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Stochastic Modelling and Analysis of Driver Behaviour

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Abstract: Driver behaviour is considered a key factor in the majority of car accidents. As a consequence driver behaviour has been receiving vast attention in different domain areas, such as psychology, transport engineering and computer science. Computer scientists are primarily interested in what and how computing means can be applied to understand the relation between driver behaviour and transport systems. In this paper, we adopt a stochastic approach to conduct a quantitative investigation of driver behaviour. We use the Markovian process algebra PEPA (Performance Evaluation Process Algebra) to describe the overall system model. The system component describing the topology and dynamic of the traffic is composed in parallel with the system component describing the driver state and its evolution due to experience. We illustrate our approach using a three-way junction as an example and present the numerical results of the system analysis.

Keywords: stochastic model, quantitative analysis, Markovian process algebra, Performance Evaluation Process Algebra (PEPA), driver behaviour.

1 Introduction

Driver behaviour has been widely studied for several decades, during which a plethora of empirical models have been developed. Although throughout the years there was a progression from skill-based models to motivational models, risk models [Sum96] up to more articulated models characterised by the cooperation of complementary cognitive processes, such as automaticity versus attention [Ran94], perception of task difficulty versus driver capability [Ful05] and intentionality versus actual behaviour [DT11], no convergence has been achieved toward a comprehensive driving model yet. Recently, empirical models of driver behaviour have been integrated with formal approaches. Shaikh and Krishnan [SK12] model the interactions between a driver and a semi-autonomous vehicle using the C programming language and verify the safety of such interaction using model checking. In another formal approach, Cerone [Cer11] proposes a process algebraic framework to characterise the interplay between automaticity and attention and apply it to driver behaviour.

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While the above models aim primarily to driver behaviour prediction for accident prevention, some recent models also investigate behavioural adaptation to be considered in the human-centered design of Intelligent Transport Systems (ITS). In this context, Schaap *et al.* [SHAB08] use a driving simulator to investigate the role of unexpected events in driver behavioural adaptation, while Dia [Dia02] proposes an agent-based approach to model response of drivers to real-time travel information and the consequent effect on driver behaviour.

This paper proposes a formal approach to modelling driver behavioural adaptation as the effect of driver experience. We adopt a stochastic approach and focus on experiences, such as a possible collision, that tend to have just temporary effects on driver behaviour. We use the Markovian process algebra PEPA (Performance Evaluation Process Algebra) to describe the overall system model.

The remainder of this paper is organised as follows. In Section 2 we briefly introduce Markovian process algebra PEPA. In Section 3 we use a three-way junction as a case study to illustrate our modelling approach. In Section 4 we present and discuss numerical results of stochastic simulation and steady state analysis conducted using the PEPA Eclipse plug-in toolset [Tri07]. Finally, in Section 5 we draw our conclusion and propose directions for future work.

2 PEPA

PEPA [Hil96] is a Markovian Process Algebra. It supports actions that occur with rates that are negatively exponentially distributed. Specifications written in PEPA represent Markov processes and can be mapped to a continuous time Markov chain (CTMC). Population level metrics can be derived by a set of scalable analysis techniques.

Systems are specified in PEPA in terms of *activities* and *components*. An activity (α, r) is described by an action α, i.e. the type of the activity, and the rate r of the associated negative exponential distribution. This rate may be either specified by a positive real number or left unspecified by using symbol $\top$.

The syntax for describing components is as follows.

$$(\alpha, r).P \mid P + Q \mid P/L \mid P \underset{\mathscr{L}}{\bowtie} Q \mid A$$

Component $(\alpha, r).P$ performs action α at rate r and with delay or duration $1/r$ and then behaves like P. Component $P + Q$ behaves either like P or like Q, the resultant behaviour being given by the first activity to complete. Component P/L behaves exactly like P except that the activities in the set L are concealed, their type is not visible and instead appears as the unknown type τ.

Concurrent components can be synchronised, $P \underset{\mathscr{L}}{\bowtie} Q$, such that activities in the cooperation set $\mathscr{L}$ involve the participation of both components. In PEPA the shared activity occurs at the slowest of the rates of the participants and, if a rate is unspecified in a component, the component is passive with respect to activities of that type.

Definition $A \overset{def}{=} P$ gives the constant A the behaviour of the component P. We employ a further shorthand that has been commonly used in the study of large parallel systems: $A[N]$ denotes that there are N instances of A in parallel, i.e. $A || \ldots || A$, but we are not concerned with the state of each individual component, rather with the number of components in each state.

