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Macrodynamics and Pollution in Open Economy: An IS-LM Analysis

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ABSTRACT *- The economy of the environment is traditionally the field of micro-economy. Yet Providing an analysis purely macro-economic is from a theoretical and praxeological point of view possible. With this in mind, we change the model IS-LM so that it incorporates the pollution. This extension of the model has allowed us to show the ecological and economic effects of different monetary or budgetary policies depending on the type of small open economy considered (with or without different kind of control pollution activities).*

Introduction

Even if there is a debate about the exact meaning to be attached to the term sustainable development, it appears that whatever their opinions underlying theory, the authors involved in this debate agree that the concept of sustainable development implies managing and maintaining an inventory of resources with a view to equity between generations and between countries[1]. Indeed the economics of the environment is traditionally treated as a sub division of the micro-economy. It therefore appears to us, after authors like Daly[2], there is a place also for a macro-economy of the environment, where the macro-economy would be regarded as a sub-system open to the ecosystem and totally dependent on the latter, both for the source of inputs into low entropy of matter and energy, and as a receptacle outputs for high entropy of matter and energy. The macroeconomy of environment should focus on the volume of trade that cross the boundaries between the system and the subsystem. As Daly points out if the optimal allocation of a given level of resource flows within an economy is a micro-economic problem, the optimal scale of the economy relative to the ecosystem is an entirely different problem in fact a macroeconomic one.

Moreover, to try to establish the foundations of a macro-economy environment, and measure their lessons, we will incorporate into the model IS-LM, but in its dynamic form, function Pollution in the form of stock.

A dynamic IS-LM model extended to the case of pollution

In its original version, the IS-LM model relies on comparative statics, but the principle of correspondence of Samuelson stipulates that the properties of comparative statics of a model relies on its dynamic properties[3]. As we intend to modify the model by adding new variables in order to introduce pollution, we therefore must study the dynamical properties of the corresponding model before to make any comparative statics analysis.

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[1] Pearce D, Markandya A, Barbier E.B (1989), *Blueprint for a Green economy*, London, Earthscan Publications.

[2] Daly H. E.,"Elements of Environmental Macroeconomics", in Costanza R , *Ecological Economics, The science and Management of Sustainability*, Columbia University Press, New York, pp 32-45

[3]In fact, in its comparative statics form, the assumption of automatic balancing of the market for goods and services is made. But in its dynamic form, one wonders about the existence of this equilibrium and the process of adjustment of the economy toward it.

In the dynamic model IS-LM we consider an economy in which national income (Y) responds to excess investment (I) on savings (S). Thus, the first dynamic equation based on the IS curve is given by the following:

$$\dot{Y} = I - S$$

In case you introduce budgetary expenditure (G), the environmental budget (G_e), taxation (T), productive investment (I) and pollution (I_e) and exports (X) and imports (M), the previous dynamic equation becomes:

$$\dot{Y} = I + I_e + G + G_e - S - T + X - M$$

The function of exports of the economy is $X = \left(\overline{X}/\Omega\right)$ with the exchange rate $\Omega = \$ / €$.We suppose that when the exchange rate increases ceteris paribus, imports rise, and when the level of household income increases imports are also increasing, hence the function of imports $M = mY(\Omega)$, where m is the propensity to import of the economy.

The investment in this economy is divided into a productive investment aimed at increasing the production capacity of the economy, I, and investment in pollution control which does not have this capacity, Ie[4]. Regarding the investment function, $I = \bar{I} - ar$, the level of productive investment depend on the level of interest, r, where the parameter "a" reflects the incentive to invest from rate interest and $\bar{I}$ an autonomous level. Note that The firm's calculation of profitability integrates expected demand, through the assessment of the marginal efficiency of capital and its comparison to the rate of interest. This assessment process will take a special importance with the function of Investment in pollution control. In fact we suppose that interest rate plays a lower role in the pollution control investment function than in the function of investment in other sectors, resulting in a coefficient "b" very low and less than "a"[5]. In this case, we write the investment function in pollution control $I_e = \bar{I}_e - br$. Thus a fundamental element in our model will be to know if the expectations of the firms in the sector of the pollution would be resolutely optimistic about the behaviour of the State's environmental standard.

In this economy public expenditure, is assumed exogenous, with the request of the State in consumer goods and services and property investment, G, and public investment in pollution control, Ge, respectively $G = \overline{G}$ and $G_e = \overline{G}_e$. With all these assumptions, IS equation is rewritten :

$$(1)\quad \dot{Y} = -s(1-t)Y - tY - c\overline{T} + \overline{C} + \overline{G} + \overline{G}_e + \bar{I} - ar + \bar{I}_e - br + \overline{X} - mY$$

In our model IS-LM dynamic interest rate (r) responds to excess demand for money (L) on the money supply determined exogenously (M). But in open economy the money supply will vary depending on changes in foreign exchange reserves and the in monetary base, H, $\dot{r} = \left[hY + \overline{L} - lr\right] - \left[\overline{H} + R\right]$, where $R = X - M + f(r)$. Thus, a fixed exchange rate regime, the LM curve dynamics is given by the relationship (2) where hY is the demand for money for purposes of transactions, and $\overline{L} - lr$ is the demand money for purposes of speculation, and finally the monetary base exogenous:

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[4] From a strictly accounting on a perfectly entitled to present the overall investment in the form of two sectors since these equations are equations balance ex-post. And we know all prices in the economy, since this is the prerequisite for the aggregation of goods produced for obtaining the equation of balance between accounting income and expenditure.

