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A ROUGH STRENGTH RELATIONAL DEMATEL MODEL FOR ANALYSING THE KEY SUCCESS FACTORS OF HOSPITAL SERVICE QUALITY

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Abstract. *Successful management of hospital service quality (HSQ) is increasingly becoming strategic perspective of hospitals to excel in medical care within reasonable prices which is among the customers'/patients' primary needs. Several key success factors (KSFs) can control the proper HSQ management and make it a complex problem. For these reasons, it is vital to set hospitals' goals and detect KSFs via customers' feedback and viewpoints. The preceding researches discussed not much about the effect of internal strength impact on the interdependencies between KSFs of HSQ management. To resolve these issues, a rough strength relational- decision making and trial evaluation laboratory (RSR-DEMATEL) model is developed to analyse the individual priorities of KSF of hospital's performance measures. The RSR-DEMATEL method reflects completely the internal as well as external total influences between the KSFs. Additionally, the proposed model has intelligence and flexibility in manipulating the inherent uncertainty due to the subjective ang vague information in KSF analysis for managing HSQ. The validity and efficiency of the RSR-DEMATEL model are examined by applying it to a hospital data available in the literature. The result analysis shows that "medical staff with professional abilities" (KSF5) is the most significant KSF in HSQ management, i.e., recruiting more skilled doctors and nurses will eventually increase the performance of hospital medical services. Finally, a comparative analysis is conducted to cross check the obtained results with other two methods from literature.*

Key words: *Strength-relation analysis, Rough numbers, DEMATEL, HSQ, KSFs.*

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1. Introduction

The hospital service quality management (HSQM) in todays' competitive marketplace is a complex decision making problem where organizations, people, information, and resources take part in achieving some predetermined objectives. Although in the past few decades HSQM referred to merely limited to medical services, nowadays, the success of a hospital broadly depends on HSQM and its business process management (BPM) (Shieh et al., 2010). To achieve the preset goals, keeping the hospital competitive, sustaining its growth rate, and raising profits not only at a local but also a global level, the hospital authority must implement BPM and simultaneously improve HSQ. Although BPM is known as business concept for decades, its strategic and operational roles with in organizations is still an important issue requiring investigation from various perspectives such as operations and information management (Bai & Sarkis, 2013). However, BPM in HSQ can be a risky intent with the possibility of huge investments and uncertain consequences. Many authors (Bai & Sarkis, 2013; Abdolvand et al., 2008; Bandara et al., 2005) warned and suggested about the failure rate of BPM. Thus, in order to implement BPM in HSQ successfully, and it is necessary to know the resulting initiatives and identify key success factors (KSFs) in HSQ incorporating BPM. Several papers sought to identify KSFs of HSQM. Most of these papers focused conceptual elements or performed qualitative analyses.

Many authors (Bowers et al., 1993; Babakus & Mangold, 1992; Koerner, 2000; Andaleeb, 2001; Youssef et al., 1995; Parasuraman et al., 1985) have studied development and implementation of BPM after maintaining HSQ strategies, but only a few used robust methodologies to conduct systematic evaluation of HSQM key/critical factors as well as BPM employment. Hospital managers should push its boundaries and implement a well-organized BPM strategy that facilitates identifying and analyzing KSFs in HSQM. Hence, BPM to improve HSQ can be well accomplished as multiple criteria decision making (MCDM) problems that consider several complex and usually conflicting or interacting factors. As mentioned earlier HSQM includes factors from organizations, people, information, and resources, it is necessary to narrow down the factors'/criteria set. Such minimal collection of KSFs in HSQM will reduce the complexity of decision making process. This is why, hospitals should pay more attention these KSFs while implementing BPM. With this, hospital managers will be able to understand better of BPM in HSQM and regulate the corresponding KSFs to successful management of HSQ.

Now, in order to assess the interactions between the KSFs, a well-organized method for quantitative analysis may provide valuable insights of cause and effect relationships. It is difficult to make actionable strategies directly from opinions given by experts and managers since they lack clear visions about the cause/effect relationships amongst the KSFs, which play vital roles in the HSQM. Hence, a systematic exploration tool, decision making trial and evaluation laboratory (DEMATEL) method can be applied to depict complex cause/effect relations through matrices (Shieh et al. 2010). Additionally, this particular tool makes use of cognitive maps to draw digraphs which portray the inter-relationships between KSFs. The DEMATEL method is beneficial in illuminating the relations amongst KSFs and ordering them depending on the cause/effect relations and prominence of their influences on other factors. But it becomes difficult to describe those relations if uncertainty exists in the data to be used for decision making process.

In response to the uncertainty, rough set theory (RST) is an excellent choice to manipulate subjective and vague data involved in the analysis of KSFs of HSQM. RST has a rich theoretical background in analyzing vague information and incomplete data. Thus, rough strength relational decision making trial and evaluation laboratory (RSR-DEMATEL) method can efficiently handle uncertainty due to subjectivity and vagueness. RSR-DEMATEL uses to rough numbers to manipulate the strengths and inter-relationships among the KSFs of HSQM under uncertainty caused by decision makers' knowledge based linguistic (qualitative) assessment.

This paper aims to assess KSFs of HSQM, with the objectives of this study being: (1) to apply a flexible and unique method that appraises KSFs of HSQM and obtain the structure of complicated causal relationships and the influence level of these factors; and (2) to help hospital managers having better control of KSFs while implementing BPM in HSQ and examine several capacities of BPM implementation practices.

To meet the abovementioned goals, this paper is organized as follows: We briefly introduce the basic elements of rough numbers and rough arithmetic in section 2. In Section 3 we provide the step-wise description of the RSR-DEMATEL model while section 4 deals with an empirical case example. The major implications of our study are articulated in section 5. The final section concludes the paper and tells about the limitations and future research directions.

2. Rough numbers and its operations

In group decision making problems, the priorities are defined on multi-expert's aggregated decision and process subjective evaluation of expert's decisions. Rough numbers consisting of upper, lower and boundary interval respectively, determine intervals of their evaluations without requiring additional information by relying only on original data (Zhai et al., 2008). Hence, obtained expert decision makers (DMs) perceptions objectively present and improve their decision making process. According to Zhai et al. (2009), the definition of rough number is shown below.

Let's U be a universe containing all objects and X be a random object from U. Then we assume that there exists set build with k classes representing DMs preferences, $R=(J_1,J_2,\dots,J_k)$ with condition $J_1<J_2<,\dots,<J_k$. Then, $\forall\ X\in U,\ J_q\in R,\ 1\le q\le k$ lower approximation $\underline{Apr}(J_q)$, upper approximation $\overline{Apr}(J_q)$ and boundary interval $Bnd(J_q)$ are determined, respectively, as follows:

$$\underline{Apr}(J_q)=\cup\left\{X\in U/R(X)\le J_q\right\} \quad (1)$$

$$\overline{Apr}(J_q)=\cup\left\{X\in U/R(X)\ge J_q\right\} \quad (2)$$

$$Bnd(J_q)=\cup\left\{X\in U/R(X)\ne J_q\right\}=\left\{X\in U/R(X)>J_q\right\}\cup \left\{X\in U/R(X)<J_q\right\} \quad (3)$$

The object can be presented with rough number (RN) defined with lower limit $\underline{Lim}(J_q)$ and upper limit $\overline{Lim}(J_q)$, respectively:

$$\underline{Lim}(J_q)=\frac{1}{M_L}\sum R(X)\big|X\in\underline{Apr}(J_q) \quad (4)$$

$$\overline{Lim}(J_q)=\frac{1}{M_U}\sum R(X)\big|X\in\overline{Apr}(J_q) \quad (5)$$

where M_L and M_U represent the sum of objects contained in the lower and upper object approximation of J_q, respectively. For object J_q, rough boundary interval $\left(IRBnd(J_q)\right)$ presents interval between lower and upper limit as:

$$IRBnd(J_q) = \overline{Lim}(J_q) - \underline{Lim}(J_q) \tag{6}$$

Rough boundary interval presents measure of uncertainty. The bigger $IRBnd(J_q)$ value shows that variations in experts' preferences exist, while smaller values show that experts had harmonized opinions without major deviations.

In $IRBnd(J_q)$ are comprised all objects between lower limit $\underline{Lim}(J_q)$ and upper limit $\overline{Lim}(J_q)$ of rough number $RN(J_q)$. That means that $RN(J_q)$ can be presented using $\underline{Lim}(J_q)$ and $\overline{Lim}(J_q)$.

$$RN(J_q) = \left[\underline{Lim}(J_q), \overline{Lim}(J_q)\right] \tag{7}$$

Since rough numbers belong to the group of interval numbers, arithmetic operations applied in interval numbers is also appropriate for rough numbers.

