

Application of the Levenberg-Marquardt Algorithm in Solving the Economic Emission Dispatch Problem Integrating Renewable Energy

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Abstract-The Economic Emission Dispatch (EED) is a multi-objective optimization problem that seeks to find the optimal balance between the reduction of the generation costs and the pollutant emissions of power thermal plants while respecting power balance and several operational restrictions. This balance could be carried out by proper scheduling power generation of the committed units to fulfill the power demands considering emissions. This paper presents a novel application of the conventional Levenberg-Marquardt Algorithm (LMA) optimization approach to solve the EED problem with the integration of renewable energy. Wind and solar energy were chosen to be injected into the system's power balance constraint. The combined EED objective function with Valve Point Effects (VPE) consideration was modeled using price penalty and weight factors. This study showed the effectiveness of the chosen optimization technique and the influence of injecting renewable energy along with traditional power resources on reducing total cost and pollutant emissions. The proposed method was applied to the IEEE 9-bus test system and tested in Matlab.

Keywords-economic emission dispatch; Levenberg-Marquardt algorithm; wind; solar; valve point effects

I. INTRODUCTION

More than 80% of total consumed energy is derived from fossil fuels such as oil, coal, and gas [1]. Fossil fuels are expensive to extract, finite, and their use emits a variety of harmful pollutants [2]. Thermal power plants can respond to sharp and sudden power demands quickly. Economic Dispatch (ED) is a necessary task in fuel-based power production systems. It is a computational optimization problem where the overall required generation is distributed among the committed thermal power units to minimize the total generation cost, no matter the harmful emissions produced. However, alternative

strategies are required to respond to the increasing demands for environmental protection [3]. Economic Emission Dispatch (EED) is one of these strategies that has received considerable attention. EED aims to simultaneously reduce total generation costs and pollutant emissions, such as SO₂, NO_x, and CO₂. Emissions could be considered within the economic dispatch problem by being incorporated as a constraint [4, 5], or as a weighted quantity within the objective function of the problem.

Renewable energy is environment-friendly and sustainable with low production cost. Once built, renewable facilities cost very little to operate. However, it is difficult for renewable energy to generate power on the same large scale as fossil fuels. The most common renewable resources are solar and wind. Both these energy sources are intermittent, as they depend on climate parameters such as solar radiation, temperature, and wind speed and they are challenging to schedule [6]. Several tools have been explored considering renewable energy in the EED problem. In [7], EED with solar and wind power was solved using harmony search. The results were carried out for one level of power demand and the influence of integrating renewable energy was studied. Artificial neural networks were applied in [8] to predict and solve EED with the integration of renewable energy, reaching an optimum in real-time. The effectiveness of the PSO-based strategy was shown in [9] by solving the EED problem by considering VPE and including wind energy sources for a 10-unit system. The results were investigated for both cases, with and without wind power penetration.

Among the various existing optimization techniques to solve power system problems, this study investigated the use of the Levenberg-Marquardt Algorithm (LMA) [10-12]. LMA is a hybrid numerical optimization approach that uses both the

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Gauss-Newton algorithm and the steepest descent search to converge to the optimal solution. This technique is particularly effective in solving systems with non-linear equations [12]. This paper proposes a novel application to LMA for solving the static lossy non-convex multi-objective EED problem with solar and wind power penetration. The results showed the influence of integrating renewable energy to reduce total production costs and pollutant emissions. This method was tested and simulated on an IEEE 9-bus system via the Matlab environment.

II. EED FORMULATION WITH WIND AND SOLAR POWER

Two objective functions were considered: minimization of the total generation cost and of the NO_x emissions of the committed power units.

A. Objective Functions

The EED problem is defined as a constrained multi-objective optimization problem that minimizes simultaneously total power cost and emissions of pollutant gases while satisfying power balance and operational constraints.

1) Cost Function with VPE

The basic form of the thermal generation units' cost function was approximated by a smooth quadratic function, as in [13]. However, in practice, multiple steam valves exist in large turbines, whose role is to maintain the power balance in thermal units by being opened and closed to reach a specific load. This operation contributes to the non-convexity of the fuel cost function, which is known as Valve-Point loading Effects (VPE) and is modeled as an additional sinusoidal component [14]. The fuel cost curve with VPE is illustrated in Figure 1.

