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THE PRODUCTION STRATEGY OF A LABOR-MANAGED FIRM AND PRICE UNCERTAINTY

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ABSTRACT

This paper extends the analysis of the production strategies of monopolistic and competitive labor-managed firms with a production function homogenous of degree ∂ (> 0) to the uncertain case where output price uncertainty exists. Although is has been mentioned that under certainty their production strategies are just perverse, we show that a non-trivial (i.e., interior) solution is more prevalent than ever thought. Hence, the more prevalent existence of the non-trivial solution has a meaning that it is possible to conduct comparative static analyses.

1. Introduction

Since Ward's pioneering work (1958), a large number of papers concerning the labor-managed firm or worker's enterprise have been published and hence many interesting results peculiar to the labor-managed firm (henceforth LMF) have been obtained. Among others, two results are very famous as being bothersome. One of them was derived by Ward and the other by Furubotn and Pejovich (1970). The former is that in the short run the labor-managed firm decreases employment and output when output price rises. This result caused a lot of arguments about both whether the objective function of Ward-Domar-Vanek (henceforth WDV) style is pertinent from a viewpoint of the worker's cooperative and whether its perverse result can be cured. The re-examination of Ward's outcome has been attempted by such researchers as Steinherr and Thisse (1979), Sertel (1972, 1987), and Miyazaki and Neary (1983). By introducting a worker-partnership deed and/or an external labor markets, Sertel showed that the LMF's pathological short-run response to an increase in output price disappears.

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On the other hand, Steinherr and Thisse and Miyazaki and Neary modify the objective function of the WDV type so that the workers are concerned with the risk of layoff, and consequently demonstrate that the LMF does not reduce employment and output when price increases.[1] The second troublesome issue is chronic underinvestment. As a way to solve that issue, Sertel (1982) proposed the acceptance of external finance by the LMF restricted to self-financing.

The first problem is a short-run one and the second is a long-run one. Besides, the existence of the third defect of the WDV model was demonstrated by some writers, including Vanek (1971), Pestieau and Thisse (1979), Landsberger and Subotnik (1981) (henceforth LS), Meza (1983) and Haruna (1985). First, Vanek noticed that in the long run, if its production function is homogeneous of arbitrary degree, the output behavior of the Illyrian LMF differs a lot from that of its capitalist equivalent. The same problem was taken up by Pestieau and Thisse, LS, Meza and Haruna.[2] According to them, no interior solution exists for a competitive LMF or a monopolistic LMF, depending on whether its production is homogeneous of degree $\rho\,(\neq 1)$ or of degree one, respectively. This means an anomalous situation in which the Illyrian LMF produces no output even in the technological conditions where the capitalist firm produces just output. This anomalous production strategy (or a corner solution) appears in the long run but not in the short run. Therefore, both Ward's pathological problem and this one do not occur coincidently.

The problem of absence of an interior solution is, to be sure, less well known in comparison with the above two defects, but it, anyway, seems to be of use to inquire further into this problem. The studies always conducted are those under certainty and ignore the presence of uncertainty. Therefore, in this paper, we incomporate uncertainty and reexamine whether the Illyrian LMF's perverse output behavior appears.[3] We use particularly the objective function of the traditional WDV type, also used by LS and Haruna, to maintain continuity with previous analyses. In the following sections we shed light on the production strategies of competitive and monopolistic LMFs, respectively. The paper will show that as a result of the introduction of uncertainty, especially output price uncertainty, a non-trivial (interior) solution is more prevalent than ever expected, and provides a base for conducting various comparative static analyses without also falling into the pitfall

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[1] Although they have a common idea, their results are quite different. For example, Steinherr and Thisse (1979) derive an inelastic (vertical) supply curve for the LMF in the short run but Miyazaki and Neary (1983) a usual upward-sloping supply curve, except in the range of very low output price.

[2] The possibility is very high that they did not know that Vanek (1970) had already treated that problem, so it seems that they considered it independent of his discussion.

[3] Haruna (1987) treats the production decision of the competitive firm facing an uncertain factor price. According to his paper, even if the firm is non-neutral to risk, there is no interior solution except for constant returns to scale technology.

of a corner solution. Moreover, we can establish that the form of the firm's utility function or its attitude toward risk plays an important role in determining whether or not an interior solution exists.

