# 3D Numerical Analysis of the Effects of an Advancing Tunnel on an Existing Loaded Pile Group

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*Abstract*—**Tunnels are often preferred for underground transportation in densely populated areas. In these areas, it is almost inevitable for tunnels to run close to some existing pile foundations. Since tunnelling activities induce stress relief and soil movement in the ground, existing piles may suffer from additional axial and lateral forces, bending moments, settlements and lateral deflections. Most of the previous researches on the responses of pile foundations due to tunnel construction were carried out under the plane strain condition. In this paper, a three-dimensional, elasto-plastic and coupled-consolidation finite element parametric study has been carried out to investigate the effects of a 6 m open-face advancing tunnel on a two by two pile group in saturated stiff clay. The influence of different cover-to-diameter (C/D) tunnel ratios (namely 2.0, 2.5 & 3.0) was studied. The objectives of this study are to determine the changes in axial load distribution, changes in shaft resistance along the shaft of pile group and settlement of pile cap due to an advancing open-face tunnel.**

***Keywords-finite element analysis; open face tunnelling; settlement of pile cap***

## I. Introduction

A piled foundation transfers the load of the structure into the ground, generating stresses in the surrounding soil. In contrast, tunnel excavation induces stress relief to the surrounding soil and results in ground movements around the tunnel, which propagate through the soil to the ground surface. Nowadays, tunnelling is a very popular technique to facilitate congested urban traffic systems in big cities like London, Hong Kong and Singapore. Since tunnel construction inevitably induces soil movement and stress changes in the ground, it may cause additional settlement and tilting to nearby existing piled foundations. Tunnelling remains a big challenge for geotechnical engineers, particularly when a tunnel is to be excavated in soft ground. To understand the pile–soil–tunnel interaction mechanism, many researchers have conducted field monitoring studies and centrifuge model tests [1-9]. Moreover, this problem has also been studied by the proposing of analytical solutions and numerical modelling [10-19]. All authors in [10-19] concluded that tunneling adjacent to existing pile foundations caused pile settlement, additional axial load on piles and induced bending moments along piles, which is unfavourable for piled foundations. The magnitudes of which likely depended on the relative locations of tunnels and piles.

Authors in [20] described the effects of a 7.5 m tunnel excavated in stiff London clay on a bored piled foundation. The clear spacing between the springline of the tunnel and the nearest 1.2 m diameter pile was only 1 m. The measured horizontal pile and ground movements were similar. Maximum horizontal displacement of 10 mm of the nearest pile to the tunnel was reported. Authors in [21] reported the measured settlement of a building due to the excavation of a 7.9 m diameter tunnel in Hong Kong. The building was supported by 2 m diameter bored piles varying in length from 41 m to 64 m. The tunnel depth was above the level of the pile toe. Maximum building settlement due to tunnel construction was recorded to be 12 mm. Authors in [22] reported measured results of the effects of 6.5 m diameter shield tunneling on the adjacent piled foundations of bridge piers in Singapore. The piers were supported by 2×2 pile group of 1.2 m diameter and 62 m long piles. The piles were embedded in completely weathered material (residual soil) with SPT values varying from 15 to 100. The nearest tunnel-pile foundation distance was 1.6 m. Tunnel depth was located at about the mid pile depth (i.e. 21 m).The maximum volume loss due to the tunnel excavation was reported as high as 1.5 %. The monitoring results showed that the piles were subjected to large dragload, and the maximum induced bending moment in the piles was at the tunnel springline due to the tunnel advancement. Authors in [23] carried out a 3D elasto-palstic analysis to study the effects of a tunnel on a single pile and a 2×2 pile group pile group at different cover-to-tunnel-diameter (C/D) ratios. Computed results showed that there was significant reduction of the induced axial force and bending moment on the pile furthest away from the tunnel (i.e., the rear pile) due to the group effect.

However, the settlement, tilting of the pile group and the load transfer mechanism between piles was not reported [23].

In previous studies, researchers were mostly interested in the tunnelling-induced axial forces and bending moments in piles due to tunnel excavation. The load transfer within the piles of a group and the induced tilting due to the tunnel advancement has not been reported. Besides this, the location of a tunnel relative to a pile foundation has not been studied systematically. With the prime objective of investigating the tunnel location effects relative to the piled foundation, 3D parametric coupled-consolidation finite element analyses were carried out at different cover-to-tunnel-diameter ratios (C/D = 1.5, 2.5 and 3.5). The effects of an advancing, open face, 6m diameter tunnel on a 2×2 pile group in stiff saturated clay were investigated. Settlement, pile group tilting and load transfer among piles at various tunneling stages were studied and discussed.

