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THE COMPLETE OR TOTAL COEFFICIENT OF AGGREGATE DEMAND ELASTICITY ON A SMOOTH ARC OF THE HYPER CURVE

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INTRODUCTION

The set

$$\Gamma = \left\{ P = [\vec{r}(t)] \in E^n \, : \, t \in [a,b] \right\}, \quad (\Gamma \subset E^n) \,, \tag{1}$$

where :

1.° t = scalar argument which takes values $t \in [a,b]$,

2.° $\vec{r} = \vec{r}(t) = \sum_{i=1}^{n} x_i(t)\,\vec{e}_i = \sum_{i=1}^{n} f_i(t)\,\vec{e}_i =$ position vector [1] of the (2)
moving point $P = [\vec{r}(t)] \in \Gamma$, i.e. vector function of the scalar argument t with the corresponding scalar equations in the parametric form, i.e.

$$\left. \begin{aligned} x_1 &= x_1(t) = f_1(t) \\ x_2 &= x_2(t) = f_2(t) \\ &\vdots \\ x_n &= x_n(t) = f_n(t) \end{aligned} \right\} , \tag{3}$$

3.° vector function (2) that is, its scalar equations in the parametric form (3) are differentiable on the closed interval $[a,b]$, and hence continuous on the closed interval $[a,b]$.

represents in $n-$ dimensional Euclidian space E^n the continuous arc of the curve (hyper curve).

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Let the vector funcion (2) be injective mapping . i.e.

$$(\forall t_1, t_2 \in [a,b])\ t_1 \neq t_2 \Longrightarrow P_1 = [\vec{r}(t_1)] \in \Gamma \neq P_2 = [\vec{r}(t_2)] \in \Gamma \qquad (4a)$$

and let the vector function (2) be surjective mapping, i.e.

$$(\forall P \in \Gamma)\, \exists t \in [a,b] : P = [\vec{r}(t)], \qquad (4b)$$

(4)

then it is said to be bijective mapping (one-to-one mapping).

The set (1) with the condition (4) represents in $n-$ dimensional Euclidian space, E^n , a continuous arc of the curve which does not intersect itself (Jordan curve).

Let vector function $\frac{d\vec{r}(t)}{dt}$ be continuous on segment $[a,b]$, i.e. let its scalar components

$$\frac{dx_1(t)}{dt} = f_1'(t), \frac{dx_2(t)}{dt} = f_2'(t), \ldots, \frac{dx_n(t)}{dt} = f_n'(t) \qquad (5a)$$

be continuous on $[a,b]$, and let

$$\frac{d\vec{r}(t)}{dt} \neq \vec{0} \Longleftrightarrow \sum_{i=1}^{n} [f_i'(t)]^2 \neq 0 \text{ za } (\forall t) t \in [a,b], \qquad (5b)$$

(i.e. let all derivatives not be simultaneously equal to zero for $t \in [a,b]$).

The Jordan curve, satisfying conditions (5a) and (5b), represents in $n-$ dimensional Euclidian space E^n a smooth arc of the curve Γ. The smooth arc of the curve Γ has at each point $P = [\vec{r}(t)] \in \Gamma$ a unique tangent which perpetually changes as point P moves along the curve Γ.

The movement of the point P, the components of which we regard as the price changes of n competitive products conditional to time t, along a smooth arc of the curve Γ will be the subject under discussion.

1. THE AVERAGE CHAIN INDEX ON A SMOOTH ARC OF THE HYPER CURVE

1.1. CONSTRUCTION OF THE INDEX

Let us compute the mean value for each scalar function (3) on the interval $[a,b]$, taking into consideration that these mean values may in general, but not necessarily, belong to the range of the corresponding scalar functions $x_i = f_i(t)$ $(i = 1, 2, \ldots, n)$. But the assumption that scalar functions (3) are differentiable on the interval $[a,b]$ hence continuous on the interval $[a,b]$ implies that the mean value of each scalar funtion $x_i = x_i(t)$ $(i = 1, 2, \ldots, n)$ belongs to its range. Hence, we may say that on the interval $[a,b]$ there are $\xi^{(i)} \in [a,b], (i = 1,2,\ldots,n)$ such that, for all $\xi^{(i)} \in [a,b]$, the functions $x_i = x_i(t)$ $(i = 1,2,\ldots,n)$ take their mean values which are

$$\left.\begin{aligned}
\bar{x}_1^{(\circ)} &= f_1[\xi^{(1)}] = \frac{\int_a^b f_1(t)dt}{b-a} = \frac{\int_a^b x_1(t)dt}{b-a}, \quad \xi^{(1)} \in [a,b] \\
\bar{x}_2^{(\circ)} &= f_2[\xi^{(2)}] = \frac{\int_a^b f_2(t)dt}{b-a} = \frac{\int_a^b x_2(t)dt}{b-a}, \quad \xi^{(2)} \in [a,b] \\
&\vdots \\
\bar{x}_n^{(\circ)} &= f_n[\xi^{(n)}] = \frac{\int_a^b f_n(t)dt}{b-a} = \frac{\int_a^b x_n(t)dt}{b-a}, \quad \xi^{(n)} \in [a,b]
\end{aligned}\right\}. \tag{6}$$

The mean values (6) represent the average prices of n competitive products in time interval $[a,b]$, and in that respect they are really generalisations of the averages we computed for m specified values of the scalar argument (time) t from interval $[a,b]$[2]. Note that, in addition to continuity the functions $x_i = x_i(t), (i = 1,2,\ldots,n)$ are non-negative and not equal to zero, which implies

$$\int_a^b x_i(t)dt > 0 \Longrightarrow \bar{x}_i^{(\circ)} > 0 \ ,(a < b) \ ,(i = 1,2,\ldots,n) \tag{6a}$$

The vector, whose components are given by the relation (6) represents the mean value of the vector function $t \mapsto \vec{r}(t)$ of the scalar argument $t \in [a,b]$, and we denote it in the following way

$$\bar{\vec{r}}(t) = \sum_{i=1}^{n} \frac{\int_a^b f_i(t)dt}{b-a}\, \vec{e}_i \ , \quad (\bar{\vec{r}}(t) \in \Gamma) \tag{7}$$

while the vector the components of which are the reciprocal values from (6) has the expression

$$\hat{\vec{r}}(t) = \sum_{i=1}^{n} \frac{1}{\bar{x}_i^{(\circ)}} \vec{e}_i = \sum_{i=1}^{n} \frac{b-a}{\int_a^b f_i(t)dt} \vec{e}_i \tag{8}$$

Let us note two positions of the moving point P on a smooth arc of the curve Γ: $P = [\vec{r}(t)] \in \Gamma$ and $P' = [\vec{r}(t + \Delta t)] \in \Gamma$ which correspond to two values of the scalar argument (time), $t \in [a, b]$ and $(t + \Delta t) \in [a, b]$, respectively.
The difference

$$\overrightarrow{PP'} = \vec{r}(t + \Delta t) - \vec{r}(t) = \Delta\rho_n(t)\ \vec{e}_n \tag{9}$$

wherein:

– $$\Delta\rho_n(t) = d(P, P') = |\ \vec{r}(t + \Delta t) - \vec{r}(t)\ | = \sqrt{\sum_{i=1}^{n} [x_i(t + \Delta t) - x_i(t)]^2} \tag{9a}$$

the distance between points P and P'.