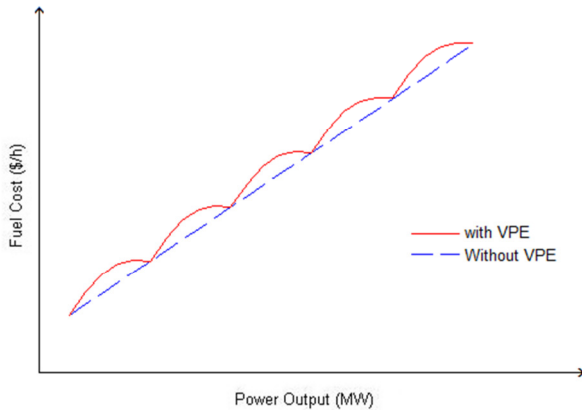


Fig. 1. Fuel cost curve with and without valve point effects.

The total generation cost of thermal power units can be expressed as:

$$\text{Minimize}(F_T = \sum_{i=1}^{N_G} C(P_{Gi})) \quad (1)$$

where F_T is the total generation cost function, $C(P_{Gi})$ is the cost of the i^{th} generation unit, P_{Gi} is the real power output of the i^{th} generation unit, and N_G is the number of dispatchable generation units.

$$C(P_{Gi}) = a_i + b_i P_{Gi} + c_i P_{Gi}^2 + |d_i \sin[e_i(P_{Gi}^{min} - P_{Gi})]| \quad (2)$$

where a_i , b_i , and c_i are the cost coefficients of the i^{th} generation unit, and d_i , e_i are the valve-point effects coefficients of fuel cost for the i^{th} generation unit.

2) Emission Function

NO_x is a serious global concern. In this study, it is one of the numerous dangerous gases generated by the production units. The emission function is a quadratic function, described as [15]:

$$\text{Minimize}(E_T = \sum_{i=1}^{N_G} E_{\text{NO}_x}(P_{Gi})) \quad (3)$$

where E_T is the total emission function measured in kg/h, and $E_{\text{NO}_x}(P_{Gi})$ is the NO_x emission of the i^{th} generation unit.

$$E_{\text{NO}_x}(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2 \quad (4)$$

where α_i , β_i , and γ_i are the NO_x emission coefficients of the i^{th} generation unit. The above bi-objective combined economic emission dispatch problem was converted into a single optimization problem by introducing a price penalty factor. The weighted objective function is represented as:

$$FO = \omega F_T(P_{Gi}) + h(1 - \omega) E_T(P_{Gi}) \quad (5)$$

where ω is the weight factor in the range of [0,1], used to balance cost and emissions. h is the price penalty factor which is the ratio between the maximum fuel cost and the maximum emissions of the corresponding generator [16]:

$$h_i = \frac{C(P_{Gi}^{max})}{E_{\text{NO}_x}(P_{Gi}^{max})} \quad (6)$$

B. Modeling Renewable Energy

1) Solar Power

The maximum power a solar panel can deliver (P_s) is determined as [7-8]:

$$P_s = P_1 E_c [1 + P_2 (T_j - T_{jref})] \quad (7)$$

where E_c is solar radiation, T_{jref} is the reference temperature of the panels in 25°C , and T_j is the cell junction temperature ($^\circ\text{C}$). k_1 represents the characteristic dispersion of the panels and is between 0.095 to 0.105, and $k_2 = 0.47\%/^\circ\text{C}$ is the drift in panels temperature. Including a third parameter P_3 in the solar power equation improves the results:

$$P_s = P_1 E_c [1 + P_2 (T_j - T_{jref})] (P_3 + E_c) \quad (8)$$

Having only 3 constant factors P_1 , P_2 , and P_3 and a simple equation, this simplified model can predict the maximum power generated by a group of solar panels, given the panel temperature. A thermal solar power plant is made up of a solar system that generates heat and feeds it to turbines in a thermal cycle to generate electricity.

