

A MULTI-OBJECTIVE APPROACH BASED ON MARKOWITZ AND DEA CROSS-EFFICIENCY MODELS FOR THE INTUITIONISTIC FUZZY PORTFOLIO SELECTION PROBLEM

Mehrdad Rasoulzadeh¹, Seyyed Ahmad Edalatpanah^{2*}, Mohammad
Fallah¹, and Seyyed Esmaeil Najafi³

¹ Department of Industrial Engineering, Central Tehran Branch, Islamic Azad
University, Tehran, Iran.

² Department of Applied Mathematics, Ayandegan Institute of Higher Education,
Tonekabon, Iran

³ Department of Industrial Engineering, Science and Research Branch, Islamic Azad
University, Tehran, Iran.

Received: 11 April 2022;

Accepted: 23 June 2022;

Available online: 3 July 2022.

Original scientific paper

Abstract: Nowadays, investors' main concerns are choosing the best portfolio so that the highest possible investment return can be achieved by accepting the least risk. In this regard, the classical Markowitz model is one of the most widely used models which helps investors get closer to their goals. Data envelopment analysis (DEA) is also a practical technique that can analyze the efficiency of firms. Few models can address companies' internal performance simultaneously in addition to considering the goals of Markowitz models. Also, we study the return and price fluctuations of assets in the market with the intuitionistic fuzzy numbers for the first time. Therefore, in this paper, we combine all these tools with returns of intuitionistic fuzzy numbers, proposing a new combined Markowitz and the cross DEA models. Furthermore, to get the best portfolio of assets, this model obtains the efficiency of all companies and, simultaneously, fully covers all constraints of the Markowitz model. To show the model's practicality, we studied a case study based on information from 50 active enterprises on Tehran Stock Exchange. We solved the proposed model using the Non-Dominated Sorting Genetic Algorithm II (NSGA-II). The obtained results and the comparisons made with the existing approaches show the effectiveness of the proposed model.

* Corresponding author

E-mail addresses: Mehrdad.ra@gmail.com (M. Rasoulzadeh), s.a.edalatpanah@aihe.ac.ir (S. A. Edalatpanah), mohammad.fallah43@yahoo.com (M. Fallah), Najafi1515@yahoo.com (S. E. Najafi)

Key words: *Markowitz mean-variance model, Data Envelopment Analysis (DEA), Intuitionistic fuzzy set, Cross-efficiency.*

1. Introduction

An increment in wealth is always the most imperative goal for every investment. Increasing wealth in any investment is entwined with the two concepts of risk and returns. Risk and returns have a direct correlation; the higher the returns expectations, the risk of investment will also be accompanied by a more elevated risk. Investors have always sought to utilize varied available models to select investments with maximum returns and the lowest risk. Several types of research have been performed so far in this respect to seek the optimum combination of assets by using mathematical models. In a multi-objective optimization model, Markowitz (1952) was capable of simultaneously selecting the highest returns and lowest risk in the mathematical approach, through which the maximum stock investment portfolio came into effect. In this model, the investor chooses from amidst the responses present on the efficient frontier, where there are responses in the most desirable conditions possible, from the viewpoint of risk and returns, thus, selecting the answer under consideration. The Markowitz model, which has been introduced, takes the historical data relative to the corporate returns and investigates the variance and their mean (averages) as a basis of calculation. After presenting the Markowitz model, numerous supplementary researches were performed on it. Konno and Yamazaki (1991) employed the risk criterion of absolute deviation from the mean instead of risk. Kellerer et al. (2000) developed it by incrementing new constraints: fixed cost and the minimum amount of transactions to this model. Chang et al. (2000) added the maximum number of constraints to the shares present in the portfolio (cardinality). By adding the boundary constraints and cardinality Fernandez and Gomez (2007) used the neural network algorithm and solved the model. Soleimani et al. (2009) presented a selection of the Markowitz stock portfolio model, with constraints consisting of the minimum number of transactions, minimizing cardinality.

Similarly, in another study, Chang et al. (2009) applied diverse risk conditions, such as the semi-variance, absolute deviation variance, and variance with skewness, imposed in the model and solved them using the genetic algorithm. In a paper in 2014, Aouni and Colapinto rendered appropriate management approaches for the stock portfolio by utilizing the concepts of goal programming (GP). Algarvio et al. (2017) dealt with and managed the portfolio in relevance with risk and the optimization of retailers operating in the electrical market and presented a model for optimizing portfolios created by the final consumers using the Markowitz hypothesis. Zhao (2018) utilized the Markowitz model to select options for the stock market. Hunjra et al. (2020) compared the performance of risk models (mean-variance, semi-variance, mean absolute deviation, and conditional value-at-risk) in different economic scenarios, namely crisis, recovery, and growth. They implemented their investigations on the stock exchange of Pakistan, Bombay, and Dhaka. The results indicated that conditional value-at-risk presented better results for each scenario in each country and portfolio performance was inconsistent in different methods.

Alongside these issues, DEA was also introduced by Charnes and Cooper (1978) and was used for optimization and attaining the optimum combination for the portfolio concerning assets (Edirisinghe and Zhang 2015a). Huang (2015) offered an integrated method for optimizing the stock portfolio, which comprises decision-making in relevance to the screening of stocks, stock selection, and the allocation of

A multi-objective approach based on Markowitz and DEA cross-efficiency models for ... investment. Edirisinghe and Zhang (2015b) discussed using fiscal ratios to survey the efficiencies of companies. They investigated the datum of budgetary statements and employed DEA techniques to determine RFS. This index indicates the competitive acceptance of a company in comparison with other companies. By compiling DEA and the multi-criteria decision-making (MCDM) method, Goodarzi et al. (2017) dealt with optimizing the stock portfolio. In the initial step, they utilized ratios as inputs and outputs in DEA; then, they computed the cross-efficiency for each unit by using the optimal weights. Hoe et al. (2017) employed DEA and financial ratios to evaluate and compare technology companies listed in Malaysia. Subsequently, the inefficient companies attained alleviation or improvement criteria by taking advantage of the efficient companies. Puri et al. (2017) presented a multi-component data envelopment analysis (MC-DEA) model with inaccurate data. They proposed a new standard weights approach by applying interval arithmetic and unified production frontier to find unique weights for measuring these interval efficiencies. Jin et al. (2020) proposed a decision-making model based on DEA and the concept of probabilistic hesitant fuzzy numbers to construct a decision-making approach with probabilistic hesitant fuzzy preference relations (PHFPRs) to determine optimal selection among alternatives. Chang et al. (2021) used the envelopment analysis of nested dynamic network data to evaluate the portfolio. The effect of the alternative optimized solutions on the DEA cross-efficiency for portfolio selection was studied by Amin and Hajjani (2021). They revealed that these optimal solutions produce cross-efficiency matrices and portfolios of low risk with a higher expected return than the conventional cross-efficiency matrix for the portfolios.

A few researchers also utilized uncertain parameters for the problem. Huang (2008) described and rendered a new description of risk for selecting a stock portfolio in the fuzzy environment; On the basis of this, a new model was proposed, and a combined intelligent (smart) algorithm was proposed to solve it. The genetic algorithm solved an optimized portfolio model with cardinality constraints and uncertain data by Sadjadi et al. (2012). Guo (2012) used the fuzzy set theory to solve the mean-variance Markowitz model and expand it to a fuzzy portfolio selection model. Carlsson (2017) suggested an approved and mixed fuzzy programming by considering the future cash flows, in the form of a single trapezoidal fuzzy number, to select the prime research and development profile. In an attempt to choose a stock portfolio, Zhou et al. (2018) took advantage of the qualitative data presented by experts and investors, which are contemplated as uncertain elements for stock selection. They chose two groups of investors, namely, the public and those prone to risk acceptance. Two models were selected for portfolios, and scores were proposed based on maximum ranking and the deviation norm. In 2018, Chen et al. evaluated and dealt with the efficiency of a stock portfolio in a fuzzy environment with several risk criteria (probable variance, probable semi-variance, and a probable absolute semi-deviation). They demonstrated that, in fuzzy conditions, the portfolio efficiency is more precise and offers better responses.