– $\vec{e}_n =$ vector of the unit magnitude . (9b)

represents the increment of the vector function $\vec{r} = \vec{r}(t)$.
The vector equation (9) has the following corresponding scalar equations

$$\left.\begin{aligned} x_1(t + \Delta t) - x_1(t) &= \Delta\rho_n(t) \cos\alpha_1 \\ x_2(t + \Delta t) - x_2(t) &= \Delta\rho_n(t) \cos\alpha_2 \\ &\vdots \\ x_n(t + \Delta t) - x_n(t) &= \Delta\rho_n(t) \cos\alpha_n \end{aligned}\right\},\quad t \in [a, b] \tag{10}$$

or

$$\left.\begin{aligned} x_1(t + \Delta t) &= x_1(t) + \Delta\rho_n(t) \cos\alpha_1 \\ x_2(t + \Delta t) &= x_2(t) + \Delta\rho_n(t) \cos\alpha_2 \\ &\vdots \\ x_n(t + \Delta t) &= x_n(t) + \Delta\rho_n(t) \cos\alpha_n \end{aligned}\right\},\quad t \in [a, b]. \tag{11}$$

Multiplying separately each equation in expression (11) by the respective magnitudes,

$$\frac{1}{\bar{x}_i^{(o)}} = \frac{1}{\frac{\int_a^b x_i(t)dt}{b-a}} = \frac{b - a}{\int_a^b x_i(t)dt}\ ,\quad (i = 1, 2, \ldots, n)\ , \tag{12}$$

we obtain

$$\left.\begin{aligned}
\frac{x_1(t+\Delta t)}{\frac{\int_a^b x_1(t)dt}{b-a}} &= \frac{x_1(t)}{\frac{\int_a^b x_1(t)dt}{b-a}} + \Delta\rho_n(t)\frac{\cos\alpha_1}{\frac{\int_a^b x_1(t)dt}{b-a}} \\
&\vdots \\
\frac{x_n(t+\Delta t)}{\frac{\int_a^b x_n(t)dt}{b-a}} &= \frac{x_n(t)}{\frac{\int_a^b x_n(t)dt}{b-a}} + \Delta\rho_n(t)\frac{\cos\alpha_n}{\frac{\int_a^b x_n(t)dt}{b-a}}
\end{aligned}\right\} \tag{13}$$

while on summing of the equations (13) we have

$$(b-a)\sum_{i=1}^{n}\frac{x_i(t+\Delta t)}{\int_a^b x_i(t)dt} = (b-a)\sum_{i=1}^{n}\frac{x_i(t)}{\int_a^b x_i(t)dt} + (b-a)\Delta\rho_n(t)\sum_{i=1}^{n}\frac{\cos\alpha_i}{\int_a^b x_i(t)dt} \tag{14}$$

After dividing the equation (14) by the expression

$$(b-a)\sum_{i=1}^{n}\frac{x_i(t)}{\int_a^b x_i(t)dt},$$

we finally obtain

$$I_{tl}^{(\Gamma)} = \frac{\left[\sum_{i=1}^{n}\frac{x_i(t+\Delta t)}{\int_a^b x_i(t)dt}\right]}{\left[\sum_{i=1}^{n}\frac{x_i(t)}{\int_a^b x_i(t)dt}\right]} = 1 + \Delta\rho_n(t)\frac{\left[\sum_{i=1}^{n}\frac{\cos\alpha_i}{\int_a^b x_i(t)dt}\right]}{\left[\sum_{i=1}^{n}\frac{x_i(t)}{\int_a^b x_i(t)dt}\right]} \tag{15}$$

where $I_{tl}^{(\Gamma)}$ **is the average chain index defined on a smooth arc of the curve** Γ.

If in expression (15) a fixed value of scalar argument (time) t is given, and different increments Δt are given such that always $(t+\Delta t)\in[a,b]$, then relation (15) also represents the average base index. We shall henceforth regard the expression (15) as the average chain index, $I_{tl}^{(\Gamma)}$. In vector-analytical interpretation the expression $I_{tl}^{(\Gamma)}$ is also the ratio of the corresponding scalar products. It is easily shown. Namely

$$I_{tl}^{(\Gamma)} = \frac{\vec{r}(t+\Delta t)\cdot\hat{\vec{r}}(t)}{\vec{r}(t)\cdot\hat{\vec{r}}(t)}, \quad t\in[a,b] \tag{16}$$

$$I_{tl}^{(\Gamma)} = \frac{\sum_{i=1}^{n} x_i(t+\Delta t)\frac{b-a}{\int_a^b x_i(t)dt}}{\sum_{i=1}^{n} x_i(t)\frac{b-a}{\int_a^b x_i(t)dt}}, \quad t \in [a,b] \tag{17}$$

$$I_{tl}^{(\Gamma)} = \frac{\sum_{i=1}^{n} \frac{x_i(t+\Delta t)}{\int_a^b x_i(t)dt}}{\sum_{i=1}^{n} \frac{x_i(t)}{\int_a^b x_i(t)dt}}, \qquad t \in [a,b]. \tag{18}$$

Note that expression (16) can also be written in the following form

$$I_{tl}^{(\Gamma)} = \frac{\vec{r}(t+\Delta t)\cdot\hat{\vec{r}}(t)}{\vec{r}(t)\cdot\hat{\vec{r}}(t)} \tag{19a}$$

$$= \frac{|\,\vec{r}(t+\Delta t)\,|\,|\,\hat{\vec{r}}(t)\,|\ \cos\angle(\vec{r}(t+\Delta t),\hat{\vec{r}}(t))}{|\,\vec{r}(t)\,|\,|\,\hat{\vec{r}}(t)\,|\ \cos\angle(\vec{r}(t),\hat{\vec{r}}(t))} \tag{19b}$$

$$= \frac{proj_{\hat{\vec{r}}(t)}\vec{r}(t+\Delta t)}{proj_{\hat{\vec{r}}(t)}\vec{r}(t)}, \tag{19c}$$

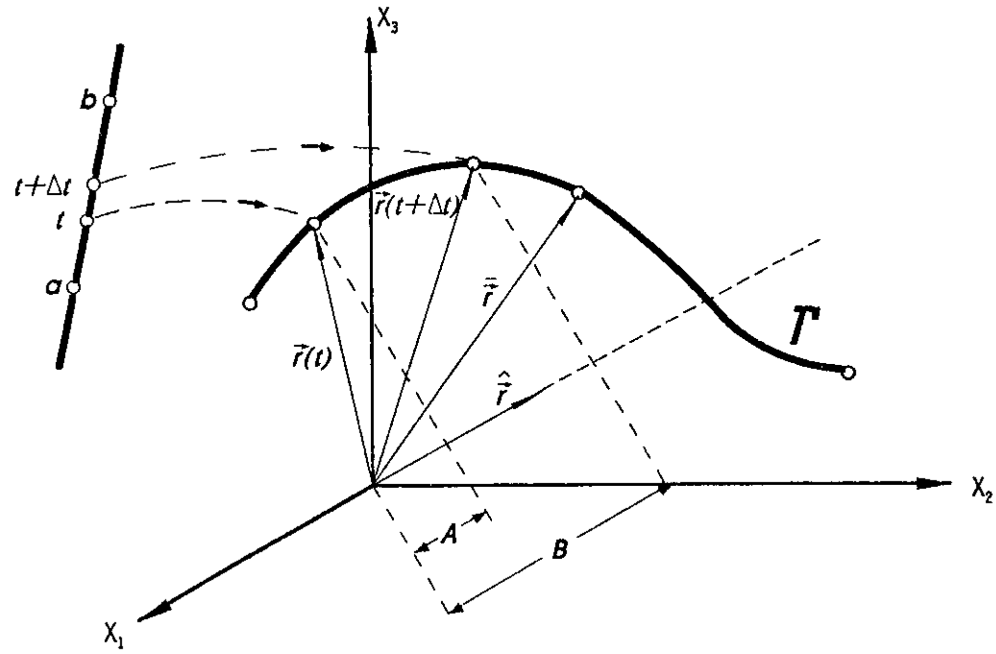
i.e.

$$I_{tl}^{(\Gamma)} = \frac{B}{A}, \qquad \begin{pmatrix} proj_{\hat{\vec{r}}(t)}\vec{r}(t+\Delta t) & = B \\ proj_{\hat{\vec{r}}(t)}\vec{r}(t) & = A \end{pmatrix}^{3)} . \tag{19d}$$

Based on (15) and (19d) the extended equation is valid

$$I_{tl}^{(\Gamma)} = \frac{\left[\sum_{i=1}^{n} \frac{x_i(t+\Delta t)}{\int_a^b x_i(t)dt}\right]}{\left[\sum_{i=1}^{n} \frac{x_i(t)}{\int_a^b x_i(t)dt}\right]} = 1 + \Delta\rho_n(t)\frac{\left[\sum_{i=1}^{n} \frac{\cos\alpha_i}{\int_a^b x_i(t)dt}\right]}{\left[\sum_{i=1}^{n} \frac{x_i(t)}{\int_a^b x_i(t)dt}\right]} = \frac{B}{A}, \tag{20}$$

based on which the geometrical interpretation of the average chain index $I_{tl}^{(\Gamma)}$, which is defined on a smooth arc of the curve Γ, is clear. In Fig. 1 the interpretation of $I_{tl}^{(\Gamma)}$ in three dimensional Euclidian space is illustrated . It is the ratio of the projections of vectors $\vec{r}(t+\Delta t)$ and $\vec{r}(t)$ onto vector $\hat{\vec{r}}$.



$$A = proj_{\hat{\vec{r}}}\vec{r}(t)$$

$$B = proj_{\hat{\vec{r}}}\vec{r}(t+\Delta t)$$

Figure 1.

