The Effect of Measurement Error on $\tilde{X}$-$\tilde{R}$ Fuzzy Control Charts

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Abstract— **Control charts are tools used for monitoring manufacturing processes. Fuzzy Set Theory has found its way in control charts and new types of fuzzy control charts, with different capabilities, has been introduced. In this paper, a process in which the result of the measuring of each piece is imprecise is studied, and a $\tilde{X}$-$\tilde{R}$ fuzzy control chart is used for monitoring. The aim is to study the effect of measurement error on the effectiveness of the fuzzy control chart to detect out of control situations. The model used in this research is a linear covariate model. ARL parameters are used to study the performance of the fuzzy control chart when the parameters of covariate model is increased or decreased.**

Keywords- process control; fuzzy control chart; average run length (ARL); measurement error.

I. INTRODUCTION

Statistical process control includes a set of upcoming problems solving tools which is useful for creating stability in the process by decreasing the variability. Control charts are powerful tools for monitoring manufacturing processes and the first samples of these type of tools were introduced by Shewhart who introduced $\overline{X}-\overline{R}$ control charts to monitor a process with continuous specifications. However, the application of control charts for monitoring continuous data is defined for situations in which the size of each piece is precisely determined and the result of measuring process can be presented as a crisp number.

Suppose that more than one expert are used for measuring each piece and all of them use a unique measurement tools or machine. If the mean of the values measured by the experts, through different measuring procedures is different, a measurement error is introduced, caused by the experts' performance. If a system is subjected to such an error, control charts can not be used for monitoring the process. We can model the gauge or impreciseness in the size of product as a fuzzy number. Some models of the fuzzy control charts are to be used for monitoring the processes with impreciseness or fuzzy data, and there is no limit in the amount of impreciseness or error. Senturk and Erginel [1] presented $\tilde{X}$-$\tilde{R}$ fuzzy control charts. They aggregated all the results of different experts' measuring in a fuzzy number and monitored the fuzzy number on an $\tilde{X}$-$\tilde{R}$ fuzzy control chart. Kaya and Khahraman [2] introduced two different structures of control charts to monitor a process in which the size of each piece is expressed as a fuzzy number.

Assume that when a process is operating under an in-control situation the true value of X index is dependant on the normal distribution with a known mean of μ and known variance of σ^2_p. But, for some reasons, the actual value can not me measured, and instead Y index is monitored which has a linear relation with the X index. Also assume that on the basis of the achievements of Linna and Woodall [3], the result of measuring process by each expert is determined by a linear covariate model as shown in (1)

$$Y = A + BX + \varepsilon \qquad (1)$$

In this relation, A and B are constants and ε is the random error in the measurement system which is independent from X and is dependant on the normal distribution with a mean of zero and known a variance of σ^2_m. Thus, variance of the observed value is equal to $\sigma^2_{p} + \sigma^2_m$.

The results of undesirable effects of Measurement Error on control charts operation have been studied by different researchers. Bennet [4] suggested that the variance of the Measurement Error (σ^2_m) can be considered insignificant if it is smaller than the variance of the process (σ^2_p). He studied the power of Shewhart control charts when the measurement error variance is equal to the process variance and when it is twice the previous amount and he compared the achieved results with a state in which there is no Measurement error. Kanazuka [5] studied the power of $\overline{X}-\overline{R}$ control chatrs when the measurement error affects the system. He showed that significant variance of the measurement error can decrease significantly the effectiveness of the control chart. He suggested that a bigger sample could solve the problem.

Walden [6] measured the power of $\overline{X}, \overline{R}$ & $\overline{X} - \overline{R}$ control charts using Average Run Length parameter, when the measurement error affects the system. Linna [7] studied the effect of increasing the measurement error variance and slope of covariate model on Shewhart control charts. Maravelakis and et al. [8] studied the effect of measurement error on EWMA charts.

The purpose of this paper is to study the effect of measurement error on X̃-R̃ fuzzy control charts [1], when the measured values by different experts are different. Also, to show the effect of increasing the slope of linear covariate model on power of this kind of fuzzy control charts to detect undesirable changes.

In the second part, the structure of X̃-R̃ fuzzy control charts is determined when there is an error in the measured values by the experts and measurement system influenced by Linear Covariate Model simultaneously. Simulation details are explained in the third part. The different conditions that may occur by changing noticeable parameters, are described in section four. Finally in the fifth part, the effect of increasing the critical measurement parameter is estimated, using ARL.