Lamb et al. (2012) unveiled the uncertainty in estimating the efficiency of DEA. They employed bootstrap to develop random DEA models for funds, the extraction of confidence intervals, and the development of techniques for comparing and ranking funds and indicating ratings. In research, Lim et al. (2014) utilized the perception of DEA cross-efficiency to select a stock portfolio. In addition to using the mean cross-efficiency scores, they also took advantage of the modifications in the (variance) cross-efficiency scores. The achieved model was implemented on stock from the stock market of Korea, illustrating that the proposed model can be an optimistic tool for stock portfolio selection. In another research (2016), in developing the previous

model, they accounted for the returns of assets in the form of a single trapezoidal fuzzy number, thence proceeded to solve the model with the NSGA II algorithm and viewpoint and compared the responses. Omrani and Mashayekhi (2017) introduced a hybrid model based on the Markowitz mean-variance model to select the stock portfolio. In addition to the risk and returns, the portfolio's efficiency is also taken into account by them simultaneously. Their model was a four-objective model, which synchronously maximizes the mean stock returns and efficiency; it simultaneously minimizes the risk of the stock portfolio. To appraise the efficiency, they employed the DEA cross-efficiency approach. Next, the model was applied to the multi-objective genetic algorithm with a non-dominating sorting (NSGA II). They implemented this model on 52 companies on the Tehran Stock Exchange and compared the responses with those of the Markowitz model. Chen et al. (2020) rendered a multi-objective model in a fuzzy environment by combining the semi-variance, variance, and cross-efficiency DEA models. In their model, the cross-efficiency was on the basis and debated upon the 'Sharp Ratio.' They solved the proposed model with the Firefly algorithm. Likewise, in another paper, Chen (2021) implicated and selected the optimal portfolio; with a hybrid of models, such as the semi-variance, variance, and the cross-efficiency DEA model, with non-dominating fuzzy inputs and outputs. For more details regarding the portfolio optimization with DEA, see Rasoulzade and Fallah (2020).

Although there are numerous investigations in the field of portfolio optimization using fuzzy data envelopment analysis, however, there are only a few studies on this topic using the fuzzy set extensions, such as intuitionistic fuzzy sets, neutrosophic sets, etc; (Yang et al. 2020, Mao et al. 2020, Edalatpanah 2018 and Edalatpanah 2020).

In recent years, data with uncertainty have been considered in numerous researches with varied formats. One of these types of uncertain data is the intuitionistic fuzzy, which is employed in various DEA studies; and has been contemplated upon by researchers for portfolio selection. Hajiagha et al. (2013) have presented a DEA model with intuitionistic fuzzy inputs and outputs. It was for the first time that Puri et al. (2015) analyzed the efficiencies of the optimistic and pessimistic inputs and outputs data from the intuitionistic fuzzy outlook. Edalatpanah (2019) rendered a developed DEA model in a triangular intuitionistic fuzzy environment. In this study, he proposed a new ranking function that considers the interaction between the membership and non-membership function in the diverse intuitionistic fuzzy sets. Javaherian et al. (2021) proposed a new DEA model to evaluate the efficiency of decision-making units by using two structures and triangular intuitionistic fuzzy data. Yu et al. (2021) developed a unified intuitionistic fuzzy multi-objective linear programming (IFMOLP) model for such portfolio selection problems. The non-membership functions were made by the pessimistic, optimistic, and mixed approaches to perfect the traditional intuitionistic fuzzy (IF) inequalities and IF theory.

In the current paper, we seek to attain a model by combining the Markowitz model with returns of the trapezoidal intuitionistic fuzzy type, including the cross-efficiency DEA model, to achieve a model where the optimum portfolio of assets comes to hand. By being attentive to the nonlinear structure of the model attained, the NSGA II algorithm will be utilized to solve it, and the responses shall be compared with the fuzzy and classical models.

The NSGA II algorithm was introduced by Deb (2002). These algorithms are based upon two cross efficiencies performed like the traditional and ordinary genetic algorithms. However, its arrangement and sorting of responses are not on the fundamentals of a lower amount or more significant amount of the objective function.

A multi-objective approach based on Markowitz and DEA cross-efficiency models for ...

In this algorithm, sorting is performed based on non-dominance; responses that do not dominate each other are grouped in the same class. In recent years, the utilization of the NSGA II algorithm to solve problems has been such that the utmost portfolio is sought after and has been used in numerous ways; this can be illustrated in (Kaucic et al. 2019, Pal et al. 2021, Karimi 2021, Eftekharian et al. 2017).

The categorizing of information in the present paper is as given. In the next section, we shall concisely present some of the required concepts. Next, the developed model and the intuitionistic fuzzy returns shall be introduced. Then, a numerical example will be solved based on the available data for the companies actively operating on the Tehran Stock Exchange. Eventually, we shall render the results that have come to hand.

2. Methodology

2.1. The Markowitz mean-variance model

The matter of selecting an efficient portfolio is one of the concepts that Markowitz discussed. An efficient portfolio signifies a portfolio's selection is from assets, of steady returns, the minimum of risk, or in a given risk, the maximum of returns (Kazemi 2012).

Model 1:

$$\text{Min } \sigma_p^2 = \sum_{i=1}^N \sum_{j=1}^N w_i w_j \text{cov}(R_i, R_j) \quad (1)$$

s.t.

$$\bar{R}_p = E(R_p) = \sum_{i=1}^N w_i \bar{R}_i \geq R \quad (2)$$

$$\sum_{i=1}^N w_i = 1 \quad (3)$$

$$w_i \geq 0 \quad i = 1, 2, 3, \dots, N \quad (4)$$

In the abovementioned model, we have the following definitions:

\bar{R}_i : Mean or average returns on the i^{th} assets

$\text{cov}(R_i, R_j)$: The covariance of the i^{th} and j^{th} asset returns

N : Number of assets having the capacity to be invested

R : Minimum return expected by investors in the investment under consideration

w_i : The i^{th} asset weight in the investment portfolio

In the model mentioned above, constraint (2) imposes the minimum of returns cases expected by the investor as a constraint in the model. Similarly, constraint (3) sets the total of the portfolio weights to be equivalent to 1, where the relative constraint is a total budget constraint. This model can be in the form of a dual-objective form and the form of a simultaneous increase of returns and a decrease in the risk portfolio as well, and taken into consideration as given below:

Model 2:

$$\text{Max } \sum_{i=1}^N w_i \bar{R}_i \tag{5}$$

$$\text{Min } \sigma_p^2 = \sum_{i=1}^N \sum_{j=1}^N w_i w_j \text{cov}(R_i, R_j) \tag{6}$$

s.t.

$$\sum_{i=1}^N w_i = 1 \tag{7}$$

$$w_i \geq 0 \quad i = 1, 2, 3, \dots, N \tag{8}$$

2.2. Data Envelopment Analysis Model (DEA) with Cross-Efficiency

To calculate the efficiency of each DMU, Charnes and Cooper (1978) rendered the following:

Model 3:

$$\text{Max } \frac{\sum_{r=1}^s u_r y_{ro}}{\sum_{i=1}^m v_i x_{io}} \tag{9}$$

s.t.

$$\frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1 \quad j = 1, \dots, n \tag{10}$$

$$u_r, v_i \geq 0 \quad ; \quad r = 1, \dots, s \quad i = 1, \dots, m \tag{11}$$

In the abovementioned model x_{ij} and y_{rj} are the inputs and outputs of the j^{th} DMU respectively. u_r and v_i are the weights of the inputs and outputs respectively, which the model is seeking to bring to hand. Likewise, the objective function is to maximize the weighted sum of the outputs to the weighted sum of the inputs.

