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GEOMORPHOLOGY

The Modified Quasi-geostrophic Barotropic Models Based on Unsteady Topography

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ABSTRACT

New models using scale analysis and perturbation methods were derivated starting from the shallow water equations based on barotropic fluids. In the paper, to discuss the irregular topography with different magnitudes, especially considering the condition of the vast terrain, some modified quasi-geostrophic barotropic models were obtained. The unsteady terrain is more suitable to describe the motion of the fluid state of the earth because of the change of global climate and environment, so the modified models are more rational potential vorticity equations. If we do not consider the influence of topography and other factors, the models degenerate to the general quasi-geostrophic barotropic equations in the previous studies.

Keywords: Quasi-geostrophic; Potential vorticity; Scale analysis; Topography.

Modelos Semigeostróficos Barotrópicos Modificados con Base en Topografía Inestable

RESUMEN

Este trabajo deduce nuevos modelos con el uso de los métodos de análisis a escala y de perturbación a partir de las ecuaciones de aguas poco profundas con base en fluidos barotrópicos. En este artículo se obtuvieron algunos modelos semigeostróficos barotrópicos para aplicar en zonas de topografía inestable con diferentes magnitudes y considerar especialmente la condición del extenso terreno. La topografía inestable es más propicia para describir el movimiento del estado fluido de la tierra debido al cambio del clima y ambiente, por lo tanto los modelos modificados son ecuaciones de vorticidad potenciales más razonables. Si no se considera la influencia de la topografía y otros factores, los modelos se reducirían a las ecuaciones generales semigeostróficas barotrópicas de estudios anteriores.

Palabras clave: Semigeostrofía; vorticidad potencial; análisis a escala; topografía.

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1. Introduction

In recent decades, many scholars have conducted extensive research on large-scale atmospheric and oceanic dynamics. Among them, the topographic effect has great influences on the dynamic mechanism of potential vorticity equations (Pedlosky, 1974; Collings, 1980; Pedlosky, 1980; Treguier, 1989). In the atmosphere, the topographic height plays an importance role in atmospheric cyclone and anticyclone changes. Luo (1990) pointed out the topographic effect was also an important factor of forming atmospheric blocking, and the global atmospheric circulation would even get affected by the topographic effect. Lu (1987) discussed the effects of topographic height and shaped on Rossby wave activation and the influences of topographic south-north and east-west slopes on waveform and energy propagation. Chen (1998) and Jiang (2000) derived the quasi-geostrophic potential vorticity equation with large-scale topography, friction, and heating under the barotropic model, and the large-scale effects of Qinghai-Tibet Plateau on atmosphere were discussed. In addition, the oceanic topography is very complex, such as The North West Shelf of Australia (Holloway, 1997), Portugal Shelf Sea Area (Sherwin, 2002), etc. The relationship between topography and ocean circulation was pointed out in the literature (Roslee et al., 2017b; Kamsani, 2017; La, 1990; Marshall, 1995; Alvarez, 1994; Sou, 1996). Cessi (1986) discussed the important role of topography in ocean circulation. Holloway (1992) introduced the interaction of eddies with seafloor topography and argued that ocean circulations would be significant interaction between turbulent vortices and topography rather than gravity wave drag. Then, general expressions for the eddy-topographic force, eddy viscosity, and stochastic backscatter, as well as a residual Jacobian term, are derived for barotropic flow over mean topography by Frederiksen (1999). All the above researches, Actually the topography height also changes with time in the earth fluid. Changes in topography can lead to tsunamis, floods and natural disasters(Abdullah, 2017; Elfithri, 2017; Rahim, 2017). Yang (2011,2012) and Song (2012, 2013) discussed topography changes over time discussed topography changes over time *t*, the influence of the nonlinear long wave amplitude and waveform. Da (2013) discussed the shallow water equation forms when underlying surface slowly changes with time and obtained the vorticity equation with an underlying surface. This model considered the actual circumstances that the topography changes with time-space (Erfen et al., 2017; Roslee, 2017a) . In this paper, we discuss different magnitude topography under spatial-temporal variable and obtain some modified models, which have important effects on the future discussion about the waveform changes of nonlinear long waves.