3 Case Study

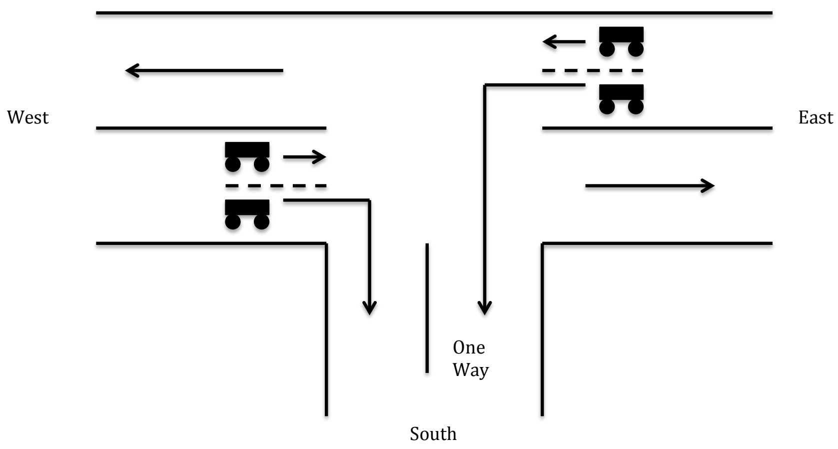

Figure 1: Three-Way Junction

Let us consider the three-way junction in Figure 1, in which we assume right-hand traffic. The three-way junction consists of one two-way main road along the east-west direction and a south-ward one-way road. Cars from the West have two options when they arrive at the junction: go straight or turn right. Similarly, cars from the East have two options: go straight and turn left.

The junction has no traffic light. Cars travelling along the east-west main road do not need to give way to turning traffic. Cars from the West turning right also do not need to give way. Cars from the East turning left must give way to cars travelling straight from the West along the east-west main road.

We assume that the probability for the driver from the East turning left to follow the give way rule depends on various factors such as maturity of the driver (e.g., novice or expert), situational conditions (e.g., being in a hurry or distracted), physical conditions (e.g., fatigue, stress, effects of alcohol [Ful05]).

3.1 Basic Model

In our basic model we consider a fixed number of cars for each of the two directions (from the East and from the West) and we "feed" each car back to the initial state after a certain time.

Cars travelling from the West on the main road are modelled in PEPA as follows.

$$\begin{aligned}
Car_W &\stackrel{def}{=} (arrival_W, P_{straight} * r_{arrival_W}).Car_W_straight \\
&+ \;\; (arrival_W, (1 - P_{straight}) * r_{arrival_W}).Car_W_right \\
Car_W_right &\stackrel{def}{=} (turnright_W, r_{turnright_W}).Car_W_passed \\
Car_W_straight &\stackrel{def}{=} (enter_W, r_{enter_W}).Car_W_exit \\
Car_W_exit &\stackrel{def}{=} (exit_W, r_{exit_W}).Car_W_passed \\
Car_W_passed &\stackrel{def}{=} (nop, r_{nop_W}).Car_W
\end{aligned}$$

Initially cars coming from the West are in state Car_W. Action $arrival_W$ denotes a car approaching the junction from the West with probability $P_{straight}$ to go straight by performing a transition to state $Car_W\_straight$, and with probability $1 - P_{straight}$ to turn right by performing a transition to state Car_W_right.

In state Car_W_right, a car turning right is modelled by action $turnright_W$ with transition to state Car_W_passed. This action models the fact that such a car turns right directly, without basically entering the junction.

In state $Car_W\_straight$, a car going straight does not need to give way when entering the junction (action $enter_W$ with transition to state Car_W_exit). However, the driver is aware of cars from the West waiting to turn left, and will drive through the junction with some degree of caution. The extent of caution depends on the value of r_{enter_W}: the higher the r_{enter_W} rate, the lower the driver's caution.

In state Car_W_exit, the car exits the junction (action $exit_W$), with transition to state Car_W_passed, in which action nop models all unspecified actions performed before the next approach to the junction (transition back to initial state Car_W). Rate r_{exit_W} associated with action $exit_W$ models the average time $1/r_{exit_W}$ spent by the car within the junction; rate r_{nop} associated with action nop models the period $1/r_{nop}$ with which the car approaches the crossing again.

Cars travelling from the East on the main road are modelled in PEPA as follows.

$$
\begin{aligned}
Car_E &\stackrel{def}{=} (arrival_E, (1 - P_{left}) * r_{arrival_E}).Car_E\_straight \\
&\quad + (arrival_E, P_{left} * r_{arrival_E}).Car_E\_left \\
Car_E\_straight &\stackrel{def}{=} (gostraight_E, r_{gostraight_E}).Car_E\_passed \\
Car_E\_left &\stackrel{def}{=} (enter_E, r_{enter_E}).Car_E\_exit \\
Car_E\_exit &\stackrel{def}{=} (exit_E, r_{exit_E}).Car_E\_passed \\
Car_E\_passed &\stackrel{def}{=} (nop, r_{nop_E}).Car_E
\end{aligned}
$$

Initially cars coming from the East are in state Car_E. Action $arrival_E$ denotes a car approaching the junction from the East, with probability $1 - P_{left}$ to go straight by performing a transition to state $Car_E\_straight$, and with probability P_{left} to turn left by performing a transition to state Car_E_left.