Since rough numbers belong to the group of interval numbers, arithmetic operations applied in interval numbers is also appropriate for rough numbers. If A and *B* presents two rough numbers $RN(A) = \left[\underline{Lim}(A), \overline{Lim}(A)\right]$ and $RN(B) = \left[\underline{Lim}(B), \overline{Lim}(B)\right]$, k denotes constant, $k > 0$, then the arithmetic operations with $RN(A)$, $RN(B)$ and k are as follows:

(1) Addition of rough numbers "+"

$$\begin{gathered} RN(A) + RN(B) = \left[\underline{Lim}(A), \overline{Lim}(A)\right] + \left[\underline{Lim}(B), \overline{Lim}(B)\right] = \\ \left[\underline{Lim}(A) + \underline{Lim}(B), \overline{Lim}(A) + \overline{Lim}(B)\right] \end{gathered} \tag{8}$$

(2) Subtraction of rough numbers "-"

$$\begin{gathered} RN(A) - RN(B) = \left[\underline{Lim}(A), \overline{Lim}(A)\right] - \left[\underline{Lim}(B), \overline{Lim}(B)\right] = \\ \left[\underline{Lim}(A) - \overline{Lim}(B), \overline{Lim}(A) - \underline{Lim}(B)\right] \end{gathered} \tag{9}$$

(3) Multiplication of rough numbers "×"

$$\begin{gathered} RN(A) \times RN(B) = \left[\underline{Lim}(A), \overline{Lim}(A)\right] \times \left[\underline{Lim}(B), \overline{Lim}(B)\right] = \\ \left[\underline{Lim}(A) \times \underline{Lim}(B), \overline{Lim}(A) \times \overline{Lim}(B)\right] \end{gathered} \tag{10}$$

(4) Dividing of rough numbers "/"

$$\begin{gathered} RN(A) / RN(B) = \left[\underline{Lim}(A), \overline{Lim}(A)\right] / \left[\underline{Lim}(B), \overline{Lim}(B)\right] = \\ \left[\underline{Lim}(A) / \overline{Lim}(B), \overline{Lim}(A) / \underline{Lim}(B)\right] \end{gathered} \tag{11}$$

(5) Scalar multiplication of rough numbers, where $k > 0$

$$k \times RN(A) = k \times \left[\underline{Lim}(A), \overline{Lim}(A) \right] = \left[k \times \underline{Lim}(A), k \times \overline{Lim}(A) \right] \tag{12}$$

Ranking rule of rough numbers:

Any two rough numbers, $RN(A) = \left[\underline{Lim}(A), \overline{Lim}(A) \right]$ and $RN(B) = \left[\underline{Lim}(B), \overline{Lim}(B) \right]$, where $\underline{Lim}(A)$ and $\underline{Lim}(B)$, and $\overline{Lim}(A)$, $\overline{Lim}(B)$ represent their lower and upper limits, respectively, are ranked by the use of the following rules:

If the rough boundary interval of a rough number is not strictly bound by another, then the ranking order is easily determined, i.e.

(a) If $\begin{cases} \underline{Lim}(A) \geq \underline{Lim}(B) \text{ and } \overline{Lim}(A) > \overline{Lim}(B) \\ \underline{Lim}(A) > \underline{Lim}(B) \text{ and } \overline{Lim}(A) \geq \overline{Lim}(B) \end{cases}$ then $RN(A) > RN(B)$

(b) If $\underline{Lim}(A) = \underline{Lim}(B)$ and $\overline{Lim}(A) = \overline{Lim}(B)$, then $RN(A) = RN(B)$.

If the rough boundary interval of a rough number is strictly bound by another, then ranking becomes awkward and medians $M(A)$ and $M(B)$ of $RN(A)$ and $RN(B)$ respectively, are used in ranking.

(a) If $\underline{Lim}(B) > \underline{Lim}(A)$ and $\overline{Lim}(B) < \overline{Lim}(A)$ then $\begin{cases} \text{if } M(A) \leq M(B) \text{ then } RN(A) < RN(B) \\ \text{if } M(A) > M(B) \text{ then } RN(A) > RN(B) \end{cases}$

(b) Similar rules can be derived if $\underline{Lim}(A) > \underline{Lim}(B)$ and $\overline{Lim}(A) < \overline{Lim}(B)$.

3. The rough strength-relation DEMATEL method

The DEMATEL method is a comprehensive method used in both the design and analysisof structural method characterized by the causal relations between complex factors (Fontela & Gabus, 1976). The method is based on graph theory, which enables visual planning and problem solving so thatall relevant factors can be classified into causal and consequential factors,for better understanding of their interrelations. This method makes it possible to better understand the complex structure of a problem and define the relations between factors (Gigović et al., 2017).

For the purpose of accepting the imprecision in the collective decision making process, this paper modifies the DEMATEL method by applying rough numbers strength-relation analysis (RSR-DEMATEL). The application of rough numbers eliminates the necessity for additional information for defining uncertain number intervals. In such a way, the quality of the existing data in the collective decision making process can be retained, as well as the experts' perception, which is expressed through the aggregation matrix. The text below shows the steps governing the RSR-DEMATEL method, which was used in the group decision making process.

Step 1. Evaluate internal strength of factors with linguistic scale. When considering the interactions between two factors, the interaction not only depends on the intensity of influencing but also on the strength of the factors that exerts (Song et al. 2017). The decision maker (expert) can evaluate the internal strength of all factors using the 5-point verbal scale 0 –No strength (NS); 1 –Low strength (LS); 2 –Medium strength (MS); 3 –High strength (HS); 4 –Very high strength (VHS).

Assuming that there are *m* experts in the research and *n* observed factors, each expert should determine the degree of intrenal strength of all factors. The evaluation matrix of *k* $(1 \le k \le m)$ expert is presented as a non-negative matrix of $n \times 1$ rank, and each element of the *k* matrix in equation $Y^k=[y^k_i]_{n\times 1}$ denotes a non-negative number y^k_{ij}, where $1 \le k \le m$.

$$Y^k = \begin{bmatrix} y_1^k \\ y_2^k \\ \vdots \\ y_n^k \end{bmatrix}_{nx1} ;1 \le i \le n;\ \ 1 \le k \le m \tag{13}$$

where y_i^k represent linguistic variable taken from the preliminary defined linguistic scale used by expert *k* for the purpose of intrenal strength evaluation. In accordance with this, $Y^1, Y^2, \ldots, Y^m$ matrices are evaluation matrices of each of *m* experts.

Step 2. Determination of experts' weight coefficients (w_i). Experts' weight coefficients are determined using three parameters: An objective expert's evaluation (w_o) which is determined on the basis of experience that an expert possesses in the field of research; mutual expert's evaluation (w_u) which is determined on the basis of mutual assessment of the experts participating in the study; and subjective expert's evaluation (w_s) which is determined on the basis of expert's assessment of their own competence for participation in the study. Reviews on all three weight parameters (w_o, w_u, w_s) are awarded on the basis of pre-defined linguistic scale: 0 – No influence (NI); 1 –Low influence (LI); 2 –Medium influence (MI); 3 –High influence (HI); 4 –Very high influence (VHI).

The obtained weighting coefficient (w_i) is calculated from the score representing the sum of individual assessment parameters (w_o, w_u and w_s). Since the requirement $\sum_{i=1}^{m} w_i = 1$ needs to be satisfied, the final w_i values are calculated using equation (14) where $W_i = w_{o_i} + w_{u_i} + w_{s_i}$ is the weighting coefficient of expert *m* (*i=1,2,...,m*).

$$w_i = \frac{W_i}{\sum_{i=1}^{m} W_i} \tag{14}$$

Step 3. Determine the aggregated internal strength of factors. On the basis of the step 1, we receive $Y^1, Y^2, \ldots, Y^m$ matrices of each of *m* experts

$$Y^k = \begin{bmatrix} y_1^1, y_1^2, \ldots, y_1^m \\ y_2^1, y_2^2, \ldots, y_2^m \\ \vdots \\ y_n^1, y_n^2, \ldots, y_n^m \end{bmatrix}_{nx1} \tag{15}$$

where $y_i = \left\{ y_i^1, y_i^2, \ldots, y_i^m \right\}$ denote the sequences used to describe the intrenal strength of the factor *i*. By applying equations (1) through (7), each sequence x_i^k is converted to rough sequences $RN\left(y_i^k\right) = \left[\underline{Lim}(y_i^k), \overline{Lim}(y_i^k)\right]$, where $\underline{Lim}(y_i^k)$ and $\overline{Lim}(y_i^k)$ represent the lower limit and upper limit of rough sequence $RN\left(y_i^k\right)$, respectively.