Let us note here that the similarity of this price index to Laspeyres' and Paasche's price index, as well as to the price index with **typical year** quantity weights, lies in the same "logic" of projecting different period price vectors onto one fixed **support** vector. The role of **support** vector in Laspeyres' price index is played by the vector whose components are base period (year) quantity weights; in Paasche's price index by the vector with given period (year) quantity weights, and in the **typical year method** by the vector with typical year quantity weights. Unlike the aforementioned indexes, the **support** vector role in the average chain index, $I_{tl}^{(\Gamma)}$ is played by expression (8). The rate of change, i.e. the relative change based on the expression (20) has the form

$$I_{tl}^{(\Gamma)}-1=\frac{\left[\sum_{i=1}^{n}\frac{x_i(t+\Delta t)}{\int_a^b x_i(t)dt}\right]}{\left[\sum_{i=1}^{n}\frac{x_i(t)}{\int_a^b x_i(t)dt}\right]}-1=\Delta\rho_n(t)\frac{\left[\sum_{i=1}^{n}\frac{\cos\alpha_i}{\int_a^b x_i(t)dt}\right]}{\left[\sum_{i=1}^{n}\frac{x_i(t)}{\int_a^b x_i(t)dt}\right]}=\frac{B-A}{A}, \tag{21}$$

which represents the relative change of the average price level of homogenous (competitive) products in relation to their average level from the period immediately preceding the given period.

1. 2. THE PROBLEM OF WEIGHTS WITH THE AVERAGE CHAIN INDEX. $I_{tl}^{(\Gamma)}$

We shall begin the analysis and clarification of this problem with a brief survey of some known indexes. Simple aggregate price index has the formula

$$\frac{\sum_{i=1}^{n} x_{it}}{\sum_{i=1}^{n} x_{io}} \tag{22}$$

wherein:

- $\sum_{i=1}^{n} x_{it}$ = sum of corresponding. n, commodity prices in the given or current year and
- $\sum_{i=1}^{n} x_{io}$ = sum of corresponding. n, commodity prices in the base or reference year.

Spiegel (see [2],p. 317) states two disadvantages which make a simple aggregate price index unfit for practical purposes:

1) "It does not take into account the relative importance of the various commodities. Thus, according to this method, equal weight or importance would be attached to milk and shaving cream in computing a cost of living index".

2) "The particular units used in price quotations, such as gallons, bushels, pounds, etc., affect the value of the index."

These disadvantages have been overcome by weighting the price of each commodity in expression (22) by its quantity. Depending on whether base given or typical year quantity weights (typical year method, see [2], p. 318) are used, we have:

a) Laspeyres' index or base year method,

b) Paasche's index or given year method,

c) The typical year method which is really a generalisation of the weighting options [4]. Thus, by this method, under the assumption that only changes of prices $x_1(t)$, $x_2(t), \ldots, x_n(t)$, (on a smooth arc of the hyper curve Γ) of n competitive goods are observed, we have

$$\frac{\sum_{i=1}^{n} x_i(t+\Delta t) q_{iT}}{\sum_{i=1}^{n} x_i(t) q_{iT}} = \left\{ \begin{array}{l} \text{Weighted aggregate price index with} \\ \text{typical year quantity weights.} \end{array} \right. \tag{23}$$

where $(t, T \in [a,b])$, $t \neq T$.

The average chain index, $I_{tl}^{(\Gamma)}$ and its form of appearance (15) do not reveal the presence of any form of weighting. Nevertheless, the weighting is included implicitly in this index. The form of the weighting and the conditions of its validity will be demonstrated by proving the following theorem. Thus, we shall more precisely define the position of the average chain index in relation to the typical year method.

<u>Theorem.</u> *If the incomes of n competitors, based on the average quantities $\bar{q}_1, \bar{q}_2, \ldots, \bar{q}_n$ from time interval $[a, b]$ and the average prices $\bar{x}_1^{(\circ)}, \bar{x}_2^{(\circ)}, \ldots, \bar{x}_n^{(\circ)}$ from the same time interval, are equal, i.e.*

$$\bar{q}_1 \cdot \bar{x}_1^{(\circ)} = \bar{q}_2 \cdot \bar{x}_2^{(\circ)} = \ldots = \bar{q}_n \cdot \bar{x}_n^{(\circ)} \tag{24}$$

then the average chain index, defined on a smooth arc of the hyper curve Γ *, assumes the form of a weighted aggregate price index where the weights are based on the average quantities from the time interval* $[a,b]$ *, i.e.*

$$\text{for } (\forall t)\ (t\in[a,b]) \implies I_{tl}^{(\Gamma)} = \frac{\sum_{i=1}^{n} x_i(t+\Delta t)\cdot \bar{q}_i}{\sum_{i=1}^{n} x_i(t)\cdot \bar{q}_i} \tag{24a}$$

Proof. The mean values (6) can also be expressed as

$$\left.\begin{array}{l} \bar{x}_1^{(\circ)} = \dfrac{\int_a^b x_1(t)dt}{b-a} = \dfrac{\bar{q}_1\cdot\int_a^b x_1(t)dt}{\bar{q}_1\cdot(b-a)} = \dfrac{L_1}{M_1} \\ \bar{x}_2^{(\circ)} = \dfrac{\int_a^b x_2(t)dt}{b-a} = \dfrac{\bar{q}_2\cdot\int_a^b x_2(t)dt}{\bar{q}_2\cdot(b-a)} = \dfrac{L_2}{M_2} \\ \vdots \\ \bar{x}_n^{(\circ)} = \dfrac{\int_a^b x_n(t)dt}{b-a} = \dfrac{\bar{q}_n\cdot\int_a^b x_n(t)dt}{\bar{q}_n\cdot(b-a)} = \dfrac{L_n}{M_n} \end{array}\right\} \iff \bar{x}_i^{(\circ)} = \frac{L_i}{M_i},\ (i=1,2,\ldots,n) \tag{25}$$

where:

– $\bar{q}_i$ = average quantity of the $i-th$ competitive good from time interval $[a,b]$. (25a)

– $L_i = \bar{q}_i\cdot\int_a^b x_i(t)dt\ ,\ (i=1,2,\ldots,n)$ (25b)

– $M_i = \bar{q}_i\cdot(b-a)\ ,\ (i=1,2,\ldots,n).$ (25c)

Now we have

$$I_{tl}^{(\Gamma)} = \frac{\left[\sum_{i=1}^{n}\frac{x_i(t+\Delta t)}{\bar{x}_i^{(\circ)}}\right]}{\left[\sum_{i=1}^{n}\frac{x_i(t)}{\bar{x}_i^{(\circ)}}\right]} = \frac{\left[\sum_{i=1}^{n}\frac{x_i(t+\Delta t)\cdot M_i}{L_i}\right]}{\left[\sum_{i=1}^{n}\frac{x_i(t)\cdot M_i}{L_i}\right]}\ , \tag{26}$$

or

$$I_{tl}^{(\Gamma)} = \frac{\dfrac{x_1(t+\Delta t)\cdot\bar{q}_1\cdot(b-a)\left(\prod\limits_{i=2}^{n} L_i\right) + \ldots + x_n(t+\Delta t)\cdot\bar{q}_n\cdot(b-a)\left(\prod\limits_{i=1}^{n-1} L_i\right)}{\prod\limits_{i=1}^{n} L_i}}{\dfrac{x_1(t)\cdot\bar{q}_1\cdot(b-a)\left(\prod\limits_{i=2}^{n} L_i\right) + \ldots + x_n(t)\cdot\bar{q}_n\cdot(b-a)\left(\prod\limits_{i=1}^{n-1} L_i\right)}{\prod\limits_{i=1}^{n} L_i}}, \qquad (27)$$

or

$$I_{tl}^{(\Gamma)} = \frac{x_1(t+\Delta t)\cdot\bar{q}_1\cdot\left(\prod\limits_{i=2}^{n} L_i\right) + \ldots + x_n(t+\Delta t)\cdot\bar{q}_n\cdot\left(\prod\limits_{i=1}^{n-1} L_i\right)}{x_1(t)\cdot\bar{q}_1\cdot\left(\prod\limits_{i=2}^{n} L_i\right) + \ldots + x_n(t)\cdot\bar{q}_n\cdot\left(\prod\limits_{i=1}^{n-1} L_i\right)} . \qquad (28)$$