[5] Several reasons require that choice. First reason is that firms that invest in clean-up will do so only if the State has encouraged the market for mitigating pollution by enacting laws requiring in all other sectors the use of new products of pollution control : the pollution sector can not declare unilaterally an increase of its production unless it has correctly anticipated a strengthening of pollution standards. The second reason lies in subsidies granted by the government which are merely transfers therefore making private investment in pollution less dependent on interest rates. Finally, in this case we must also consider the fact that environmental standards, in the first place, determine the use of cleaner products. Once internalized this information the rate of interest takes the second place.

(2) $\dot{r} = hY + \overline{L} - lr - \overline{H} - \left[\overline{X} - mY + f(r) \right]$

We will now introduce the environmental dimension in this model showing the evolution equation stock pollution widely used in theoretical models of sustainability[6]. Thus Strom[7] in his article assumes that the stock waste is the appropriate measure of environmental the rate of decline in the density of waste, reflecting the rateδquality. With of increase in the assimilative capacity of the natural environment due to capital investment, and W the waste was then the following equation: $\dot{Z} = W - \tilde{h} I_r - Z\delta$. In transposing this equation to adapt to the dynamic model IS-LM, we can then write the equation dynamic equilibrium macro-environment in the form of the equation (3), namely:

(3), $\dot{E} = (\alpha)Y - \beta(\overline{G}_e) - \gamma(\bar{I}_e) + \gamma(br) - \delta(E)$

where E is the stock of pollution or waste, αY is the emission of pollution emitted by the productive activities, β G_e the reduction of the stock of pollution caused by government spending on pollution, and $\gamma(\bar{I}_e) - \gamma(br)$ the reduction of the stock of pollution due to the clean-up activities of the private sector, δ the rate of decline of natural waste stock E. Therefore, the dynamic system can be written by the following matrix:

$$\begin{pmatrix} \dot{Y} \\ \dot{r} \\ \dot{E} \end{pmatrix} = \begin{pmatrix} -s(1-t)-t-m & -(a+b) & 0 \\ (h+m) & -(l+f) & 0 \\ \alpha & b\gamma & -\delta \end{pmatrix} \begin{pmatrix} Y \\ r \\ E \end{pmatrix} + \begin{pmatrix} \overline{I_e} + \overline{G_e} + \bar{I} + \overline{G} + \overline{C} + \overline{X} - c\overline{T} \\ \overline{L} - \overline{H} - \overline{X} \\ -\beta(\overline{G}_e) - \gamma(\bar{I}_e) \end{pmatrix}$$

In the mathematical appendix, we demonstrate the stability of this model. We can therefore study its behaviour near of the equilibrium. Doing so the previous equations become:

$$\begin{pmatrix} (s(1-t)+t+m) & (a+b) & 0 \\ (-m-h) & (f+l) & 0 \\ \alpha & b\gamma & -\delta \end{pmatrix} \begin{pmatrix} Y \\ r \\ E \end{pmatrix} = \begin{pmatrix} \overline{I_e} + \overline{G_e} + \bar{I} + \overline{G} + \overline{C} - c\overline{T} + \overline{X} \\ \overline{L} - \overline{H} - \overline{X} \\ \beta(\overline{G}_e) + \gamma(\bar{I}_e) \end{pmatrix}$$

This system is in fact the structural form of a small open economy subject to a pollution problem. We examine this structural form deducting the reduced form in the next section.

Dynamic behaviour analysis of the model

The behavioural analysis of the previous dynamic model is identical to study the behaviour of the structural form of a Keynesian model of a short-term applied to the local pollution. We will study the structural form by deducting the reduced form.

With activities for pollution reduction private non-autonomous $I_e = \bar{I}_e - br$, and public expenditure shocks $G_e = \overline{G}_e$, the IS curve, in open economy, is given by the relationship (1), i.e.:

$$Y(1-t) = cY - ctY - c\overline{T} - tY + \overline{C} + \overline{G} + \bar{I} + \overline{G}_e + \bar{I}_e - ar - br + \overline{X} - mY$$

The LM curve is given by the relationship (2), namely:

$$\overline{H} + \overline{X} - mY + f(r) = hY + \overline{L} - lr$$

The equation environmental equilibrium, taking into account the public and private spending on pollution control is given by (3):

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[6] See for example Gradus R., Smulders S. (1993), The Trade-off between environmental care and long-term growth, pollution in three prototype growth models, *Journals of Economics*, *58(1)*, pp25-51.

