B*, but unable to provide the rank as given in situation 3, when both the membership and non-membership values are equal. Motivated by the ranking procedure in IFS, Wang et al. (2017) proposed the comparison technique between two PFSs with the help of score and accuracy function. But their proposed method cannot discriminate the PFNs when the score and accuracy values are same. We observed that the neutral membership grade could be considered to contribute in positive membership grade as well as negative membership grade. This motivated us to propose a new ranking method for the PFNs even when the score and accuracy values are equal. The proposed approach is given below in a stepwise manner.

Let $f_i=(\mu_i, \eta_i, \nu_i), i=1,2,\ldots,n$ be the set of PFNs, where μ_i, η_i *and* ν_i respectively denote the positive, neutral and negative membership degree.

Step 1: The positive ideal solution (PIS) $f^+ = \left(\mu^+, \eta^+, \nu^+\right)$ of the PFNs $f_i = \left(\mu_i, \eta_i, \nu_i\right)$, $\left(i = 1, 2.......n\right)$ is determined, where

$$f^+ = \left(\mu^+, \eta^+, \nu^+\right) = \left(\max_i \mu_i, \min_i \eta_i, \min_i \nu_i\right) \tag{15}$$

Step 2: The positive goal difference (PGD) μ_i^* and negative goal difference (NGD) ν_i^* of each of the PFNs $f_i = \left(\mu_i, \eta_i, \nu_i\right), \left(i = 1, 2, ..., n\right)$ are computed by $\mu_i^* = \mu^+ - \mu_i$ and $\nu_i^* = \nu_i - \nu^+$.

Step 3: Absolute score of each PFN is calculated as

$$p_i = \left(1 - \mu_i^*\right) - \nu_i^* \tag{16}$$

The absolute score of a PFN is computed using the membership and non-membership grade only. It completely ignores the neutral membership grade. However the neutral membership grade has an important contribution in finding the score which is narrated in the following steps.

Step 4: Next the average neutral degree $\bar{\eta}$ is computed, where $\bar{\eta} = \left(\sum_{i=1}^{n} \eta_i\right) / \eta.$.

Step 5: Estimate the actual score S_i (given below) of the PFNs $f_i = \left(\mu_i, \eta_i, \nu_i\right)$, $\left(i = 1, 2, ..., n\right)$ using the average neutral degree. When actual scores of the two PFNs f_i and f_j are the same, then go to *Step 6.*

$$S_i = \frac{p_i}{1 - \left(\bar{\eta} - \eta_i\right)} \tag{17}$$

Here the actual score will be always a finite value because the difference between average neutral degree and individual neutral degree of an PFN is never equal to 1, i.e., $\left(\bar{\eta} - \eta_i\right) \neq 1$.

Step 6: I) If $\mu_i > \mu_j$ *and* $\eta_i \geq \eta_j$ then $S_i > S_j$.

II) If $\mu_i \geq \mu_j$ *and* $\eta_i < \eta_j$ then

III) If $\nu_i \leq \nu_j$ then $S_i > S_j$ otherwise $S_i < S_j$.

Remark 3.1. Absolute score *p_i =1* if $\mu^+ = \mu_i$ and $\nu^+ = \nu_i$, i.e., when membership degree highest and non-membership degree is lowest, then the absolute score will be at most.

Remark 3.2. The actual score basically depends on the neutral degree. If the neutral degree of all PFNs are same then actual score equal to absolute score. Similarly, the actual score increases if the neutral degree decreases alternatively actual score decrease when the neutral degree increases.

4. Practical Example

In this section, we present a practical example to demonstrate the evaluation of the students and their ranking concerned with the multiple-choice questions (MCQs) based examination system with picture fuzzy information to illustrate the proposed method. Suppose n be the number of students who are appearing in a competitive examination where the question paper is composed of multiple-choice questions. During the evaluation process, normally this kind of exams assign some positive marks opting for the correct choice and negative marks for opting the wrong choice. This exam system does not consider the non-attempted questions in the evaluation process. In the exam, some students may attempt all the questions while some other students may not attempt all the questions. Now, among the attempted questions, two cases may arise. In the first case, all answers may be correct and in the second case, some answers may be correct while the rest are wrong. Let's consider, μ_i , ν_i and η_i be the percentage of correct answers, wrong answers, and not attempted questions respectively for the i^{th} candidate. In the examination system, we consider that there are no wrong questions or out of syllabus questions. So, $\mu_i + \eta_i + \nu_i = 1$. We assume that there is no refusal membership value of the students in this examination system. The result of the i^{th} student can be presented by a PFN $f_i = (\mu_i, \eta_i, \nu_i)$ where *i=1,2,..,n*. Table 1 shows the results of seven students (T_1, T_2, T_3, T_4, T_5, T_6, T_7) for a particular MCQ based exam using PFNs.

Table 1. Students' results using PFNs

Students	T1	T2	T3	T4	T5	T6	T7
Result	(0.64, 0.22, 0.14)	(0.74, 0.15, 0.11)	(0.72, 0.19, 0.09)	(0.82,0.1, 0.08)	(0.82,0.1 4,0.04)	(0.9,0.05, 0.05)	(0.68,0.1, 0.22)

To find out the ranking of the students, we illustrate the proposed approach as given below.

Step1: Calculate the PIS $f^+ = (0.9, 0.22, 0.03)$ is calculated using Eq. (15) and Table 1.

Step 2: The positive and negative goal differences of individual students are given in Table 2.