## II. 3D COUPLED-CONSOLIDATION ANALYSES

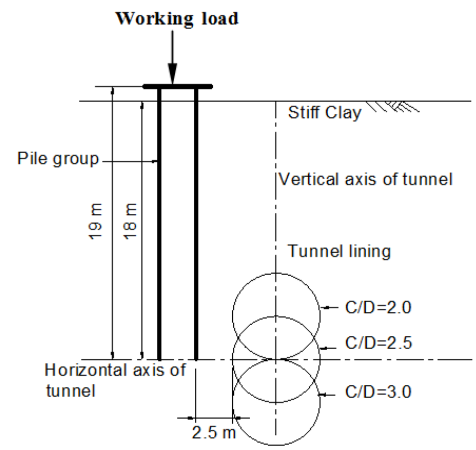
A hypothetical tunnel excavation in a stiff, non-homogeneous and over consolidated saturated clay is modeled. Figure 1 shows tunnel geometry and pile group size and location relative to the tunnel. Tunnel diameter (D) is taken as 6 m with different cover depths (C) of 12 m, 15 m and 18 m. A (2×2) pile group with 2.5 m center to center distance, 19 m length (1 m above ground surface) and each pile, with 0.8 m diameter, is located at a distance of 5.5 m from tunnel center line. For the ease of descriptions and discussion, P1 and P2 are referred as front and rear piles, respectively (see Figure 1b). The purpose of this numerical study is to investigate the effect of tunneling on the nearby pile group at different C/D ratios, while keeping the pile group length unchanged. The finite element program, ABAQUS, was used to carry out these numerical analyses.

### A. Finite Element Mesh and Boundary Conditions

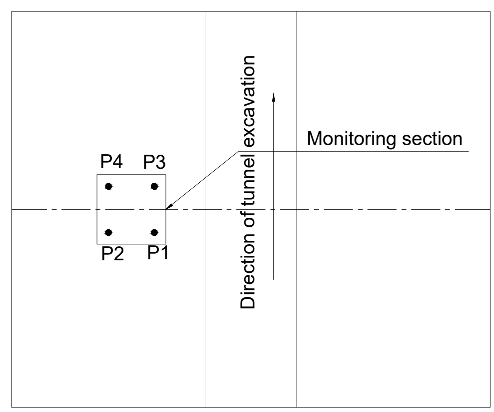
Figure 2 shows a three-dimensional finite element mesh (soil, pile group and pile cap), adopted for all the numerical runs. Taking advantage of symmetry, only the half of the domain is simulated at x=0. The finite element mesh is 60 m long, 60 m wide and 36 m high. This mesh consists of 12,276 elements and 15,331 nodes. The eight-noded brick elements, four-noded shell elements and two-noded beam elements are used to model the soil, lining and pile cap and pile group, respectively. A monitoring section is selected at pile group center line for reference. Roller supports and pin supports are applied on vertical sides and mesh base respectively. Therefore, the movement to normal direction of vertical sides of mesh and movement in all directions of mesh base are restrained. The water table is assumed to be located at ground surface. At the first step pore water pressure distribution profile is assumed to be hydrostatic. Free drainage is allowed at the top of the mesh. Tunnel lining is assumed to be impervious.

### B. Constitutive Models and Model Parameters

An elasto-plastic soil model using Drucker-Prager failure criterion with non-associated flow rule is used to model the behaviour of stiff clay for these numerical analyses. The strength parameters effective cohesion (c’), effective angle of friction (ϕ’) and angle of dilation (ψ) for stiff clay are assumed as 5kPa, $20^0$ and $11^0$ respectively [12]. Stiffness parameters for stiff clay are assumed as anisotropic and increase linearly with depth [1]. All the soil parameters used for the numerical analyses are summarized in Table 1. The concrete piles, cap and tunnel lining are assumed to be linear elastic with Young’s modulus of 35 GPa and Poisson’s ratio as 0.25. Lining thickness and pile cap are taken as 0.25m and 1m respectively. The unit weight of concrete is assumed 24 kN/m$^3$. Piles are assumed to be rigidly connected with cap.