In state $Car_E\_straight$, a car going straight is modelled by action $gostraight_E$, with transition to state Car_E_passed. This action models the fact that such a car goes straight without basically entering the junction.

In state Car_E_left, a car turning left is modelled by action $enter_E$, with transition to state Car_E_exit.

In state Car_E_exit, the car exits the junction (action $exit_E$) by performing a transition to state Car_E_passed, in which action nop models all unspecified actions performed before the next approach to the junction (transition back to initial state Car_E). Rates r_{exit_E} and r_{nop} play the same roles as rates r_{exit_W} and r_{nop} in the model of the car from the West.

Processes Car_W and Car_E describe the behaviour of the cars just in terms of their directions with choices of routes dictated by probabilities ($P_{straight}$ and P_{left}). We now need to model topology and capacity of the junction as well as how a driver reacts to traffic conditions, also depending on driver status, which may include level of maturity and physical as well as situational conditions. Therefore we introduce process *Junction* modelled as follows.

$$
\begin{aligned}
Junction &\stackrel{def}{=} (enter_W, r_{enter_W}).Junction\_busy_W \\
&\quad + (enter_E, (1 - P_{giveway}) * r_{enter_E}).Junction\_busy_E \\
Junction\_busy_W &\stackrel{def}{=} (exit_W, r_{exit_W}).Junction \\
&\quad + (enter_E, (1 - P_{brake}) * r_{enter_E}).PCollision \\
Junction\_busy_E &\stackrel{def}{=} (exit_E, r_{exit_E}).Junction \\
&\quad + (enter_W, (1 - P_{brake}) * r_{enter_W}).PCollision \\
PCollision &\stackrel{def}{=} (exit_W, r_{exit_W}).(exit_E, r_{exit_E}).Junction \\
&\quad + (exit_E, r_{exit_E}).(exit_W, r_{exit_W}).Junction
\end{aligned}
$$

Initially the junction is in state *Junction*. The junction can be either entered by a car from the West with action $enter_W$ and transition to state $Junction\_busy_W$ or by a car from the East with action $enter_E$ and transition to state $Junction\_busy_E$. Since a car turning left must give way when entering the junction, rate r_{enter_E}, which is associated with action $enter_E$, is decreased by factor $1 - P_{giveway}$, where $P_{giveway} \in (0, 1)$ is the probability that the driver actually gives way. Which action, between $enter_W$ and $enter_E$, is more likely to occur (win the competition) depends on the rates r_{enter_W} and $(1 - P_{giveway}) * r_{enter_E}$. Thus, the higher the probability $P_{giveway}$ that the car from the East gives way, the lower the probability that action $enter_E$ wins the competition, that is, the lower the probability that the car from the East enters the junction first and the higher the time spent by that car in the queue for turning left. There may be various reasons why the driver fails to give way: driver's general attitude, being in a hurry or distracted, or abnormal physical conditions of the drivers such as stress, fatigue or effect of alcohol.

We assume that at most one car from each of the two directions, from the West going straight and from the East turnig left, can be inside the junction at one time. Alternatively we could model the junction as a buffer with a limited capacity, but this would make the model more complex without adding anything in terms of methodology and would possibly obscure important aspects of the model.

Under this assumption, in state $Junction\_busy_W$, either the car from the West exits the junction (action $exit_W$), with transition to the initial state *Junction*, or a car from the East enters the junction (action $enter_E$), with transition to state *PCollision*. The second choice is less likely than the first one and is due to the driver failing to brake after realising that there is a car from West already inside the junction. Such a behaviour of the driver may be due to distraction or abnormal physical conditions of the drivers such as stress, fatigue or effect of alcohol; certainly it cannot be due to driver's attitude or being in a hurry. Moreover, automatic reaction by the driver normally tend to increase the probability to brake. Therefore, if $P_{brake} \in (0, 1)$ is the probability that the driver brakes, we normally assume $P_{brake} > P_{giveway}$. Thus rate r_{enter_E} is decreased by factor $1 - P_{brake}$, that is, the probability that the driver fails to brake.