Thus we obtain rough matrices $Y^1, Y^2, ..., Y^m$, where m denotes the number of experts. Therefore for each rough matrix ($Y^1, Y^2, ..., Y^m$) in position ($i,1$) we obtain rough sequence $RN(y_{i1}) = \left\{ \left[\underline{Lim}(y_{i1}^1), \overline{Lim}(y_{i1}^1) \right], \left[\underline{Lim}(y_{i1}^2), \overline{Lim}(y_{i1}^2) \right], ..., \left[\underline{Lim}(y_{i1}^m), \overline{Lim}(y_{i1}^m) \right] \right\}$.

By applying equation (16) the rough aggregated internal strength of factor i is obtained

$$RN(y_i) = \left[\underline{Lim}(y_i), \overline{Lim}(y_i) \right] = \begin{cases} \underline{Lim}(y_i) = \sum_{e=1}^{m} \underline{Lim}(y_i^e) \cdot w_i^e \\ \overline{Lim}(y_i) = \sum_{e=1}^{m} \overline{Lim}(y_i^e) \cdot w_i^e \end{cases} \tag{16}$$

where $\underline{Lim}(y_i)$ and $\overline{Lim}(y_i)$ represent the lower limit and upper limit of the rough aggregated internal strength $RN(y_i)$, respectively.

Thus, the agregated matrix of internal strengths Y is obtained

$$Y = \begin{bmatrix} RN(y_1) \\ RN(y_2) \\ \vdots \\ RN(y_n) \end{bmatrix}_{nx1} = \begin{bmatrix} [\underline{Lim}(y_1), \overline{Lim}(y_1)] \\ \underline{Lim}(y_2), \overline{Lim}(y_2) \\ \vdots \\ \underline{Lim}(y_n), \overline{Lim}(y_n) \end{bmatrix}_{nx1} \tag{17}$$

Step 4. Analysis ofexpert's response matrix for the factors. Assuming that there are *m* experts in the research and *n* observed factors (criteria), each expert should determine the degree to which criterion *i* affects criterion *j*. Comparative analysis of the i^{th} and j^{th} pairwise by *k* expert is denoted as x_{ij}^e, where: $i=1,...,n; j=1,...,n$. The value of each x_{ij}^k pair is an integer, where: 0 –No influence (NI); 1 –Low influence (LI); 2 –Medium influence (MI); 3 –High influence (HI); 4 –Very high influence (VHI). The judgment of *k* expert is presented as a non-negative matrix of $n\times n$ rank, and each element of the *k* matrix in equation $X^e=[x^k_{ij}]_{n\times n}$ denotes a non-negative number x^e_{ij}, where $1 \leq k \leq m$.

$$X^k = \begin{bmatrix} 0 & x_{12}^k & \cdots & x_{1n}^k \\ x_{21}^k & 0 & \cdots & x_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1}^k & x_{n2}^k & \cdots & 0 \end{bmatrix}_{nxn} ; \; 1 \leq i,j \leq n; \; 1 \leq k \leq m \tag{18}$$

where x_{ij}^e arepresent linguistic variable taken from the preliminary defined linguistic scale.

In accordance with this, $X^1, X^2, ..., X^m$ matrices are judgment matrices of each of *m* experts. The diagonal elements of the judgment matrix are all set to zero since the same factors do not influence each other.

Step 5. Determine the aggregated rough direct-relation matrix. Based on response matrices $X^k=[x^k_{ij}]_{n\times n}$ obtained from each *m* expert, we built the integrated rough direct relation matrix X^*.

$$X^* = \begin{bmatrix} x_{11}^1, x_{11}^2, \ldots, x_{11}^k & x_{12}^1; x_{12}^2; \ldots; x_{12}^k, & \ldots, & x_{1n}^1; x_{1n}^2, \ldots, x_{1n}^k \\ x_{21}^1, x_{21}^2, \ldots, x_{21}^k & x_{22}^1; x_{22}^2; \ldots; x_{22}^k, & \ldots, & x_{2n}^1; x_{2n}^2, \ldots, x_{2n}^k \\ \cdots & \cdots & \cdots & \cdots \\ x_{n1}^1, x_{n1}^2, \ldots, x_{n1}^k & x_{n2}^1; x_{n2}^2; \ldots; x_{n2}^k, & \ldots, & x_{nn}^1; x_{nn}^2, \ldots, x_{nn}^k \end{bmatrix} \tag{19}$$

where $x_{ij} = \{x_{ij}^1, x_{ij}^2, \ldots, x_{ij}^k\}$ denote the sequence used to describe the relative importance of criterion *i* in relation to criterion *j*. By applying equations (1) through (7), sequence x_{ij}^k is converted to rough number $RN(x_{ij}^k) = [\underline{Lim}(x_{ij}^k), \overline{Lim}(x_{ij}^k)]$, where $\underline{Lim}(x_{ij}^k)$ and $\overline{Lim}(x_{ij}^k)$ represent the lower limit and upper limit of rough number $RN(x_{ij}^k)$, respectively. Thus we obtain $X^1, X^2, \ldots, X^m$ rough matrices (where *m* denotes the number of experts).

By applying equation (20) the aggregated rough element $RN(x_{ij})$ of the aggregated rough direct-relation matrix is obtained

$$RN(x_{ij}) = [\underline{Lim}(x_{ij}), \overline{Lim}(x_{ij})] = \begin{cases} \underline{Lim}(x_{ij}) = \sum_{e=1}^{m} \underline{Lim}(x_i^e) \cdot w_i^e \\ \overline{Lim}(x_{ij}) = \sum_{e=1}^{m} \overline{Lim}(x_i^e) \cdot w_i^e \end{cases} \tag{20}$$

where $\underline{Lim}(x_{ij})$ and $\overline{Lim}(x_{ij})$ represent the lower limit and upper limit of the rough aggregated rough element $RN(x_{ij})$, respectively.

Finally, we get the agregated rough direct-relation matrix, $X = [RN(x_{ij})]_{n\times n} = [\underline{Lim}(x_{ij}), \overline{Lim}(x_{ij})]_{n\times n}$

$$X = \begin{bmatrix} 0 & RN(x_{12}) & \cdots & RN(x_{1n}) \\ RN(x_{21}) & 0 & \cdots & RN(x_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ RN(x_{n1}) & RN(x_{n2}) & \cdots & 0 \end{bmatrix} =$$

$$\begin{bmatrix} [0,0] & [\underline{Lim}(x_{12}), \overline{Lim}(x_{12})] & \cdots & [\underline{Lim}(x_{1n}), \overline{Lim}(x_{1n})] \\ [\underline{Lim}(x_{21}), \overline{Lim}(x_{21})] & [0,0] & \cdots & [\underline{Lim}(x_{2n}), \overline{Lim}(x_{2n})] \\ \vdots & \vdots & \ddots & \vdots \\ [\underline{Lim}(x_{n1}), \overline{Lim}(x_{n1})] & [\underline{Lim}(x_{n2}), \overline{Lim}(\widehat{x_{n2}})] & \cdots & [0,0] \end{bmatrix} \tag{21}$$

Step 6. Construct the group direct strength-relation matrix. The rough numbers representing the strength of factors, eq. (17), are inserted into the principal diagonal of the group direct-relation matrix (21). The group direct strength-relation matrix *D* is obtained as

$$D=\begin{bmatrix} 0 & RN(d_{12}) & \cdots & RN(d_{1n}) \\ RN(d_{21}) & 0 & \cdots & RN(d_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ RN(d_{n1}) & RN(d_{n2}) & \cdots & 0 \end{bmatrix} \tag{22}$$

where

$$RN(d_{ij})=\left[\underline{Lim}(d_{ij}),\overline{Lim}(d_{ij})\right]=$$

$$\begin{cases} \text{If } i=j \text{ then } RN(d_{ij})=RN(y_i)=\left[\sum_{e=1}^{m}\underline{Lim}(x_i^e)\cdot w_i^e,\sum_{e=1}^{m}\overline{Lim}(y_i^e)\cdot w_i^e\right] \\ \text{If } i\neq j \text{ then } RN(d_{ij})=RN(x_{ij})=\left[\sum_{e=1}^{m}\underline{Lim}(x_i^e)\cdot w_i^e,\sum_{e=1}^{m}\overline{Lim}(x_i^e)\cdot w_i^e\right] \end{cases} \tag{23}$$

Matrix *D* shows the first effects that particular factor causes, as well as the initial effects one gets from other factors. The sum of each *i*-th matrix row *D* represents total direct effects which factor i has caused to other factors, and the sum of each *j*-th column of matrix *D* represents total direct effects which factor *j* has received from other factors.