The standard input-axis model of return-to-scale (CRS) Data Envelopment Analysis is as follows:

Model 4:

$$\text{Max } \sum_{r=1}^s u_r y_{ro} \tag{12}$$

s.t.

$$\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j = 1, \dots, n \tag{13}$$

$$\sum_{i=1}^m v_i x_{io} \leq 1 \tag{14}$$

$$x_i, y_r \geq \varepsilon \quad \forall i, r \tag{15}$$

Where,

n : The number of decision-making units (DMUs),

m and s : the number of inputs and outputs, respectively,

x_{ij} and y_{rj} : the amounts for the i^{th} inputs and r^{th} outputs, respectively, for the j^{th} DMUs.

v_i and u_r : the weights allocated to the i^{th} inputs and r^{th} outputs, respectively, which the model computes.

The efficiency scores of each DMU are obtained by solving model (3) and bringing the optimum responses to hand. Let us assume that, u_r^* and v_i^* are the optimum responses, which have been achieved from the model (3) for the k^{th} unit. Through equation (16), the cross-efficiency of the other DMUs, which are evaluated by the k^{th} unit, can be calculated.

$$e_{kl} = \frac{\sum_{r=1}^s u_r^* y_{rl}}{\sum_{i=1}^m v_i^* x_{il}} \tag{16}$$

A multi-objective approach based on Markowitz and DEA cross-efficiency models for ...

Now, by computing the cross-efficiency for the entire units of DMUs; and then placing them in a $n \times n$ matrix, which is reputedly known (and brought to hand) as the (e_{ij}) , $i, j = 1, \dots, n$, cross-efficiency matrix. In this matrix, each element of (e_{ij}) is the cross-efficiency of the j^{th} unit, which has been evaluated by the i^{th} unit. In other words, in column (l) of the abovementioned matrix, $(e_{.l})$ is the cross-efficiency vector of unit l. Now, for every DMU like l, the mean is taken from every column (such as l), from which we can obtain and conclude the cross-efficiency ranking of that unit which is represented as \bar{e}_l (Lim et al., 2014):

$$\bar{e}_l = \frac{1}{n} \sum_{k=1}^n e_{kl} \quad (17)$$

For each DMU to achieve the highest efficiency score in DEA, it designates the highest weights to its points of strength; and the lowest weights to its points of weakness. In other words, each DMU in DEA does not require considering the selected set of weights for the other DMUs, and only utilizes its weights. Though, when DEA is used to assign an asset portfolio option within a multi-criteria decision-making (MCDM) framework, the weights to measure efficiency are determined externally and may modify over time. So this mechanism is no longer appropriate for use, although it proves valid for evaluating efficiency. To eliminate the aspect of risk due to weight changes, the utilization of DEA cross-efficiency is one of the modes to confront the problem.

Based on evaluating cross-efficiency, all the DMUs are ranked at a desirable level, and their efficiency for all the indexes is relatively good. They are resilient to weight change, and the variance of their cross-efficiency is relatively small. However, units that have performed well in a series of criteria have a lower score, are exposed to modifications in weight, and have a high cross-efficiency performance. Due to the reasons mentioned, the cross-efficiency evaluation helps to select a stock portfolio in which the DMUs are stable (Lim et al. 2014).

2.3. Mean and Variance Cross-Efficiency Models

In selecting an asset portfolio, the risk of weight modifications consists of two parts: the sole and single risk of each DMU unit; and the risk between the units of DMUs. The risk of individual DMUs could be illustrated by the variance of the cross-efficiency, of each distinctive unit of DMU, in the stock portfolio, and the risk between the units of DMUs, can be demonstrated by the covariance between each pair of units. An uncomplicated application of evaluating cross-efficiency successfully reduces risks for each of the units, though it is incapacitated in considering the risk between the DMUs (Lim et al. 2014). To conduct this matter, returns and risk for each DMU are regarded with a cross-efficiency and variance (performance) score, respectively. Similarly, for the stock portfolio Ω ; whereas, with the unique DMUs, the returns and risk are described as $V_\Omega = w^T \Sigma w$ and $E_\Omega = w^T \bar{e}$, where, Σ is the matrix of the cross-efficiency variance; and (k, l) is the consistent element of the covariance between the cross-efficiency of the k^{th} unit and the l^{th} unit. Likewise, the w 's are considered a weight vector, which is contemplated as a sum of 1. In this case, an optimum stock portfolio is achieved by solving the quadratic optimization model, given hereunder, that comes to hand (Schaerf 2002).

Model 5:

$$\begin{aligned} \text{Min } V_\Omega & \quad (18) \\ \text{s.t.} & \quad (19) \end{aligned}$$

$$E_{\Omega} \geq (1 - \gamma)E_{\Omega}^b$$

$$e^T w = 1 \tag{20}$$

$$w \geq 0 \tag{21}$$

In which γ is the return risk swap-over parameter. E_{Ω}^b is the maximum of returns attainable of the stock portfolio and e is an appropriate return vector, the elements of which are similar. In models where the inputs and outputs can take adverse or negative values, the utilization of radial DEA/CRS models is inappropriate. Collective VRS models offer the scopes to accommodate negative data as inputs and outputs (Pastor 2007). Amidst the varied collective DEA models, we take advantage of the VRS collective model compared to other models, as there are numerous features, such as comprehensiveness and stability, in relevance with transfer and the change of interest rate. The DEA -VRS cumulative model with inefficient criteria is as below (Cooper et al. 1999):

Model 6:

$$\text{Min } \frac{1}{(m+s)} (R^- s^- + R^+ s^+) \tag{22}$$

$$\text{s.t.} \tag{23}$$

$$X\lambda + s^- = x_k \tag{23}$$

$$Y\lambda - s^+ = y_k \tag{24}$$

$$e^T \lambda = 1 \tag{25}$$

$$\lambda, s^-, s^+ \geq 0 \tag{26}$$

That, $X = x_{ij} \in R^{n \times m}$ and $Y = y_{rj} \in R^{n \times s}$ demonstrates the inputs and outputs, data matrix respectively. Here, each column represents one of the units, and each row displays a level of one of the aspects of the factors of the relative DMU. R^- and R^+ is also described in the following form:

$$R^- = \left(\frac{1}{R_1^-} \cdot \frac{1}{R_2^-} \cdot \frac{1}{R_3^-} \cdot \dots \cdot \frac{1}{R_m^-} \right) \tag{27}$$

$$R^+ = \left(\frac{1}{R_1^+} \cdot \frac{1}{R_2^+} \cdot \frac{1}{R_3^+} \cdot \dots \cdot \frac{1}{R_s^+} \right) \tag{28}$$

$$R_i^- = \max_{j=1, \dots, n} \{x_{ij}\} - \min_{j=1, \dots, n} \{x_{ij}\} \quad i = 1, \dots, m \tag{29}$$

$$R_r^+ = \max_{j=1, \dots, n} \{y_{rj}\} - \min_{j=1, \dots, n} \{y_{rj}\} \quad r = 1, \dots, s \tag{30}$$

The dual model (6) is as given hereunder:

Model 7:

$$\text{Max } e_k^d = py_k - qx_k + \xi \tag{31}$$

$$\text{s.t.} \tag{32}$$

$$pY - qX + \xi e < 0 \tag{32}$$

$$p \geq \frac{1}{m+s} R^+ \tag{33}$$

$$q \geq \frac{1}{m+s} R^- \tag{34}$$

In the abovementioned model, vectors p and q are weights of the inputs and outputs. In the case that, for the unit (1), the optimum response is illustrated by * the cross-efficiency for the l^{th} unit, which is appraised by the k^{th} unit, signifies that, e_{kl}^* is computed as follows:

$$e_{kl}^* = p_k^* y_k - q_k^* x_k + \xi_k \tag{35}$$

A multi-objective approach based on Markowitz and DEA cross-efficiency models for ...