In state $Junction\_busy_E$, either the car from the East exits the junction (actions $exit_E$), with transition to the initial state *Junction* or a car from the West enters the junction (action $enter_W$), with transition to state *PCollision*. Rate r_{enter_W} is decreased by factor $1 - P_{brake}$.

State *PCollision* denotes that there is the possibility of a collision between a car from the East turning left and a car from the West going straight. The two cars potentially involved in the collision exit the junction in any order as modelled by the interleaving of actions $exit_W$ and $exit_E$ in state $PCollision$, with a transition back to initial state $Junction$. We are not modelling what would happen in case of collision; in fact our goal is to describe a continuous behaviour which includes states where collisions are possible and analyse the probabilities of these "unwanted" states to be reached under a variety of conditions.

The overall system consists of N cars from the West and M cars from the East that synchronise with the $Junction$ process over actions $enter_W$, $exit_W$, $enter_E$ and $exit_E$.

$$System_1 \stackrel{def}{=} (Car_W[N] || Car_E[M]) \underset{\mathscr{L}}{\bowtie} Junction$$

where $\mathscr{L} = \{enter_W, exit_W, enter_E, exit_E\}$.

In terms of evaluation, we are interested in the probability of possible collisions, the average waiting time in a queue from arrival at the junction to finally passing the junction and the average number of cars waiting in a queue (traffic load). Those metrics are derived respectively by analysing the probability of state $PCollision$, the average time from the occurrence of action $enter_W$ (or $enter_E$) and action $exit_W$ (or $exit_E$), and the average number of cars in states $Car_W\_straight$ and Car_E_left.

3.2 Modelling Driver's Experience

The basic model described in Section 3.1 takes into account the status of the driver only in terms of fixed probabilities of giving way and failing to brake. In this section we extend the model by considering how experiences encountered while driving may affect the behaviour of the driver. In particular, we model the effect of an event in which the driver from the West is not given way by a driver from the East turning left and experiences therefore a possible collision. We can assume that the future behaviour of the driver from the West is affected by such an event with probability $P_{affected}$. In general the effect is that the driver will be more cautious when approaching the junction next time. This is modelled by process $DriverState_W$ as follows.

$$\begin{array}{rcl}
DriverState_W & \stackrel{def}{=} & (enter_W, r_{enter_W}).FoundFree_W \\
 & + & (enter_E, (1-P_{giveway}) * r_{enter_E}).FoundBusy_W \\
FoundFree_W & \stackrel{def}{=} & (exit_W, r_{exit_W}).DriverState_W \\
 & + & (enter_E, P_{affected} * (1-P_{giveway}) * r_{enter_E}).Affected_W \\
 & + & (enter_E, (1-P_{affected}) * (1-P_{giveway}) * r_{enter_E}).NotAffected_W \\
FoundBusy_W & \stackrel{def}{=} & (exit_E, r_{exit_E}).DriverState_W \\
 & + & (enter_W, P_{affected} * (1-P_{brake}) * r_{enter_W}).Affected_W \\
 & + & (enter_W, (1-P_{affected}) * (1-P_{brake}) * r_{enter_W}).NotAffected_W \\
NotAffected_W & \stackrel{def}{=} & (exit_E, r_{exit_E}).(exit_W, r_{exit_W}).DriverState_W \\
 & + & (exit_W, r_{exit_W}).(exit_E, r_{exit_E}).DriverState_W \\
Affected_W & \stackrel{def}{=} & (exit_E, r_{exit_E}).(exit_W, r_{exit_W}).Evaluate_W \\
 & + & (exit_W, r_{exit_W}).(exit_E, r_{exit_E}).Evaluate_W
\end{array}$$

$$
\begin{array}{rcl}
Evaluate_W & \stackrel{def}{=} & (enter_W, (1-P_{cautious}) * r_{enter_W}).FoundFree_W \\
& + & (enter_E, (1-P_{giveway}) * r_{enter_E}).Alert_W \\
Alert_W & \stackrel{def}{=} & (exit_E, r_{exit_E}).Evaluate_W \\
& + & (enter_W, (1-P_{cautious}) * P_{affected} * (1-P_{brake}) * r_{enter_W}).Affected_W \\
& + & (enter_W, (1-P_{cautious}) * (1-P_{affected}) * (1-P_{brake}) * r_{enter_W}).NotAffected_W
\end{array}
$$

Initially the driver from the West is in state $DriverState_W$, may found the junction free and enter it with action $enter_W$ by performing a transition to state $FoundFree_W$. Alternatively, a driver from the East may enter the junction first with action $enter_E$, making it busy — hence a transition to state $FoundBusy_W$.