Step 7. Normalize the group direct strength-relation matrix. Based on matrix *Z*, a normalized initial direct-relation matrix $Z=\left[IRN(z_{ij})\right]_{n\times n}$ is obtained, equation (24). By normalization, each element in matrix *Z* is assigned a value between zero and one. The *Z* matrix is obtained when each element $RN(d_{ij})$ of matrix *D* is divided by number *s*, as shown in equations (25) and (26)

$$Z=\begin{bmatrix} RN(z_{11}) & RN(z_{12}) & \cdots & RN(z_{1n}) \\ RN(z_{21}) & RN(z_{22}) & \cdots & RN(z_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ RN(z_{n1}) & RN(z_{n2}) & \cdots & RN(z_{nn}) \end{bmatrix} \tag{24}$$

where $RN(z_{ij})$ is obtained by applying equation (25)

$$RN(z_{ij})=\frac{RN(d_{ij})}{s}=RN\left(\frac{\underline{Lim}(d_{ij})}{s},\frac{\overline{Lim}(d_{ij})}{s}\right) \tag{25}$$

where

$$\begin{aligned} s&=\max\left(\sum_{j=1}^{n}RN(d_{ij})\right)=\max\left(\sum_{j=1}^{n}\underline{Lim}(d_{ij}),\sum_{j=1}^{n}\overline{Lim}(d_{ij})\right) \\ &=\max\left[\max\left(\sum_{j=1}^{n}\underline{Lim}(d_{ij})\right),\max\left(\sum_{j=1}^{n}\overline{Lim}(d_{ij})\right)\right] \end{aligned} \tag{26}$$

Step 8. Determine the total strength-relation matrix. By applying equations (27) and (28), the total- strength-relation matrix $T=\left[RN(t_{ij})\right]_{n\times n}$ of rank *n×n* is calculated, where I denotes the identity matrix of the nxn rank. The element $RN(t_{ij})$ denotes a direct influence of factor *i* on factor *j*, while *T* matrix denotes total strength-relations among each pair of factors.

Since each rough number is composed of two sequences (upper and lower approximation), then the normalized matrix of average perception $Z=\left[RN(z_{ij})\right]_{n\times n}$ can be divided into two sub-matrices, i.e. $Z=\left[Z^L,Z^U\right]$, where $Z^L=\left[\underline{Lim}(z_{ij})\right]_{n\times n}$ and $Z^U=\left[\overline{Lim}(z_{ij})\right]_{n\times n}$. Moreover, $\lim_{m\to\infty}\left(Z^L\right)^m=O$ and $\lim_{m\to\infty}\left(Z^U\right)^m=O$, where O denotes a zero matrix.

$$\left.\begin{array}{l}\lim_{m\to\infty}\left(I+Z^L+Z^{2L}+\ldots+Z^{mL}\right)=\left(I-Z^L\right)^{-1}\\ \text{and}\\ \lim_{m\to\infty}\left(I+Z^U+Z^{2U}+\ldots+Z^{mU}\right)=\left(I-Z^U\right)^{-1}\end{array}\right\} \tag{27}$$

Therefore, the matrix of the total influences *T* will be obtained by calculating of the following elements

$$\left.\begin{array}{l}T^L=\lim_{m\to\infty}\left(I+Z^L+Z^{2L}+\ldots+Z^{mL}\right)=\left(I-Z^L\right)^{-1}=\left[\underline{Lim}(t_{ij}^L)\right]_{n\times n}\\ \text{and}\\ T^U=\lim_{m\to\infty}\left(I+Z^U+Z^{2U}+\ldots+Z^{mU}\right)=\left(I-Z^U\right)^{-1}=\left[\underline{Lim}(t_{ij}^U)\right]_{n\times n}\end{array}\right\} \tag{28}$$

where $Z^L=\left[\underline{Lim}(z_{ij})\right]_{n\times n}$ and $Z^U=\left[\overline{Lim}(z_{ij})\right]_{n\times n}$.

Sub-matrices T^L and T^U together represent the rough matrix of the total influences $T=\left(T^L,T^U\right)$. Based on equations (27) and (28), a total strength-relation matrix is defined:

$$T=\begin{bmatrix}RN(t_{11}) & RN(t_{12}) & \cdots & RN(t_{1n})\\ RN(t_{21}) & RN(t_{22}) & \cdots & RN(t_{2n})\\ \vdots & \vdots & \ddots & \vdots\\ RN(t_{n1}) & RN(t_{n2}) & \cdots & RN(t_{nn})\end{bmatrix} \tag{29}$$

where $RN(t_{ij})=\left[\underline{Lim}(t_{ij}),\overline{Lim}(t_{ij})\right]$ is the overall influence rating of the decision maker for each factor i on factor j, thus reflecting mutual dependence of each factor pair.

Step 9. Calculating the sum of rows and columns of total strength-relation matrix T. In matrix *T*, the sum of rows and sum of columns are denoted as vectors *R* and *C*, rank *n×1*:

$$RN(R_i)=\left[\sum_{j=1}^{n}RN(t_{ij})\right]_{n\times 1} \tag{30}$$

$$RN(C_i)=\left[\sum_{i=1}^{n}RN(t_{ij})\right]_{1\times n} \tag{31}$$

The value R_i denotes the sum of the *i*-th row of matrix *T* and shows the total direct and indirect effects that criterion *i* delivers to other factors. Similarly, the value C_i is the sum of the *j*-th column of matrix *T*, and represents the total direct and indirect effects that factor *j* receives from other factors. In cases where *i=j*, equation (R_i+C_i) indicates the impact of the factors and equation (R_i-C_i) indicates the intensity of the factors compared to others (Pamučar & Ćirović, 2015).

To effectively determine the "Prominence" and the "Relation", the sum of rows R_i to the sum of columns C_i in the total strength-relation matrix *T* need to be converted into the crisp forms R_i^{crisp} and C_i^{crisp} by applying equations (32)-(34)

$$RN(R_i)=\left[\underline{Lim}(R_i),\overline{Lim}(R_i)\right]=\begin{cases}\underline{Lim}(R_i)=\dfrac{\underline{Lim}(R_i)-\min_i\left\{\underline{Lim}(R_i)\right\}}{\max_i\left\{\overline{Lim}(R_i)\right\}-\min_i\left\{\underline{Lim}(R_i)\right\}}\\ \overline{Lim}(R_i)=\dfrac{\overline{Lim}(R_i)-\min_i\left\{\underline{Lim}(R_i)\right\}}{\max_i\left\{\overline{Lim}(R_i)\right\}-\min_i\left\{\underline{Lim}(R_i)\right\}}\end{cases} \tag{32}$$

where $\underline{Lim}(R_i)$ and $\overline{Lim}(R_i)$ represent the lower limit and upper limit of the rough number $RN(R_i)$, respectively; $\underline{Lim}(R_i)$ and $\overline{Lim}(R_i)$ are the normalized forms of $\underline{Lim}(R_i)$ and $\overline{Lim}(R_i)$.

After normalization we obtain a total normalized crisp value

$$\beta_i=\frac{\underline{Lim}(R_i)\cdot\left\{1-\underline{Lim}(R_i)\right\}+\overline{Lim}(R_i)\cdot\overline{Lim}(R_i)}{1-\underline{Lim}(R_i)+\overline{Lim}(R_i)} \tag{33}$$

Finally crisp form R_i^{crisp} for $RN(R_i)$ is obtained by applaying eq. (34)

$$R_i^{crisp}=\min_i\left\{\underline{Lim}(R_i)\right\}+\beta_i\cdot\left[\max_i\left\{\overline{Lim}(R_i)\right\}-\min_i\left\{\underline{Lim}(R_i)\right\}\right] \tag{34}$$

The final crisp form C_i^{crisp} for $RN(C_i)$ can be obtained similarly.