2.4. A combined Markowitz and a cross-efficiency DEA model

To evaluate the risk, returns, and efficiency of the model rendered below, Omrani and Mashayekhi (2017) proposed the said model.

Model 8:

$$\text{Max } \sum_{i=1}^N w_i \bar{R}_i \quad (36)$$

$$\text{Min } \sum_{i=1}^N \sum_{j=1}^N w_i w_j \text{Cov}(R_i, R_j) \quad (37)$$

$$\text{Max } \sum_{i=1}^N w_i \bar{e}_i \quad (38)$$

$$\text{Min } \sum_{i=1}^N \sum_{j=1}^N w_i w_j \text{Cov}(e_i, e_j) \quad (39)$$

$$\text{s.t. } \sum_{i=1}^n z_i \leq h \quad (40)$$

$$l_i z_i \leq w_i \leq u_i z_i \quad i=1, \dots, N \quad (41)$$

$$\sum_{i=1}^N w_i = 1 \quad (42)$$

$$w_i \geq 0 \quad i = 1, \dots, N \quad (43)$$

In the abovementioned model, z_i is a binary variable; when the i^{th} asset takes place in the stock portfolio, it sums it as (1); if this is not the case, it is (0). Variables l_i and u_i are respectively in relevance with the minimum and maximum percentage of investments of the total budget, which pertains to the i^{th} variable in the stock portfolio. In contrast, h is the maximum number of stocks selected in the stock portfolio.

In model (8), the objective functions (36 and 37) are the same ones present in the Markowitz model. The mentioned is employed to maximize returns and minimize risk returns. The objective functions (38 and 39) optimize portfolio efficiency and reduce portfolio risk. Constraint (40) restricts the number of stocks present in the portfolio. Constraint (41) demonstrates the maximum and minimum of the total budget deficit, which is liable for allotment to each share. The values h , l_i and u_i can vary and be following the investor's opinion.

3. A combined Markowitz model with intuitionistic fuzzy returns and a cross-efficiency model

As a development of the model rendered in the prior section (Model 8), we seek to contemplate the absence of certainty in the form of trapezoidal intuitionistic fuzzy numbers for returns. The new model combines the Markowitz and cross-efficiency DEA models with intuitionistic fuzzy numbers.

In this relevance, permit us to initially present and denote the definitions we will require in this section.

\tilde{A}^I Which is a fuzzy intuitionistic trapezoidal number, can be contemplated upon as $A = (a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4)$, where the membership and non-membership functions $\mu_{\tilde{A}^I}$ and $\nu_{\tilde{A}^I}$, respectively, are denoted as given below (Puri and Yadav 2015)

$$\mu_{\tilde{A}^I} = \begin{cases} f_A(x) & a_1 \leq x < a_2 \\ 1 & a_2 \leq x \leq a_3 \\ g_A(x) & a_3 < x \leq a_4 \\ 0 & \text{otherwise} \end{cases} \quad (44)$$

$$v_{\tilde{A}^I} = \begin{cases} h_A(x) & b_1 \leq x < b_2 \\ 0 & b_2 \leq x \leq b_3 \\ k_A(x) & b_3 < x \leq b_4 \\ 1 & \text{otherwise} \end{cases} \quad (45)$$

Such that, $0 \leq \mu_{\tilde{A}^I}(x) + v_{\tilde{A}^I}(x) \leq 1$ and $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 \in \mathbb{R}$ in a way that $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$.

Functions f_A and k_A are continuous piecewise, Non-descending functions, respectively, in the intervals of $[a_1, a_2)$ and $(b_3, b_4]$ and functions g_A and h_A are continuous piecewise non-descending functions, respectively, in the intervals of $(a_3, a_4]$ and $[b_1, b_2)$.

The expected distance of a fuzzy intuitionistic number \tilde{A}^I , in the above form, is a precise $EI(\tilde{A}^I)$ interval, which has been shown as $EI(\tilde{A}^I) = [E_L(\tilde{A}^I), E_R(\tilde{A}^I)]$ and can be computed as given hereunder (Grzegorzewski 2003).

$$E_L(\tilde{A}^I) = \frac{b_1+a_2}{2} + \frac{1}{2} \int_{b_1}^{b_2} h_A(x) dx - \frac{1}{2} \int_{a_1}^{a_2} f_A(x) dx \quad (46)$$

$$E_R(\tilde{A}^I) = \frac{a_3+b_4}{2} + \frac{1}{2} \int_{a_3}^{a_4} g_A(x) dx - \frac{1}{2} \int_{b_3}^{b_4} k_A(x) dx \quad (47)$$

On the fundamentals of which, the expected value is calculated as follows:

$$EV(\tilde{A}^I) = \frac{E_L(\tilde{A}^I) + E_R(\tilde{A}^I)}{2} \quad (48)$$

Assuming:

$$f_A(x) = \frac{x-a_1}{a_2-a_1} \quad (49)$$

$$g_A(x) = \frac{x-a_4}{a_3-a_4} \quad (50)$$

$$h_A(x) = \frac{x-b_2}{b_1-b_2} \quad (51)$$

$$k_A(x) = \frac{x-b_3}{b_4-b_3} \quad (52)$$

In this case, we have:

$$EV(A) = \frac{1}{8} (\sum_{i=1}^4 a_i + \sum_{i=1}^4 b_i) \quad (53)$$

For calculating the variance, we have:

$$VAR(x) = E(x^2) - (E(x))^2 \quad (54)$$

$$E_L(x^2) = \frac{b_1+a_2}{2} + \frac{1}{2} \int_{b_1}^{b_2} h_A(x^2) dx - \frac{1}{2} \int_{a_1}^{a_2} f_A(x^2) dx \quad (55)$$

$$E_U(x^2) = \frac{a_3+b_4}{2} + \frac{1}{2} \int_{a_3}^{a_4} g_A(x^2) dx - \frac{1}{2} \int_{b_3}^{b_4} k_A(x^2) dx \quad (56)$$

$$E_L(x^2) = -\frac{1}{3} (a_1^2 + a_2^2 + b_1^2 + b_2^2 + a_1 a_2 + b_1 b_2) + a_1 + a_2 + b_1 + b_2 \quad (57)$$

$$E_U(x^2) = -\frac{1}{3} (a_3^2 + a_4^2 + b_3^2 + b_4^2 + a_3 a_4 + b_3 b_4) + a_3 + a_4 + b_3 + b_4 \quad (58)$$

$$E(x^2) = \frac{E_L(x^2) + E_U(x^2)}{2} \quad (59)$$

By inserting the variance formula and a simplification, we shall have:

$$VAR(\tilde{A}^I) = \frac{1}{4} \left(\sum_{i=1}^4 a_i + \sum_{i=1}^4 b_i \right) - \frac{1}{12} (a_1 a_2 + a_3 a_4 + b_1 b_2 + b_3 b_4 + \sum_{i=1}^4 a_i^2 + \sum_{i=1}^4 b_i^2) - \frac{1}{64} \left(\sum_{i=1}^4 a_i + \sum_{i=1}^4 b_i \right)^2 \quad (60)$$

A multi-objective approach based on Markowitz and DEA cross-efficiency models for ...