In state $FoundFree_W$ there are three possible choices: the driver from the West exits the junction (action $exit_W$), with transition to the initial state $DriverState_W$; a driver from the East enters the junction (action $enter_E$) with probability $P_{affected}$, affecting the future behaviour of the driver from the West through transition to state $Affected_W$; a driver from the East enters the junction (action $enter_E$) with probability $1-P_{affected}$ and transition to state $NotAffected_W$, without any effect on the future behaviour of the driver from the West.

In state $FoundBusy_W$ there are three possible choices: the driver from the East exits the junction (action $exit_E$), with transition to the initial state $DriverState_W$; the driver from the West enters the junction (action $enter_W$) with probability $P_{affected}$ and has an experience of possible collision affecting future behaviour — hence a transition to state $Affected_W$; a driver from the West enters the junction (action $enter_E$) with probability $1-P_{affected}$ and has an experience of possible collision without any effect on future behaviour — hence a transition to state $NotAffected_W$.

In states $NotAffected_W$ and $Affected_W$ drivers exit the junction in any order through the interleaving of actions $exit_W$ and $exit_E$ with transition to state $DriverState_W$ and state $Evaluate_W$, respectively.

In state $Evaluate_W$, the next entry of the driver from the West to the junction is modelled by associating action $enter_W$ with a decrease of rate r_{enter_W} by factor 1-$P_{cautious} \in (0,1)$, which represents the extra caution of the driver (due to the possible collision experienced the previous time). Such extra caution consists in evaluating whether it is safe to allow a driver turning left to go first. If this driver is allowed to go first, then action $enter_E$ occurs with transition to state $Alert_W$; otherwise the driver from the West enters the free junction with an occurrence of action $enter_W$ and a transition to state $FoundFree_W$.

In state $Alert_W$, the junction is busy with the driver from the East turning left. The cautious driver from the West may enter the junction with action $enter_W$ associated with a strong decrease of rate r_{enter_W} by (1-$P_{cautious}$) $*(1-P_{brake}) \in (0,1)$, which represents the extra caution of the driver (due to the possible collision experienced the previous time). Such extra caution consists in being alert to check for a possible driver turning left who is already in the junction. If the driver from the West manages to allow the turning driver to exit the junction first, then action $exit_E$ occurs with transition to state $Evaluate_W$, in which the driver from the West may possibly cautiously enter the junction.

The transition from state $Evaluate_W$ to state $FoundFree_W$ ensures that the effect of the previous experience lasts only for one re-entry to the junctions. Moreover, state $Alert_W$ also models the fact that a driver from the West, who enters the junction cautiously due to a previous experience of possible collision and, at the same time, experiences another possible collision, may (transition to state $Affected_W$) or may not (transition to state $NotAffected_W$) be affected next time.

More realistically, we could assume that the effect of experiencing a possible collision lasts for a few re-entries to the junction, but gradually decreases at each re-entry until it totally disappears after a number k of re-entries without any further experiences of possible collisions. Therefore, we replace probability $P_{cautious}$ with a set of probabilities $\{P_i\}_{i=1,\dots k} \subseteq (0,1)$ such that $P_i > P_{i+1}$ for each $i = 1,\dots k-1$ and processes $FoundFree_W$, $Evaluate_W$ and $Alert_W$ with k triples of processes $FoundFree(i)_W$, $Evaluate(i)_W$ and $Alert(i)_W$, with $i = 1,\dots k$.

Processes $DriverState_W$ is re-defined as follows.

$$\begin{aligned} DriverState_W &\stackrel{def}{=} (enter_W, r_{enter_W}).FoundFree(1)_W \\ &+ (enter_E, (1-P_{giveway}) * r_{enter_E}).FoundBusy_W \end{aligned}$$

Processes $FoundBusy_W$ is unchanged and process $FoundFree(1)_W$ is defined as follows.

$$\begin{aligned} FoundFree(1)_W &\stackrel{def}{=} (exit_W, r_{exit_W}).DriverState_W \\ &+ (enter_E, P_{affected} * (1-P_{giveway}) * r_{enter_E}).Affected_W \\ &+ (enter_E, (1-P_{affected}) * (1-P_{giveway}) * r_{enter_E}).NotAffected_W \end{aligned}$$

Process $NotAffected_W$ is unchanged and process $Affected_W$ is re-defined as follows.

$$\begin{aligned} Affected_W &\stackrel{def}{=} (exit_E, r_{exit_E}).(exit_W, r_{exit_W}).Evaluate(1)_W \\ &+ (exit_W, r_{exit_W}).(exit_E, r_{exit_E}).Evaluate(1)_W \end{aligned}$$

For each $i = 1,\dots k-1$, process $Evaluate(i)_W$ is defined as follows.

$$\begin{aligned} Evaluate(i)_W &\stackrel{def}{=} (enter_W, (1-P_i) * r_{enter_W}).FoundFree(i+1)_W \\ &+ (enter_E, (1-P_{giveway}) * r_{enter_E}).Alert(i)_W \end{aligned}$$

After the driver from the West enters the junction (action $enter_W$) with caution characterised by probability $1-P_i$, there is a transition to $FoundFree(i+1)_W$ in which the driver from the West may exit the junction (action $enter_W$) with transition to state $Evaluate_W(i+1)$, in which decreased caution ($P_{i+1} < P_i$) is characterised by a higher probability ($1-P_{i+1} > 1-P_i$) of entering the junction.