This paper is organized as follows: In Section 2, starting from the rotating shallow water equation set, we give the bottom topography which is not smooth boundary and simplify the equation set with unsteady topography. Section 3 is given scale analysis and perturbation methods; then we obtain new equation set by topographic conditions with different magnitudes. Later we derive the equations with different orders and get some modified models in Section 4. Finally, make the relevant conclusions in Section 5.

2. Basic equations

Assuming the static equilibrium condition, the fluid can be regarded as barotropic, incompressible, frictionless state. The upper boundary height is $h(x,y,t)$, and $w_h = w|_{z=h} = \frac{dh}{dt} = \frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y}$, free surface pressure intensity $p_h = p|_{z=h} = p_0$ is constant. The basic equation set can be written as (Pedlosky, 1987)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{1a}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} \tag{1b}$$

$$-g - \frac{1}{\rho}\frac{\partial p}{\partial z} = 0 \tag{1c}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1d}$$

especially, the bottom topography is about spatial-temporal variable $h_B = h_B(x,y,t)$,

the coriolis parameter is

$$f(y) = 2\Omega[\sin\varphi_0 + \frac{\cos\varphi_0}{a}y - \frac{1}{2}\sin\varphi_0(\frac{y}{a})^2 + \cdots] \tag{2}$$

first of all, we integrate Eq. (1c)

$$\int_z^h \frac{\partial p}{\partial z}dz = -\int_z^h g\rho dz \tag{3}$$

and get

$$p = p_0 + \rho g(h - z) \tag{4}$$

so

$$\frac{\partial p}{\partial x} = \rho g\frac{\partial h}{\partial x} \tag{5}$$

$$\frac{\partial p}{\partial y} = \rho g\frac{\partial h}{\partial y} \tag{6}$$

Eqs. (5) and (6) indicate that the horizontal pressure gradient under the barotropic model can be expressed by the gradient of the free surface gravitational potential (Taharin & Roslee, 2017).

We assume that the preliminary horizontal velocity is independent of the z , Eqs. (1a) and (1b) can be rewritten as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -g\frac{\partial h}{\partial x} \tag{7a}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu = -g\frac{\partial h}{\partial y} \tag{7b}$$

then integrate continuity Eq. (1d) from $z = h_B(x,y,t)$ to the free surface $z = h(x,y,t)$, we can get

$$\frac{\partial(h-h_B)}{\partial t} + u\frac{\partial(h-h_B)}{\partial x} + v\frac{\partial(h-h_B)}{\partial y} + (h-h_B)(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = 0 \tag{7c}$$

assuming that the free surface height H is constant when the fluid is static (Pedlosky, 1987).

$$h(x,y,t) = H + \hat{h}, \quad \hat{h} << H \tag{8}$$

the barotropic equation set with unsteady topography can be written as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -g\frac{\partial(H+\hat{h})}{\partial x} \tag{9a}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-fv=-g\frac{\partial(H+\hat{h})}{\partial x} \quad (9b)$$

$$\frac{\partial(H+\hat{h}-h_B)}{\partial t}+u\frac{\partial(H+h-h_B)}{\partial x}+v\frac{\partial(H+h-h_B)}{\partial y}+(H+\hat{h}-h_B)(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}) \quad (9c)$$

If

$$\phi=gh \quad \phi_0=gH=c_0^2 \quad \hat{\phi}=g\hat{h} \quad \phi_B=gh_B \quad (10)$$

the equation set can be written as

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-fv=-\frac{\partial\hat{\phi}}{\partial x} \quad (11a)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+fu=-\frac{\partial\hat{\phi}}{\partial y} \quad (11b)$$

$$\frac{\partial(\hat{\phi}-\phi_B)}{\partial t}+u\frac{\partial(\phi-\phi_B)}{\partial x}+v\frac{\partial(\phi-\phi_B)}{\partial y}+(c_0^2+\hat{\phi}-\phi_B)(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y})=0 \quad (11c)$$