By inserting computations in the model, we shall gain access to the model rendered below:

Model 9:

$$\text{Max } E \left(\sum_{i=1}^N \tilde{R}_i^1 w_i \right) = \frac{1}{8} \left(\sum_{i=1}^N (a_{i1} + a_{i2} + a_{i3} + a_{i4} + b_{i1} + b_{i2} + b_{i3} + b_{i4}) w_i \right) \quad (61)$$

$$\text{Min } \sigma^2 \left(\sum_{i=1}^N \tilde{R}_i^1 w_i \right) = \frac{1}{4} \sum_{i=1}^N \left((\sum_{j=1}^4 a_{ij} + \sum_{j=1}^4 b_{ij}) - \frac{1}{12} (a_{i1}a_{i2} + a_{i3}a_{i4} + b_{i1}b_{i2} + b_{i3}b_{i4} + \sum_{j=1}^4 a_{ij}^2 + \sum_{j=1}^4 b_{ij}^2) - \frac{1}{64} (\sum_{j=1}^4 a_{ij} + \sum_{j=1}^4 b_{ij})^2 w_i \right) \quad (62)$$

$$\text{Max } \sum_{i=1}^N w_i \bar{e}_i \quad (63)$$

$$\text{Min } \sum_{i=1}^N \sum_{j=1}^N w_i w_j \text{cov}(e_i, e_j) \quad (64)$$

s.t.

$$\sum_{i=1}^N z_i \leq h \quad (65)$$

$$l_i z_i \leq w_i \leq u_i z_i \quad i = 1, \dots, N \quad (66)$$

$$\sum_{i=1}^N w_i = 1 \quad (67)$$

$$w_i \geq 0 \quad i = 1, \dots, N \quad (68)$$

In the abovementioned model, $(a_{1j}, a_{2j}, a_{3j}, a_{4j}, b_{1j}, b_{2j}, b_{3j}, b_{4j})$ illustrates the j^{th} asset returns, which has been considered a trapezoidal fuzzy number and the other parameters equate to the parameters defined in the fuzzy form.

With due attention to the fact that in the abovementioned model, such as the upper and lower limits of investments per share; and likewise, the number of maximum shares present; and have come to hand in the portfolio, is determined by the investor. We shall eliminate the maximum constraints of the number of shares in this model; it will be abolished. Thereby, we shall deal with solving the following model in this paper as follows:

Model 10:

$$\text{Max } E \left(\sum_{i=1}^N \tilde{R}_i^1 w_i \right) = \frac{1}{8} \left(\sum_{i=1}^N (a_{i1} + a_{i2} + a_{i3} + a_{i4} + b_{i1} + b_{i2} + b_{i3} + b_{i4}) w_i \right) \quad (69)$$

$$\text{Min } \sigma^2 \left(\sum_{i=1}^N \tilde{R}_i^1 w_i \right) = \frac{1}{4} \sum_{i=1}^N \left((\sum_{j=1}^4 a_{ij} + \sum_{j=1}^4 b_{ij}) - \frac{1}{12} (a_{i1}a_{i2} + a_{i3}a_{i4} + b_{i1}b_{i2} + b_{i3}b_{i4} + \sum_{j=1}^4 a_{ij}^2 + \sum_{j=1}^4 b_{ij}^2) - \frac{1}{64} (\sum_{j=1}^4 a_{ij} + \sum_{j=1}^4 b_{ij})^2 w_i \right) \quad (70)$$

$$\text{Max } \sum_{i=1}^N w_i \bar{e}_i \quad (71)$$

$$\text{Min } \sum_{i=1}^N \sum_{j=1}^N w_i w_j \text{cov}(e_i, e_j) \quad (72)$$

s.t.

$$l_i z_i \leq w_i \leq u_i z_i \quad i = 1, \dots, N \quad (73)$$

$$\sum_{i=1}^N w_i = 1 \tag{74}$$

$$w_i \geq 0 \quad i = 1, \dots, N \tag{75}$$

4. Solving a numerical example

So to solve a numerical example and compare the responses that have come to hand, we have utilized data and information relative to the companies of the Tehran Stock Exchange. For this objective, we have selected 50 companies from those present in the Tehran Stock Exchange from 2018 to 2021 in the tableau of the Tehran Stock Exchange. By paying heed to the fact that banks, insurance enterprises, and investment companies have a diverse fiscal structure from that of other companies, these categories of companies are not incorporated in the list of companies. Similarly, the information published in the fiscal statements protracting to 19/03/2021 has been used to survey companies' efficiency.

4.1. Value Returns

The returns are considered a trapezoidal intuitionistic fuzzy number by considering the company's efficiency for three years (2016 to 2019). These returns, which are being considered, are for 50 companies; and have been rendered in Table (1).