(a) Sectional view




(b) Plan view

Fig. 1. Geometry of problem studied

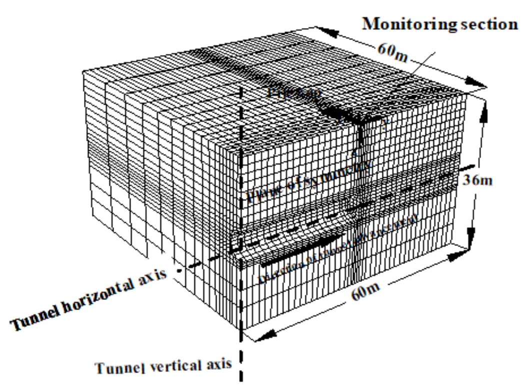



Fig. 2. Three-dimensional finite element mesh

TABLE I. SOIL PARAMETERS USED IN FINITE ELEMENT ANALYSES.

| Soil Parameter | |
|---|---|
| Dry density ($\gamma_d$) | 1500 kg/m3 |
| Void ratio (i) | 1.0 |
| Vertical effective Young's modulus ($E'_v$) | 7500+3900·Z kPa |
| Horizontal effective Young's modulus ($E'_h$) | 12000+6240·Z kPa |
| Shear modulus in vertical plane ($G_{vh}$) | 0.44E'v kPa |
| n= $E'_h/E'_v$ | 1.6 |
| Poisson's ratio ($\nu$) | 0.125 |
| Coefficient of permeability (k) | 1×10-9 m/s |
| Effective cohesion (c') | 5 kPa |
| Effective angle of friction ($\phi'$) | $22^0$ |
| Angle of dilation ($\psi$) | $11^0$ |
| Coefficient of lateral earth pressure at rest ($K_0$) | 1.0 |

### *C. Numerical Modeling Procedure*

Two separate numerical runs were carried out on two different finite element meshes. The first numerical run was conducted to determine the ultimate axial load carrying capacity of pile group and the second numerical run was conducted to determine the effects on pile group. The numerical modeling procedure steps are summarized as follows:

1. Establish the initial stress condition using $K_0$=1.0.
2. Determine the axial load carrying capacity of wished-in-place pile group from numerical load test by conducting first numerical run.
3. Then create another mesh with same initial stress condition as in Step 1. Carry out second numerical run after applying working load, computed from first numerical run, on pile group with FOS=3 at initial stress condition.
4. Allow the excess pore pressure to dissipate, developed in response of working load on pile group.
5. Then start to excavate tunnel with 3m (D/2) unsupported length.
6. Apply the 250 mm thick lining to exposed surface of tunnel.
7. Advance the tunnel excavation, repeating the same procedure until the tunnel is completed.

An open face tunnel sequential excavation was modeled in this study. Tunnel excavation was simulated by deactivating the elements located in tunnel zone and tunnel lining was simulated by activating elements of lining.

### *D. Determination of Pile Group Axial Load Capacity*

Prior to tunnel excavation, it is desirable to determine pile group axial load carrying capacity. For this purpose, a numerical pile load test was carried out. In pile test, load was increased from 0 to 24,000 kN on the pile cap middle for over a period of 24 hours. Figure 3 shows the load settlement curve obtained from the pile load test. The ultimate axial load capacity was determined based on new displacement-based failure criterion proposed in [10]. The failure criterion is expressed as follows:

$$\delta_{ph,\max} \cong 0.045 d_p + \frac{1}{2}\frac{P_h L_p}{A_p E_p} \qquad (1)$$

where $P_h$ = pile head load, $L_p$= pile length, $E_p$= pile shaft elastic modulus, $A_p$= Cross sectional area of pile and $d_p$ = pile diameter. As shown in Figure 3, the ultimate load capacity of the simulated pile group was determined to be 21,210 kN. With a factor of safety 3.0, a working load of 7,070 kN was applied on the mid of pile cap. An initial pile cap settlement of 6.5 mm (0.8% dp) was calculated due to applied working load. The excess pore pressure, generated due to working load prior to tunnel excavation, was allowed to fully dissipate.