2. THE MODEL

Consider the problem of decision-making in LMFs, especially a competitive LMF and a monopolistic one. Each firm produces a single output q using two inputs, capital K and labor L, and the firm's production function is exhibited by $q = q(K, L)$, which is assumed to be characterized by positive but diminishing marginal products of capital and labor, and further to be homogeneous of degree ρ in K and L. It is also assumed that the firm must determine inputs and then output prior to the price of output p being known. That is, the firm must operate under output price uncertainty. The demand function which the monopolistic LMF faces is given by

$$p = p(q, u), \frac{\partial p(q, u)}{\partial q} = p' < 0, \frac{\partial p(q, u)}{\partial u} > 0, \tag{1}$$

where u stands for a random variable with known density function f(u) and mean μ. In contrast, for the competitive LMF (1) becomes

$$p = p(u), \frac{\partial p(u)}{\partial u} > 0.$$

We suppose particularly that there is no other uncertainty than price uncertainty.

Treating the problem of the LMF's production strategy under uncertainty, we follow Ward's Illyria in the formulation of the objective function. Now, in the presence of uncertainty the objective of the Illyrian LMF is to choose the quantities of capital and labor so as to maximize the expected utility of dividend per member:

$$EU(Y) = EU\left[\frac{pq - rK}{L}\right],$$

where U(Y) is a von Neumann-Morgenstern utility function, $U(Y)' > 0$ and $U''(Y) \gtreqless 0$, depending on whether the firm is risk averse, risk neutral or risk loving, and r is the competitive rental price of capital. Of course, although there still remains a small question about the propriety of that objective, we adopt it to maintain consisency with the previous studies and to elucidate properties of Ward's Illyrian firm.[4] The

[4] The Illyrian LMF's objective function differs from those which Steinherr and Thisse and Miyazaki and Neary employed in analysing the LMF. Besides, in Inselbag and Sertel (1979) and Sertel (1982) the LMF was considered by use of a member's specific utility function.

first-order conditions for maximization are given by

$$EU'(Y)\,(q_K MR - r) = 0 \tag{2}$$

$$EU'(Y)\,(q_L MR - Y) = 0, \tag{3}$$

where MR = p + p'q for the monopolistic LMF and MR = p for the competitive one. (2) and (3) are the long-run equilibrium conditions.

3. THE PRODUCTION STRATEGY OF THE FIRM FACING UNCERTAINTY

LS have described that in the long-run equilibrium the output behavior of the monopolistic firm with a homogeneous production technology is quite strange compared to that without it; that is, when the production function exhibits non-increasing returns to scale, the firm's optimal solution becomes a corner one. Alternatively, it has been presented by Haruna (1985) that for the competitive firm a similar conclusion holds. In this case, the technolgical condition that the firm produces output is restricted at least to a consant returns to scale technology. In order to see whether the firm in uncertain circumstances behaves as mentioned by LS and Haruna, we concentrate our discussion on the problem of the long-run production strategy of the monopolistic and competitive firms operating under output price uncertainty. That is, we discuss whether the optimal solution of each firm degenerates into a trivial one as expected.

3.1. *The Monopolistic Firm*

First, we take the monopolistic firm. Immediately, we can obtain the following results:

Proposition 1: Let us assume that the firm has a production function homogeneous of degree ρ (> 0) and faces random demand, p(q, u):

(a) If the firm is risk averse or risk loving, then it produces positive output for any homogeneous production function of degree ρ, and moreover the amount of output itself, as usual, depends on ρ, the distribution of the random variable, the factor price and the firm's attitude toward risk.

(b) If the firm is risk neutral, then it produces output for the case of $\rho > 1$, but not for the case of $\rho \leqq 1$, and then its amount is generally dependent on the distribution of the random variable and ρ, but determined, independent of the factor price.

Proof: (a) By use of first-order conditions (2) and (3) we can derive

$$\rho EU'(Y)\,MR = EU'(Y)\,p, \tag{4}$$

where EU'(Y) MR > 0 and EU'(Y) p > 0. It is now apparent that under risk aversion or risk seeking condition (4) the decision of output holds

for a homogeneous production function of arbitrary degree. Besides, through (4) we can easily see what ingredients influence the quantity of output.

(b) For the risk neutral firm, (4) is rewritten as

$$\rho \bar{p} q \left(\frac{p-1}{\rho} - \frac{1}{e} \right) = 0, \tag{5}$$

where $\bar{p} = E(p)$ and $e = -(d\bar{q}/\bar{q}) / (dp/p)$ (> 0) denotes the price elasticity of demand. By the way, in order for (5) to be satisfied ρ must be more than unity because of $e > 0$, while if $\rho \leqq 1$, then equilibrium condition (5) holds only at $q = 0$ because of $(\rho - 1) / \rho - 1/e < 0$. In addition, in the case of $\rho > 1$, as expressed by (5), the level of output is determined by ρ as well as the form of the distribution of u, however r is not obviously included in (5).