2) Wind power

The power provided by a wind turbine P_w is expressed as [7-8]:

$$P_w = \frac{1}{2} \rho C_p A v^3 10^{-3} \quad (9)$$

where A is the area traversed by the wind (m^2), ρ is the air density ($1.225kg/m^3$), V is the wind speed (m/s), and C_p is the efficiency factor which depends on the wind speed and the architecture of the system. Wind turbines should be built considering a mechanical adjustment to develop nominal power from a nominal wind speed in a way to avoid mechanical overloads.

C. Problem Constraints

The objective function is minimized under the following operational constraints.

1) Power Balance Constraint

The total generation power must meet demand while also accounting for transmission losses. Solar and wind power are injected into the system power balance constraint, where P_{ren} is the sum of solar and wind power and P_d is the power demand.

$$\sum_{i=1}^{N_G} P_{Gi} + P_{ren} - P_D - P_L = 0 \quad (10)$$

The transmission lines' network power loss P_L could be calculated by solving the power flow problem [17]. The total real power losses can be calculated using the total net injected real power at all buses using [18]:

$$P_L = \sum_{k=1}^{Nl} g_k [V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j)] \quad (11)$$

where Nl is the number of transmission lines, g_k is the conductance of the k^{th} line that connects bus i to bus j , V_i is the voltage magnitude at bus i , and δ_i is the voltage angle at bus i .

2) Real Power Operating Limits

To ensure reliable operation, each unit's power generation should be provided concerning its minimal and maximal boundaries.

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \quad (12)$$

Using the Lagrange function, the above-constrained optimization problem was converted into an unconstrained problem. The objective function henceforth is expressed as:

$$L(P_{Gi}, \lambda) = FO + \lambda \sum_{i=1}^{N_G} (P_{Gi} + P_{ren} - P_D - P_L) \quad (13)$$

III. LEVENBERG-MARQUARDT ALGORITHM

LMA is a mathematical-based optimization approach based on a combination of the directions of the Gauss-Newton algorithm and the gradient descent search. $V(x)$ is assumed to be the function to minimize considering the parameters vector x . Newton's update for this vector is [19]:

$$x_{k+1} = x_k - [\nabla^2 V(x)]^{-1} \nabla V(x) \quad (14)$$

where $\nabla^2 V(x)$ is the Hessian matrix and $\nabla V(x)$ is the gradient. The function $V(x)$ is assumed to be a sum of squares function:

$$V(x) = \sum_{i=1}^n e_i^2(x) \quad (15)$$

where $e(x)$ is the difference between the target and the network output. Then it can be shown that:

$$\nabla V(x) = J^T(x)e(x) \quad (16)$$

and

$$\nabla^2 V(x) = J^T J + S(x) \quad (17)$$

The Jacobian matrix $J(x)$ is:

$$J(x) = \begin{bmatrix} \frac{\partial e_1(x)}{\partial x_1} & \frac{\partial e_2(x)}{\partial x_2} & \dots & \frac{\partial e_n(x)}{\partial x_n} \\ \frac{\partial e_2(x)}{\partial x_1} & \frac{\partial e_2(x)}{\partial x_2} & \dots & \frac{\partial e_n(x)}{\partial x_n} \\ \vdots & \vdots & \dots & \vdots \\ \frac{\partial e_n(x)}{\partial x_1} & \frac{\partial e_n(x)}{\partial x_2} & \dots & \frac{\partial e_n(x)}{\partial x_n} \end{bmatrix} \quad (18)$$

and

$$S(x) = \sum_{i=1}^n e_i(x) \nabla^2 e_i(x) \quad (19)$$

The Gauss-Newton method assumed that $S(x)=0$, so the equation becomes:

$$x_{k+1} = x_k - [J^T(x)J(x)]^{-1} J^T(x)e(x) \quad (20)$$

The Levenberg-Marquardt modification to the Gauss-Newton method is modeled as follows, where the characteristic μ_k is generally set to 0.01 as a starting point:

$$x_{k+1} = x_k - [J^T(x)J(x) + \mu_k I]^{-1} J^T(x)e(x) \quad (21)$$

$$x_{k+1} = x_k - [H(x) + \mu_k]^{-1} \nabla V(x) \quad (22)$$

This algorithm sets μ_k at 0.01 as a starting point and then it's multiplied by 10 whenever a step results in an increased $V(x)$, otherwise, if $V(x)$ decreases μ_k is divided by 10. To adapt LMA to this problem, $x = [P_{Gi}, \lambda]$ and $V(x) = L(P_{Gi}, \lambda)$.