After dividing the numerator and the denominator in expression (28) by the expression, say $\prod\limits_{i=2}^{n} L_i$, the relevance of

$$I_{tl}^{(\Gamma)} = \frac{x_1(t+\Delta t)\cdot\bar{q}_1 + x_2(t+\Delta t)\cdot\bar{q}_2\cdot\left(\frac{L_1}{L_2}\right) + \ldots + x_n(t+\Delta t)\cdot\bar{q}_n\cdot\left(\frac{L_1}{L_n}\right)}{x_1(t)\cdot\bar{q}_1 + x_2(t)\cdot\bar{q}_2\cdot\left(\frac{L_1}{L_2}\right) + \ldots + x_n(t)\cdot\bar{q}_n\cdot\left(\frac{L_1}{L_n}\right)}, \qquad (29)$$

and taking into consideration that

$$\left.\begin{aligned} \frac{L_1}{L_2} &= \frac{\bar{q}_1\cdot\int_a^b x_1(t)\,dt}{\bar{q}_2\cdot\int_a^b x_2(t)\,dt} = \frac{\bar{q}_1\cdot\bar{x}_1^{(\circ)}}{\bar{q}_2\cdot\bar{x}_2^{(\circ)}} = \eta_2 \\ &\vdots \\ \frac{L_1}{L_n} &= \frac{\bar{q}_1\cdot\int_a^b x_1(t)\,dt}{\bar{q}_n\cdot\int_a^b x_n(t)\,dt} = \frac{\bar{q}_1\cdot\bar{x}_1^{(\circ)}}{\bar{q}_n\cdot\bar{x}_n^{(\circ)}} = \eta_n \end{aligned}\right\}, \qquad (30)$$

then substituing (30) in (29), we finally obtain

$$I_{tl}^{(\Gamma)} = \frac{x_1(t+\Delta t)\cdot\bar{q}_1 + \left[\sum_{i=2}^{n} x_i(t+\Delta t)\cdot\bar{q}_i\cdot\eta_i\right]}{x_1(t)\cdot\bar{q}_1 + \left[\sum_{i=2}^{n} x_i(t)\cdot\bar{q}_i\cdot\eta_i\right]} \quad , \qquad t\in[a,b]. \tag{31}$$

Expression (31) is equivalent to expression (15). It represents another form of its appearance. Based on the assumption of the theorem we have

$$\bar{q}_1\cdot\bar{x}_1^{(\circ)} = \bar{q}_2\cdot\bar{x}_2^{(\circ)} = \ldots = \bar{q}_n\cdot\bar{x}_n^{(\circ)}$$

implying $\eta_i = 1$, $(i=2,3,\ldots,n)$ so that expression (31) assumes the form (24a), i.e.

$$I_{tl}^{(\Gamma)} = \frac{\sum_{i=1}^{n} x_i(t+\Delta t)\cdot\bar{q}_i}{\sum_{i=1}^{n} x_i(t)\cdot\bar{q}_i} \quad , \qquad t\in[a,b] \tag{32}$$

which was the purpose of the proof.

Let us note that even more general formulation of the preceding theorem is valid (it can be easily proven):

The competitive goods vector, $(q_{1T}, q_{2T}, \ldots, q_{nT})$, whose components specify the quantities of each competitive good from the time interval $[a,b]$, satisfying the condition

$$q_{1T}\cdot\bar{x}_1^{(\circ)} = q_{2T}\cdot\bar{x}_2^{(\circ)} = \ldots = q_{nT}\cdot\bar{x}_n^{(\circ)} \quad , \qquad T\in[a,b] \tag{32a}$$

determines a form of weighting the average chain index.

Therefore, if condition (24) is not satisfied, it still does not mean that the quantities $q_{iT}, (i=1,2,\ldots,n), T\in[a,b]$ which could satisfy the more general condition (32a) do not exist. If, for instance, this was the case with the competitive goods vector, $(q_{1\circ}, q_{2\circ}, \ldots, q_{n\circ})$, the components of which specify the quantities of each competitive good from the base period (the beginning of interval $[a,b]$), then the average chain index, $I_{tl}^{(\Gamma)}$ would assume a form of Laspeyres' price index, i.e.

$$\text{for}(\forall t)(t \in [a,b]) \Longrightarrow I_{tl}^{(\Gamma)} = \frac{\sum_{i=1}^{n} x_i(t+\Delta t)\cdot q_{io}}{\sum_{i=1}^{n} x_i(t)\cdot q_{io}} = \text{Laspeyres' price index.}$$

1. 3. THE INFINITESIMAL FORM OF THE AVERAGE CHAIN INDEX WHICH IS DEFINED ON A SMOOTH ARC OF THE CURVE Γ

The vector equation of the tangent to a smooth arc of the curve (hyper curve) Γ at point $P = [\vec{r}(t)] \in \Gamma$ has the form

$$\vec{v}(t+\Delta t) = \vec{r}(t) + \frac{d\vec{r}(t)}{dt}\Delta t \quad , \qquad (\Delta t \equiv dt) \quad , \qquad t \in [a,b] \tag{33}$$

which can also be given the following form

$$\vec{v}(t+\Delta t) = \vec{r}(t) + \vec{u}(t)\cdot ds_n \ , \qquad t \in [a,b] \tag{34}$$

wherein :

– $\vec{u}(t) = \dfrac{\frac{d\vec{r}(t)}{dt}}{\left|\frac{d\vec{r}(t)}{dt}\right|}$ = unit tangent vector, i.e. the vector of unit length in the direction of the tangent to a smooth arc of the curve Γ at point $P = [\vec{r}(t)] \in \Gamma$,

– $ds_n = \sqrt{\sum_{i=1}^{n}\left[\frac{dx_i(t)}{dt}\right]^2} dt = \sqrt{\sum_{i=1}^{n}[dx_i]^2} =$ { infinitesimal tangential distance, i.e. the length of each infinitesimal segment of the curve Γ in the direction of the tangent to a smooth arc of the curve Γ at point $P = [\vec{r}(t)] \in \Gamma$. (34a)

Let us start from the equation

$$\vec{v}(t+\Delta t) = \vec{r}(t+\Delta t) \ , \tag{35}$$

which is satisfied only for **infinitesimal increments of the scalar argument (parameter)** t. Expression (34) can now be written as

$$\vec{r}(t+\Delta t) = \vec{r}(t) + \vec{u}(t) \cdot ds_n \tag{36}$$

and is also valid only for infinitesimal increments of the parameter t. The scalar expression of equation (36) is

$$\left.\begin{aligned} x_1(t+\Delta t) &= x_1(t) + \cos\beta_1 \cdot ds_n \\ x_2(t+\Delta t) &= x_2(t) + \cos\beta_2 \cdot ds_n \\ &\vdots \\ x_n(t+\Delta t) &= x_n(t) + \cos\beta_n \cdot ds_n \end{aligned}\right\} , \quad t \in [a,b] \tag{37}$$

while

$$\cos\beta_i := \frac{\dfrac{dx_i(t)}{dt}}{\sqrt{\sum\limits_{i=1}^{n}\left[\dfrac{dx_i(t)}{dt}\right]^2}} = \frac{dx_i}{\sqrt{\sum\limits_{i=1}^{n}[dx_i]^2}} = \frac{dx_i}{ds_n}, \quad (i=1,2,\ldots,n), \quad t \in [a,b]. \tag{38}$$

Arranging equations (37) as done to equations (11) we obtain the expression, a form which resembles relation (15), i.e.