II. The Structure of Fuzzy Control Chart for Covariate Value

We assume such a process that, under a controlled situation, the real value of X index follows a normal distribution with a known mean and variance of μ and σ^2_p, respectively. To design the structure of a control chart, under a controlled situation, we select some samples and ask experts to measure all of pieces. Because of the fact that experts can only measure the value of Y, we monitor Y instead of X. But the values expressed by different experts are not necessarily equal. For each piece, all values expressed by different experts will be converted to a fuzzy number (Y_a, Y_b, Y_c). Because of the measurement system error, we will have:

$$Y_a = A + BY_a + \varepsilon, Y_b = A + BY_b + \varepsilon,$$
$$Y_c = A + BY_c + \varepsilon \quad (2)$$

A. The structure of fuzzy control chart

When the process is operating under in-control situation, we choose m samples that each contains n pieces. Y_{kij} is the result of measuring the ith piece of the jth sample, by the kth expert. The mean for the jth sample will be calculated as follows:

$$\overline{Y}_{aj} = \frac{\sum_{i=1}^{n} Y_{aij}}{n}, \overline{Y}_{bj} = \frac{\sum_{i=1}^{n} Y_{bij}}{n}, \overline{Y}_{cj} = \frac{\sum_{i=1}^{n} Y_{cij}}{n},$$
$$j = 1, \ldots, m. \quad (3)$$

where, n is the sample size. Total mean for m samples, can be calculated as:

$$\overline{\overline{Y}}_k = \frac{\sum_{j=1}^{m} Y_{kj}}{m}, k = a, b, c, \quad (4)$$

Also, R_{aj}, R_{bj} and R_{cj} for each sample are defined as:

$$R_{aj} = Y_{max,aj} - Y_{min,cj} \quad (5)$$
$$R_{bj} = Y_{max,bj} - Y_{min,bj} \quad (6)$$
$$R_{cj} = Y_{max,cj} - Y_{min,aj} \quad (7)$$

$Y_{max,aj}$ and $Y_{min,aj}$ denotes the maximum and minimum values of Y_a in the sample. $Y_{max,bj}$, $Y_{min,bj}$, $Y_{max,cj}$ and $Y_{min,cj}$ also can be defined in the same way. The place of these parameters is shown on figure 1.

For an n-pieces sample the general mean of $\overline{R}_a$, $\overline{R}_b$ and $\overline{R}_c$ will be calculated as follows:

$$\overline{R}_k = \frac{\sum_{j=1}^{m} R_{kj}}{m}, k = a, b, c \quad (8)$$

Finally the control limits of X̃ fuzzy chart will be calculated as follows:

$$CL^{\alpha}_{mr-\bar{x}} = \frac{\overline{\overline{Y}}^{\alpha}_a + \overline{\overline{Y}}^{\alpha}_c}{2} \quad (9)$$

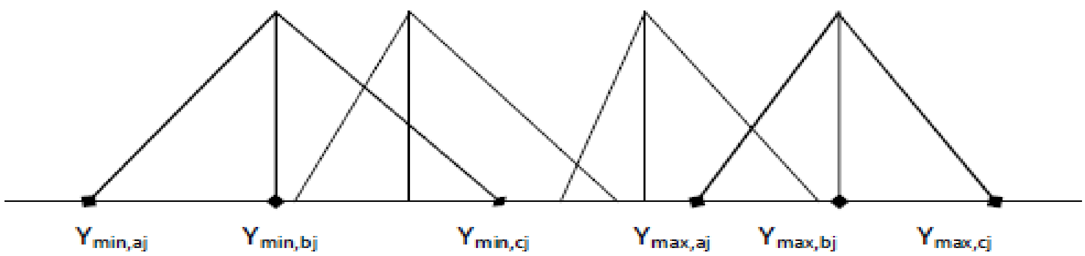

Fig. 1. Place of $Y_{max,aj}$, $Y_{min,aj}$, $Y_{max,bj}$, $Y_{min,bj}$, $Y_{max,cj}$, $Y_{min,cj}$.

$$UCL^{\alpha}_{mr-\bar{X}} = CL^{\alpha}_{mr-\bar{X}} + A\left(\frac{\bar{R}^{\alpha}_{a} + \bar{R}^{\alpha}_{c}}{2}\right) \quad (10)$$

$$LCL^{\alpha}_{mr-\bar{X}} = CL^{\alpha}_{mr-\bar{X}} - A\left(\frac{\bar{R}^{\alpha}_{a} + \bar{R}^{\alpha}_{c}}{2}\right) \quad (11)$$

In which:

$$\bar{\bar{Y}}^{\alpha}_{a} = \bar{\bar{Y}}_{a} + \alpha(\bar{\bar{Y}}_{b} - \bar{\bar{Y}}_{a}) \quad (12)$$

$$\bar{\bar{Y}}^{\alpha}_{c} = \bar{\bar{Y}}_{c} + \alpha(\bar{\bar{Y}}_{c} - \bar{\bar{Y}}_{b}) \quad (13)$$

and:

$$\bar{R}^{\alpha}_{a} = \bar{R}_{a} + \alpha(\bar{R}_{b} - \bar{R}_{a}) \quad (14)$$

$$\bar{R}^{\alpha}_{c} = \bar{R}_{c} + \alpha(\bar{R}_{c} - \bar{R}_{b}) \quad (15)$$