Step 10. Calculate "Prominence"/"Relation" and prioritize factors. The vector P_i named "Prominence" is made by adding R_i^{crisp} to C_i^{crisp}. The vector F_i named "Relation" is made by subtracting R_i^{crisp} to C_i^{crisp}.

$$P_i=R_i^{crisp}+C_i^{crisp} \tag{35}$$

$$F_i=R_i^{crisp}-C_i^{crisp} \tag{36}$$

The vector P_i combines the interrelations of both directions (the horizontally exerted and the vertically received influence) of the factor *i* and therefore is interpreted as an overall influence intensity of that factor. It reveals how much importance the factor has. The larger value of P_i the greater overall importance/influence of factor *i* in terms of overall relationships with other factors. All the factors can then be prioritized based on the P_i (Song et al., 2017).

The vector F_i shows the difference between the exerted and received influence, and it is a basis for classification of the factors. When the value F_i is positive, the factor *i* belongs to the cause group. The factor *i* is a net cause for other factors. If the value F_i is negative, the factor *i* belongs to the effect group.

Step 11. Determine the cause and effect relationships between factors. Based on the P_i and F_i the cause-and-effect diagram (CED) can be acquired by mapping the dataset of the (P_i, F_i). In the CED the prominence axis shows how important a criterion relative to the available set of factors, whereas the relation axis will divide the factors into cause and effect groups (Song et al., 2017). The construction of a CED visualizes the complex interrelationship and provides information in order to determine the most important factors and how they influence the affected factors. Factors with a value higher than threshold value α are selected and shown in the CED.

$$\alpha = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\left[RN(t_{ij})\right]}{N} \tag{37}$$

where *N* denotes the number of matrix elements (29).

The rough numbers in the matrix (29) should be converted into crisp numbers. Using eqs. (32)-(34) the crisp total strength-relation matrix $T^* = \left[t_{ij}^{crisp}\right]_{n\times n}$ can be obtained.

Elements of matrix T^* with values higher than the threshold α arese lected and shown in the diagram where the *x*–axis represents P_i, and the *y* – axis F_i and they are used to denote the relationship between two factors. When presenting the factor relationships, the arrow of the cause-and-effect relationship will be directed from the factor with the value lower than threshold value *α* to the element with the value higher than threshold value *α*.

4. An emperical case study

4.1. Case background

To validate the feasibility and efficiency of the proposed RSR-DEMATEL method for analyzing the key success factors (KSFs) of hospital service quality (HSQ), the method is applied to the case example from Shieh et al. (2010). Due to the increased competition, health care institutions/hospitals face huge challenge to attract and retain patients, fulfill their several needs, like high quality medical care in reasonable price, proper health insurance etc. Successful management of hospital service quality is the only way to meet those goals. In order to do that the hospital authority should identify key criteria and their importance by conducting a survey from the viewpoints of patients and or their families. For further analysis, the authority needs feedbacks from several departmental personnel (we call them decision makers (DMs)) in the hospital. It's quite evident that the DMs would possess different opinions about the importance of the key success factors. Some of them also may have conflicts with others regarding the interrelationships and the mutual influences between the key success factors, which shall help mitigating the priorities of these factors. Thus, in this case study, the proposed method is utilized for evaluating and analyzing the critical success factors a successful hospital as well as examining their interrelationships. Twenty one managerial personnel (DMs) having knowledge in networking with medical services from diverse occupations in the hospital are invited. More details on the DMs can be found in Shieh et al. (2010).

In the data collection phase, the key success factors are deducted from the operational process (asking patients'/their family's responses) in the hospital and literature review. The research team then organized a focused group discussion lasting an hour to understand and validate the key success factors identified from the literature. The decision panned finalize all the seven key success factors (see Table 1) which are significant for their work, and thus decide to provide the necessary inputs to be used in this research based on the seven key success factors in Table 1.

Table 1. The internal strength of factors evaluated by decision makers (experts) and expert weights

Experts	Factors							Expert weights
	C1	C2	C3	C4	C5	C6	C7	
DM1	HS	MS	MS	MS	MS	HS	HS	0.059
DM2	HS	MS	MS	HS	MS	VHS	HS	0.049
DM3	VHS	VHS	VHS	VHS	HS	MS	MS	0.053
DM4	VHS	HS	HS	MS	HS	VHS	VHS	0.044
DM5	VHS	VHS	VHS	VHS	VHS	VHS	VHS	0.048
DM6	MS	MS	MS	LS	MS	HS	LS	0.058
DM7	MS	MS	LS	LS	MS	VHS	HS	0.057
DM8	HS	MS	MS	LS	LS	VHS	HS	0.058
DM9	HS	LS	MS	HS	MS	MS	MS	0.052
DM10	MS	MS	LS	LS	LS	VHS	MS	0.058
DM11	HS	VHS	HS	HS	MS	VHS	MS	0.044
DM12	VHS	MS	MS	HS	HS	MS	HS	0.057
DM13	MS	MS	LS	MS	MS	MS	MS	0.054
DM14	VHS	VHS	VHS	VHS	HS	VHS	HS	0.048
DM15	VHS	VHS	VHS	VHS	VHS	VHS	VHS	0.048
DM16	HS	MS	MS	LS	MS	MS	MS	0.046
DM17	HS	LS	MS	HS	MS	MS	MS	0.058
DM18	MS	MS	LS	LS	LS	VHS	MS	0.051
DM19	HS	VHS	HS	HS	MS	VHS	MS	0.058

4.2. Implementation

Step 1 to 2. Internal strength of each KSFs of HSQ is assessed with verbal language. In this stage, the nineteen DMs are requested to appraise the internal strength of different KSFs of HSQ according to pre-defined linguistic scale: 0 –No strength (NS); 1 –Low strength (LS); 2 –Medium strength (MS); 3 –High strength (HS); 4 –Very high strength (VHS). All the internal strength of KSFs are provided in form of linguistic scales in Table 1. The evaluation set of the KSF1 (well-equipped medical equipment) can be denoted as Y_{KSF1} ={HS, HS, VHS, VHS, VHS, MS, MS, HS. HS, MS, HS, VHS, MS, HS, VHS, HS, HS, MS, HS }={3, 3, 4, 4, 4, 2, 2, 3, 3, 2, 3, 4, 2, 4, 4, 3, 3, 2, 3}. For manipulating the impreciseness, subjectivity and vagueness due to the decision makers' verbal information in the internal strength of KSF1, Y_{KSF1} is transformed into the rough interval number according to Eqs. (1)- (7) as follows:

$$\underline{\text{Lim}}(3) = \frac{2+2+2+2+2+3+3+3+3+3+3+3+3}{13} = 2.62,$$

$$\overline{\text{Lim}}(3) = \frac{3+3+3+3+3+3+3+3+4+4+4+4+4+4}{14} = 3.43.$$

Similarly, $\underline{\text{Lim}}(4) = 3.05$, $\overline{\text{Lim}}(4) = 4.00$, $\underline{\text{Lim}}(2) = 2.00$, $\overline{\text{Lim}}(2) = 3.05$

Thus, Y_{KSF1} can then be transformed into a set of rough intervals as

$\tilde{Y}_{KSF1}$ = {[2.62, 3.43], [2.62, 3.43], ..., [2.00, 3.05], [2.62, 3.43]}

The other internal strengths of KSFs can be obtained similarly in terms of rough interval numbers.

Step 3. Allowing for different background of the DMs, different weights from Table 1 are assigned to them for calculating the rough aggregated internal strength of factor i according to Eq. (16). The rough aggregated internal strength of KSFs are shown in Table 2.

Table 2. The rough internal strength of factors

Factor	Internal strength of factor
C1	[2.575, 3.494]
C2	[1.967, 3.159]
C3	[1.698, 3.023]
C4	[1.648, 2.875]
C5	[1.746, 2.771]
C6	[2.739, 3.746]
C7	[2.078, 3.067]

Step 4 to 5. Evaluate influence between KSFs to construct direct-relation matrix. The nineteen decision experts evaluate the direct impacts among the seven KSFs with the help of the vocal ratings: 0 –No influence (NI); 1 –Low influence (LI); 2 –Medium influence (MI); 3 –High influence (HI); 4 –Very high influence (VHI). Based on these ratings, the influence evaluations in Table 3 can be transformed into non-negative integers from 0 to 4. All the direct-relation matrices X_k *(k=1, 2, ..., 19)* of KSFs of HSQ could be obtained according to Eq. (18) and then the individual direct-relation matrices are then blended consecutively to generate a group direct-relation matrix (see Table 3).