$$\frac{\left[\sum\limits_{i=1}^{n} \dfrac{x_i(t+\Delta t)}{\int_a^b x_i(t)dt}\right]}{\left[\sum\limits_{i=1}^{n} \dfrac{x_i(t)}{\int_a^b x_i(t)dt}\right]} \approx 1 + ds_n \frac{\left[\sum\limits_{i=1}^{n} \dfrac{\cos\beta_i}{\int_a^b x_i(t)dt}\right]}{\left[\sum\limits_{i=1}^{n} \dfrac{x_i(t)}{\int_a^b x_i(t)dt}\right]} = MI_{tl}^{(\Gamma)} \qquad t \in [a,b] \tag{39}$$

yet its validity, unlike expression (15), is constrained to inifnitesimal increments of the parameter t. Expression (39) which we term **the inifinitesimal form of the average chain index**, is defined on a smooth arc of curve Γ. From expression (39) we obtain the infinitesimal rate of change, i.e. the infinitesimal relative change

$$\frac{\left[\sum_{i=1}^{n} \frac{x_i(t+\Delta t)}{\int_a^b x_i(t)\,dt}\right]}{\left[\sum_{i=1}^{n} \frac{x_i(t)}{\int_a^b x_i(t)\,dt}\right]} - 1 \approx ds_n \frac{\left[\sum_{i=1}^{n} \frac{\cos\beta_i}{\int_a^b x_i(t)dt}\right]}{\left[\sum_{i=1}^{n} \frac{x_i(t)}{\int_a^b x_i(t)dt}\right]} = MI_{tl}^{(\Gamma)} - 1\,, \qquad t \in [a,b] \tag{40}$$

2. INDEX OF REAL INCOME FROM WHICH DEMAND FOR COMPETITIVE PRODUCTS IS FINANCED

2.1. CONSTRUCTION OF THE INDEX ON A SMOOTH ARC OF THE CURVE $\hat{\Gamma}$

Given a set

$$\hat{\Gamma} = \{P = [\vec{r}(t)] \in E^{n+1} : t \in [a,b]\} \quad , \qquad \left(\hat{\Gamma} \subset E^{n+1}\right) \tag{41}$$

which has properties of a smooth arc of the curve (hyper curve) $\hat{\Gamma}$ in $(n+1)-$ dimensional Euclidian space , E^{n+1} . The increment of the vector function $\vec{r}(t)$ between points $P = [\vec{r}(t)] \in \hat{\Gamma}$ and $P' = [\vec{r}(t+\Delta t)] \in \hat{\Gamma}$ is

$$\vec{r}(t+\Delta t) - \vec{r}(t) = \Delta\rho_{n+1}(t) \cdot \vec{e}_{n+1}. \tag{42}$$

From equation (42) we have

$$\vec{r}(t+\Delta t) = \vec{r}(t) + \Delta\rho_{n+1}(t) \cdot \vec{e}_{n+1} \tag{43}$$

where :

– $\Delta\rho_{n+1}(t) = \sqrt{\sum_{i=1}^{n+1} [x_i(t+\Delta t) - x_i(t)]^2}$, (43a)

– $\vec{e}_{n+1} =$ unit vector .

Vector equation (43) has the following scalar equations in the parametric form (the component functions)

$$\left.\begin{aligned} x_1(t+\Delta t) &= x_1(t) + \Delta\rho_{n+1}(t)\cos\alpha_1 \\ x_2(t+\Delta t) &= x_2(t) + \Delta\rho_{n+1}(t)\cos\alpha_2 \\ &\vdots \\ x_n(t+\Delta t) &= x_n(t) + \Delta\rho_{n+1}(t)\cos\alpha_n \\ x_{n+1}(t+\Delta t) &= x_{n+1}(t) + \Delta\rho_{n+1}(t)\cos\alpha_{n+1} \end{aligned}\right\}, \qquad t\in[a,b]. \tag{44}$$

The first n equations from relation (44) we regard as the price changes of n competitive goods, while the last, $(n+1)-th$ equation, we regard as reflecting the changes of a nominal income from which aggregate demand for competitive products is financed. We obtain, on dividing $(n+1)-th$ equation (in which the variable $x_{n+1}(t)$ represents nominal income as a function of the time) in the system (44) by $x_{n+1}(t)$, the following

$$\frac{x_{n+1}(t+\Delta t)}{x_{n+1}(t)} = 1 + \Delta\rho_{n+1}(t)\frac{\cos\alpha_{n+1}}{x_{n+1}(t)}, \qquad t\in[a,b] \tag{45}$$

The left side of the equation (45) is clear in meaning. It is the index of a nominal income from which demand for the subtitutes is financed. We shall denote it by $I_{nd}^{(\hat{\Gamma})}$ emphasizing by superscript that the changes of a nominal income are observed on a smooth arc of the hyper curve $\hat{\Gamma}$. Now we can write (45) as

$$I_{nd}^{(\hat{\Gamma})} = 1 + \Delta\rho_{n+1}(t)\frac{\cos\alpha_{n+1}}{x_{n+1}(t)}, \qquad t\in[a,b] \tag{46}$$

If, on the other hand, we arrange the first n equations in system (44) as done to equations (11) , we obtain the expression

$$I_{tl}^{(\hat{\Gamma})} = \frac{\left[\sum\limits_{i=1}^{n}\frac{x_i(t+\Delta t)}{\int_a^b x_i(t)dt}\right]}{\left[\sum\limits_{i=1}^{n}\frac{x_i(t)}{\int_a^b x_i(t)dt}\right]} = 1 + \Delta\rho_{n+1}(t)\cdot\frac{\left[\sum\limits_{i=1}^{n}\frac{\cos\alpha_i}{\int_a^b x_i(t)dt}\right]}{\left[\sum\limits_{i=1}^{n}\frac{x_i(t)}{\int_a^b x_i(t)dt}\right]}, \quad t\in[a,b] \tag{47}$$

which represents the average chain index defined on a smooth arc of the curve $\hat{\Gamma}$. Finally, dividing equation (46) by the equation (47) we obtain the index of real income from which aggregate demand for the competitive products is financed. This index is defined on a smooth arc of the curve $\hat{\Gamma}$, i.e.

$$I_{rd}^{(\hat{\Gamma})} = \frac{I_{nd}^{(\hat{\Gamma})}}{I_{tl}^{(\hat{\Gamma})}} = \frac{1 + \Delta\rho_{n+1}(t)\dfrac{\cos\alpha_{n+1}}{x_{n+1}(t)}}{1 + \Delta\rho_{n+1}(t)\cdot\dfrac{\left[\sum\limits_{i=1}^{n}\dfrac{\cos\alpha_i}{\int_a^b x_i(t)dt}\right]}{\left[\sum\limits_{i=1}^{n}\dfrac{x_i(t)}{\int_a^b x_i(t)dt}\right]}}, \quad t\in[a,b] \tag{48}$$

The rate of change, i.e. the relative change based on expression (48), after arranging, has the expression

$$I_{rd}^{(\hat{\Gamma})} - 1 = \frac{I_{nd}^{(\hat{\Gamma})}}{I_{tl}^{(\hat{\Gamma})}} - 1 = \frac{\Delta\rho_{n+1}(t)\left\{\dfrac{\cos\alpha_{n+1}}{x_{n+1}(t)} - \dfrac{\left[\sum\limits_{i=1}^{n}\dfrac{\cos\alpha_i}{\int_a^b x_i(t)dt}\right]}{\left[\sum\limits_{i=1}^{n}\dfrac{x_i(t)}{\int_a^b x_i(t)dt}\right]}\right\}}{1 + \Delta\rho_{n+1}(t)\cdot\dfrac{\left[\sum\limits_{i=1}^{n}\dfrac{\cos\alpha_i}{\int_a^b x_i(t)dt}\right]}{\left[\sum\limits_{i=1}^{n}\dfrac{x_i(t)}{\int_a^b x_i(t)dt}\right]}}, \quad t\in[a,b] \tag{49}$$

2.2. INFINITESIMAL OR MARGINAL FORM OF $I_{rd}^{(\hat{\Gamma})}$ INDEX WHICH IS DEFINED ON A SMOOTH ARC OF THE HYPER CURVE $\hat{\Gamma}$

Under this index we understand the approximation the $I_{rd}^{(\hat{\Gamma})}$ index, which is valid only for the infinitesimal increments of the scalar argument (time) t. Since derivation of this index is analogous to the procedure which was implemented to obtain $I_{rd}^{(\hat{\Gamma})}$, provided we start from the scalar equations (component functions) of the corresponding vector equation of a tangent to a smooth arc of the hyper curve $\hat{\Gamma}$ at point $P = [\vec{r}(t)] \in \hat{\Gamma} \subset E^{n+1}$, we shall give only its final form

$$MI_{rd}^{(\hat{\Gamma})} = \frac{MI_{nd}^{(\hat{\Gamma})}}{MI_{tl}^{(\hat{\Gamma})}} = \frac{1 + ds_{n+1}\dfrac{\cos\beta_{n+1}}{x_{n+1}(t)}}{1 + ds_{n+1}\cdot\dfrac{\left[\sum\limits_{i=1}^{n}\dfrac{\cos\beta_i}{\int_a^b x_i(t)dt}\right]}{\left[\sum\limits_{i=1}^{n}\dfrac{x_i(t)}{\int_a^b x_i(t)dt}\right]}}, \quad t \in [a, b] \tag{50}$$

In expression (50) we have :

– $MI_{rd}^{(\hat{\Gamma})}$ = marginal (approximate) form of $I_{rd}^{(\hat{\Gamma})}$ index.