3. Scale analysis, perturbation methods

Making dimensionless analysis on the Eqs. (11a), (11b) and (11c), for the large-scale motions

$$(x,y)=L(x_1,y_1) \quad t=(\frac{L}{U})t_1 \quad (u,v)=U(u_1,v_1) \quad \hat{\phi}=f_0UL\phi_1$$

$$f=f_0f_1 \qquad \phi_B=\lambda f_0UL\phi_{B1} \quad (12)$$

where λ is an undetermined parameter.
Substituting Eq. (12) into Eqs. (11a), (11b) and (11c), then

$$\frac{U^2}{L}(\frac{\partial u_1}{\partial t_1}+u_1\frac{\partial u_1}{\partial x_1}+v_1\frac{\partial u_1}{\partial y_1})-f_0U(f_1v_1)=f_0U(-\frac{\partial\hat{\phi}_1}{\partial x_1}) \quad (13a)$$

$$\frac{U^2}{L}(\frac{\partial v_1}{\partial t_1}+u_1\frac{\partial v_1}{\partial x_1}+v_1\frac{\partial v_1}{\partial y_1})+f_0U(f_1u_1)=f_0U(-\frac{\partial\hat{\phi}_1}{\partial y_1}) \quad (13b)$$

$$f_0U^2\frac{d(\hat{\phi}_1-\lambda\phi_{B1})}{dt_1}+[c_0^2+f_0UL(\hat{\phi}_1-\lambda\phi_{B1})]\frac{U}{L}(\frac{\partial u_1}{\partial x_1}+\frac{\partial v_1}{\partial y_1})=0 \quad (13c)$$

Eqs. (13a) and (13b) divided by f_0U, (13c) divided by $\frac{c_0^2U}{L}=\frac{gHU}{L}$, we get

$$Ro\frac{du_1}{dt_1}-f_1v_1=-\frac{\partial\hat{\phi}_1}{\partial x_1} \quad (14a)$$

$$Ro\frac{dv_1}{dt_1}+f_1u_1=-\frac{\partial\hat{\phi}_1}{\partial y_1} \quad (14b)$$

$$Ro\mu_0^2\frac{d(\hat{\phi}_1-\lambda\phi_{B1})}{dt_1}+[1+Ro\mu_0^2(\hat{\phi}_1-\lambda\phi_{B1})](\frac{\partial u_1}{\partial x_1}+\frac{\partial v_1}{\partial y_1})=0 \quad (14c)$$

where

$$Ro=\frac{U}{f_0L},\ \mu_0^2=(\frac{f_0L}{c_0})^2,\ \frac{d}{dt_1}=\frac{\partial}{\partial t_1}+u_1\frac{\partial}{\partial x_1}+v_1\frac{\partial}{\partial y_1} \quad (15)$$

Let

$$u_1=u_1^{(0)}+Rou_1^{(1)}+\cdots$$
$$v_1=v_1^{(0)}+Rov_1^{(1)}+\cdots \quad (16)$$
$$\hat{f}_1=f_1^{(0)}+Rof_1^{(1)}+\cdots$$

meantime $f=f_0f_1=f_0+\beta_0y$, $\beta_1=\frac{L^2}{U}\beta_0$, so

$$f_1=1+Ro\beta_1y_1 \quad (17)$$

then we get

$$Ro\frac{d}{dt_1}(u_1^{(0)}+Rou_1^{(1)}+\cdots)-(1+Ro\beta_1y_1)(v_1^{(0)}+Rov_1^{(1)}+\cdots)$$
$$=-\frac{\partial}{\partial x_1}(\hat{\phi}_1^{(0)}+Ro\phi_1^{(1)}+\cdots) \quad (18a)$$

$$Ro\frac{d}{dt_1}(v_1^{(0)}+Rov_1^{(1)}+\cdots)+(1+Ro\beta_1y_1)(u_1^{(0)}+Rou_1^{(1)}+\cdots)$$
$$=-\frac{\partial}{\partial y_1}(\hat{\phi}_1^{(0)}+Ro\hat{\phi}_1^{(1)}+\cdots) \quad (18b)$$