Table 1. Trapezoidal intuistic-fuzzy returns

| Asset ID | Intuitionistic Fuzzy Returns | Asset ID | Intuitionistic Fuzzy Returns |
|----------|--|----------|---|
| 1 | (0.04,0.09,0.14,0.19,-0.13,0.08,0.15,0.61) | 26 | (0.05,0.1,0.16,0.21,-0.21,0.09,0.17,0.71) |
| 2 | (0.03,0.1,0.16,0.23,-0.21,0.08,0.18,0.63) | 27 | (0.05,0.08,0.11,0.13,-0.14,0.07,0.11,0.4) |
| 3 | (0.05,0.09,0.12,0.15,-0.17,0.08,0.13,0.52) | 28 | (0.04,0.07,0.09,0.12,-0.16,0.06,0.1,0.38) |
| 4 | (0.01,0.05,0.08,0.12,-0.21,0.04,0.09,0.44) | 29 | (0.09,0.11,0.14,0.16,-0.11,0.11,0.14,0.43) |
| 5 | (0.07,0.1,0.12,0.15,-0.13,0.09,0.13,0.56) | 30 | (0.03,0.08,0.13,0.18,-0.28,0.07,0.14,0.58) |
| 6 | (0.04,0.06,0.09,0.11,-0.18,0.06,0.09,0.35) | 31 | (0.03,0.09,0.15,0.22,-0.22,0.08,0.17,0.71) |
| 7 | (0.05,0.12,0.18,0.25,-0.23,0.1,0.2,0.74) | 32 | (0.08,0.13,0.17,0.22,-0.16,0.12,0.18,0.64) |
| 8 | (0.04,0.09,0.14,0.19,-0.23,0.08,0.15,0.66) | 33 | (0.07,0.09,0.1,0.11,-0.04,0.08,0.1,0.4) |
| 9 | (0.04,0.1,0.16,0.22,-0.22,0.09,0.17,0.64) | 34 | (0.02,0.05,0.08,0.11,-0.14,0.04,0.09,0.34) |
| 10 | (0.07,0.1,0.13,0.16,-0.16,0.09,0.13,0.56) | 35 | (0.03,0.08,0.13,0.17,-0.24,0.07,0.14,0.54) |
| 11 | (0.04,0.08,0.11,0.14,-0.18,0.07,0.12,0.51) | 36 | (-0.04,0.08,0.2,0.31,-0.2,0.05,0.22,0.72) |
| 12 | (0.08,0.11,0.14,0.17,-0.13,0.11,0.15,0.46) | 37 | (0.03,0.1,0.18,0.26,-0.18,0.08,0.2,0.89) |
| 13 | (0.04,0.07,0.11,0.14,-0.2,0.07,0.12,0.52) | 38 | (-0.01,0.09,0.18,0.27,-0.21,0.06,0.2,0.72) |
| 14 | (0.08,0.11,0.14,0.16,-0.16,0.1,0.14,0.43) | 39 | (0.06,0.11,0.16,0.22,-0.18,0.1,0.18,0.67) |
| 15 | (-0.01,0.03,0.07,0.11,-0.2,0.02,0.08,0.32) | 40 | (0.04,0.07,0.11,0.14,-0.18,0.07,0.12,0.54) |
| 16 | (0.07,0.1,0.14,0.17,-0.13,0.09,0.15,0.53) | 41 | (0.07,0.1,0.12,0.14,-0.12,0.09,0.13,0.48) |
| 17 | (0.05,0.1,0.14,0.19,-0.22,0.08,0.15,0.56) | 42 | (0.06,0.13,0.21,0.28,-0.12,0.12,0.22,0.84) |
| 18 | (0.07,0.1,0.14,0.17,-0.22,0.09,0.14,0.55) | 43 | (0.05,0.07,0.09,0.11,-0.17,0.07,0.1,0.42) |
| 19 | (0.04,0.08,0.12,0.16,-0.15,0.07,0.13,0.59) | 44 | (-0.02,0.08,0.17,0.26,-0.24,0.05,0.19,0.78) |
| 20 | (0.06,0.11,0.17,0.22,-0.22,0.1,0.18,0.7) | 45 | (0.1,0.12,0.13,0.14,-0.03,0.11,0.13,0.37) |
| 21 | (0.04,0.07,0.09,0.12,-0.12,0.06,0.1,0.33) | 46 | (0.06,0.09,0.12,0.15,-0.16,0.08,0.13,0.35) |
| 22 | (0.05,0.12,0.19,0.26,-0.19,0.1,0.21,0.7) | 47 | (0.05,0.08,0.11,0.14,-0.14,0.07,0.12,0.57) |
| 23 | (0.06,0.1,0.13,0.16,-0.17,0.09,0.14,0.44) | 48 | (0.03,0.08,0.12,0.17,-0.19,0.06,0.14,0.5) |
| 24 | (0.04,0.09,0.15,0.2,-0.26,0.08,0.16,0.71) | 49 | (0.13,0.16,0.19,0.22,-0.13,0.15,0.2,0.52) |
| 25 | (0.06,0.12,0.18,-0.45,0.04,0.13,0.47) | 50 | (0.04,0.08,0.11,0.15,-0.24,0.07,0.12,0.49) |

A multi-objective approach based on Markowitz and DEA cross-efficiency models for ...
 We have finally solved the model attained by the NSGA II algorithm and the MATLAB 2014 Software.

4.2. The Input and Output Values

Seven criteria from the input and 7 criterions from the output have been used to survey the efficiency. The list of criteria has been given in Table (2). The information relative to each issue inserted in the audited financial statements, prolonging to 19/03/2020, has been extracted and computed.

Table 2. Inputs and outputs

| Type | Parameter | Description | Perspective |
|--------|---|--|---------------|
| Input | Turnover of accounts receivable | Period income divided by accounts receivable | Productivity |
| | Inventory of materials and goods | Period income divided by inventories | |
| | Asset turnover | Period income divided by assets | |
| | Current Ratio | Current assets over current debts | Liquidity |
| | Instantaneous Ratio | Quick assets as to the current debts | |
| | Ratio of debt to the shareholders' equity | Total debt divided by shareholders' equity | |
| | Debt Ratio | Total debts over total assets | |
| Output | Return on the shareholders' equity | Net profit on the shareholders' equity | Profitability |
| | Return on assets | Net profit on assets | |
| | Net profit margin | Net profit on sales | |
| | Earnings per share | Net profit on the number of shares | Growth |
| | Income growth rate | Current period income divided by the previous period income minus one | |
| | Net profit growth rate | Net profit for the current period divided by the net profit for the prior period minus one | |
| | Growth rate of earnings per share | Current period EPS divided by previous period EPS minus one | |

4.3. Model Parameters

To solve the model with the NSGA II algorithm is as follows:
 The population has been contemplated as (N_{pop}), 100, $p_m = 0.1$, $p_c = 0.8$; the maximum number of iterations is equivalent to 200, $\mu = 2$; the model has been coded by the MATLAB 2014 (mechanism). Similarly, the minimum investment per the (l_i) share by the investor is 10 percent, and the maximum investment for each (u_i) share has been considered as and equates to 30 percent.

4.4. Results

Two specifications, such as those given hereunder, are considered to select the stock portfolio. Concerning each of the two, we have computed the optimal portfolio, based on the information of the companies, including the returns, from the existing shares; eventually, we have compared the responses.

Portfolio-1: A combination of the Markowitz and the cross-efficiency model with fuzzy returns (Mashayekhi and Omrani 2016).

Portfolio-2: A combination of the Markowitz and the cross-efficiency model, with intuitionistic fuzzy returns (Model 11).

The model results based on the weights assigned to each share are given in Table (3).

The conceptions of the expected rate of returns, portfolio efficiency, and portfolio risk on the fundamentals of the rate of returns, as well as the portfolio risk in terms of efficiency, are considered, respectively, following equations (76 to 79) (Mashayekhi and Omrani 2016).

$$E(\sum_{i=1}^N \tilde{R}_i^I w_i) \tag{76}$$

$$\sum_{i=1}^N w_i \bar{e}_i \tag{77}$$

$$\sigma^2(\sum_{i=1}^N \tilde{R}_i^I w_i) \tag{78}$$

$$\sum_{i=1}^N \sum_{j=1}^N w_i w_j cov(e_i, e_j) \tag{79}$$

In implementing the model, a set of optimum Pareto responses come to hand. The investor can select one of these Pareto responses, as an investment portfolio, based on criteria. Various criteria can be contemplated for choosing the desired portfolio. We have considered the highest returns, and in each model, the response with the highest returns is considered in this paper. The reactions achieved following these criteria are shown in Table (3) And Tabel (4).

Table 3. Weights of the assets in the selected portfolio for the intuitionistic fuzzy returns model

| | | | | |
|-----------------------|------|------|------|------|
| Asset ID | 7 | 37 | 42 | 44 |
| weights of the assets | 0.23 | 0.23 | 0.23 | 0.31 |

Table 4. Weights of the assets in the selected portfolio for the fuzzy Returns model

| | | | | |
|-----------------------|------|------|------|------|
| Asset ID | 18 | 32 | 42 | 49 |
| weights of the assets | 0.20 | 0.20 | 0.32 | 0.28 |

Likewise, the responses in relevance to the objective function in each model have been rendered in Table (5).