Table 3. The verbal scores of direct-relations between factors

	C1	C2	C3	...	C7
C1	0;0;0;0;0;0;0;0...0;0	1;0;1;2;2;2;1;1...2;1	2;2;2;2;2;3;1;3...3;2		1;0;2;2;0;2;1;2...2;1
C2	2;2;0;2;2;2;1;2...0;2	0;0;0;0;0;0;0;0...0;0	2;1;1;2;3;2;3;3...3;2		1;1;1;2;3;2;2;3...1;1
C3	2;1;1;2;3;2;1;3...2;2	2;0;1;1;3;2;2;3...3;2	0;0;0;0;0;0;0;0...0;0		2;1;2;2;2;2;2;3...3;2
C4	1;1;2;2;2;0;1;2...1;1	1;3;2;2;3;2;2;3...2;1	2;1;1;2;3;2;3;3...2;2	...	1;1;1;2;2;2;1;2...2;1
C5	3;1;1;2;2;0;2;1...1;3	2;0;1;2;3;1;2;3...2;2	3;2;2;2;2;2;2;3...2;3		1;1;1;2;1;1;2;2...0;1
C6	3;2;2;2;2;0;1;2...2;3	2;0;1;1;3;1;2;3...2;2	3;2;2;2;3;2;2;3...1;3		1;0;1;2;3;2;2;2...1;1
C7	1;0;0;1;1;0;1;1...1;1	2;0;1;1;3;0;2;3...2;2	2;1;1;2;2;1;2;3...3;2		0;0;0;0;0;0;0;0...0;0

Rough numbers are utilized for manipulating the imprecision and subjectivity in data inputs from DMs. According to Eqs. (20)-(21), the aggregated rough direct-relation matrix ($\hat{X}$) (shown in Table 4) of different expert can be obtained.

Table 4. The group direct-relation matrix in the rough interval form

	C1	C2	C3	C4	...	C7
C1	[0.00, 0.00]	[1.13, 1.90]	[1.69, 2.54]	[1.35, 2.42]		[0.78, 1.98]
C2	[1.03, 2.11]	[0.00, 0.00]	[1.68, 2.45]	[2.02, 2.61]		[1.13, 1.92]
C3	[1.55, 2.35]	[1.51, 2.38]	[0.00, 0.00]	[1.73, 2.37]		[1.62, 2.28]
C4	[0.79, 1.92]	[1.75, 2.53]	[1.65, 2.45]	[0.00, 0.00]	...	[1.26, 1.89]
C5	[1.28, 2.50]	[1.51, 2.45]	[1.88, 2.54]	[1.33, 2.13]		[1.04, 1.80]
C6	[1.62, 2.55]	[1.42, 2.36]	[1.79, 2.55]	[1.50, 2.28]		[1.19, 2.06]
C7	[0.63, 1.63]	[1.15, 2.31]	[1.41, 2.30]	[1.09, 2.08]		[0.00, 0.00]

Step 6. The aggregated group direct strength-relation matrix (D) (shown in Table 5) is obtained using Eqs. (22)-(23). In this stage, the rough intervals denoting the strength of KSFs of HSQ (calculated in Step 3, Table 2) are implanted into the main diagonal of the aggregated group direct-relation matrix (attained in Step 5).

Table 5. The group direct strength-relation matrix

	C1	C2	C3	C4	...	C7
C1	[2.58, 3.49]	[1.13, 1.90]	[1.69, 2.54]	[1.35, 2.42]		[0.78, 1.98]
C2	[1.03, 2.11]	[1.97, 3.16]	[1.68, 2.45]	[2.02, 2.61]		[1.13, 1.92]
C3	[1.55, 2.35]	[1.51, 2.38]	[1.70, 3.02]	[1.73, 2.37]		[1.62, 2.28]
C4	[0.79, 1.92]	[1.75, 2.53]	[1.65, 2.45]	[1.65, 2.87]	...	[1.26, 1.89]
C5	[1.28, 2.50]	[1.51, 2.45]	[1.88, 2.54]	[1.33, 2.13]		[1.04, 1.80]
C6	[1.62, 2.55]	[1.42, 2.36]	[1.79, 2.55]	[1.50, 2.28]		[1.19, 2.06]
C7	[0.63, 1.63]	[1.15, 2.31]	[1.41, 2.30]	[1.09, 2.08]		[2.08, 3.07]

Step 7. To convert the interface scales of KSFs of HSQ into equivalent scales and confirm the existence of the total strength-relation matrix T, the group direct-relation matrix ($\hat{X}$) is normalized according to Eqs. (24)-(26). The normalized rough direct-relation matrix (Z) is shown in Table 6.

Table 6. Normalized the group direct strength-relation matrix

	C1	C2	C3	C4	...	C7
C1	[0.02, 0.03]	[0.01, 0.02]	[0.01, 0.02]	[0.01, 0.02]		[0.01, 0.02]
C2	[0.01, 0.02]	[0.02, 0.03]	[0.01, 0.02]	[0.02, 0.02]		[0.01, 0.02]
C3	[0.01, 0.02]	[0.01, 0.02]	[0.01, 0.03]	[0.01, 0.02]		[0.01, 0.02]
C4	[0.01, 0.02]	[0.01, 0.02]	[0.01, 0.02]	[0.01, 0.02]	...	[0.01, 0.02]
C5	[0.01, 0.02]	[0.01, 0.02]	[0.02, 0.02]	[0.01, 0.02]		[0.01, 0.01]
C6	[0.01, 0.02]	[0.01, 0.02]	[0.01, 0.02]	[0.01, 0.02]		[0.01, 0.02]
C7	[0.01, 0.01]	[0.01, 0.02]	[0.01, 0.02]	[0.01, 0.02]		[0.02, 0.03]

Step 8. The total strength-relation matrix T, shown in Table 7, can be computed by applying Eqs. (27)-(29). The components in Table 7 specify the overall influence grades of decision makers for the KSF(i) against the KSF(j) bearing in mind their internal strengths.

Table 7. The total strength-relation matrix

	C1	C2	C3	C4	...	C7
C1	[0.02, 0.03]	[0.01, 0.02]	[0.02, 0.02]	[0.01, 0.02]		[0.01, 0.02]
C2	[0.01, 0.02]	[0.02, 0.03]	[0.02, 0.02]	[0.02, 0.03]		[0.01, 0.02]
C3	[0.01, 0.02]	[0.01, 0.02]	[0.02, 0.03]	[0.02, 0.02]		[0.01, 0.02]
C4	[0.01, 0.02]	[0.02, 0.02]	[0.02, 0.02]	[0.02, 0.03]	...	[0.01, 0.02]
C5	[0.01, 0.02]	[0.01, 0.02]	[0.02, 0.02]	[0.01, 0.02]		[0.01, 0.02]
C6	[0.01, 0.02]	[0.01, 0.02]	[0.02, 0.02]	[0.01, 0.02]		[0.01, 0.02]
C7	[0.01, 0.02]	[0.01, 0.02]	[0.01, 0.02]	[0.01, 0.02]		[0.02, 0.03]

Step 9. The sum of rows ($RN(R_i)$ and columns ($RN(C_j)$) of rough total strength-relation matrix *T* are calculated using Eqs. (30)-(31) and presented in Table 8. In order to effectively rank the KSFs of HSQ and investigate the cause-effect relations between them, it is necessary to remove roughness from data according to Eqs. (32)-(34). The final crisp form (R_i^{crisp}) and (C_j^{crisp}), which are also provided in Table 8.

Table 8. The sum of rows, sum of columns, “Prominence” and “Relation”

Factor	RN(Ri)	RN(Ci)	Ricrisp	Cicrisp	Pi	Fi	Rank	Cause /effect
C1	[0.097, 0.169]	[0.087, 0.162]	0.135	0.122	0.257	0.014	6	Cause
C2	[0.105, 0.170]	[0.097, 0.166]	0.141	0.131	0.273	0.010	4	Cause
C3	[0.113, 0.175]	[0.109, 0.172]	0.151	0.144	0.294	0.007	2	Cause
C4	[0.101, 0.162]	[0.097, 0.163]	0.133	0.130	0.263	0.004	5	Cause
C5	[0.098, 0.163]	[0.109, 0.179]	0.132	0.149	0.281	-0.017	3	Effect
C6	[0.109, 0.176]	[0.122, 0.181]	0.149	0.159	0.308	-0.011	1	Effect
C7	[0.079, 0.150]	[0.081, 0.144]	0.109	0.106	0.215	0.003	7	Cause

Step 10. The “Prominence” (Pi) and the “Relation” (F_i) vectors are computed via Eqs. (35) and (36), respectively, and shown in Table 8. Now, depending on “Prominence” and “Relation” vectors, the impact-relation map of KSFs of HSQ can be accomplished by plotting the dataset of (P_i, F_i) in Fig. 1.