[7] Strom S, "Economic Growth and Biological Equilibrium", Swedish Journal of Economics, Vol. 75, N°2, June 1973. Strom S, (June 1973) Economic Growth and Biological Equilibrium, *Swedish Journal of Economics*, *75(2)*.

$$E\delta = \alpha Y - \beta(\overline{G}_e) - \gamma(\bar{I}_e) + \gamma(br)$$

Presented in matrix form the previous three equations give us the following system:

$$\begin{pmatrix} (s(1-t)+t+m) & a+b & 0 \\ -m-h & (f+l) & 0 \\ \alpha & b\gamma & -\delta \end{pmatrix} \begin{pmatrix} Y \\ r \\ E \end{pmatrix} = \begin{pmatrix} \overline{I_e} + \overline{G_e} + \overline{I} + \overline{G} + \overline{C} - c\overline{T} + \overline{X} \\ \overline{L} - \overline{H} - \overline{X} \\ \beta(\overline{G}_e) + \gamma(\bar{I}_e) \end{pmatrix}$$

With the determinant $\Lambda = -\delta\left[\left(s(1-t)+t+m\right)(f+l)+(a+b)(h+m)\right]$, and with $\overline{M} = \begin{pmatrix} \overline{I_e} + \overline{G_e} + \bar{I} + \overline{G} + \overline{C} - c\overline{T} + \overline{X} \\ \overline{L} - \overline{H} - \overline{X} \\ \beta(\overline{G}_e) + \gamma(\bar{I}_e) \end{pmatrix}$.the structural system is written in its reduced form in the following manner:

$$\begin{pmatrix} Y \\ r \\ E \end{pmatrix} = \left(\frac{1}{\Lambda}\right) \begin{pmatrix} -(f+l)\delta & (a+b)\delta & 0 \\ -(h+m)\delta & -(s(1-t)+t+m)\delta & 0 \\ -(m+h)b\gamma - (f+l)\alpha & (a+b)\alpha - (s(1-t)+t+m)b\gamma & -\Lambda/\delta \end{pmatrix} \overline{M}$$

Starting from this reduced form model we will deduct the behaviour of this model in the several cases. To do this we will calculate the various multipliers corresponding to this model.

Comparative statics in the simplified case without pollution control

In the simplest case we make the assumption that no expenditure pollution, neither public nor private, shall be undertaken in the economy ($\bar{I}_e = \overline{G}_e = 0$ and b=0). In sum, it's only the assimilative capacity of the environment that converts waste produced by the economic process. The equation of environmental equilibrium is given by: $E\delta = \alpha Y$.

Multipliers

The multipliers are obtained by differentiating the reduced form model:

$$\frac{\Delta Y}{\Delta \overline{C}} = \frac{\Delta Y}{\Delta \bar{I}} = \frac{\Delta Y}{\Delta \overline{G}} = k_{Y,1} > 0 \quad \text{and} \quad \frac{\Delta Y}{\Delta \overline{T}} = -ck_{Y,1} > 0$$

$$\frac{\Delta Y}{\Delta \overline{H}} = -k_{Y,2} > 0 \quad \frac{\Delta Y}{\Delta \overline{L}} = k_{Y,2} < 0$$

where $k_{Y,1} = \dfrac{\left(f+l\right)}{\left[\left(s(1-t)+t+m\right)\left(f+l\right)+a\left(m+h\right)\right]}$, and

$$k_{Y,2} = -\frac{a}{\left[\left(s(1-t)+t+m\right)\left(f+l\right)+a\left(m+h\right)\right]}$$

The monetary and budgetary multipliers are:

$$\frac{\Delta r}{\Delta \overline{C}} = \frac{\Delta r}{\Delta \bar{I}} = \frac{\Delta r}{\Delta \overline{G}} = k_{r,1} > 0 \quad \text{et} \quad \frac{\Delta r}{\Delta \overline{T}} = -ck_{r,1} > 0$$

$$\frac{\Delta r}{\Delta \overline{H}} = -k_{r,2} < 0 \quad \frac{\Delta r}{\Delta \overline{L}} = k_{r,2} < 0$$

where $k_{r,1} = \dfrac{\left(m+h\right)}{\left[\left(s(1-t)+t+m\right)\left(f+l\right)+a\left(m+h\right)\right]}$

and $k_{r,2} = \dfrac{(s(1-t)+t+m)}{[(s(1-t)+t+m)(f+l)+a(m+h)]}$

The equation of ecological equilibrium allows us to calculate the environmental multipliers:

$$\Delta E = k_{E,1}[\Delta\overline{C} + \Delta\overline{I} + \Delta\overline{G} - c\Delta\overline{T}] + k_{E,2}[\Delta\overline{L} - \Delta\overline{H}]$$

$$\frac{\Delta E}{\Delta\overline{C}} = \frac{\Delta E}{\Delta\overline{I}} = \frac{\Delta E}{\Delta\overline{G}} = k_{E,1} > 0 \quad \text{and} \quad \frac{\Delta E}{\Delta\overline{T}} = -ck_{E,1} > 0$$

$$\frac{\Delta E}{\Delta\overline{H}} = -k_{E,2} > 0 \qquad \frac{\Delta E}{\Delta\overline{L}} = k_{E,2} < 0$$

where $k_{E,1} = \dfrac{(f+l)\alpha}{\delta[(s(1-t)+t+m)(f+l)+a(m+h)]}$ and

$$k_{E,2} = \frac{-a\alpha}{\delta[(s(1-t)+t+m)(f+l)+a(m+h)]}$$

The sign of ecological multipliers are the same as monetary and budgetary multipliers compared to Y. This is perfectly normal. Indeed in this model no place has been given to expenditure on pollution control, so the emission level is equal to the difference between the level of gross pollution and assimilative capacity of the natural environment, assumed fixed. So in this case, emissions of pollution are only proportional to the volume of production and hence the national income of the economy and in a linear fashion. Therefore, any economic stimulus of income through budgetary and monetary policies corresponds to an increase in national income and thus and thus to a proportional increase in the same sense of the level of waste emissions. The main reason is that any distribution of income, wages, or simply any stimulation of demand creates ceteris paribus, a stimulation in the same direction of pollution (more wages mean more consumption more Investment and so on. And therefore more consumption and thus more waste).