$$Ro\mu_0^2\frac{d(\hat{\phi}_1-\lambda\phi_{B1})}{dt_1}+[\frac{\partial(u_1^{(0)}+Rou_1^{(1)}+\cdots)}{\partial x_1}+\frac{\partial(v_1^{(0)}+Rov_1^{(1)}+\cdots)}{\partial y_1}]$$

$$+Ro\mu_0^2(\hat{\phi}_1-\lambda\phi_{B1})(\frac{\partial(u_1^{(0)}+Rou_1^{(1)}+\cdots)}{\partial x_1}+\frac{\partial(v_1^{(0)}+Rov_1^{(1)}+\cdots)}{\partial y_1})=0 \quad (18c)$$

where

$$\frac{d}{dt_1}=\frac{\partial}{\partial t_1}+(u_1^{(0)}+Rou_1^{(1)}+\cdots)\frac{\partial}{\partial x_1}+(v_1^{(0)}+Rov_1^{(1)}+\cdots)\frac{\partial}{\partial y_1} \quad (19)$$

4. Derive the barotropic models

Making classified discussion on the magnitude of λ.

4.1. λ ~ 1 magnitude

The scale of the ϕ_B is consistent with the small amplitude function ϕ, most of the topography parameters following with this situation in the real world.
Ro^0 order approximation

$$u_1^{(0)}=-\frac{\partial\hat{\phi}_1^{(0)}}{\partial y_1},\ v_1^{(0)}=\frac{\partial\hat{\phi}_1^{(0)}}{\partial x_1},\ \frac{\partial v_1^{(0)}}{\partial y_1}+\frac{\partial u_1^{(0)}}{\partial x_1} \quad (20)$$

or a dimensional form

$$u_1^{(0)}=-\frac{\partial\hat{\phi}_1^{(0)}}{\partial y_1},\ v_1^{(0)}=\frac{\partial\hat{\phi}_1^{(0)}}{\partial x_1},\ \frac{\partial v_1^{(0)}}{\partial y_1}+\frac{\partial u_1^{(0)}}{\partial x_1} \quad (21)$$

Ro^1 order approximation

$$(\frac{\partial}{\partial t_1}+u_1^{(0)}\frac{\partial}{\partial x_1}+v_1^{(0)}\frac{\partial}{\partial y_1})u_1^{(0)}-\beta_1 y_1 v_1^{(0)}-v_1^{(1)}=-\frac{\partial\hat{\phi}_1^{(1)}}{\partial x_1} \quad (22a)$$

$$(\frac{\partial}{\partial t_1}+u_1^{(0)}\frac{\partial}{\partial x_1}+v_1^{(0)}\frac{\partial}{\partial y_1})v_1^{(0)}+\beta_1 y_1 u_1^{(0)}+u_1^{(1)}=-\frac{\partial\hat{\phi}_1^{(1)}}{\partial y_1} \quad (22b)$$

$$\mu_0^2(\frac{\partial}{\partial t_1}+u_1^{(0)}\frac{\partial}{\partial x_1}+v_1^{(0)}\frac{\partial}{\partial y_1})(\hat{\phi}_1^{(0)}-\lambda\phi_{B1})+(\frac{\partial u_1^{(1)}}{\partial x_1}+\frac{\partial v_1^{(1)}}{\partial y_1})=0 \quad (22c)$$

after calculation, the dimensionless vorticity equation is derived

$$\frac{f_0 L}{U}(\frac{\partial}{\partial t}+u^{(0)}\frac{\partial}{\partial x}+v^{(0)}\frac{\partial}{\partial y})\zeta^{(0)}+\beta_0 v^{(0)}=-f_0(\frac{\partial u^{(1)}}{\partial x}+\frac{\partial v^{(1)}}{\partial y}) \quad (23a)$$