For each $i = 1,\dots k$, process $Alert(i)_W$ is defined as follows.

$$\begin{aligned} Alert_W(i) &\stackrel{def}{=} (exit_E, r_{exit_E}).Evaluate(i)_W \\ &+ (enter_W, (1-P_i) * P_{affected} * (1-P_{brake}) * r_{enter_W}).Affected_W \\ &+ (enter_W, (1-P_i) * (1-P_{affected}) * (1-P_{brake}) * r_{enter_W}).NotAffected_W \end{aligned}$$

Note that the transitions from $Alert_W(i)$ to $Affected_W$ and then to $Evaluate(1)_W$ reset the probability of being cautious to the highest probability factor P_1.

For each $i = 2, \ldots k-1$, processes $FoundFree(i)_W$ is defined as follows.

$$\begin{aligned} FoundFree(i)_W &\stackrel{def}{=} (exit_W, r_{exit_W}).Evaluate(i+1)_W \\ &+ (enter_E, P_{affected} * (1 - P_{giveway}) * r_{enter_E}).Affected_W \\ &+ (enter_E, (1 - P_{affected}) * (1 - P_{giveway}) * r_{enter_E}).NotAffected_W \end{aligned}$$

Finally, process $Evaluate(k)_W$ is defined as follows.

$$\begin{aligned} Evaluate(k)_W &\stackrel{def}{=} (enter_W, P_k * r_{enter_W}).FoundFree(1)_W \\ &+ (enter_E, (1 - P_{giveway}) * r_{enter_E}).Alert(k)_W \end{aligned}$$

In state $Evaluate(k)_W$ action $enter_W$ is associated with the lowest probability factor P_k and causes a transition to state $FoundFree(1)_W$ in which the driver from the West may exit the junction (action $exit_W$) with transition to state $DriverState$, in which the caution induced by the experienced possible collision has totally disappeared.

We must remark that we do not need to associate drivers with specific cars because we are investigating the average behaviour of drivers rather than behaviours of individual drivers.

The overall system that includes driver experience is defined as follows.

$$System_2 \stackrel{def}{=} DriverState_W[N] \underset{\mathscr{L}}{\bowtie} (Car_W[N] || Car_E[M]) \underset{\mathscr{L}}{\bowtie} Junction$$

where $\mathscr{L} = \{enter_W, exit_W, enter_E, exit_E\}$.

We can observe that we could remove driver reactions from the *Junction* process by using pure rates (without probability factors $1 - P_{giveway}$ and $1 - P_{brake}$ associated with rates). In this way, process *Junction* would just model topology and capacity of the junction, while process $DriverState_W$ would model all aspects of driver reaction and experience, but the overall system behaviour would not change. Finally, we can observe that the first version of process $DriverState_W$ presented at page 6 is equivalent to the instantiation with $k = 1$ of the general model presented at page 8.

4 Numerical Results and Discussion

In this section we present the evaluation of the overall model $System_2$, which includes driver's experience, defined in Section 3.2. In all experiments we instantiate the general model of process $DriverState_W$ presented at page 8 with $k = 3$.

We use PEPA Eclipse plug-in [Tri07], a toolset that supports time series analyses, using either the ODE (Ordinary Differential Equation) method or stochastic simulation, and steady state analysis. We derive the probability of possible collision (as the probability to be in state $PCollision$) through steady state analysis. However, in this case the model suffers from state space explosion when the number of cars from each direction exceeds 3. Hence, we run simulations for 30 cars from each direction (with 0.05 confidence interval and 6000 replications) to find the average number of cars waiting in a queue and the average waiting time at the junction. Numerical outputs from the analysis are transferred to Microsoft Excel to generate plots. Values of all rates are set to 1 with the exception of r_{nop_W} and r_{nop_E}, which are set to 0.01. This low rate models the fact that the time for cars to re-approach the junction is much longer than the duration of all actions performed to drive across the junction.

Probability factors are set as follows.