Table 2. PGD and NGD of students

Students	T_1	T_2	T_3	T_4	T_5	T_6	T_7
PGD	0.26	0.16	0.18	0.08	0.08	0.0	0.22
NGD	0.11	0.08	0.06	0.05	0.00	0.02	0.19

Step 3: Table 3 shows the absolute score of each student using Eq. (16).

Table 3. Absolute score of each student

Students	T_1	T_2	T_3	T_4	T_5	T_6	T_7
Absolute Score	0.63	0.76	0.76	0.87	0.92	0.98	0.59

Step 4: Table 4 shows an actual score of each student using Eq. (17).

Table 4. Actual score of each student

Students	T_1	T_2	T_3	T_4	T_5	T_6	T_7
Actual Score	0.68	0.77	0.80	0.84	0.92	0.98	0.59

Now the ranking list of students is found as $T_6 > T_5 > T_4 > T_3 > T_2 > T_1 > T_7$.

5. Comparative Study

To present the comparative study, we have compared the proposed method with the right marks (RM) method (Lesage et al., 2013), multiple mark question (MMQ) method (Tarasowaand Auer, 2013) and score and accuracy function based method (Wang et al., 2017) with the same example stated above in Section 4. As presented in (Lesage et al., 2013), the degree of correctness (μ_i) considers the score of the students which is presented in Table 5. According to the RM method, the more be the score, the higher be the rank. This method does not consider the negative marking for the wrong answers. So the students are privileged to guess the answers to the unknown questions and attempt those questions. At this moment the students utilize the ambiguity of this technique. This technique does not measure the actual knowledge of the students. In Table 5, both the students T_4 and T_5 score 0.82 along with 0.08 and 0.04 incorrect answers respectively. This method does not give any penalty for incorrect answers, and there is no proper method to handle when scores are the same.

Table 5. Score of students according to the right marks method

Students	T_1	T_2	T_3	T_4	T_5	T_6	T_7
Actual Score	0.64	0.74	0.72	0.82	0.82	0.9	0.68

To overcome the drawbacks of right mark method, the authors introduced negative marking (NM) method (Lesage et al., 2013) which measures the score of students as the difference between the degree of correctness (μ_i) and degree of incorrectness (ν_i), which is presented in Table 6. This technique incorporates some penalty for the wrong answers. So, the students try to attempt their known questions to maximize their scores. But the categories of students, who attempt as many questions as possible without adequate knowledge can get the advantage of this technique. This method attempt to minimize the guessing tendency of the students with a penalty of the wrong answers but to guess advantage remains. In this negative marking method, there's no provision for handling the situation when the score of two or more students are the same. In the following table (Table 6), one can find that the score of two students T_2 and T_3 are similar. As a result, we are unable to get the proper rank of the students using the methods presented in (Lesage et al., 2013). But the proposed method is capable of ranking the students even when the score values are equal. The example described in Section 4 illustrates that ranking of the student (T_3) is higher than the rank of the student (T_2) since the actual score (0.80) of the student (T_3) is more than that of the student (T_2) which is (0.77).

Table 6. Score of students according to negative marks method

Students	T_1	T_2	T_3	T_4	T_5	T_6	T_7
Actual Score	0.50	0.63	0.63	0.74	0.78	0.85	0.46

Next multiple mark question (MMQ) (Tarasowa and Auer, 2013) method has been compared with the proposed method. In MMQ method, there are options to mark more than one right choices for a particular question for the checking the depth of knowledge of the students. But in case of mathematical problems, there's generally one correct answer. So, MMQ method is suitable for medical entrance examination but not appropriate for all types of examinations. In our proposed method with a single right answer for every question, we mainly consider the guessing tendency of the examinees and calculate the actual score based on the positive and negative answers and not attempted questions. In this method, if an examinee cleverly attempts more questions based on his assumptions and if these are found wrong, he will lag behind regarding score. Table 7 shows the score value and accuracy value of the individual PFNs associated with the seven students using the score and accuracy function (Wang et al., 2017). Comparison method of two IFNs using score and accuracy functions is mentioned in Section 2.4, where the alternatives are ranked based on their score and accuracy values. But there is no proper clarification when the alternatives have the same score value. In table 7, student T_2 and T_3 have the same score and accuracy values. Therefore we can't compare those two students. But our proposed method can compare two PFNs even if the score and accuracy values are equal. As per the proposed method, the rank of T_3 is higher than that of T_2, i.e., $T_3 > T_2$ since the actual scores of the students T_3 and T_2 are 0.80 and 0.77 respectively.

Table 7. Score and accuracy value according to score and accuracy function

Students	T_1	T_2	T_3	T_4	T_5	T_6	T_7
Score value($\mu_i - \nu_i$)	0.50	0.63	0.63	0.74	0.78	0.85	0.46
Accuracy value ($\mu_i + \eta_i + \nu_i$)	1	1	1	1	1	1	1

6. Conclusion

In this article, a new approach is presented to rank the PFNs. The new approach is different and improved from the existing score and accuracy function based approaches in the sense that we can achieve the ranking of alternatives in spite of having an equal score and accuracy values. We consider the neutral membership of the picture fuzzy sets to be the key element to determine the actual score. In the proposed method, we observe that when all neutral degrees of some picture fuzzy numbers are equal, then the score depends only on goal differences. In future, the proposed method can be used to obtain the ranking for the various extensions of fuzzy sets.

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