– $MI_{nd}^{(\hat{\Gamma})} = 1 + ds_{n+1}\frac{\cos\beta_{n+1}}{x_{n+1}(t)}$ = marginal form of $I_{nd}^{(\hat{\Gamma})}$ index. (50a)

– $$MI_{tl}^{(\hat{\Gamma})} = 1 + ds_{n+1}\cdot\frac{\left[\sum\limits_{i=1}^{n}\dfrac{\cos\beta_i}{\int_a^b x_i(t)dt}\right]}{\left[\sum\limits_{i=1}^{n}\dfrac{x_i(t)}{\int_a^b x_i(t)dt}\right]} \text{ = marginal form of } I_{tl}^{(\hat{\Gamma})} \text{ index,} \tag{50b}$$

$$- \quad ds_{n+1} = \sqrt{\sum_{i=1}^{n+1}\left[\frac{dx_i(t)}{dt}\right]^2} dt = \sqrt{\sum_{i=1}^{n+1}[dx_i]^2} = \left\{ \begin{array}{l} \text{infinitesimal tangential} \\ \text{distance, i.e. the length} \\ \text{of each infinitesimal} \\ \text{segment of the curve } \hat{\Gamma} \\ \text{in direction of the} \\ \text{tangent to a smooth arc} \\ \text{of the curve } \hat{\Gamma} \text{ at point} \\ P = [\vec{r}(t)] \in \hat{\Gamma}. \end{array} \right. \tag{50c}$$

$$- \quad \cos\beta_i = \frac{\frac{dx_i(t)}{dt}}{\sqrt{\sum_{i=1}^{n+1}[\frac{dx_i(t)}{dt}]^2}} = \frac{dx_i}{\sqrt{\sum_{i=1}^{n+1}[dx_i]^2}} = \frac{dx_i}{ds_{n+1}}, \quad \begin{pmatrix} i = 1, 2, \ldots, n+1 \\ t \in [a, b] \end{pmatrix}. \tag{50d}$$

3. CONSTRUCTION OF THE COMPLETE OR TOTAL COEFFICIENT OF AGGREGATE DEMAND ELASTICITY ON A SMOOTH ARC OF THE HYPER CURVE $\hat{\Gamma}$

Here we want to answer how in terms of elasticity, the aggregate demand for competitive products responds not only to simultaneous changes of the prices $x_1(t), x_2(t), \ldots, x_n(t)$ of these substitutes but also to nominal income $x_{n+1}(t)$ changes, from which the aggregate demand is financed. The assumption is that changes of all variables $x_1(t), x_2(t), \ldots,$-$x_n(t), x_{n+1}(t)$ occur on a smooth arc of the hyper curve $\hat{\Gamma}$; therefore a differentiable scalar function (the aggregate demand) of the position vector of a moving point $P = [\vec{r}(t)] \in \hat{\Gamma}$ is defined over a smooth arc of the hyper curve $\hat{\Gamma}$, that is

$$\left. \begin{aligned} q &= f(P) = f[\vec{r}(t)] \\ &= f(x_1, x_2, \ldots, x_n, x_{n+1}) \\ &= f[x_1(t), x_2(t), \ldots, x_n(t), x_{n+1}(t)] \\ q &= F(t) \end{aligned} \right\} . \tag{51}$$

The requirement is that the relative change of demand (51) must become a reflection of the relative change of the magnitude that synthesizes in itself the simultaneous changes of prices

$x_1(t), x_2(t), \ldots, x_n(t)$ and nominal income $x_{n+1}(t)$. That synthetic magnitude is defined by expression (48), i.e. (49). We define the complete or total coefficient of aggregate demand elasticity on a smooth arc of the hyper curve $\hat{\Gamma}$, denoted by $E^{(\hat{\Gamma})}_{q,(x_1,x_2,\ldots,x_{n+1})}$; by the expression

$$E^{(\hat{\Gamma})}_{q,(x_1,x_2,\ldots,x_{n+1})} = \frac{\dfrac{\Delta q(t)}{q(t)}}{I^{(\hat{\Gamma})}_{rd} - 1} = \frac{\dfrac{\Delta q(t)}{q(t)}}{\dfrac{I^{(\hat{\Gamma})}_{nd}}{I^{(\hat{\Gamma})}_{tl}} - 1}, \tag{52}$$

or at a point

$$E^{(\hat{\Gamma})}_{q,(x_1,x_2,\ldots,x_{n+1})} = \lim_{\Delta\rho_{n+1}(t)\to 0} \left\{ I^{(\hat{\Gamma})}_{tl} \cdot \frac{1}{q(t)} \cdot \frac{\Delta q(t)}{\left(I^{(\hat{\Gamma})}_{nd} - I^{(\hat{\Gamma})}_{tl}\right)} \right\}. \tag{53}$$

In expression (53) the difference, $I^{(\hat{\Gamma})}_{nd} - I^{(\hat{\Gamma})}_{tl}$ is obtained by substracting (47) from (46), therefore

$$I^{(\hat{\Gamma})}_{nd} - I^{(\hat{\Gamma})}_{tl} = \Delta\rho_{n+1}(t) \cdot \underbrace{\left\{ \frac{\cos\alpha_{n+1}}{x_{n+1}(t)} - \frac{\left[\sum\limits_{i=1}^{n} \dfrac{\cos\alpha_i}{\int_a^b x_i(t)dt}\right]}{\left[\sum\limits_{i=1}^{n} \dfrac{x_i(t)}{\int_a^b x_i(t)dt}\right]} \right\}}_{Q} = \Delta\rho_{n+1}(t) \cdot Q, \tag{54}$$

so that on substituting (54) into (53) we can write

$$E^{(\hat{\Gamma})}_{q,(x_1,x_2,\ldots,x_{n+1})} = \left\{ \lim_{\Delta\rho_{n+1}(t)\to 0} I^{(\hat{\Gamma})}_{tl} \right\} \cdot \left\{ \lim_{\Delta\rho_{n+1}(t)\to 0} \frac{1}{Q} \right\} \cdot \frac{1}{q(t)} \cdot \left\{ \lim_{\Delta\rho_{n+1}(t)\to 0} \frac{\Delta q(t)}{\Delta\rho_{n+1}(t)} \right\}. \tag{55}$$

Let us analyse each process of limit contained in expression (55) . Firstly, we shall find the limit of

$$\lim_{\Delta\rho_{n+1}(t)\to 0}\frac{\Delta q(t)}{\Delta\rho_{n+1}(t)}.$$

Based on the formula for the finite increment of a function, the increment of function (51) can be expressed by

$$\Delta q(t)=\sum_{i=1}^{n+1}\frac{\partial f(P)}{\partial x_i}\cdot\Delta x_i(t)+\sum_{i=1}^{n+1}\varepsilon_i\Delta x_i(t)\quad , \tag{56}$$

which on dividing by $\Delta\rho_{n+1}(t)$ has the expression

$$\frac{\Delta q(t)}{\Delta\rho_{n+1}(t)}=\sum_{i=1}^{n+1}\frac{\partial f(P)}{\partial x_i}\cdot\frac{\Delta x_i(t)}{\Delta\rho_{n+1}(t)}+\sum_{i=1}^{n+1}\frac{\varepsilon_i\Delta x_i(t)}{\Delta\rho_{n+1}(t)}\quad , \tag{57}$$

or

$$\frac{\Delta q(t)}{\Delta\rho_{n+1}(t)}=\sum_{i=1}^{n+1}\frac{\partial f(P)}{\partial x_i}\cdot\cos\alpha_i+\sum_{i=1}^{n+1}\frac{\varepsilon_i\Delta x_i(t)}{\Delta\rho_{n+1}(t)}\quad , \tag{58}$$

where

$$\cos\alpha_i=\frac{\Delta x_i(t)}{\Delta\rho_{n+1}(t)}=\frac{x_i(t+\Delta t)-x_i(t)}{\sqrt{\sum\limits_{i=1}^{n+1}[x_i(t+\Delta t)-x_i(t)]^2}}\ ,\quad (i=1,2,\ldots,n+1). \tag{59}$$