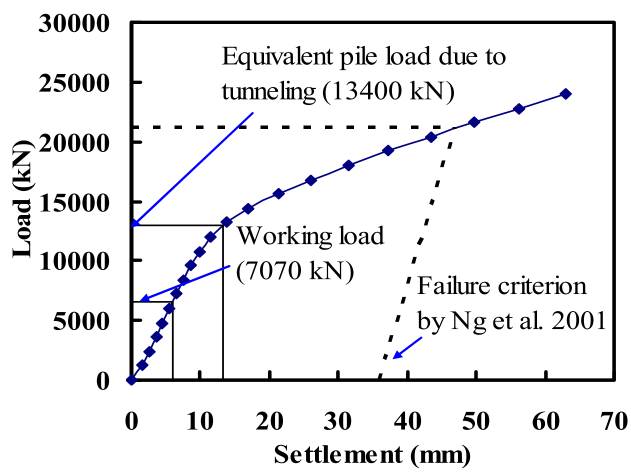



Fig. 3. Load settlement curve of simulated pile group load test

## III. COMPUTED RESULTS

In this section, computed results are presented. From the computed data, it was observed that the most critical moment is when tunnel face aligns with the pile group center line (monitoring section). However, there are not any significant changes in surface settlement as tunnel face passes beyond the y/D≥3 from the monitoring section. Therefore, it can be assumed that monitoring section attains the plane strain condition.

### *A. Transverse Surface Settlement Trough*

Figure 4 shows the normalized immediate surface settlements (S), when tunnel face reached monitoring section (y/D=0) and at the plane strain condition, when tunnel face passed beyond monitoring section at a distance of y/D≥3, for all three cases. Although the surface settlements were initially induced due to load applied on the pile group, only tunnelling-induced settlements are considered here. It can be seen that immediate settlement is less than settlement at plane strain condition. This occurs because of the dissipation of excessive pore water pressure, after the tunnel face has passed through monitoring section. The Gaussian distribution curves [11] are fitted on the basis of $S_{max}$ and i (distance from tunnel center line to the curve inflexion point) at plane strain condition, computed from each numerical analysis. The calculated values of i, at plane strain condition are 9.9 m, 10.8 m and 11.7 m from tunnel center line for C/D = 2.0, 2.5 and 3.0, respectively. The computed maximum surface settlements at tunnel center line are 10.3 mm, 10 mm and 9 mm for C/D=2.0, 2.5 and 3.0 respectively. Based on $S_{max}$ and i, it can be observed that with the increase of C/D ratio, maximum settlement decrease and settlement trough become wider. The volume loss associated due to tunnel excavation calculated on the basis of area of settlement trough is about 0.9% for each case.

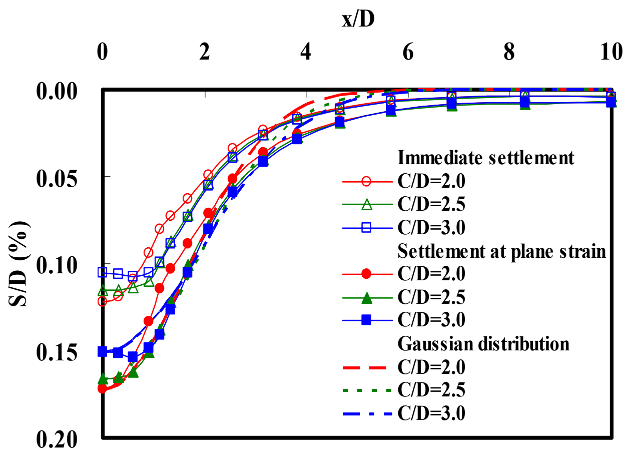



Fig. 4. Transverse settlement at monitoring section due to tunnelling

*B. Pile Group Settlement Due to Tunnel Advancement*

Figure 5 shows the incremental settlement of pile group due to tunnelling for C/D=2.0, 2.5 and 3.0. It can be observed that as tunnel advances to monitoring section, excessive settlement occurs in the pile group. In the case of C/D=2.0, the pile group settlement due to tunneling is less than in the other cases. Settlement profiles for the cases C/D=2.5 and C/D=3.0 are similar because the pile group base is subjected to stress release due to tunneling in both cases. The pile group settlements due to tunneling advancement are 5.6 mm for C/D=2.0 and 7.5 mm for both C/D=2.5 and C/D=3.0 at plane strain conditions (i.e. y/D≥3). This implies that total pile group settlement due to tunneling is 12 mm (1.5%dp) for C/D =2.0 and 14 mm (1.75%dp) for C/D=2.5 and C/D=3.0. Since there is not much difference in total pile group settlement for all three cases, therefore from load settlement curve (Figure 3), the load corresponding to 14 mm settlement is 13,400 kN. It can be considered that the pile group is subjected to an equivalent load of 13,400 kN after tunnel passes beyond monitoring section, which reduces the factor of safety from 3.0 to 1.6 in each case.