Graphical representation

The graphic representation of the environmental equilibrium equation is exactly like that of a balance of payments curve. Indeed it is written (3) $0 = \alpha Y - E\delta$, ie $Y = E\delta/\alpha$. The status of this curve is very particular. It brings together all the points which, for a given level of pollution, E, for a given level of assimilation of natural waste, δ, and finally to an intensity of pollution in the production sector aα, associates a level of income, Y. It is not a worthy that for a given objective pollution emitted, E, there is a single level of income, Y. Thus BE is a line parallel to the axis. Hence the graphical representation of the environmental equation is the following:

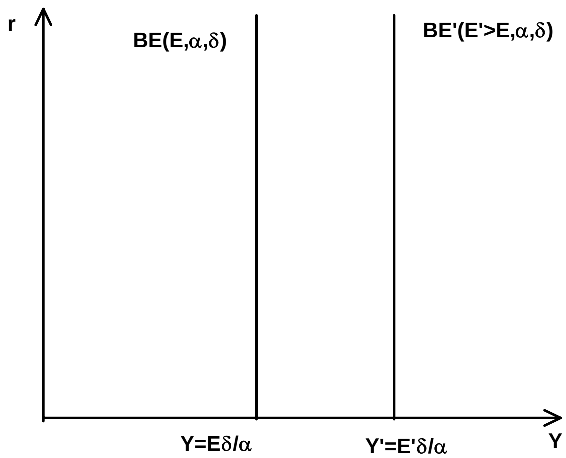

Figure 1. Environmental equilibrium curves according to the emission level of pollution

Thus you have a family of curves where the closest to the origin correspond to low levels of emissions and the furthest a higher level of pollution. In the case of this small open economy where no environmental expenditure is implemented, the environmental impact of a policy of budgetary or monetary can only be the same as that exercised by these policies on income level since we just verify that the respective multipliers are the same, as can be seen in the following graphic:

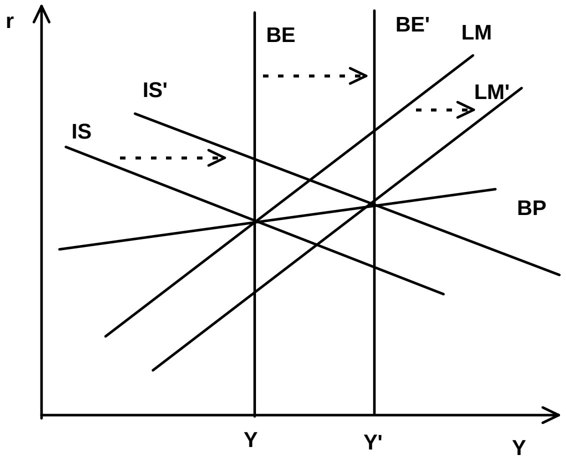

Figure 2. Effect of a budgetary stimulus in open economy on the level of income and of pollution.

Thus, in open economy and in a fixed exchange rate regime any budgetary policy moves IS to the right, thereby increasing the rate of interest. In fixed exchange rate regime, and with complete freedom of capital movements, it increases the stock of gold and currency of the central bank, and it has more than compensate for the fact that imports have increased. The balance of payments is in surplus resulting in a exogenous creation of money (capital flows). So LM move to the right. At the equilibrium the three lines intercept (IS' LM' and BP, which in fixed exchange does not move); national income has increased, with Y'> Y. From an environmental perspective, this increase in overall income implies that the level of waste has been raised and therefore the curve BE move to the right in BE '. In the new ecological and economical equilibrium, the economy is at a higher income level, which implies an emission level also higher.

Thus, from a macroeconomic view, we get the same results as those of usual IS-LM model with a fixed exchange rate regime. However all variations in the level of national income reflects a change in the same sense of the level of emissions. Therefore after adjustment we will always be in equilibrium macroeconomicaly and ecologically speaking. Note that in the latter case the notion of equilibrium should be understood as the level of pollution consistent with the level of income, given the intensity in pollution of the national economy, and bearing in mind assimilative capacity of the environmental represented by δ . It's certainly not the level of emissions that would ensure an hypothetical ecological "paradise". This is of great importance if one believes, like Daly, in the notion of size or scale of the sustainable economy. Indeed, in this case, we can consider that this "sustainable size" corresponds to a maximum amount of waste that can assimilate the ecosystem in the short term and in the long term. This leads us therefore to set an upper limit to the evolution of GNP. As stated Daly, the limit depends on the country and a whole range of factors, geographical, ecological and demographic[8]. In the next part of this article we introduce environmental public and private spending allowing more flexibility to reconcile national income and environmental constraints.

[8] In our model, we could introduce it as a theoretical form of a line parallel to the y-axis.