$$\frac{f_0 L}{U}(\frac{\partial}{\partial t}+u^{(0)}\frac{\partial}{\partial x}+v^{(0)}\frac{\partial}{\partial y})(\phi^{(0)}-\phi_B)+c_0^2(\frac{\partial u^{(1)}}{\partial x}+\frac{\partial v^{(1)}}{\partial y})=0 \quad (23b)$$

eliminating $\frac{\partial u^{(1)}}{\partial x}+\frac{\partial v^{(1)}}{\partial y}$ from the Eqs. (23a), (23b), we get

$$(\frac{\partial}{\partial t}+u^{(0)}\frac{\partial}{\partial x}+v^{(0)}\frac{\partial}{\partial y})(f+\zeta^{(0)})=\frac{f_0^2}{c_0^2}(\frac{\partial}{\partial t}+u^{(0)}\frac{\partial}{\partial x}+v^{(0)}\frac{\partial}{\partial y})(\psi-\psi_B) \quad (24)$$

namely

$$(\nabla^2-\frac{1}{\lambda_R^2})\frac{\partial\psi}{\partial t}+\frac{f_0}{H}\frac{\partial h_B(x,y,t)}{\partial t}+J(\psi,\beta_0 y+\nabla^2\psi-\frac{1}{\lambda_R^2}\psi+\frac{f_0}{H}h_B(x,y,t))]=0 \quad (25)$$

where

$$u^{(0)}=-\frac{\partial\psi}{\partial y},\ v^{(0)}=\frac{\partial\psi}{\partial x},\ \zeta^{(0)}=\nabla^2\psi,\ \psi=\frac{\hat{\phi}^{(0)}}{f_0},\ \psi_B=\frac{gh_B}{f_0},\lambda_R=\frac{\sqrt{gH}}{f_0} \quad (26)$$

λ_R is the Rossby radius of deformation, Eq. (25) is a modified model.

4.2. λ ~ 10 magnitude

Large-scale atmospheric motion of large topography is suitable for such conditions (L ~ $10^6 m$, U ~ $10 m/s$, f_0 ~ $10^{-4} s^{-1}$). For example, a case study of Tibetan Plateau topography, the height is 3 4 ×$10^3 m$ approximately which is suit for the situation.

Ro^0 order approximation

$$u_1^{(0)}=-\frac{\partial\hat{\phi}_1^{(0)}}{\partial y_1},\ v_1^{(0)}=\frac{\partial\hat{\phi}_1^{(0)}}{\partial x_1},\ \frac{\partial v_1^{(0)}}{\partial y_1}+\frac{\partial u_1^{(0)}}{\partial x_1}=0 \quad (27)$$

or a dimensional form

$$f_0 u^{(0)}=-\frac{\partial\hat{\phi}^{(0)}}{\partial y},\ f_0 v^{(0)}=\frac{\partial\hat{\phi}^{(0)}}{\partial x},\ \frac{\partial v^{(0)}}{\partial y}+\frac{\partial u^{(0)}}{\partial x}=0 \quad (28)$$

in order to get the results, we should consider the magnitude of μ_0^2 $\mu_0^2=\frac{(f_0L)^2}{gH}\sim10^{-1}$, according to the Eq. (18c), μ_0^2 can be considered as a parameter.

Ro^1 order approximation

$$(\frac{\partial}{\partial t_1}+u_1^{(0)}\frac{\partial}{\partial x_1}+v_1^{(0)}\frac{\partial}{\partial y_1})u_1^{(0)}-\beta_1 y_1 v_1^{(0)}-v_1^{(1)}=-\frac{\partial\hat{\phi}_1^{(1)}}{\partial x_1} \quad (29a)$$

$$(\frac{\partial}{\partial t_1}+u_1^{(0)}\frac{\partial}{\partial x_1}+v_1^{(0)}\frac{\partial}{\partial y_1})v_1^{(0)}+\beta_1 y_1 u_1^{(0)}+u_1^{(1)}=-\frac{\partial\hat{\phi}_1^{(1)}}{\partial y_1} \quad (29b)$$