Table 5. Optimum values of the Objective Function

| Function | Function Type | Objective | Fuzzy | Intuitionistic Fuzzy |
|----------|---------------|--|-------|----------------------|
| z1 | Maximum | Expected rate of returns | 0.16 | 0.18 |
| z2 | Minimum | Portfolio risk, based on the rate of returns | 0.003 | 0.24 |
| z3 | Maximum | Portfolio Efficiency | 0.41 | 0.72 |
| z4 | Minimum | Portfolio risk, based on efficiency | 0.16 | 0.07 |

A multi-objective approach based on Markowitz and DEA cross-efficiency models for ...

As can be observed in the abovementioned Table, the intuitionistic fuzzy model has a higher expected rate of returns than the fuzzy model. The same also exhibits a better efficiency than the fuzzy model. Regardless of the direct correlation between risk and returns, based on the fundamentals of the rate of returns, the risk portfolio and similarly, in accordance with efficiency, has also shown increment. Some other Pareto responses relevant to any model are presented in Table (6).

Table 6. Some Pareto solutions obtained from the proposed model

| Type of Model | Serial Number | Expected rate of returns | Portfolio risk, based on the rate of returns | Portfolio Efficiency | Portfolio risk, based on efficiency |
|----------------------|---------------|--------------------------|--|----------------------|-------------------------------------|
| Intuitionistic fuzzy | 1 | 0.06 | 0.11 | 0.23 | 0.04 |
| | 2 | 0.12 | 0.18 | 0.57 | 0.02 |
| | 3 | 0.16 | 0.22 | 0.83 | 0.03 |
| Fuzzy | 1 | 0.08 | 0.0009 | 0.55 | 0.04 |
| | 2 | 0.11 | 0.0002 | 0.53 | 0.16 |
| | 3 | 0.14 | 0.005 | 0.80 | 0.05 |

5. Conclusion

This paper combined Markowitz's mean-variance model and a cross-efficiency model to introduce a four-objective model that increased efficiency and decreased the covariance of cross-efficiencies besides increasing returns and reducing the portfolio risk. Although many studies have addressed portfolio optimization using Markowitz's and cross-efficiency models independently, a few studies have combined these models and benefitted from the advantages of both models. Correspondingly, the returns are assumed as intuitionistic trapezoidal fuzzy numbers to represent return uncertainty. A non-dominated sorting genetic algorithm (NSGA-II) was also used to solve the new model. Moreover, the proposed model was implemented on 50 firms listed on the Tehran Stock Exchange. The results were compared in two cases intuitionistic trapezoidal fuzzy numbers and trapezoidal fuzzy numbers. The results indicated that, despite significant improvement in portfolio efficiency in the case of intuitionistic trapezoidal fuzzy returns, the portfolio risk was increased substantially in response to a slight increase in portfolio returns. Some extensions can be considered for future studies, such as adding constraints on transaction costs and the number of stocks within the portfolio. Also, other cases of return or efficiency uncertainty can be treated. Furthermore, it is suggested to find new models by developing efficiency measurement structures, such as cross-efficiency in network DEA.

Author contributions: Research problem, M.R. and S.A.E.; Conceptualization, M.R. and S.A.E.; Methodology, S.A.E.; Software, M.R.; Validation, S.A.E. and M.F. and S.E.N.; Formal analysis, M.R. and S.A.E.; Investigation, M.R.; Resources, M.R. and S.A.E.; Data curation, M.R. and S.A.E.; Writing, M.R. and S.A.E. and S.E.N.; Reviewing and editing, M.F.; Visualization, M.R. and S.A.E.; Supervision, S.A.E. and M.F.; All authors have read and approved the published version of the manuscript.

Acknowledgments: The authors would like to express their gratitude to the editors and anonymous referees for their informative, helpful remarks and suggestions to improve this paper as well as the important guiding significance to our researches.

Funding: This research received no external funding.

Conflict of Interest: The authors declare that they have no conflict of interest.

References

Afshar, K. M., Khaliliaraghi, M., & Sadat, K. A. (2012). Stock selection of Tehran stock exchange investors with hybrid of data envelopment analysis (DEA) and goal programming (GP), 49-63.

Algarvio, H., Lopes, F., Sousa, J., & Lagarto, J. (2017). Multi-agent electricity markets: Retailer portfolio optimization using Markowitz theory. *Electric Power Systems Research*, 148, 282-294.

Amin, G. R., & Hajjami, M. (2021). Improving DEA cross-efficiency optimization in portfolio selection. *Expert Systems with Applications*, 168, 114280.

Aouni, B., Colapinto, C., & La Torre, D. (2014). Financial portfolio management through the goal programming model: Current state-of-the-art. *European Journal of Operational Research*, 234(2), 536-545.

Carlsson, C., & Fullér, R. (2001). On possibilistic mean value and variance of fuzzy numbers. *Fuzzy sets and systems*, 122(2), 315-326.

Carlsson, C., Fuller, R., Heikkilä, M., & Majlender, P. (2007). A fuzzy approach to R&D project portfolio selection. *International Journal of Approximate Reasoning*, 44(2), 93-105.

Carlsson, C., Fullér, R., & Majlender, P. (2002). A possibilistic approach to selecting portfolios with highest utility score. *Fuzzy sets and systems*, 131(1), 13-21.

Chang, T. J., Meade, N., Beasley, J. E., & Sharaiha, Y. M. (2000). Heuristics for cardinality constrained portfolio optimisation. *Computers & Operations Research*, 27(13), 1271-1302.

Chang, T. S., Tone, K., & Wu, C. H. (2021). Nested dynamic network data envelopment analysis models with infinitely many decision making units for portfolio evaluation. *European journal of operational research*, 291(2), 766-781.

Chang, T. J., Yang, S. C., & Chang, K. J. (2009). Portfolio optimization problems in different risk measures using genetic algorithm. *Expert Systems with applications*, 36(7), 10529-10537.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European journal of operational research*, 2(6), 429-444.

Chen, W., Li, S. S., Zhang, J., & Mehlawat, M. K. (2020). A comprehensive model for fuzzy multi-objective portfolio selection based on DEA cross-efficiency model. *Soft computing*, 24(4), 2515-2526.

Chen, W., Li, S. S., Mehlawat, M. K., Jia, L., & Kumar, A. (2021). Portfolio Selection Using Data Envelopment Analysis Cross-Efficiency Evaluation with Undesirable Fuzzy Inputs and Outputs. *International Journal of Fuzzy Systems*, 23(5), 1478-1509.

Chen, W., Gai, Y., & Gupta, P. (2018). Efficiency evaluation of fuzzy portfolio in different risk measures via DEA. *Annals of Operations Research*, 269(1), 103-127.