And the control limits for the R̃ fuzzy chart will be defined as follows:

$$CL^{\alpha}_{mr-\bar{R}} = \frac{\bar{R}^{\alpha}_{a} + \bar{R}^{\alpha}_{c}}{2} \quad (16)$$

$$UCL^{\alpha}_{mr-\bar{R}} = D_4 * CL^{\alpha}_{mr-\bar{R}}$$
$$LCL^{\alpha}_{mr-\bar{R}} = D_3 * CL^{\alpha}_{mr-\bar{R}} \quad (17)$$

Where D_3 and D_4 are control chart coefficients [9].

The S^{α}_{mr-yj} for the jth sample is calculated as follows for the X̃ control cart:

$$S^{\alpha}_{mr-yj} = \frac{(\bar{Y}_{aj} + \bar{Y}_{cj}) + \alpha[(\bar{Y}_{bj} - \bar{Y}_{aj}) - (\bar{Y}_{cj} - \bar{Y}_{bj})]}{2} \quad (18)$$

The S^{α}_{mr-Rj}, for the jth sample is calculated as follows for the R̃ control cart:

$$S^{\alpha}_{mr-Rj} = \frac{(R_{aj} + R_{cj}) + \alpha[(R_{bj} - R_{aj}) - (R_{cj} - R_{bj})]}{2} \quad (19)$$

In case that the following limitations are effective, the process is under control, otherwise, the process is out of control:

$$\begin{cases} LCL^{\alpha}_{mr-\bar{X}} \le S^{\alpha}_{mr-yj} \le UCL^{\alpha}_{mr-\bar{X}} \\ and \\ LCL^{\alpha}_{mr-\bar{R}} \le S^{\alpha}_{mr-Rj} \le UCL^{\alpha}_{mr-\bar{R}} \end{cases} \quad (20)$$

III. SIMULATION

Our aim is to simulate a condition in the system, in which there exists an error on the measured values by the experts and the measurement system has the linear covariate error model. For simplification, we have supposed that only three experts, measure the size of each product. Let the mean of the true value of the product be μ and the mean of results achieved by experts a, b and c be μ_a, μ_b and μ_c, respectively such that $\mu_a \le \mu_b \le \mu_c$. In such a condition, the difference between the measured values of experts which is named d, averagely would be equal to (μ_c-μ_b). Also, because of the presence of the measurement error, the observed value by each expert is calculated as Y=A+BX+ε.

The detection power of control charts is estimated by the Average Run Length (ARL) parameter. This parameter represents the average number of SPC points (subgroups) that will pass before a signal or special cause is encountered. When the process is out-of-control, the aim is that the control chart can signal as soon as possible. Therefore, smaller ARL is a sign of better performance of a control chart.

The model is simulated using MATLAB software for n=5, α=0.65 and 20000th loops.

IV. STUDY OF DIFFERENT STATES

A different view to the structure of the X̃-R̃ fuzzy control charts shows that the notification to impreciseness or error resulted from the expert's measurements can be decreased or increased by changing the value of α. The structure of control limits of X̃ & R̃ control charts, shows that the ratio of significance of the opinion of expert b is α, and the ratio of significance of the opinion of experts a and c (together) is equal to (1-α). So:

- If α =1, only the results achieved by expert b are used, and the opinions of experts a and c are not considered.
- If α=1, A=0, B=1, σ^2_m=0, the chart will act as a traditional $\bar{X} - \bar{R}$ control chart.
- If α=1, A≠0 or B≠1 and σ^2_m≠0 the chart acts like a Shewhart control chart, when the measurement error with linear covariate model affects the process.
- Finally, when α=0, the opinions of experts a and c is considered and the opinion of expert b is believed insignificant.

V. EFFECT OF σ^2_m AND B ON X̃-R̃ CONTROL CHART

Table (I) illustrates the effect of increase of (σ^2_m/ σ^2_p) on the power of the fuzzy X̃-R̃ chart, when the results of measurements by different experts are varied about 0.2 unit (d=0.2) in average. The left side column shows the change of mean and the first row shows the amount of σ^2_m/ σ^2_p. The ARL changes in rows show that there is a decreasing effect on power of fuzzy X̃-R̃ control chart to detect the upcoming changes in the mean as the ratio of σ^2_m/ σ^2_p increases.