$$\mu_0^2(\frac{\partial}{\partial t_1}+u_1^{(0)}\frac{\partial}{\partial x_1}+v_1^{(0)}\frac{\partial}{\partial y_1})(\hat{\phi}_1^{(0)}-\lambda\phi_{B1})$$
$$+(1-\mu_0^2 Ro\lambda\phi_{B1})(\frac{\partial u_1^{(1)}}{\partial x_1}+\frac{\partial v_1^{(1)}}{\partial y_1})=0 \quad (29c)$$

or dimensional form

$$\frac{f_0 L}{U}[(\frac{\partial}{\partial t}+u^{(0)}\frac{\partial}{\partial x}+v^{(0)}\frac{\partial}{\partial y})u^{(0)}-\beta_0 y v^{(0)}]-f_0 v^{(1)}=-\frac{\partial\hat{\phi}^{(1)}}{\partial x} \quad (30a)$$

$$\frac{f_0 L}{U}[(\frac{\partial}{\partial t}+u^{(0)}\frac{\partial}{\partial x}+v^{(0)}\frac{\partial}{\partial y})v^{(0)}+\beta_0 y u^{(0)}]+f_0 u^{(1)}=-\frac{\partial\hat{\phi}^{(1)}}{\partial y} \quad (30b)$$

$$\frac{f_0 L}{U}(\frac{\partial}{\partial t}+u^{(0)}\frac{\partial}{\partial x}+v^{(0)}\frac{\partial}{\partial y})(\hat{\phi}^{(0)}-\phi_B)+(c_0^2-\phi_B)(\frac{\partial u^{(1)}}{\partial x}+\frac{\partial v^{(1)}}{\partial y})=0 \quad (30c)$$

Eqs. (30a), (30b) and (30c) can be transformed into the vorticity equations

$$\frac{f_0 L}{U}[(\frac{\partial}{\partial t}+u^{(0)}\frac{\partial}{\partial x}+v^{(0)}\frac{\partial}{\partial y})\zeta^{(0)}+\beta_0 v^{(0)}]=-f_0(\frac{\partial u^{(1)}}{\partial x}+\frac{\partial v^{(1)}}{\partial y}) \quad (31a)$$

$$\frac{f_0 L}{U}(\frac{\partial}{\partial t}+u^{(0)}\frac{\partial}{\partial x}+v^{(0)}\frac{\partial}{\partial y})(\hat{\phi}^{(0)}-\phi_B)+(c_0^2-\phi_B)(\frac{\partial u^{(1)}}{\partial x}+\frac{\partial v^{(1)}}{\partial y})=0 \quad (31b)$$

Eqs. (31a), (31b) are quasi-geostrophic barotropic models, so

$$\frac{f_0 L}{U}[(\frac{\partial}{\partial t}+u^{(0)}\frac{\partial}{\partial x}+v^{(0)}\frac{\partial}{\partial y})(f+\zeta^{(0)})]=-f_0(\frac{\partial u^{(1)}}{\partial x}+\frac{\partial v^{(1)}}{\partial y}) \quad (32a)$$

$$\frac{f_0 L}{U}(\frac{\partial}{\partial t}+u^{(0)}\frac{\partial}{\partial x}+v^{(0)}\frac{\partial}{\partial y})[f_0(\psi-\psi_B)]+(c_0^2-f_0\psi_B)(\frac{\partial u^{(1)}}{\partial x}+\frac{\partial v^{(1)}}{\partial y})=0 \quad (32b)$$

where

$$u^{(0)}=-\frac{\partial\psi}{\partial y},v^{(0)}=\frac{\partial\psi}{\partial x},\zeta^{(0)}=\nabla^2\psi,\psi=\frac{\hat{\phi}^{(0)}}{f_0},\psi_B=\frac{gh_B}{f_0} \quad (33)$$

assuming $c_0^2 - f_0\psi_B \neq 0$, we eliminate $\frac{\partial u^{(1)}}{\partial x} + \frac{\partial v^{(1)}}{\partial y}$ from Eqs. (32a) and (32b), then