A. Levenberg-Marquardt Algorithm

- Step 1: Read the given data cost coefficients (a_i, b_i, c_i), VPE coefficients (d_i, e_i), NO_x coefficients ($\alpha_i, \beta_i, \gamma_i$), power demand (P_D), and unit's power limits ($P_{Gi}^{min}, P_{Gi}^{max}$).
- Step 2: Read the forecasted wind and solar power (P_{ren}), where $P_{ren} < 0.3P_D$.
- Step 3: Run power flow analysis and calculate transmission lines' power losses (P_L).
- Step 4: Set the initial values of the Lagrangian multiplier λ^0 , the active generated power P_{Gi}^0 , and the Levenberg-Marquardt characteristic μ_0 .
- Step 5: Calculate the Lagrange function $L(P_{Gi}, \lambda)$.
- Step 6: Calculate the Jacobian matrix:

$$J(P_{gi}, \lambda) = \begin{bmatrix} \frac{\partial L_1}{\partial P_{g1}} & \frac{\partial L_1}{\partial P_{g2}} & \dots & \frac{\partial L_1}{\partial P_{gn}} & \frac{\partial L_1}{\partial \lambda} \\ \frac{\partial L_2}{\partial P_{g1}} & \frac{\partial L_2}{\partial P_{g1}} & \dots & \frac{\partial L_2}{\partial P_{gn}} & \frac{\partial L_2}{\partial \lambda} \\ \vdots & \vdots & \dots & \vdots & \vdots \\ \frac{\partial L_n}{\partial P_{g1}} & \frac{\partial L_n}{\partial P_{g1}} & \dots & \frac{\partial L_n}{\partial P_{gn}} & \frac{\partial L_n}{\partial \lambda} \end{bmatrix} \quad (23)$$

- Step 7: Calculate the Hessian matrix:

$$H(P_{gi}) = J^T(P_{gi})J(P_{gi}) \quad (24)$$

- Step 8: Update the power generation following (23):

$$\begin{bmatrix} P_{gi(k+1)} \\ \lambda_{(k+1)} \end{bmatrix} = \begin{bmatrix} P_{gi(k)} \\ \lambda_{(k)} \end{bmatrix} - \Delta P \begin{bmatrix} P_{gi} \\ \lambda \end{bmatrix} \quad (25)$$

- Step 9: Calculate the new value of $L_{k+1}(P_{Gi}, \lambda)$ for each generating unit.
- Step 10: Update the Levenberg-Marquardt characteristic μ_k :
If $L_{k+1} \geq L_k$ set $\mu = \mu \times 10$
else if $L_{k+1} < L_k$ set $\mu = \mu / 10$.

- Step 11: Check generation limits for each unit:

$$\text{Set } P_{Gi} = P_{Gi}^{min} \text{ if } P_{Gi} < P_{Gi}^{min}$$

$$\text{Set } P_{Gi} = P_{Gi}^{max} \text{ if } P_{Gi} > P_{Gi}^{max}$$

- Step 12: Calculate the power mismatch ΔP_g

$$\Delta P_g = \sum_{i=1}^{N_g} P_{gi} - P_{ren} - P_d - P_L \quad (26)$$

- Step 13: If $\Delta P_g \leq \epsilon$, where ϵ is the convergence criteria set to 0.01, then stop the calculation and move to step 15. Otherwise, go to the next step.
- Step 14: Repeat the procedure from step 5.
- Step 15: Calculate the cost function and the emission quantity using the optimal values of P_{Gi} and λ_{opt} .

IV. RESULTS AND DISCUSSIONS

Dispatch simulation was carried out for 3 different cases:

- Case study 1: ED with VPE consideration.
- Case study 2: EED without VPE consideration.
- Case study 3: EED with VPE consideration in the presence of renewable energy.