Since the scalar functions $x_i = x_i(t), (i = 1,2,\ldots,n,n+1)$ satisfy the conditions of Lagrange's Mean Value Theorem on the closed interval $[a,b]$, they satisfy the conditions of this Theorem on each subinterval $[t,t+\Delta t]\subset[a,b]$, so that

$$\begin{aligned}\cos\alpha_i&=\frac{x_i'[t+\theta_i\Delta t]\Delta t}{\sqrt{\sum\limits_{i=1}^{n+1}\{x_i'[t+\theta_i\Delta t]\}^2\Delta t}}\ ,\\ &=\frac{x_i'[t+\theta_i\Delta t]}{\sqrt{\sum\limits_{i=1}^{n+1}\{x_i'[t+\theta_i\Delta t]\}^2}}\ ,\qquad \begin{pmatrix} i=1,2,\ldots,n+1\\ 0<\theta_i<1\\ t\in[a,b]\end{pmatrix}\end{aligned} \tag{60}$$

where θ_i is certainlv a function of observed subinterval $[t,t+\Delta t]$. Let us resume the process of limit in expression (58) . We have

$$\lim_{\Delta\rho_{n+1}(t)\to 0}\frac{\Delta q(t)}{\Delta\rho_{n+1}(t)}=\sum_{i=1}^{n+1}\frac{\partial f(P)}{\partial x_i}\cdot\lim_{\Delta\rho_{n+1}(t)\to 0}\cos\alpha_i+\lim_{\Delta\rho_{n+1}(t)\to 0}\sum_{i=1}^{n+1}\frac{\varepsilon_i\Delta x_i(t)}{\Delta\rho_{n+1}(t)}\quad . \tag{61}$$

If we demonstrate the process of limit (61) by use of the logical connectives then the variables are connected through compound statement

$$[(\Delta\rho_{n+1}(t)\to 0)\iff(\Delta t\to 0)]\iff\left[\bigwedge_{i=1}^{n+1}(\Delta x_i(t)\to 0)\Longrightarrow\bigwedge_{i=1}^{n+1}\varepsilon_i\to 0\right]\quad , \tag{62}$$

then

$$\lim_{\Delta\rho_{n+1}(t)\to 0}\cos\alpha_i=\lim_{\Delta t\to 0}\frac{x_i'[t+\theta_i\Delta t]}{\sqrt{\sum_{i=1}^{n+1}\{x_i'[t+\theta_i\Delta t]\}^2}}\quad , \tag{63}$$

that is

$$\lim_{\Delta\rho_{n+1}(t)\to 0}\cos\alpha_i=\frac{x_i'(t)}{\sqrt{\sum_{i=1}^{n+1}[x_i'(t)]^2}}\quad , \tag{63a}$$

$$\lim_{\Delta\rho_{n+1}(t)\to 0}\cos\alpha_i=\frac{dx_i}{\sqrt{\sum_{i=1}^{n+1}[dx_i]^2}}=\frac{dx_i}{ds_{n+1}}\quad , \tag{63b}$$

or

$$\lim_{\Delta\rho_{n+1}(t)\to 0}\cos\alpha_i=\cos\beta_i\quad ,\qquad (i=1,2,\ldots,n+1) \tag{63c}$$

while

$$\left\{\lim_{\Delta\rho_{n+1}(t)\to 0}\sum_{i=1}^{n+1}\frac{\varepsilon_i\Delta x_i(t)}{\Delta\rho_{n+1}(t)}\right\}\longrightarrow 0$$

is a higher - order infinitesimal which can be omitted. We can take that

$$\lim_{\Delta\rho_{n+1}(t)\to 0}\frac{\Delta q(t)}{\Delta\rho_{n+1}(t)}=\sum_{i=1}^{n+1}\frac{\partial f(P)}{\partial x_i}\cdot\cos\beta_i\quad , \tag{64}$$

or in another form of appearance

$$\lim_{\Delta\rho_{n+1}(t)\to 0}\frac{\Delta q(t)}{\Delta\rho_{n+1}(t)}=\frac{1}{ds_{n+1}}\left[\sum_{i=1}^{n+1}\frac{\partial f(P)}{\partial x_i}\cdot dx_i\right] \tag{64a}$$

Let us find the limit of

$$\lim_{\Delta\rho_{n+1}(t)\to 0} \frac{1}{Q} \quad .$$

Since we have already shown (expression $(63c)$) that

$$\lim_{\Delta\rho_{n+1}(t)\to 0} \cos\alpha_i = \cos\beta_i \quad , \qquad (i = 1, 2, \ldots, n+1)$$

then the following holds true

$$Q \longrightarrow Q^{(\circ)} = \left\{ \frac{\cos\beta_{n+1}}{x_{n+1}(t)} - \frac{\left[\sum_{i=1}^{n} \frac{\cos\beta_i}{\int_a^b x_i(t)dt}\right]}{\left[\sum_{i=1}^{n} \frac{x_i(t)}{\int_a^b x_i(t)dt}\right]} \right\} \quad , \tag{65}$$

when $\Delta\rho_{n+1}(t) \to 0$. Now

$$\lim_{\Delta\rho_{n+1}(t)\to 0} \frac{1}{Q} = \frac{1}{Q^{(\circ)}} \quad , \tag{66}$$

and quite obviously

$$\lim_{\Delta\rho_{n+1}(t)\to 0} I_{tl}^{(\hat{\Gamma})} = 1 \quad . \tag{67}$$

On substituting (64), (66) and (67) into (55) we obtain

$$\boxed{E^{(\hat{\Gamma})}_{q,(x_1,x_2,\ldots,x_{n+1})} = \frac{1}{Q^{(\circ)}} \cdot \frac{1}{q(t)} \cdot \left[\sum_{i=1}^{n+1} \frac{\partial f(P)}{\partial x_i} \cdot \cos\beta_i\right]} \quad , \qquad t \in [a, b] \tag{68}$$

If, conversely, we substitute $(64a)$, (66) and (67) into (55) then we obtain the expression

$$E^{(\hat{\Gamma})}_{q,(x_1,x_2,\ldots,x_{n+1})} = \frac{1}{Q^{(\circ)}} \cdot \frac{1}{q(t)} \cdot \frac{1}{ds_{n+1}} \left[\sum_{i=1}^{n+1} \frac{\partial f(P)}{\partial x_i} \cdot dx_i\right] \quad . \tag{69}$$