When the firm is risk averse or risk loving, it operates even if the production function exhibits not only increasing returns to scale but also non-increasing returns to scale. This involves there being an interior solution above the class of homogeneous production functions, including the Cobb-Douglas and CES production functions. This result differs greatly from that of LS. Further, LS showed that its level in the riskless firm is never dependent on that price, but on the contrary relies on that price. Thus, uder risk aversion and risk seeking the anomalies of the firm's production strategy would disappear. Let us turn to the risk neutral firm. Then, we see from Proposition 1(b) that if the firm is risk neutral, it behaves with respect to output just as the riskless firm does. That is to say, the strange situation emerges again that the optimal output is zero under the non-increasing returns to scale technology where the capitalist firm produces output properly. In contrast, although the firm operates when $\rho > 1$, the amount of it does not rely on the factor price, as in the case of LS. In effect, as seen through the comparison concerning the production strategies of both the risk averse (seeking) and risk neutral firms, there exists an important difference between them. It is now worth noticing that the difference is based only on the firm's attitude toward risk.

Now let us specify the demand function as follows:

$$p(q, u) = g(q) + h(q)\, u, \quad \frac{\partial p}{\partial q} = g' + h'u < 0. \tag{6}$$

It is said that the demand function is additive when $h = 1$ and multiplicative when $g = 0$. Focusing the multiplicative demand function, we get $E(p) = \bar{p} = \mu h$ and $\bar{p}' = \mu h'$, where $E(u) = \mu\ (> 0)$. We in turn substitute these terms into e so that $e = -h/gh'$; that is, u is eliminated from term e. This involves the distribution of the random variable having no influence on the choice of output level. Thus we obtain the following corollary:

If the demand function is multiplicative, then the risk neutral firm chooses the level of output independent of the distribution of the random variable, and hence its level depends solely on ρ.

The corollary demonstrates that in a risk neutral firm facing the multiplicative demand curve, its production strategy is not entirely affeced by uncertainty; namely, the firm behaves as if there is no uncertainty. Consequently, the production strategy of the firm completely accords with that of the riskless firm.

3.2. *The Competitive Firm*

As described by Haruna 1985), even if the competitive firm operates under a non-increasing returns to scale technology, it is not always general for its optimal solution to be an interior one. Rather, that is rare and the interior solution is obtained only if its technology exhibits constant returns to scale. Let us now consider the optimization problem of output of the competitive firm facing output uncertainty.

Proposition: 2: Let us assume that the firm has the production function homogeneous of degree ρ and faces random output price:

(a) If the firm is risk averse, then it produces output regardless of the value of ρ, and the quantity of output is further determined by the probability distribution of output price, the factor price and the firm's attitude toward risk, but not by ρ.

(b) If the firm is risk neutral or risk averse, then it produces no output except for the restricted case of a constant returns to scale technology, and moreover the quantity of output could not be determined without further conditions.

Proof: To begin with, when utilizing (4) and setting p' in it equal to zero, as the optimal condition for output choice of the firm we have

$$q\,(\rho - 1)\,[EU'(Y)\,E(p) + cov\,(U'(Y), p)] = 0, \tag{7}$$

where cov denotes covariance. Let us see the sign of cov (U'(Y), p). Differentiating U'(Y) with respect to p, we have $\partial U'(Y) / \partial p = qU''(Y)/L$. so that cov (U'(Y), p) is negative, zero or positive, according to whether the firm is risk averse, risk neutral or risk loving. Hence, if the firm is risk averse, then (7) is met, while if the firm is risk neutral or risk loving, then to meet (7) either $\rho = 1$ or $q = 0$ must hold. Thus, if the production function does not exhibit constant returns to scale, then the risk neutral and risk loving firms produce no output to attain the maximum of their expected utility. In the case of risk neutrality or risk seeking, when $\rho = 1$, the firm operates but we cannot, anyway, know its amount, because the distribution of random price, the factor price and the firm's attitude toward risk do not participate in the choice of output at all.

The results of this proposition indicate that the firm's attitude toward risk fundamentally has a great impact on its production strategy, as presented in the previous subsection. When comparing the pro-

duction behavior under uncertainty with that under centainty, we can immediately recognize that there is a crucial difference. Namely, although in the absence of uncertainty the existence of an interior solution is limited solely to the class of linearly homogeneous production functions, under risk aversion its existence is expanded to cover the entire class of homogeneous production functions of degree ρ. Alternatively, as in the case of the monopolistic firm, under risk neutrality the competitive firm undoubtedly operates as long as the technology is constant returns to scale, although we cannot know the level of output to be would be chosen, without adding one or more assumptions.