The performance of LMA was tested on an IEEE 9-bus system for these cases. The considered network system consists of 9 buses, 3 power generators, 3 power transformers, 6 transmission lines, and 3 loads, as shown in Figure 2. Table I shows the units' data for all cases. The proposed approach was programmed using Matlab.

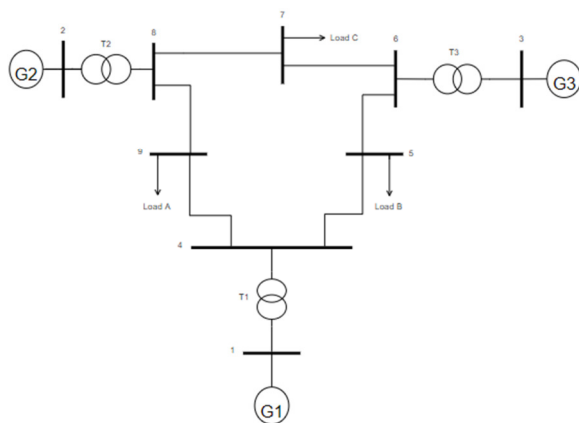


Fig. 2. IEEE 9-bus system.

TABLE I. IEEE-9 BUS COST AND EMISSION DATA [20]

Network data		Generating Unit		
		1	2	3
Cost coefficients	a	0.001562	0.001940	0.004820
	b	7.920	7.85	7.97
	c	561	310	78
	d	300	200	150
	e	0.0315	0.0420	0.0630
NO _x coefficients	α_i	0.043732540	0.055821713	0.027731524
	β_i	-9.4868099 e-05	-9.7252878 e-05	-3.5373734 e-04
	γ_i	1.4721848 e-07	3.0207577 e-07	1.9338531 e-06
Unit's limit (MW)	P_{Gi}^{min}	100	100	50
	P_{Gi}^{max}	600	400	200

A. Case Study 1

In this case, the single-objective ED problem was solved by setting the weight factor to $\omega=1$, thus considering only the cost function. The power demand to be met by the 3 generating units was 850MW, and the transmission lines' power losses were neglected. The ED results using the LMA and other methods are presented in Table I. LMA was compared with SDE [21], GA [22], and MPSO [23]. LMA succeeded in finding the generation outputs that give the minimum cost while meeting the total power demand. From Table II, it is clear that the proposed approach reported the global optimum solution over other algorithms.

TABLE II. RESULT COMPARISON FOR CASE STUDY 1

Unit	Method			
	SDE [21]	GA [22]	MPSO [23]	LMA
P_{G1} [MW]	400.0000	400.000	400.0000	402.4788
P_{G2} [MW]	300.2669	300.000	300.2700	312.5117
P_{G3} [MW]	149.7331	150.000	149.7400	135.0095
Cost [\$]	8241.50	8234.60	8234.07	8232.50

B. Case Study 2

In this case, the multi-objective EED problem was solved without considering VPE, which means that the cost function was considered in its convex form. The power demand to be met by the 3 generating units was 850MW, and the transmission lines' power losses were taken into account. Table III shows the best compromise solutions to reduce simultaneously cost and NO_x emissions found by LMA, CSA [24], and NSGA II [15].

TABLE III. RESULT COMPARISON FOR CASE STUDY 2

Unit	Method		
	CSA [24]	NSGA II [15]	LMA
P_{G1} [MW]	470.9502	470.957	442.2904
P_{G2} [MW]	280.7243	280.663	306.8524
P_{G3} [MW]	113.6211	113.675	116.1568
P_L [MW]	15.2950	15.2940	15.2996
Cost [\$]	8349.722	8349.72	8339.80
NO _x Emissions [kg/h]	0.09654	0.09654	0.09770

The best solution for LMA was determined by running the program for all possible values of the weight factor in the range of [0, 1] with a step of 0.1. LMA provided a profit of 9.92\$/h in terms of cost. CSA and NSGA reported a minimum solution

for NO_x emissions with a profit of 0.1% compared to LMA. It seems that LMA converged to a near-global solution in this scenario.