Comparative statics in the general case

Multipliers

Using the equation representing the equilibrium on the market products ie IS we get the following multipliers:

$$\frac{\Delta Y}{\Delta \overline{C}}=\frac{\Delta Y}{\Delta \overline{I}}=\frac{\Delta Y}{\Delta \overline{G}}=\frac{\Delta Y}{\Delta \overline{I}_e}=\frac{\Delta Y}{\Delta \overline{G}_e}=k_{Y,1}>0 \text{ and } \frac{\Delta Y}{\Delta \overline{T}}=-ck_{Y,1}<0$$

$$\frac{\Delta Y}{\Delta \overline{H}}=-k_{Y,2}>0 \quad \frac{\Delta Y}{\Delta \overline{L}}=k_{Y,2}<0$$

with $k_{Y,1}=\dfrac{(f+l)}{\left[(s(1-t)+t+m)(f+l)+(a+b)(m+h)\right]}$

and $k_{Y,2}=-\dfrac{(a+b)}{\left[(s(1-t)+t+m)(f+l)+(a+b)(m+h)\right]}$

Applying the same method to the equation on the money market, we have:

$$\frac{\Delta r}{\Delta \overline{C}}=\frac{\Delta r}{\Delta \overline{I}}=\frac{\Delta r}{\Delta \overline{G}}=\frac{\Delta r}{\Delta \overline{I}_e}=\frac{\Delta r}{\Delta \overline{G}_e}=k_{r,1}>0 \text{ and } \frac{\Delta r}{\Delta \overline{T}}=-ck_{r,1}<0$$

$$\frac{\Delta r}{\Delta \overline{H}}=-k_{r,2}<0 \quad \frac{\Delta r}{\Delta \overline{L}}=k_{r,2}>0$$

where $k_{r,1}=\dfrac{(m+h)}{\left[(s(1-t)+t+m)(f+l)+(a+b)(m+h)\right]}$

and $k_{r,2}=\dfrac{(s(1-t)+t+m)}{\left[(s(1-t)+t+m)(f+l)+(a+b)(m+h)\right]}$

Regarding BE we obtain:

$$\Delta E=k_{E,1}\left[\Delta\overline{C}+\Delta\overline{I}+\Delta\overline{G}+\Delta\overline{G}_e+\Delta\overline{I}_e-c\Delta\overline{T}\right]+k_{E,2}\left[\Delta\overline{L}-\Delta\overline{H}\right]+k_{E,3}\left[\beta(\Delta\overline{G}_e)+\gamma(\Delta\overline{I}_e)\right]$$

$$\frac{\Delta E}{\Delta \overline{C}}=\frac{\Delta E}{\Delta \overline{I}}=\frac{\Delta E}{\Delta \overline{G}}=k_{E,1}>0 \text{ and } \frac{\Delta E}{\Delta \overline{T}}=-ck_{E,1}<0$$

$$\frac{\Delta E}{\Delta \overline{I}_e}=k_{E,1}+k_{E,3}\gamma=(k_{E,1}-\gamma/\delta)>0 \text{ and } \frac{\Delta E}{\Delta \overline{G}_e}=k_{E,1}+k_{E,3}\beta=(k_{E,1}-\beta/\delta)>0$$

$$\frac{\Delta E}{\Delta \overline{H}}=-k_{E,2} \quad \frac{\Delta E}{\Delta \overline{L}}=k_{E,2}$$

With the multipliers

$k_{E,1}=\dfrac{(f+l)\alpha+(m+h)b\gamma}{\delta\left[(s(1-t)+t+m)(f+l)+(a+b)(m+h)\right]}$, $k_{E,3}=-1/\delta$, and

$k_{E,2}=\dfrac{(s(1-t)+t+m)b\gamma-(a+b)\alpha}{\delta\left[(s(1-t)+t+m)(f+l)+(a+b)(m+h)\right]}$.

The important point about these multipliers is to note that, from a strictly economic point of view, nothing changes compared to the usual IS-LM model without pollution. Nevertheless, from an ecological point of view the environmental public expenditure multiplier has the same economic impact that another government expenditure ($dY/DG=dY/dG_e$), but without having the same environmental consequences, since the multiplier of environmental pollution control public expenditure is less than that of an usual public expenditure ($dE/dG>dE/dG_e$). Thus, while increasing G_e lead to an increase on income identical to that of the same increasing of G but with an emission level of pollution corresponding much less important. However, it must be stressed that, for G_e, it has been implicitly supposed that environmental government spending have a real environmental effect on pollution, symbolized by the coefficient β between zero and one. But one could imagine the case of environmental public spending that would not impact assessments, administrative activities, without significant effect (a part from the distribution of wages to employees of the Ministry of Environment or writing reports with non-recycled paper etc. ..), that is to say a coefficient β equivalent to zero or very close to zero.