*C. Changes in Unit Shaft Friction Along the Length of Pile P1 and Pile P2 Due to Tunnelling.*

Figure 6 shows the changes in the unit shaft friction along the shafts of both P1 and P2, when tunnel face is at monitoring section (i.e. y/D=0). The unit shaft friction reduces significantly in P1 at the upper part of the shaft (i.e.0≤z/D≤2.5), when tunnel is located at C/D=2.0. It indicates the reduction in shaft resistance because of the stress release around P1 due to tunneling. The maximum reduction of 35 kPa in unit shaft friction of P1 is observed at the tunnel crown (i.e. z/D=2.0). However, the unit shaft friction increases along the lower part of P1 below tunnel horizontal axis (z/D≥2.5). As a result, base load of P1 is increased. Unit shaft friction also reduces along the upper part of shaft of P2 (i.e. z/D≤2.25). The increment in unit shaft friction at the lower part of shaft of P2 shows that base load is mobilized. It is clear that most of the load has transferred to the pile group base in case of C/D=2.0. When the tunnel is located at C/D=2.5, unit shaft friction increases along the upper part of the shaft (i.e. z/D≤ 1.4) of P1. However, the unit shaft friction near the base of P1 is reduced. It implies that no significant base load is mobilized which is discussed below. However, the negligible reduction in unit shaft friction is observed along the shaft of P2 except for the lower part of the shaft. It suggests that both shaft friction and base load of P2, support the additional load transferred from P1. It can be observed that unit shaft friction increases along the upper part of the shaft (z/D≤2.25) of P1 in the case of C/D=3.0. However, unit shaft friction reduces significantly at the lower part of P1 (i.e. z/D>2.25). This shows that the base load of P1 reduces significantly. The maximum reduction of 44 kPa in unit shaft friction occurs at tunnel crown (z/D=3.0). However, unit shaft friction along the upper part of the shaft of P2 (0≤z/D≤2.5) almost remained unchanged. The shaft resistance increases at the lower part (z/D≥2.5) of P2. It suggests that the load is shared by both shaft resistances and the pile base. Most of the load in P1 is supported by shaft resistance after tunnel face passes through monitoring section in the case of C/D=3.0.