$$(\frac{\partial}{\partial t} + u^{(0)}\frac{\partial}{\partial x} + v^{(0)}\frac{\partial}{\partial y})(f + \zeta^{(0)}) = \frac{f_0^2}{c_0^2 - f_0\psi_B}(\frac{\partial}{\partial t} + u^{(0)}\frac{\partial}{\partial x} + v^{(0)}\frac{\partial}{\partial y})(\psi - \psi_B) \quad (34)$$

finally

$$(\frac{\partial}{\partial t} - \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y})[\beta_0 y + \nabla^2\psi - \frac{f_0^2}{c_0^2 - gh_B}\psi + \frac{f_0}{c_0^2 - gh_B}gh_B)] = 0 \quad (35)$$

where $u^{(0)} = -\frac{\partial\psi}{\partial y}, v^{(0)} = \frac{\partial\psi}{\partial x}, \zeta^{(0)} = \nabla^2\psi, \psi = \frac{g\hat{h}}{f_0}, \psi_B = \frac{\phi_B}{f_0} = \frac{gh_B}{f_0}$

Eq. (35) is a modified model.

4.3. $\lambda \leq 10^{-1}$ magnitude

According to (18c), it is observed that the magnitude of topography expression form λ is too small, which approximately ignores the influences of topographic effect.

Now we can observe that the Eqs. (25), (35) are two quasi-geostrophic barotropic models under variable topographic conditions with time t. If the topographic effect is independent of time, equations degrade into the following forms.

Under the stable topography, Eq. (35) degenerates into

$$(\frac{\partial}{\partial t} - \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y})(f + \nabla^2\psi - \frac{f_0^2}{c_0^2 - gh_B}\psi) + \frac{f_0 g}{c_0^2 - gh_B}(\frac{\partial\psi}{\partial x}\frac{\partial h_B}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial h_B}{\partial x}) = 0 \quad (36)$$

where

$$u^{(0)} = -\frac{\partial\psi}{\partial y}, v^{(0)} = \frac{\partial\psi}{\partial x}, \ \zeta^{(0)} = \nabla^2\psi, \psi = \frac{\hat{\phi}^{(0)}}{f_0}, \psi_B = \frac{gh_B}{f_0} \quad (37)$$

the model (25) under the stable topography can be written as (Pedlosky,1987)

$$(\nabla^2 - \frac{1}{\lambda_R^2})\frac{\partial\psi}{\partial t} + J[\psi, \beta y + \nabla^2\psi - \frac{1}{\lambda_R^2}\psi + \frac{f_0}{H}h_B(x,y)] = 0 \quad (38)$$

where

$$u^{(0)} = -\frac{\partial\psi}{\partial y}, v^{(0)} = \frac{\partial\psi}{\partial x}, \zeta^{(0)} = \nabla^2\psi, \psi = \frac{\hat{\phi}^{(0)}}{f_0}, \psi_B = \frac{gh_B}{f_0}, \lambda_R = \frac{\sqrt{gH}}{f_0} \quad (39)$$

otherwise, if we don't consider the topographic effect and the divergence of the fluid, Eqs. (36) and (38) degenerate into (Liu, 1991)

$$\nabla^2\frac{\partial\psi}{\partial t} + J(\psi, \beta y + \nabla^2\psi) = 0 \quad (40)$$

Eq. (40) is a classics dynamics model used by the large-scale atmospheric and oceanic motions.

5. Conclusion

(a). Under the unsteady topography, some new modified models (25), (35) are derived. These models meet the general rule that the topography changes with time in reality. When the topography has nothing to do with the time, Eq. (35) degenerates into a dynamics model (36), Eq. (25) degenerates into Eq. (38) which is a quasi-geostrophic barotropic model under the spatial topography.

(b). The modified models under the topographic effect with different magnitudes are presented, we can see the pattern under the condition of large terrain, which is the improvement of the model. After the above-modified models are given, we will also derive the mathematical model for Rossby wave in the further study, and the further exploration of the large-scale factual influences of topography on atmosphere and ocean are required.

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