Regarding the monetary multiplier, they depend on the sign of $\left(s(1-t)+t\right)b\gamma-(a+b)\alpha$. This term is only represents the slopes of IS and BE. We stressed in the previous section that the coefficient "b" is smaller than "a". This means that the normal case is a negative sign of the previous term. In the unusual case where the term is positive, it means that the multiplier effect of private investment in pollution control is more important than for the usual investment. This could happen only in an economy where the expectations of the decision makers in the field of pollution would be resolutely optimistic on the behaviour of the State's environmental standard. Therefore, any monetary policy has a double effect in lowering the rate of interest: it facilitates investment as a whole, and hence pollution, but investment in pollution control is also encouraged. Thus, an environmental point of view, the net effect depends on expectations of the firms and hence rely on the environmental policy imposed by the State, and, hence, by the respective sizes of the pollution control sectors compared to the usual investment sector.

Graphical representation

The graphic representation of the environmental equation is given by (3) $E\delta=\alpha Y-\beta(\overline{G}_e)-\gamma(\bar{I}_e)+\gamma(br)$. This equation may be written in the following form:

$$r=-\frac{\alpha}{\gamma(b)}Y+\frac{E\delta+\beta(\overline{G}_e)+\gamma(\bar{I}_e)}{\gamma(b)}$$

The status of this curve is consistent with the previous case. Indeed it gather all the points which, for a level of public and private environmental expenditure and for an intensity of pollution in the production sector, associates to a couple of income level and interest rates a specific level of pollution E. The reverse of the slope of the curve BE is then equal to $-\alpha/\gamma(b)$, hence the graphic representation of Figure 3:

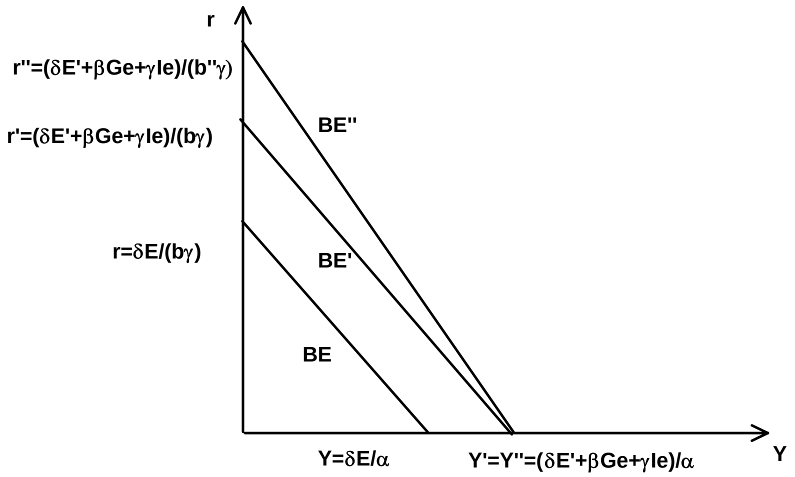

Figure 3. Curves balances macro-environment according to the emission level of environmental expenditure

and the sensitivity of private environmental investment at the rate of interest.

An examination of Figure 3 shows us that, when comparing the curve BE and the curve BE ', an increase in the level of environmental private or public expenditure lead to increased pollution level, income level and interest rates. This is perfectly consistent. A comparison of curves BE "and BE 'shows that for identical level of environmental expenditure, for a ratio b different, we obtain income and pollution levels which are identical but different interest rates (the more b decrease in passing from b ' to b ", the more interest rate increases). Thus, we conclude that lower is b, i.e. less private investment in pollution control is sensitive to interest rates, the stronger changes in r have to be in order to obtain the same level of pollution. Moreover this chart is perfectly consistent with the definition of BE given in the previous section because, when the coefficient b tends towards zero, ie when private investment becomes independent and indifferent to the rate of interest, then BE becomes vertical.

An important consequence of the shape of the curve BE is that, according to the place of the curve BE compared to IS, the effect of monetary policy on the emission level will not be the same (as indicated by the sign of the corresponding multiplier which differs depending on whether one is in the usual case or not). But this changes nothing with regard to the comparison between the effect of an environmental public expenditure versus a usual public expenditure, as can be seen in Figure 4:

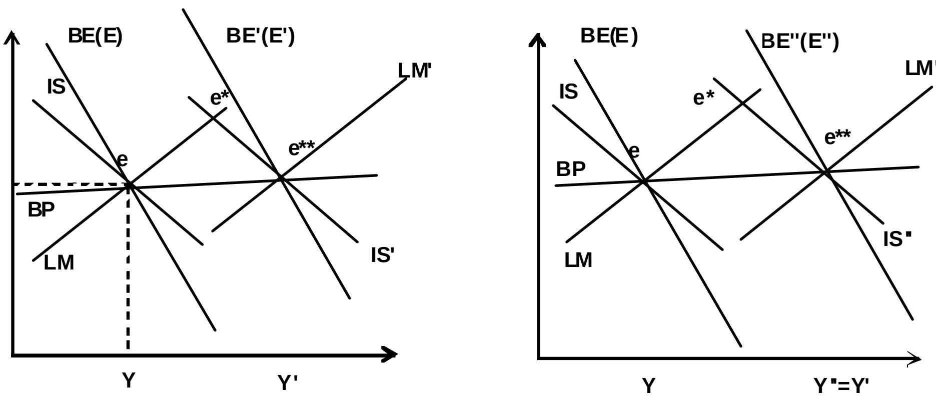

Figure 4. Effects on the level of income and of pollution compared to those of a budgetary environment.