C. Case Study 3

In this case, the multi-objective EED problem considering VPE and the presence of renewable energy was resolved for different demand levels. Hence, power demand was randomly changed at the different network load buses in a wide range of units' power limits. The first step consisted of performing Newton-Raphson power flow analysis to calculate the power losses of the transmission lines using the generators and the data of the lines from [25]. The penetration of solar and wind power was randomly generated in a way to be limited to up to 30% of power demand due to the stochastic availability of these resources. The weight factor was set to 0.5 to give the same importance to the two objective functions considered. Several levels of power demand were generated with intermittent renewable energy power. For each level of power demand, the LMA was run to converge to the optimum values of generating power, and λ_{opt} , cost, and emissions quantities were calculated. The proposed approach structure is illustrated in Figure 3. The LMA solutions for various values of power demand are presented in Table IV.

Since there is a lack of studies with the same criteria (multi-objective, VPE, Power losses, wind and solar power, IEEE-9

bus system), the simulation results were compared only with and without the presence of renewable energy.

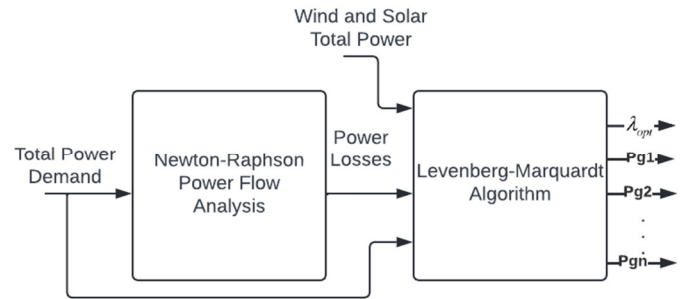


Fig. 3. The proposed LMA architecture flowchart.

Table IV shows that both cost and emissions decrease significantly when integrating renewable energy. It is obvious that when the penetration of renewable energy is high, the reduction in cost and emission is more considerable. As an example, for the case of 761MW power demand, penetration of renewable energy led to a reduction in cost and NO_x emissions of about 60% and 21%, respectively. Therefore, the enhancement of the EED problem is more efficient with the presence of renewable energy.

TABLE IV. EED WITH VPE USING LMA WITH/WITHOUT RENEWABLE ENERGY PENETRATION FOR ω=0.5

Power Demand [MW]	P_{ren} [MW]	P_{G1} [MW]	P_{G2} [MW]	P_{G3} [MW]	Cost [\$]	Profit in cost [%]	Emission [kg/h]	Profit in emissions [%]
451	-	162.1612	193.0458	102.0527	7.3932e+03	17.3%	0.0923	1.2%
	15.0481	176.1747	169.8884	96.1484	6.1163e+03		0.0912	
505	-	168.3291	224.7244	119.8273	8.5375e+03	51.8%	0.0943	2.0%
	62.8873	161.8538	203.5875	84.552	4.1135 e+03		0.0924	
615	-	229.0195	283.5452	116.0003	1.3820e+04	40.3%	0.0950	0.84%
	20.132	319.0071	141.7072	147.7186	6.2000e+03		0.0942	
761	-	241.5953	350.9251	198.3037	3.2606e+04	62.0%	0.1219	21.0%
	53.6624	347.9706	316.3517	72.8395	1.2377e+04		0.0961	
835	-	326.6593	368.3790	195.3959	5.2899e+04	38.8%	0.1219	8.3%
	195.0715	271.5196	225.4007	198.4422	3.2358e+04		0.1118	

V. CONCLUSION

This paper presented a new application of the mathematical-based method Levenberg-Marquardt for solving the static multi-objective non-convex EED problem for the IEEE 9-bus network incorporating solar and wind energy. The proposed approach was applied to two other case studies. The results provided by LMA were compared with those obtained by SDE, GA, MPSO, CSA, and NSGA II. The comparison showed that LMA provided a better solution for the case of the single-objective problem. For the multi-objective problem, LMA reported a near-global optimum in terms of emissions compared to the heuristic methods. For the EED problem with wind and solar power, LMA was investigated for several levels of power demand and compared to results provided without the penetration of renewable energy sources. LMA proved its efficiency in finding the best compromise of cost and emissions in a short execution time. However, it's important to note that LMA suffers from high sensitivity to the initial parameters. The integration of renewable energy has a significant impact on reducing both the total cost and emissions.

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