In this figure, the prominence axis tells about the relative importance of a KSF of HSQ compared to the other KSFs of HSQ under consideration, whereas the relation axis divides the KSFs of HSQ into cause and effect groups.

A KSF of HSQ will be given top most priority if it’s has the highest prominence value (visibility/importance/influence) in terms of overall relationships with other KSFs. From Table 8, we observe that C6 (medical staff with professional abilities) is the most important KSF of HSQ followed by C3 (trusted medical staff with professional competence of health care), C5 (detailed description of the patient’s condition by the medical doctor), C2 (service personnel with good communication skills), C4 (service personnel with immediate problem-solving abilities), C1 (well-equipped medical equipment) and C7 (pharmacist’s advices on taking medicine).

Based on the “Relation” vector from Table 8, all the KSFs of HSQ can be categorized into cause group and effect group, as shown in Fig. 1. Fig. 1 depicts that “Relations” of five KSFs are positive. They are C3 (trusted medical staff with professional competence of health care), C2 (service personnel with good communication skills), C4 (service personnel with immediate problem-solving abilities), C1 (well-equipped medical equipment) and C7 (pharmacist’s advices on taking medicine). These factors belong to the cause group and have net cause for other KSFs. The “Relations” of the rest of KSFs (C6, C5) are negative, and they belong to the effect group which are reliant on the change of cause KSFs of HSQ.

Step 11. Finally, it remains to explore the comprehensive interactions between KSFs of HSQ. To do so we need to plot a relationship digraph to recognize essential influencing relationships of KSFs depending upon the rough total strength-relation matrix (Table 7). The rough intervals in Table 7 are transformed to crisp numbers to form crisp total strength-relation matrix (Table 9) using Eqs. (32)-(34).

Table 9. The crisp total strength-relation matrix of factors

	C1	C2	C3	C4	C5	C6	C7
C1	0.030	0.013	0.020	0.017	0.022	0.018	0.011
C2	0.015	0.025	0.019	0.023	0.023	0.020	0.014
C3	0.019	0.019	0.023	0.020	0.024	0.024	0.018
C4	0.012	0.020	0.019	0.022	0.022	0.021	0.014
C5	0.018	0.018	0.021	0.016	0.022	0.021	0.013
C6	0.019	0.018	0.021	0.018	0.021	0.032	0.015
C7	0.009	0.015	0.016	0.014	0.014	0.014	0.025

The bold numbers indicates the relationships that exceed the threshold α=0.0189

A threshold value (α) of total strength relation can be computed according to Eq. (37) for drawing the interpretational diagraph to graphically describe the interrelationship maps between the KSFs of HSQ. Particular relationships that exceed the threshold 0.0189 (note the bold numbers in Table 9) are encompassed in the concluding interacting maps in Fig. 1.

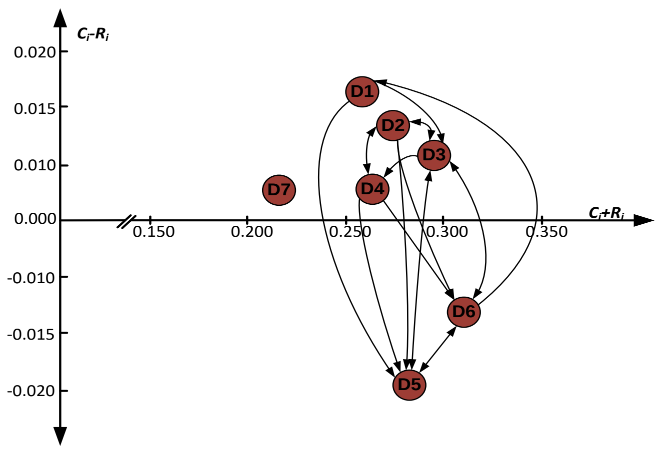

Figure 1. Cause and effect relationships between factors

4.3. Comparisons and discussion

To endorse the efficiency and powers of the rough strength relational DEMATEL model for analyzing KSFs of HSQ in this paper, a comparative exploration is accompanied to analyze the same problem. Traditional DEMATEL (Shieh et al. 2010) and fuzzy DEMATEL (Pamučar and Ćirović, 2015) are well-known in the literature. The rank priorities of the seven KSFs derived from these two methods are shown in Table 10 along with the ranking produced by rough DEMATEL model. Fig. 2 is a pictorial demonstration and relationship of the rank orders according to all those methods.

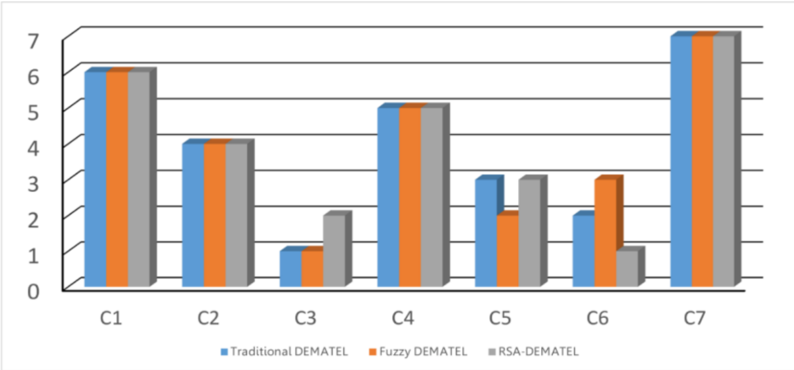

Figure 2. Comparison analysis of ranking of the KSFs using different methods

Firstly, the ranking results from the traditional DEMATEL and rough DEMATEL method are different except for C3, C5 and C6. Also, the major interrelations among the KSFs differ in traditional DEMATEL and the rough DEMATEL method. The impact of C2 on C4 (KSF2→KSF4) is reflected as one of the most precarious relations in the rough DEMATEL (Fig. 3c). But this is missing in case of the crisp DEMATEL model (Fig. 3a).

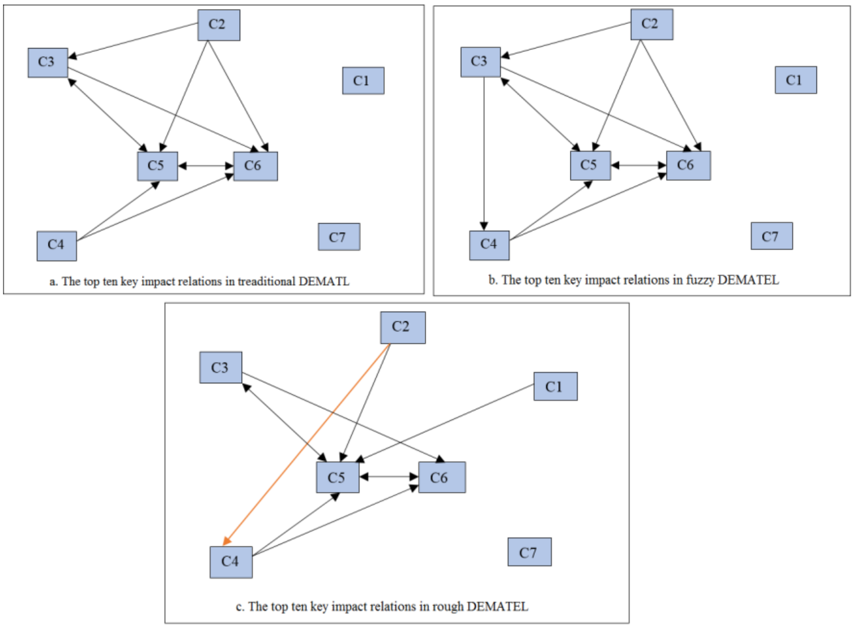

Figure 3. Comparison of the top ten impact relations in different methods

This is possibly due to rough DEMATEL ponders strength- impacts of the KSFs C2 and C4 ([1.967, 3.159] and [1.648, 2.875]) on the relation KSF2→KSF4. Note that traditional DEMATEL does not consider the strengths of C2 and C4 in examining the interactions between KSFs. The rough DEMATEL method is also able to manipulate uncertainty in the KSFs analysis based on decision makers' opinions. The key success factors input data from DMs is transformed into rough number that reflects the uncertainty in the decision making process due to the linguistic assessments of DMs. For example, nineteen DMs opined on the direct-relation between C1 (KSF1) and C5 (KSF5) as {HI, NI, LI, MI, HI, HI, LI, MI, HI, MI, HI, LI, HI, MI, MI, HI, HI, HI, HI}. Then the proposed method transforms such linguistic ratings into a sequence of rough interval numbers as {[2.26, 3], [0, 2.26], [0.75, 2.39], [1.30, 2.67], …, [2.26, 3], [2.26, 3]} which deliberates the vague information in decision making problem. The traditional DEMATEL only characterizes the linguistic assessment set, {HI, NI, LI, MI, HI, HI, LI, MI, HI, MI, HI, LI, HI, MI, MI, HI, HI, HI, HI} into crisp score set, {3, 0, 1, 2, 3, 3, 1, 2, 3, 2, 3, 1, 3, 2, 2, 3, 3, 3, 3}. Thus, to analyze the KSFs for maintaining HSQ, the rough DEMATEL can deliver more appreciated suggestion than the crisp DEMATEL method.