In this figure, we make the assumption that we are in the normal case, ie that the slope of IS is not greater than or equal to that of BE (but change this assumption would not into question the results). In the first case, a budgetary stimulus has the effect of moving IS to the right where it crosses LM in e *. But this is not a point of equilibrium in open economy. The interest rate has increased resulting in an influx of foreign capital just inflating the stock of gold and foreign exchange considering our implicit assumption of high mobility of capital (given the slope of the curve BP). This largely offsets the decline in the stock of foreign exchange resulting from increased imports. In total, there is therefore an exogenous creation of money that moves LM to the right where it crosses IS ' in e **. This is a point of economic equilibrium symbolized by the intersection of IS 'and LM. From an environmental point of view, the shift from Y to Y 'has resulted in an increase in same intensity in the level of emissions from E to E'. Hence, from the environmental point of view, the situation has worsened without ambiguity.

This figure, allows us to verify that for an increase in Environmental public expenditure of same intensity as the public expenditure of the previous case, income Y increases of the same amount as in the previous case, and passes from Y to Y ", where Y = Y '. The curves fit IS and LM is the same as in the first case. By contrast the level of emissions even though it has increased compared to its initial level E is at a level E" which is less than E'. Accordingly, a an environmental public expenditure, even if it leads to an increased pollution, is more efficient ecologically speaking than a usual public expenditure, because it causes relatively less pollution that the latter, for the same increase in national income.

By contrast, if we look to the environmental impact of a monetary economy open, there is no such dichotomy "usual/ environmental", but a dichotomy between normal and abnormal as can be seen on the figures 4 and 5:

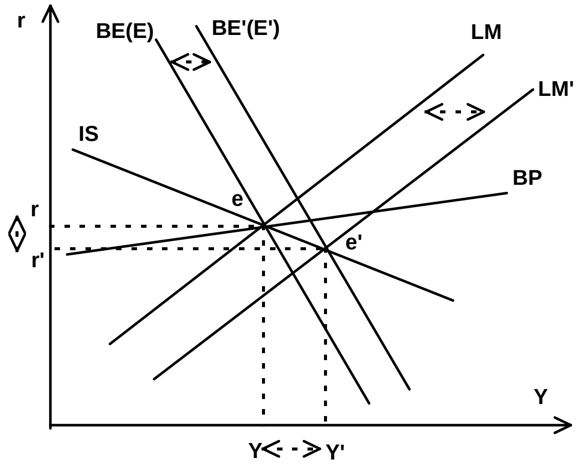

Figure 4. Effect of an active monetary policy in open economies on the level of income and that of emissions in the normal case.

In the normal case, the sign of the environmental monetary multiplier is positive. An initial increase of money supply by lowering the level of interest rates going from r to r ', stimulates the economy whose income goes to Y'. But, in open economy, e ' is a situation of unstable equilibrium because situated below the curve of the balance of payments. Indeed, with a perfect capital mobility, the declining interest rate leads to a flight of capital which, coupled with rising imports leads the stock of gold and currency of the Central Bank on the decline. So the money supply contracts. Therefore LM go back to its original position. The level of income returns to Y, and hence the economy returns to its stable equilibrium position, e. From an environmental view situation is identical. At first, when the income from Y to Y ', the level of emissions increases since the multiplier monetary environment is positive, and therefore the curve BE moves on the right (where the level of pollution emissions E ' is higher than E). Then the LM' , BE' IS 'curves intercept in e'. This position being unstable, when the level of income is declining to return to Y, the level of emissions is declining as well and hence the curve BE shift back. Indeed, the money supply experienced an exogenous destruction of money, so the multiplier monetary environmental operated but with a decline in money supply. Hence the economy has experienced a reduction in emissions in the normal case, and therefore a return to the level of emissions at its initial level E after passing through the level E ': the economy has returned to its initial level of wealth and pollution.

If we turn now to the unusual case, illustrated figure 5, the results are greatly changed.

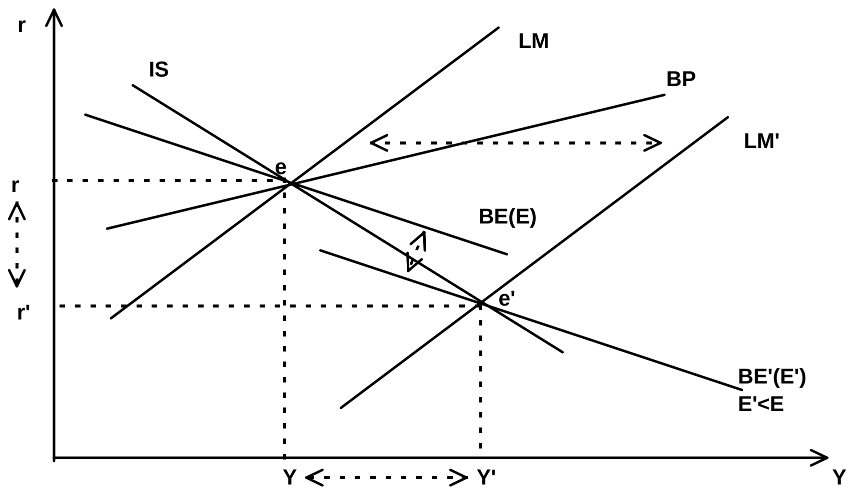

Figure 5. Effect of an active monetary policy in open economies on the level of income and that of emissions in the unusual case.