We can see that the achieved results are the same as the results Linna & Woodall achieved in their researches on Shewhart control charts [3].

TABLE I. ESTIMATED ARL PERFORMANCE OF $\tilde{X}$-$\tilde{R}$ CONTROL CHART AGAINST SEVERAL VALUES OF σ_m^2/σ_p^2, UNDER ESTIMATED LIMITS: d=0.2, A=0, B=1

$\lvert\mu_0-\mu'\rvert$ \ σ^2_m/σ^2_p	0.0	0.1	0.2	0.5	1.0
0.0	200.91	200.76	200.85	200.64	200.81
0.2	195.60	198.81	199.61	200.10	200.23
0.4	158.65	159.10	160.05	183.34	192.01
0.6	72.67	73.00	77.50	105.16	153.65
0.8	21.28	21.75	22.77	38.61	87.17
1.0	7.03	7.12	7.44	15.53	37.55
1.2	3.02	3.10	3.20	5.02	15.73
1.4	1.71	1.73	1.78	2.56	7.23
1.6	1.24	1.26	1.30	1.62	3.95
1.8	1.07	1.08	1.09	1.24	2.40
2.0	1.01	1.02	1.02	1.07	1.65

TABLE II. - ESTIMATED ARL AGAINST SEVERAL VALUES OF B, FOR $\tilde{X}$-$\tilde{R}$ CONTROL CHART UNDER ESTIMATED LIMITS d=0.2, A=0 AND $\sigma_m^2/\sigma_p^2 = 0.2$

$\lvert\mu_0-\mu'\rvert$ \ B	1	2	3	4
0.0	200.85	200.82	200.78	200.81
0.2	199.61	199.55	199.15	199.07
0.4	160.05	159.68	159.43	158.77
0.6	77.50	76.93	76.23	74.69
0.8	22.77	22.32	21.14	21.37
1.0	7.44	7.28	7.20	7.07
1.2	3.20	3.14	3.08	3.03
1.4	1.78	1.78	1.70	1.74
1.6	1.30	1.28	1.27	1.23
1.8	1.09	1.08	1.07	1.06
2.0	1.02	1.02	1.01	1.01

Table (II) reports the estimated average run length (ARL) to show the effect of the increase of B on the power of the fuzzy $\tilde{X}$-$\tilde{R}$ chart, when the results of the measured values by different experts is averagely different about 0.2 unit (d=0.2) and A=0. The first left side column shows the size of the shift in the mean value and the first row shows the value of B.

As it can be seen in [3], increasing the value of B can result in an increase in the power of the $\tilde{X}$-$\tilde{R}$ control chart in detecting the changes in the mean value.

VI. CONCLUSIONS

In this research the effect of the measurement error on the $\tilde{X}$-$\tilde{R}$ fuzzy control chart is studied. The model used is a linear covariate model. The effectiveness of the control chart in detecting the changes in the mean value is calculated using the average run length (ARL). It is shown that, when the mean values of measurements are different, a smaller value of measurements' variance results to greater effectiveness of the $\tilde{X}$ -$\tilde{R}$ fuzzy control chart. Also the ALR changes show that in similar condition, the effectiveness of the control chart increases when the slope of the linear covariate model increases.

REFERENCES

[1] S. Senturk, N. Erginel, "Development of fuzzy $\tilde{X}$_$\tilde{R}$ and $\tilde{X}$-$\tilde{S}$ control charts using α-cuts", Information Sciences, pp. 1542-1551, 2009

[2] I. Kaya, C. Kahraman, "Process capability analyses based on fuzzy measurements and fuzzy control charts", Expert Systems with Applications, Vol. 38, No. 4, pp. 3172-3184, 2011

[3] K. W. Linna, W. H. Woodall, "Effect of measurement error on Shewhart control charts", Journal of Quality technology, Vol. 33, No. 2, pp. 213-222, 2001

[4] C. A. Bennett, "Effect of measurement error on chemical process control", Industrial Quality Control, Vol. 10, No. 4, pp. 17-20, 1954

[5] T. Kanazuka, "The effect of measurement error on the power of X-R charts", Journal of Quality Technology, Vol. 18, No. 2, pp. 91-95, 1986

[6] C. T. Walden, An analysis of variables control charts in the presence of measurement error, Mississippi State University, Department of Industrial Engineering, 1990

[7] K. W. Linna, Control chart performance under linear covariate measurement processes, PhD Thesis, Uiversity of Alabama, 1991

[8] P. Maravelakis, J. Panaretos, S. Psarakis, "EWMA Chart and Measurement Error", Journal of Applied Statistics, Vol. 31, No. 4, pp. 445-455, 2004

[9] D. C. Montgomery, Introduction to Statistical Quality Control, John Wiley & Sons, Inc., 1996