- A multi-objective approach based on Markowitz and DEA cross-efficiency models for ...
- Cooper, W. W., Park, K. S., & Pastor, J. T. (1999). RAM: a range adjusted measure of inefficiency for use with additive models, and relations to other models and measures in DEA. *Journal of Productivity analysis*, 11(1), 5-42.
- Deb, K., Pratap, A., Agarwal, S., & Meyarivan, T. A. M. T. (2002). A fast and elitist multi-objective genetic algorithm: NSGA-II. *IEEE transactions on evolutionary computation*, 6(2), 182-197.
- Edalatpanah, S. A. (2020). Data envelopment analysis based on triangular neutrosophic numbers. *CAAI transactions on intelligence technology*, 5(2), 94-98.
- Edalatpanah, S. A. (2019). A data envelopment analysis model with triangular intuitionistic fuzzy numbers. *International journal of data envelopment analysis*, 7(4), 47-58.
- Edalatpanah, S. A. (2018). Neutrosophic perspective on DEA. *Journal of applied research on industrial engineering*, 5(4), 339-345.
- Edirisinghe, N. C. P., & Zhang, X. (2008). Portfolio selection under DEA-based relative financial strength indicators: case of US industries. *Journal of the operational research society*, 59(6), 842-856.
- Edirisinghe, N. C., & Zhang, X. (2007). Generalized DEA model of fundamental analysis and its application to portfolio optimization. *Journal of banking & finance*, 31(11), 3311-3335.
- Eftekharian, S. E., Shojafar, M., & Shamshirband, S. (2017). 2-phase NSGA II: An optimized reward and risk measurements algorithm in portfolio optimization. *Algorithms*, 10(4), 130.
- Fernández, A., & Gómez, S. (2007). Portfolio selection using neural networks. *Computers & Operations Research*, 34(4), 1177-1191.
- Goodarzi, M., Yakideh, K., & Mahfoozi, G. (2017). Portfolio optimization by combining data envelopment analysis and decision-making Hurwicz Method. *Modern Research in Decision Making*, 1(4), 143-165.
- Grzegorzewski, P. (2003, September). Distances and orderings in a family of intuitionistic fuzzy numbers. In *EUSFLAT Conf.* (pp. 223-227).
- Guo, H., Sun, B., Karimi, H. R., Ge, Y., & Jin, W. (2012). Fuzzy investment portfolio selection models based on interval analysis approach. *Mathematical Problems in Engineering*, 2012.
- Hajiagha, S. H. R., Akrami, H., Kazimieras Zavadskas, E., & Hashemi, S. S. (2013). An intuitionistic fuzzy data envelopment analysis for efficiency evaluation under uncertainty: case of a finance and credit institution.
- Hoe, L. W., Siew, L. W., & Fai, L. K. (2017, November). Improvement on the efficiency of technology companies in Malaysia with data envelopment analysis model. In *International Visual Informatics Conference* (pp. 19-30). Springer, Cham.
- Huang, C. Y., Chiou, C. C., Wu, T. H., & Yang, S. C. (2015). An integrated DEA-MODM methodology for portfolio optimization. *Operational Research*, 15(1), 115-136.
- Huang, X. (2008). Risk curve and fuzzy portfolio selection. *Computers & Mathematics with Applications*, 55(6), 1102-1112.

Hunjra, A. I., Alawi, S. M., Colombage, S., Sahito, U., & Hanif, M. (2020). Portfolio Construction by Using Different Risk Models: A Comparison among Diverse Economic Scenarios. *Risks*, 8(4), 126.

Javaherian, N., Hamzehee, A., & Sayyadi Tooranloo, H. (2021). Designing an intuitionistic fuzzy network data envelopment analysis model for efficiency evaluation of decision-making units with two-stage structures. *Advances in Fuzzy Systems*, 2021.

Jin, F., Garg, H., Pei, L., Liu, J., & Chen, H. (2020). Multiplicative consistency adjustment model and data envelopment analysis-driven decision-making process with probabilistic hesitant fuzzy preference relations. *International Journal of Fuzzy Systems*, 22(7), 2319-2332.

Karimi, A. (2021). Stock portfolio optimization using multi-objective genetic algorithm (NSGA II) and maximum Sharp ratio, 389-410.

Kaucic, M., Moradi, M., & Mirzazadeh, M. (2019). Portfolio optimization by improved NSGA-II and SPEA 2 based on different risk measures. *Financial Innovation*, 5(1), 1-28.

Kellerer, H., Mansini, R., & Speranza, M. G. (2000). Selecting portfolios with fixed costs and minimum transaction lots. *Annals of Operations Research*, 99(1), 287-304.

Konno, H., & Yamazaki, H. (1991). Mean-absolute deviation portfolio optimization model and its applications to Tokyo stock market. *Management science*, 37(5), 519-531.

Lamb, J. D., & Tee, K. H. (2012). Resampling DEA estimates of investment fund performance. *European Journal of Operational Research*, 223(3), 834-841.

Lim, S., Oh, K. W., & Zhu, J. (2014). Use of DEA cross-efficiency evaluation in portfolio selection: An application to Korean stock market. *European Journal of Operational Research*, 236(1), 361-368.

Mao, X., Guoxi, Z., Fallah, M., & Edalatpanah, S. A. (2020). A neutrosophic-based approach in data envelopment analysis with undesirable outputs. *Mathematical problems in engineering*, 2020.

Markowitz, H. (1952). Portfolio Selection in *The Journal of Finance* Vol. 7, 77-91.

Mashayekhi, Z., & Omrani, H. (2016). An integrated multi-objective Markowitz-DEA cross-efficiency model with fuzzy returns for portfolio selection problem. *Applied Soft Computing*, 38, 1-9.

Omrani, H., Mashayekhi, Z. (2017). Multi objective portfolio selection by combining markowitz and dea cross efficiency models. *Sharif Journal of Industrial Engineering & Management*, 33.1(1.1), 87-94. doi: 10.24200/j65.2017.5578.

Pal, R., Chaudhuri, T. D., & Mukhopadhyay, S. (2021). Portfolio formation and optimization with continuous realignment: A suggested method for choosing the best portfolio of stocks using variable length NSGA-II. *Expert Systems with Applications*, 186, 115732.

Pastor, J. T., & Ruiz, J. L. (2007). Variables with negative values in DEA. In *Modeling data irregularities and structural complexities in data envelopment analysis* (pp. 63-84). Springer, Boston, MA.

A multi-objective approach based on Markowitz and DEA cross-efficiency models for ...

Puri, J., & Yadav, S. P. (2015). Intuitionistic fuzzy data envelopment analysis: An application to the banking sector in India. *Expert Systems with Applications*, 42(11), 4982-4998.

Puri, J., Yadav, S. P., & Garg, H. (2017). A new multi-component DEA approach using common set of weights methodology and imprecise data: an application to public sector banks in India with undesirable and shared resources. *Annals of Operations Research*, 259(1), 351-388.

Rasoulzadeh, M., & Fallah, M. (2020). An overview of portfolio optimization using fuzzy data envelopment analysis models. *Journal of fuzzy extension and applications*, 1(3), 180-188.

Sadjadi, S. J., Gharakhani, M., & Safari, E. (2012). Robust optimization framework for cardinality constrained portfolio problem. *Applied Soft Computing*, 12(1), 91-99.

Schaerf, A. (2002). Local search techniques for constrained portfolio selection problems. *Computational Economics*, 20(3), 177-190.

Soleimani, H., Golmakani, H. R., & Salimi, M. H. (2009). Markowitz-based portfolio selection with minimum transaction lots, cardinality constraints and regarding sector capitalization using genetic algorithm. *Expert Systems with Applications*, 36(3), 5058-5063.

Yang, W., Cai, L., Edalatpanah, S. A., & Smarandache, F. (2020). Triangular single valued neutrosophic data envelopment analysis: application to hospital performance measurement. *Symmetry*, 12(4), 588.

Yu, G. F., Li, D. F., Liang, D. C., & Li, G. X. (2021). An Intuitionistic Fuzzy Multi-Objective Goal Programming Approach to Portfolio Selection. *International Journal of Information Technology & Decision Making*, 20(05), 1477-1497.

Zhao, L., & Palomar, D. P. (2018). A markowitz portfolio approach to options trading. *IEEE Transactions on Signal Processing*, 66(16), 4223-4238.

Zhou, W., & Xu, Z. (2018). Portfolio selection and risk investment under the hesitant fuzzy environment. *Knowledge-Based Systems*, 144, 21-31.



© 2022 by the authors. Submitted for possible open access publication under the terms and conditions of the Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>).