Since the difference, $(50a) - (50b)$, is

$$MI_{nd}^{(\hat{\Gamma})} - MI_{tl}^{(\hat{\Gamma})} = Q^{(\circ)} \cdot ds_{n+1} \quad , \tag{70}$$

expression (69) assumes the form

$$E^{(\hat{\Gamma})}_{q,(x_1,x_2,\ldots,x_{n+1})} = \frac{1}{MI^{(\hat{\Gamma})}_{nd} - MI^{(\hat{\Gamma})}_{tl}} \cdot \left[\sum_{i=1}^{n+1} \frac{1}{q(t)} \frac{\partial f(P)}{\partial x_i} \cdot dx_i \right] , \qquad (71)$$

which can also be given the following form of appearance

$$E^{(\hat{\Gamma})}_{q,(x_1,x_2,\ldots,x_{n+1})} = \frac{1}{MI^{(\hat{\Gamma})}_{nd} - MI^{(\hat{\Gamma})}_{tl}} \cdot \left[\sum_{i=1}^{n+1} \frac{x_i(t)}{q(t)} \cdot \frac{\partial f(P)}{\partial x_i} \cdot \frac{dx_i}{x_i(t)} \right] , \qquad (72)$$

$$= \frac{1}{MI^{(\hat{\Gamma})}_{nd} - MI^{(\hat{\Gamma})}_{tl}} \cdot \left[\sum_{i=1}^{n+1} \frac{dx_i}{x_i(t)} \cdot E^{(\hat{\Gamma})}_{q,x_{i(t)}} \right]_P , \qquad (73)$$

that is

$$E^{(\hat{\Gamma})}_{q,(x_1,x_2,\ldots,x_{n+1})} = \left[\sum_{i=1}^{n+1} \frac{\frac{dx_i}{x_i(t)}}{\left(MI^{(\hat{\Gamma})}_{nd} - MI^{(\hat{\Gamma})}_{tl}\right)} E^{(\hat{\Gamma})}_{q,x_{i(t)}} \right]_P \qquad t \in [a,b] \qquad (74)$$

where the partial elasticity coefficients, $E^{(\hat{\Gamma})}_{q,x_{i(t)}}$, are taken at point $P = [\vec{r}(t)] = [x_1(t),$- $x_2(t), \ldots, x_{n+1}(t)] \in \hat{\Gamma}$.

The complete or total coefficient of aggregate demand elasticity on a smooth arc of the hyper curve $\hat{\Gamma}$ in expressions (68) and (74) shows how many percentage points the aggregate demand for competitive products changes per one percent change in the real income from which this demand is financed and which is defined on a smooth arc of the hyper curve $\hat{\Gamma}$.

Let the prices $x_1(t), x_2(t), \ldots, x_n(t)$ of competitive products remain unchanged, i.e.

$$dx_i = 0 \quad , \qquad (i = 1,2,\ldots,n). \qquad (75)$$

Then the following holds true for expressions $(50d)$

$$\cos\beta_i = 0 \quad , \qquad (i = 1,2,\ldots,n) \quad ,$$

and $MI^{(\hat{\Gamma})}_{tl}$ (expression $(50b)$) becomes

$$MI^{(\hat{\Gamma})}_{tl} = 1 \quad . \qquad (76)$$

Let the nominal income change simultaneously satisfy

$$dx_{n+1} \neq 0 \quad , \tag{77}$$

which implies

$$MI_{nd}^{(\hat{\Gamma})} = 1 + ds_{n+1} \cdot \frac{\cos\beta_{n+1}}{x_{n+1}(t)} = 1 + \frac{dx_{n+1}}{x_{n+1}(t)} \quad . \tag{78}$$

Conditions (75) and (77) with implications (76) and (78) respectively, give the expression (74) a new form of appearance

$$E^{(\hat{\Gamma})}_{q,(x_1,x_2,\dots,x_{n+1})} = \left[E^{(\hat{\Gamma})}_{q,x_{n+1}(t)}\right]_P \quad , \tag{79}$$

which tells the size of the relative change of the aggregate demand for competitive products, when that change is induced solely by the one-percent change of the nominal income.

Let now $dx_{n+1} = 0$ and $dx_i \neq 0$ for $(\forall i)(i = 1,2,\dots,n)$. Then $MI_{nd}^{(\hat{\Gamma})} = 1$ while $ds_{n+1} = ds_n \Longrightarrow MI_{tl}^{(\hat{\Gamma})} = MI_{tl}^{(\Gamma)}$. The implication of this is that expression (74) can be written in the form

$$E^{(\Gamma)}_{q,(x_1,x_2,\dots,x_n)} = \left[\sum_{i=1}^{n} \frac{\frac{dx_i}{x_i(t)}}{\left(1 - MI_{tl}^{(\Gamma)}\right)} \cdot E^{(\Gamma)}_{q,x_i(t)}\right]_P \quad , \tag{80}$$

that is

$$E^{(\Gamma)}_{q,(x_1,x_2,\dots,x_n)} = -\left[\sum_{i=1}^{n} \frac{\frac{dx_i}{x_i(t)}}{\left(MI_{tl}^{(\Gamma)} - 1\right)} \cdot E^{(\Gamma)}_{q,x_i(t)}\right]_P \quad , \tag{81}$$

where the partial elasticity coefficients, $E^{(\Gamma)}_{q,x_i(t)}$, are taken at point $P = [\vec{r}(t)] = [x_1(t), x_2(t), \dots, x_n(t)] \in \Gamma$,**Expression (81) shows how many percentage points the aggregate demand for competitive products changes per one-percent change of the average price level of these products assuming an unchanged level of the nominal income.**

If,

$$E^{(\hat{\Gamma})}_{q,(x_1,x_2,\dots,x_{n+1})} = 0 \tag{82}$$

then we have the case of redistribution of the aggregate demand for competitive products. i.e. the change of market shares of the competitors within an unchanged absolute level of aggregate demand (51)[5) .

4. CONCLUDING REMARKS

Early in this paper the subject under discussion was changes of prices of competitive products on a smooth arc of the hyper curve Γ . Later, the induction of a nominal income, from which aggregate demand for the substitutes is financed, extended the subject under discussion to the changes on a smooth arc of the hyper curve $\hat{\Gamma}$. Hence, we first constructed, on a smooth arc of the hyper curve Γ, the average chain index, a magnitude which synthesized the simultaneous change of prices $x_1(t), x_2(t), \ldots, x_n(t)$. Then, the index of real income, which synthesized the simultaneous variation not only of the prices $x_1(t), x_2(t), \ldots, x_n(t)$ of competitive products but of a nominal income from which their demand is also financed , was constructed on a smooth arc of the hyper curve $\hat{\Gamma}$. Only then were we in a position to realise the primary task : construction and interpretation of the **complete or total coefficient of aggregate demand elasticity on a smooth arc of the hyper curve** $\hat{\Gamma}$. Except the general assumptions (given in the introductory exposition) regarding the functions $x_1(t), x_2(t), \ldots, x_n(t)$, the issue of their practical realisation has not been discussed. We shall leave that for some future paper.

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FOOTNOTES

1. In this expression vectors $\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n$ represent unit vectors which form an orthonormal basis in $n-$ dimensional real Euclidian space E^n .

2. See [1], p. 168.

3. $A > 0$ and $B > 0$ for the angle between the vectors $\vec{r}(t)$ and $\hat{\vec{r}}$ is always acute regarding the nature of the problem under discussion .

4. In that respect, the average quantities from observed time interval can also appear as the weights .

5. Interpretation of this case is analogous to that given in [1]. p. 181.

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POTPUNI ILI TOTALNI KOEFICIJENT ELASTIČNOSTI AGREGATNE TRAŽNJE NA GLATKOM LUKU HIPERKRIVULJE

Slobodan Sekulović

Rezime

U prvoj fazi izlaganja predmet našeg posmatranja su bile promjene cijena konkurentskih proizvodâ na glatkom luku hiperkrivulje Γ , da bi se potom taj predmet posmatranja, sa

uvođenjem nominalnog dohotka iz koga se alimentira agregatna tražnja za supstitutima, proširio na promjene na glatkom luku hiperkrivulje $\hat{\Gamma}$ .Shodno tome prvo je na glatkom luku hiperkrivulje Γ konstruisan srednji indeks lančani, veličina kojom smo sintetizovali simultanu varijaciju cijena $x_1(t), x_2(t), \ldots, x_n(t)$, a potom je na glatkom luku hiperkrivulje $\hat{\Gamma}$ konstruisan indeks realnog dohotka kojim smo sintetizovali simultanu varijaciju ne samo cijena $x_1(t), x_2(t), \ldots, x_n(t)$ konkurentskih proizvoda već i nominalnog dohotka $x_{n+1}(t)$ iz koga se alimentira njihova (agregatna) tražnja. Tek tada smo bili u mogućnosti da realizujemo primarni zadatak, tj. konstrukciju i tumačenje **potpunog ili totalnog koeficijenta elastičnosti agregatne tražnje na glatkom luku hiperkrivulje** $\hat{\Gamma}$. Osim što su u uvodnom izlaganju date opšte postavke koje se odnose na funkcije $x_1(t), x_2(t), \ldots, x_n(t)$ u problematiku njihove praktične realizacije nismo ulazili što ostavljamo za neko naredno izlaganje.