In the unusual case, the sign of the monetary multiplier environmental tells us that any change in the money supply reflects a change in direction in the level of emissions, because the declining interest rate stimulates heavily investing in pollution control that compensates well beyond or even adverse effects on investment usual. As a result, an increase of the amount of currency in the case abnormal, leads to lower emissions of pollution. Therefore, active monetary policy by lowering the level of interest rates going from r to r ', stimulates the economy which income rise from Y to Y '. But in the open economy e' is a situation of unstable equilibrium because situated below the curve of the balance of payments, which is in deficit. Indeed with a perfect capital mobility declining interest rates caused a flight of capital which therefore reduces the LM curve to its original position. The level of income returns to Y, and hence the economy returns to its stable equilibrium position, e. From an environmental point of view, the situation is reversed. At first, when the income rise from Y to Y ', the level of emissions decrease, following the stimulation of investment in pollution control, and therefore the curve BE moves left to BE' where E '<E. But this position is an unstable economic equilibrium. When the level of income return from Y' to Y the level of emissions increases in response to the declining investment in pollution control. So the curve BE ' leaves back and returned to his original position, BE, where the level of emissions regains its initial level E. Hence, after adjustments, the economy has returned to its initial level of wealth and emission pollution.

Note finally that in the unusual case we make the implicit assumption that the private sector of investment in control of pollution is an extremely important sector of the economy with the concomitant assumption that the firms, when they formulate their expectations, know that the government is committed to not to relax the laws and standards, and therefore not depress demand for goods of pollution-control equipment, explaining the important weight of this sector. It is therefore a very special case which to our knowledge doesn't exist up to now in the reality where the private sector of investment in pollution control is less important than usual private sector: that's why we consider it as an abnormal case.

Conclusion

The introduction of pollution in the form of stock in a dynamic IS-LM model, has allowed us to analyse the environmental consequences of macroeconomic policies. According to the model, an environmental public expenditure, even if it leads to increased pollution is preferable to a usual public expenditure, because it causes relatively fewer emissions of pollution that the latter, for an identical increase in national income. The environmental effect of an expansionary monetary policy depends on the type of economy involved in. In the unusual case, where the bulk of investment activities would be dedicated to clean-up, any change in money supply leads to a variation in the opposite sense in the level of pollution, for the reason that a lower interest rate stimulates investment in pollution control that compensates the much more adverse effects of investment in usual sector. In the normal case where the private sector pollution is smaller than the usual private sector, any monetary policy induced by the decline of interest rates encourages more the usual investment (with environmental standards unchanged), thereby increasing pollution and the income levels. Hence, one of the major lessons of this model is that what is important is the expectations in the sector of the pollution control and the size of this sector relatively to the rest of the economy. Also, a government anxious to make a sustainable economic growth should give priority to try to drive the expectations of these pollution control firms through environmental standards increasingly severe as long as the economy did not have a private sector of pollution control at least as important in its economic size as the usual private sector. In the meantime, environmental public policies should be preferred, from an environmental point of view, to any monetary and budgetary policy, provided that public environmental measures are concrete and truly effective remediation. Thus, our findings reinforce the arguments of post-Keynesians like Peter Bird[9], who recognized the importance of informational constraints in a state of uncertainty, and prefer maintaining standards seeking optimality.

Moreover, if one takes as relevant the criterion proposed by Daly of "carrying capacity", ie the optimal scale of the economy compared to the ecosystem behind it, we can conclude that our model, except in unusual circumstances, show that any monetary or budgetary policy increases the pollution level and therefore drive the economy a little closer to the sustainable limit. If the economy is in an unusual case, one moves more and more of this limit, then there is sustainable development in its fullest sense.

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Mathematical appendix

The resolution of the dynamic system passes through the calculation of values of the main matrix. The result is the characteristic polynomial following:

$$P(\lambda) = [-\delta - \lambda]\left[\lambda^2 + \left(s(1-t) + t + l + m + f\right)\lambda + \left((s(1-t) + t + m)(l + f) + (h + m)(a + b)\right)\right] = 0$$

The resolution of this polynomial imposes using the determinant Δ where :

[9] Bird P. W. N.(1982), Neoclassical and Environmental Post-Keynesian Economics, *Journal of Post-Keynesian Economics*, 4(4), pp586-593.

$$\Delta = (s(1-t)+t+m-l-f)^2 - 4(h+m)(a+b)$$.

If Δ is positive, then the three values, solutions of the previous polynomial equation are three real rootsl: $\lambda_3 = -\delta$ and

$$\lambda_{i=1,2} = \frac{1}{2}\left[(-s(1-t)-t-l-m-f) \pm \sqrt{\Delta}\right]$$.

If Δ is zero, then the three values of polynomial previous solutions are: $\lambda_1 = -\delta$ and a real double root $\lambda = (1/2)[-s(1-t)-t-l-m-f]$.

If Δ is negative, then the three values of polynomial previous solutions are $\lambda_3 = -\delta$ and two complex roots $\lambda_{i=1,2} = (1/2)[(-s(1-t)-t-l-m-f) \pm i\sqrt{\Delta}]$

All the real values are strictly negative, the model is stable.