4. AN EXAMPLE

For better understanding of the analysis in this paper, consider the following example. We take particularly the monopolistic firm, because the outcome relating to the competitive firm could be easily derived from those of the monopolistic one. We first assume that the distribution of the random variable u is normal with mean μ and variance σ^2. Further, let the demand function be (6) and the utility function have constant absolute risk aversion, $U(Y) = -e^{\alpha Y}$, where $\alpha\ (>0)$ is constant. Then, the measure of absolute risk aversion is $r(Y) = \alpha$. By the way, the dividend or value added per member equals.

$$Y = \frac{(g + hu)\, q - rK}{L},$$

With Y normally distributed, the maximization of the expected utility of the dividend per member is equivalent to the maximization of

$$E(Y) - \frac{\alpha var\,(Y)}{2}, \tag{8}$$

where var denotes variance (for the derivation of (8), see, e.g., Baron (1970), and Newbery and Stiglitz (1981).

First, consider the problem of production decision of the firm operating under an additive demand function. We now set $h = 1$ in (6) since the demand function (6) is additive when $h = 1$, i.e., $p(q, u) = g(q) + u$, $\partial p/\partial q = g' < 0$. Substituting this into (8), we obtain

$$q\left[\left(q + \mu - \frac{\alpha\sigma^2 q}{L}\right)(\rho - 1) + \rho g' q\right] = 0, \tag{9}$$

which corresponds to (4), and where $\mu \neq 0$. As the results in the previous section are endorsed, this evidently shows that not only does the firm produce output regardless of ρ but also in addition to ρ the distribution characteristics of u, i.e., the first and second moments, μ and

σ^2, and the degree of risk aversion, i. e., the firm's attitude toward risk, affect the level of output.

Next, we turn to the multiplicative demand function. Setting $g = 0$ in (6), we obtain it, e. e., $p(q, u) = h(q)u$, $\partial p/\partial q = uh' < 0$. By the same way with (9) we can obtain as the optimal condition for output choice:

$$q\,[\rho MR_1(q) - h]\,\left(\mu - \frac{\alpha\sigma^2 hg}{L}\right) = 0, \tag{10}$$

where $MR_1(q) = h(q) + h'(q)q$ is non-stochastic. Needless to say, this expresses that the results of Proposition 1 are supported in the case of the multiplicative demand function as well as in the case of the additive one. On the other hand, setting $\alpha = 0$ in (10), we can get the condition for the production choice of the risk neutral firm as follows:

$$\mu q\,[\rho MR_1(q) - h] = 0. \tag{11}$$

On the other hand, when the demand function is multiplicative, from (9) this condition is

$$q\,[(q + u)(\rho - 1) + \rho g' q] = 0. \tag{12}$$

From conditions (11) and (12) we find that the former and the latter correspond just to the corollary and Proposition 1(b), respectively.

5. CONCLUSION

The purpose of this paper has been to analyse the long-run production strategies of monopolistic and competitive Illyrian LMFs, whose production functions are homogeneous in the presence of output price uncertainty. In consequence, it was shown that, compared to the firms in the absence of uncertainty, the anomalous output behavior of each firm is cured, in particular, under risk aversion. This also has another meaning. Namely, the realm of a corner solution, which makes comparative statics useless, vanishes in the case of risk aversion. Thus we could provide a base for conducting comparative static analyses without falling into pitfalls pointed out by LS, Pestieau and Thisse and Haruna. However, we should bear in mind that the existence of an interior solution depends crucially on the firm's attitude toward risk.

To be sure, although it has been presented that, under uncertainty, the perverse situation of the firm with the homogeneous production function is ameliorated fairly, this is not a fundamental solution for curing the perverse attribute of the Illyrian LMF's objective function. Henceforth, it will be necessary to re-examine its objective function or Ward's Illyrian model itself. By the way, it is Sertel (1978) that gives a hint for avoiding such a perverse situation in the long run. That is, like his model, if a worker-partnership deed and an external labor market are incorporated into a model, then the perversity in the production

strategy of the Illyrian LMF would be found not to emerge. Inter alia, a fixed wage settled on the external labor market plays a crucial role. But this model includes a controversial point. That is, as he notes, long-run equilibrium conditions are nothing but traditional capitalist equilibrium ones. Therefore, there may be a possibility of the LMF's characteristic dropping out, because the dividend per member comes to be equivalent to the external fixed wage in the long-run equilibrium.

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PROIZVODNA STRATEGIJA SAMOUPRAVNOG PREDUZEĆA I CENOVNA NEIZVESNOST

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Rezime

Ovaj članak proširuje analizu proizvodne strategije monopolističkih i konkurentnih samoupravnih preduzeća s proizvodnom funkcijom stepena homogenosti $\rho (> 0)$ *u uslovima neizvesnih finalnih cena. Mada je pomenuto kako je u uslovima neizvesnosti njihova proizvodna strategija deformisana (perverzna), pokazuje se da je netrivijalno rešenje učestalije no što se obično misli. Na taj način učestalija pojava netrivijalnih rešenja ukazuje na mogućnost primene komparativne statičke analize.*