The second comparative analysis is accompanied with the outcome from the fuzzy DEMATEL method. The attained ranking grades according to fuzzy DEMATEL model are accessible from Table 10. There is notable similarity between the major interrelations among the KSFs produced by the fuzzy DEMATEL and the rough DEMATEL. All of them are exactly same except the relations C2→C3, C2→C4 and C3→C4. It is because both the rough and fuzzy approaches consider subjectivity and vagueness while making decisions. While fuzzy DEMATEL operates conventional symmetric triangular fuzzy numbers (STFNs), the rough DEMATEL makes use of rough numbers. The rough numbers can flexibly manipulate uncertainty to the highest extent when it is caused by subjective and vague information (Zhu et al., 2015). As before we consider the direct impact relation between C1 and C5 as crisp rating set {3, 0, 1, 2, 3, 3, 1, 2, 3, 2, 3, 1, 3, 2, 2, 3, 3, 3, 3}.

Table 10. Comparison analysis of the ranking results of factors

Factors	Crisp DEMATEL		Fuzzy DEMATEL		RSR-DEMATEL	
	Pi	Ranking	Pi	Ranking	Pi	Ranking
C1	16.30	6	4.97	6	0.26	6
C2	17.64	4	5.37	4	0.27	4
C3	18.93	1	5.71	1	0.29	2
C4	17.48	5	5.31	5	0.26	5
C5	18.52	3	5.61	2	0.28	3
C6	18.57	2	5.58	3	0.31	1
C7	14.80	7	4.65	7	0.22	7

The rough DEMATEL changes this decision set into {[2.26, 3], [0, 2.26], [0.75, 2.39], [1.30, 2.67], …, [2.26, 3], [2.26, 3]} and combines these intervals into [1.69, 2.77]. On the other hand, the fuzzy DEMATEL adapts the decision set as {[2, 4], [0, 1], [0, 2], [1, 3], …, [2, 4], [2, 4]} and combines the these STFNs into [1.32, 3.32] with fixed interval of 2. Now, on changing the original direct impact ratings between C1 and C5 (KSF1→KSF5) are changed to {2, 2, 1, 2, 2, 2, 2, 1, 3, 3, 2, 3, 1, 3, 1, 2, 2, 3, 1, 2}, then rough aggregation will produce the combined rough rating as [1.57, 2.57]. On contrary the fuzzy aggregation yields the combined STFN rating as [1.00, 2.93] with fixed interval of 2 which does not replicate the changes in DMs' judgments. This is due to the predetermined fuzzy membership function in fuzzy DEMATEL method (Song et al. 2017). Hence, the rough DEMATEL possesses higher flexibility and more rational than the fuzzy DEMATEL.

5. Major Implications

The result analysis of KSFs of HSQ reflects important comprehensions in the theoretical and practical perspective, hence contributes to the hospital service quality management. Depending on these outcomes, hospital management can take unambiguous actions to measure, regulate and alleviate the acknowledged KSFs of HSQ. This paper, theoretically, advances a framework that helps in identifying the key success factors of HSQ as well as the strength and impact between them. This study fills the gap of identifying KSFs of HSQ management and their strength impact inter-relationships under subjectivity and vagueness.

In real world problems, many DMs/administrators focus little on the inter-relationships of KSFs of HSQ. The proposed model for analyzing KSFs in HSQ management may help to apprehend the structure of interacting relations among these factors. A hospital can develop truly proactive services with such management and decision-making tool which delivers provision in scheduling the path of managing HSQ by regulating the influences of KSFs on each other. The rough DEMATEL method also describes the inter-dependencies among the KSFs systematically, since it deliberates the strength effect of KSFs on their inter-dependencies, which is discussed by no previous researcher.

The rough DEMATEL model aids the managers of HSQ to ensure the customers'/patients' needs are fulfilled and the hospital performs well even though stakes are high due to today's competitive market. This paper offers a practical impact in the literature hospital service quality management. The proposed model also facilitates the consciousness of KSFs in HSQ management. The decision panel includes essentially executives from several departments to establish a comprehensive deliberation of KSFs and direct impact relations in detailed analysis and prioritization of them. Numerous useful suggestions can also be deliberated as follows.

First, the most important KSF is "medical staff with professional abilities" (C6) in HSQ management, i.e., engaging more skilled doctors and nurses will help the achieving primary goals, like, attracting and retaining the patients. Failure to select skilled doctors and nurses can affect handling "well-equipped medical equipment" (C1) and "detailed description of the patient's condition by the medical staff" (C5), because trained medical staff with professional abilities plays a vital role (KSF) for a hospital to be successful. Hence, the hospital authority should pay more attention in recruiting accomplished medical professionals to serve better medical treatments in order to govern its KSFs and improve overall performance of hospital services.

Second, "well-equipped medical equipment" (C1), "trusted medical staff with professional competence of health care" (C3), "service personnel with good communication skills" (C2), "service personnel with immediate problem-solving abilities" (C4), and "pharmacist's advices on taking medicine" (C7) are among influential KSFs since it they belong to the cause group. Improvement of well-equipped medical apparatus would reflect impacts in C5 and C3, even though "well-equipped medical equipment" (C1) is ranked sixth in the final list. On contrary, "medical staff with professional abilities" (C6) is the most essential criterion and conjointly have mutual impacts with other two top KSFs-C5 and C3. This means that the managers of HSQ should focus on the interaction between medical staff and patients as this is far more important. A better interaction will help to grow higher satisfaction in the patients (Shieh et al. 2010). This is how a hospital can retain its customers who are satisfied with their care.

Finally, if the hospital wants to accomplish high performance in hospital services, it should control the "cause KSFs" (C1, C2, C3, C4 and C7 beforehand if it is willing to take care of the “effect KSFs” (C5, C6). If the hospital authority thinks to control the KSFs of “detailed description of the patient’s condition by the medical staff” (C5) and medical staff with professional abilities” (C6), it will be essential to pay more attention to the KSFs of C2, C3 and C4. *This is because the* “medical staff with professional abilities” (C6) and “detailed description of the patient’s condition by the medical staff” (C5) are the influenced KSF and can be improved, while the “trusted medical staff with professional competence of health care” (C3) and “service personnel with good communication skills” (C2) are the influencing KSFs and can dispatch influences. Hospital managing board must be aware of such relationships to control and diminish the risk KSFs in HSQ management.

6. Conclusions

To categorize the KSFs of hospital service quality, a systematic research framework grounded on rough numbers and the DEMATEL technique are proposed in this study. The theoritical and real-world importance of this paper can be listed below:

The proposed model can concurrently analyse the internal strength and external impacts of KSFs in HSQ management. This speciality serves better information for imposing key/critical decision and provides more accurate ranking orders in KSFs. The rogh DEMATEL model is also very effective in manipulating the vagueness and subjectivity in data since the rough numbers intervals flexiblely specifies the uncertain information in experts’ knowledge based decisions. Unlike the fuzzy approaches, the rough DEMATEL needs no auxiliary data (e.g., robust fuzzy membership value, data distribution) in real-world decision problems, which keeps simplier for managers to adopt it in practice. The proposed model helps practitioners to apprehend the inter-relationships among KSFs of HSQ to produce valuable perceptions and actionable trials. It can also help hospital management to concentrate on the major evolving issues of KSFs which might boost the overall performance of the hospital.

Although the proposed model serves good in both theoretical and practical perspectives, it has still some margins. The KSF internal strength analyses are totally based on final verdicts of decision experts (DMs), which can make the decision making process more difficult. It is one of the limits of our proposed model. So, for future works, we will consider probability theory to implement the internal strengths of KSFs in HSQ management and to measure their impacts on hospital’s performances more accurately. Finaaly, the different direct-impacts between the KSFs may be distinguished into positive impacts and negative impacts.

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