$P_{giveway}$	P_{brake}	$P_{affected}$	$P_{straight}$	P_{left}	$P_{cautious}$	P_1	P_2	P_3
0.1–0.8	0.1–0.9	0.5	0.5	0.5	0.8	0.8	0.5	0.1

with $P_{giveway} < P_{brake}$, normally instantiated as $P_{giveway} = 0.6$ and $P_{brake} = 0.9$.

In Figures 2 and 3 we show the results of simulations with fixed $N = M = 30$ and $P_{brake} = 0.9$,

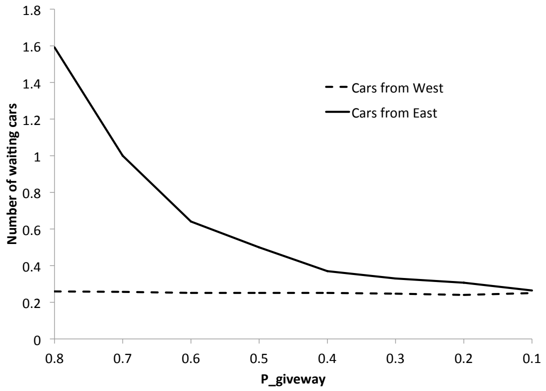

Figure 2: Average number of waiting cars with variable probability $P_{giveway}$ of giving way; fixed parameters $N = M = 30$ and $P_{brake} = 0.9$.

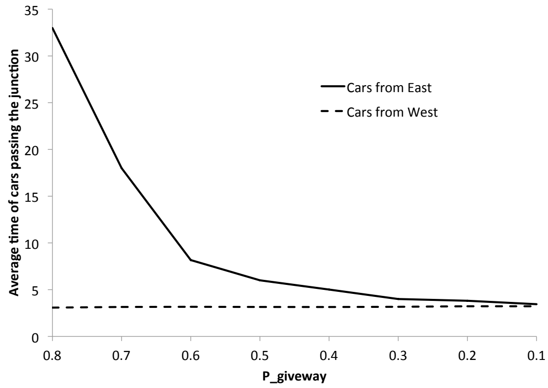

Figure 3: Average waiting time at the junction with variable probability $P_{giveway}$ of giving way; fixed paramenters $N = M = 30$ and $P_{brake} = 0.9$.

and with variable $P_{giveway}$. We can observe that

- both the average number of waiting cars (Figure 2) and the average waiting time at the junction (Figure 3) are stable and very low for cars from the West going straight,

whereas, for cars from the East turning left,

- the average number of waiting cars (Figure 2) rapidly decreases when probability $P_{giveway}$ of giving way decreases from 0.8 to about 0.4;

- the average waiting time (Figure 3) rapidly decreases when probability $P_{giveway}$ decreases from 0.8 to about 0.6, but then both only slightly decrease when $P_{giveway}$ further decreases.

In Figures 4 and 5 we show the results of simulations with fixed $N = M = 30$ and $P_{giveway} = 0.6$, and with variable P_{brake}. The average number of waiting cars (Figure 4) is

- almost stable and low for cars from the West going straight;

- moderately decreasing in an almost steady way for cars from the East turning left.

The average waiting time at the junction (Figure 5) is

- stable for all cars, but is obviously considerably lower for cars from the West.

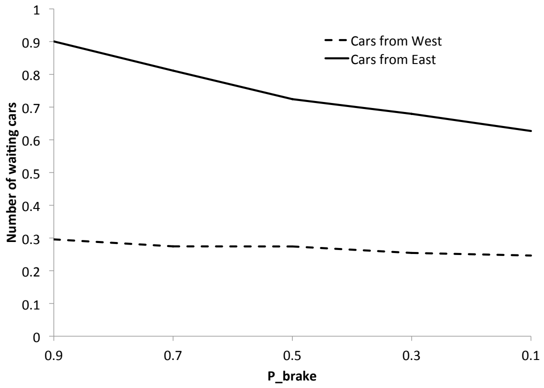

Figure 4: Average number of waiting cars with variable probability P_{brake}; fixed parameters $N = M = 30$ and $P_{giveway} = 0.6$.

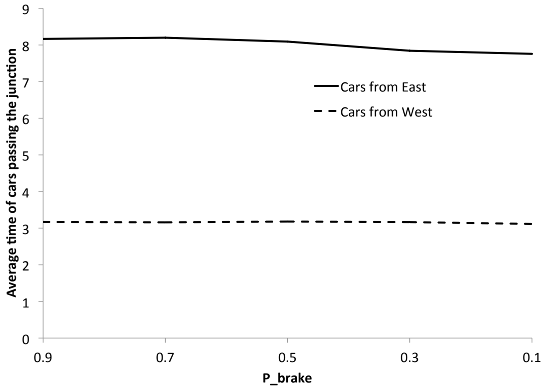

Figure 5: Average waiting time at the junction with variable probability P_{brake}; fixed paramenters $N = M = 30$ and $P_{giveway} = 0.6$.