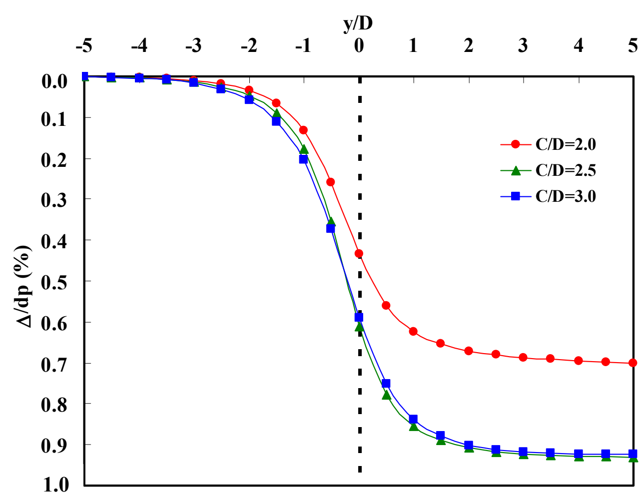



Fig. 5. Pile cap settlement due to tunnel advancement

Unit shaft friction is decreased significantly in P1 and P2 above the tunnel crown, when tunnel is located at C/D=2.0. As a result, base load of P1 is increased and pile head load is significantly decreased. Pile head load and base load of P2 increased substantially. It is clear that some part of the load is transferred to the base of P1 and some part of the load is transferred to P2. The head load of P1 reduced by 8%, whereas the head load of P2 increased by 7%. The base load of both piles P1 and P2 increases by 60% and 32%, respectively. In the case of C/D=2.5, tunnel horizontal axis is at the pile group base (i.e. z/D=3.0). Therefore, almost negligible change is observed in the base of both piles. Due to load transfer mechanism in pile group, the load transfers from P1 to P2. The reduction in pile head load is 10% and pile head load of P2 increased 10% of total load on each pile. The base load of P2 is mobilized to take additional vertical load, transferred from P1, to maintain vertical equilibrium. The base load of P1 and P2 increased 13% and 30%, respectively. When the tunnel is located at C/D=3.0, almost no change of shaft friction is observed along the shaft of P1. However, base load of P1 reduces significantly. It is because the tunnel crown is located at the pile base (i.e. z/D=2.5). To establish vertical equilibrium, the load transfers from P1 to P2. As a result of this, base load of P2 is mobilized to carry the additional load transferred from P1. The head and base load of P1 reduce 7% and 35% respectively. The head and base load of pile P2 increase 7% and 11%, respectively.

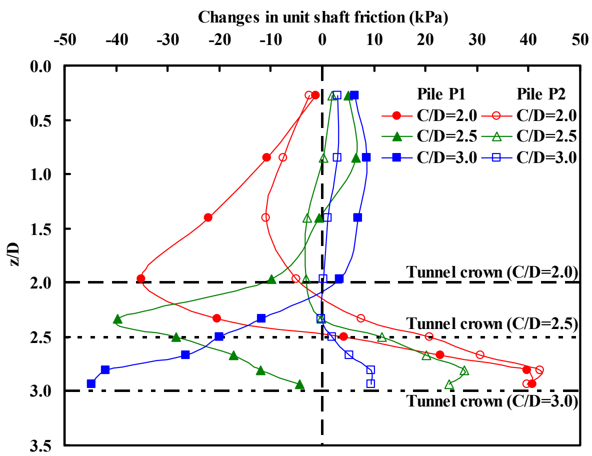



Fig. 6. Changes in unit shaft friction due to tunnelling

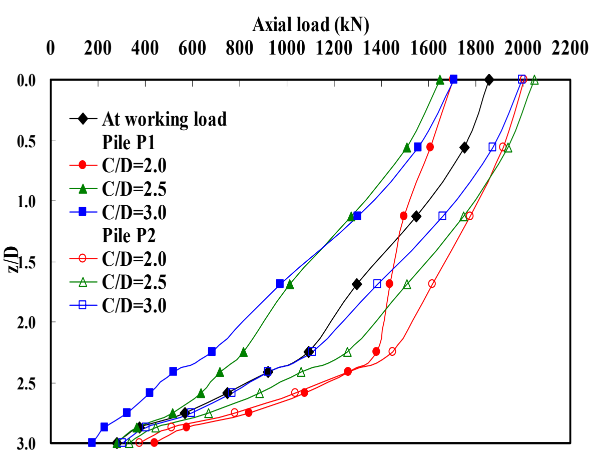



Fig. 7. Axial load changes in P1 and P2 due to tunnelling

## IV. CONCLUSIONS

Computed results make clear that surface settlement troughs at monitoring sections follow Gaussian distribution curve very well for all three cases. When the tunnel is excavated at C/D=2.0, it causes the reduction of shaft resistance in both the front (P1) and rear (P2) piles and load transfers from front pile to rear pile. This leads to an increase in base load by 60% and 32% in the front and rear piles, respectively. When tunnel is constructed at C/D=2.5, the shaft resistance and base load of front pile reduces significantly. Consequently, the load transfers from the front (P1) to the rear (P2) pile. The base load as well as shaft resistance of the rear pile is mobilized to support additional vertical load, transferred from the front pile to maintain equilibrium. Each base load of front and rear piles increases by 13% and 30%, respectively. When tunnel is excavated at C/D=3.0 (i.e. the tunnel crown is located at pile base), the excavation causes the reduction of 35% in the base load of front pile (P1) and leads to load transfer from the front to the rear piles. This results in an increase of base load of rear pile (P2) by 11%. The maximum reduction of 35 kPa, 40 kPa and 44 kPa in unit shaft friction is observed near tunnel crown in case of C/D=2.0, 2.5 and 3.0, respectively. Due to the additional settlement of pile group resulting from tunnelling, it can be deduced that the pile group is subjected to an additional equivalent load of 13,400kN. This means that the factor of safety for pile group is reduced from 3.0 to 1.6 in each case.

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