These outcomes are quite obvious: it is the fact of giving way to increase the lenght of a queue and the waiting time rather than the braking reaction when observing a car already in the junction; moreover, if giving way occurs when it is not necessary, because the car is still too far (modelled by values of $P_{giveway}$ close to 1), as in the case of a novice, lenght of the queue and waiting time increase quite rapidly.

In Figure 6 we show the results of steady state analysis with fixed $N = M = 3$ and $P_{giveway} =$

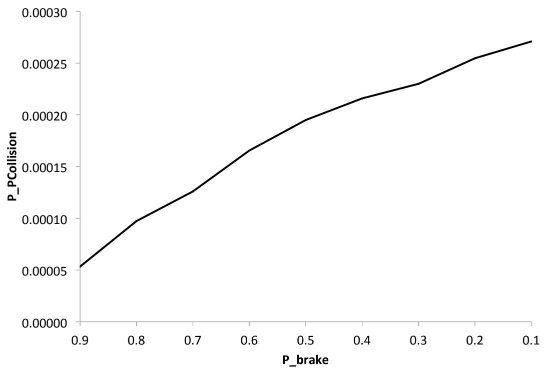

Figure 6: Probability of possible collision with variable probability P_{brake} of braking; fixed parameters $N = M = 3$ and $P_{giveway} = 0.6$.

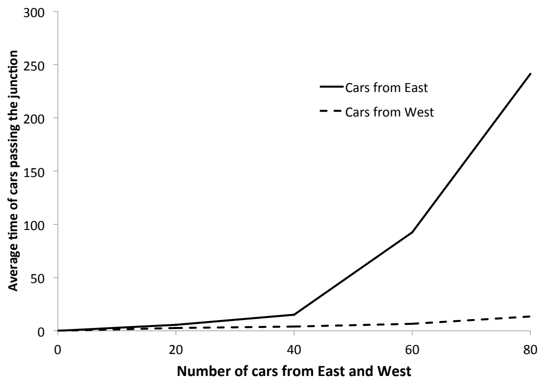

Figure 7: Average waiting time at the junction with variable number of cars M and N; fixed parameters $P_{giveway} = 0.6$ and $P_{brake} = 0.9$.

0.6, and with variable P_{brake}. We normally assume $P_{brake} = 0.9$, which corresponds to 0.00005 probability of possible collision. However, such probability strongly increases under abnormal physical conditions of the drivers such as stress, fatigue or effect of alcohol. Our experiments show that when $P_{brake} = 0.1$, the probability of possible collision increases to 0.00026, that is, over 5 times the normal value, whereas there are no significant effects of variations of probability $P_{giveway}$ of giving way (the maximum increase of probability of possible collision is only

0.00000787). This reflect the reality that abnormal physical conditions of drivers (especially effect of alcohol) have a much higher impact on the likelihood of collision than frequent failures of giving way (which is a sort of "culturally endemic" in some areas).

In Figure 7 we observe that the waiting time is not significantly affected by the increased number of cars from the East when these are less than 40, but rapidly increases for number larger than 40. This shows that, if the topology and capacity of the junction are modelled accurately, our simulation approach may provide a way to predict the level of traffic that would cause congestion.

5 Conclusion and Further Work

In this paper, we have shown how quantitative aspects of driver behaviour can be modelled with Markovian process algebra PEPA and analysed using the PEPA Eclipse plug-in toolset. The methodology was applied to a case study given by a three-way junction consisting of a two-way main road with a diverging one-way road. The analysis was performed using stochastic simulation with 30 cars from each direction and steady state analysis with 3 cars from each direction. The results show a trade-off between junction performance (reflected in number of cars in a queue and waiting time) and safety (reflected in probability of possible collision) under certain conditions on driver behaviour.

One way to overcome the state space explosion encountered in steady state analysis with more than 3 cars for each direction is to parse the output of a sufficient number of stochastic simulations and extract probabilistic data concerning a given set of states. This is part of our future work. Furthermore, we plan to improve the scalability of the analysis using model simplification and approximation and adopting some scalable analysis techniques.

We have modelled the effects of driver's experience in terms of state transitions associated with a finite number of pre-defined probability factors. In future work we aim to extend our model with the use of functional rate, that is, rate depending on the real time state of the overall system. This would support the modelling of behavioural adaptation as a continuous function of experience. Although none of the currently available tools directly support functional rate a number of techniques may be used to solve functional rate using PEPA Eclipse plug-in [ZT09].

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