Analysis of a Frictionless Electro Viscoelastic Contact Problem with Signorini Conditions

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Abstract-This study considered a mathematical model to describe the process of a quasi-static contact between a piezoelectric body and an electrically conductive foundation. The behavior of the material was modeled with a nonlinear electro-viscoelastic constitutive law, the contact was frictionless, and the result was described with the Signorini condition. A variational formulation was derived for the problem, proving the existence and uniqueness of a weak solution of the model. The proof was based on arguments for nonlinear equations with maximal monotone operators.

Keywords-elecro-viscoelastic materials; frictionless contact; Signorini conditions; maximal monotone operators; weak solution

I. INTRODUCTION

Considerable progress has been achieved recently in modeling, mathematical analysis, and numerical simulations of various contact processes and, as a result, a general mathematical theory of contact mechanics is currently emerging. Mathematical structures underlie general contact problems with different constitutive laws for materials, varied geometries, and different contact conditions, [1, 2]. The piezoelectric effect is characterized by the coupling between the mechanical and electrical properties of materials. The appearance of electric charges on some crystals submitted to the action of body forces and surface tractions was observed and their dependence on the deformation process was examined. Conversely, it was proved experimentally that the action of an electric field on the crystals may generate strain and stress. A deformable material having such behavior is called a piezoelectric material. Piezoelectric materials are used extensively as switches in many engineering systems such as radio electronics, electro acoustics, and measuring equipment. Some general models for electro-elastic materials can be found in [3, 4]. A static frictional contact problem for electro-elastic materials was considered in [5, 6].

II. NOTATIONS AND PRELIMINARIES

This study considered a body made of a piezoelectric material that occupies the domain with a smooth boundary $\partial\Omega=\Gamma$ and a unit normal υ. A body force of f_0 density acts on the body and has volume-free electric charges of q_0 density, constrained mechanically and electrically on the boundary. To describe these conditions, a partition of Γ was considered into three open disjoint parts Γ_1, Γ_2, and Γ_3. A partition of $\Gamma_1 \cup \Gamma_2$ was in two open parts Γ_a and Γ_b. It was assumed that $meas\Gamma_1 > 0$ and $meas\Gamma_a > 0$. The spatial and time variables were denoted as $x \in \Omega \cup \Gamma$ and $t \in [0,T]$ respectively. The body was clamped on Γ_1 and the displacement field vanished there. Surface tractions of density f_2 act on Γ_2. It was also assumed that the electrical potential vanishes on Γ_a and a surface-free electrical charge of density q_2 is prescribed on Γ_b. The notation S^d was used for the space of second-order symmetric tensors on $\mathbb{R}^d$, while $\cdot$ and $\|\cdot\|$ represent the inner product and the Euclidean norm on S^d and $\mathbb{R}^d$ respectively, where:

$$\begin{aligned} u.v = u_i v_i \quad , \quad \|v\| = (v.v)^{\frac{1}{2}}, \ \forall u, v \in \mathbb{R}^d \\ \sigma.\tau = \sigma_{ij}\tau_{ij} \ , \|\sigma\| = (\sigma.\sigma)^{\frac{1}{2}}, \ \forall \sigma, \tau \in S^d \end{aligned} \quad (1)$$

where i, j=1,…,d. The summation over repeated indices is used and the index which follows a comma indicates a partial derivative. Let $\Omega \subset \mathbb{R}^d$ be a bounded domain, $x = (x_i)$ a point in $\Omega \cup \Gamma$, while $\upsilon = \upsilon_i$ denotes the outward unit normal at Γ. Also, the inner products on the Hilbert spaces $L^2(\Omega)^d$ and $L^2(\Gamma)^d$ are given by:

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$$(u,v)_{L^2(\Omega)^d} = \int_\Omega u.v dx \ ; (u,v)_{L^2(\Gamma)^d} = \int_\Gamma u.v da \quad (2)$$

The associated norms will be denoted by $\|.\|_{L^2(\Omega)^d}$ and $\|.\|_{L^2(\Gamma)^d}$ respectively. The closed subspace of $H^1(\Omega)^d$ is defined by:

$$V = \{v \in H^1(\Omega)^d : v = 0 \ on \ \Gamma_1\} \quad (3)$$

$$Q = \{\tau = (\tau_{ij}) \ / \ \tau_{ij} \in L^2(\Omega)^d : \tau_{ij} = \tau_{ji}\} \quad (4)$$

which are Hilbert spaces with the following scalar products:

$$(u,v)_V = \int_\Omega \varepsilon(u).\varepsilon(v)dx, (\sigma,\tau)_Q = \int_\Gamma \sigma.\tau dx \quad (5)$$

The associated norms are $\|.\|_V$ and $\|.\|_Q$, respectively. The deformation tensor $\varepsilon : H^1(\Omega)^d \to Q$ is given by:

$$\varepsilon(v) = \left(\varepsilon_{ij}(v)\right), \ \varepsilon_{ij}(v) = \frac{1}{2}\left(v_{ij} + v_{ij}\right); \forall v \in H^1(\Omega)^d \quad (6)$$

Therefore $(V, |.|_V)$ is a real Hilbert space. Moreover, by Sobolev's trace theorem, a constant C_0 exists which depends only on Ω, Γ_1, and Γ_3 such that:

$$\|v\|_{L^2(\Gamma_3)^d} \le C_0\|v\|_V, \forall v \in V \quad (7)$$

given a Hilbert space X, for a function $\phi : X \to]-\infty, +\infty]$:

$$Dom(\phi) = \{u \in X : \ \phi(u) \neq \infty\},$$

$$\partial\phi u = \{f \in X : \ \phi(v) - \phi(u) \ge (f, v-u), \forall u, v \in X\}.$$

so, the following existence and uniqueness result can be concluded:

Lemma: Let X be a real Hilbert space and let $\phi : X \to [-\infty, +\infty]$ be a convex proper lower semicontinuous function. Then for every $f \in L^2(0,T,X)$ and $u_0 \in Dom(\phi)$, a unique function $u \in W^{1,2}(0,T,X)$ exists which satisfies:

$$\begin{gathered} \dot{u}(t) + \partial\phi u(t) \ni f(t), a.e. t \in (0,T), \\ u(0) = u_0. \end{gathered} \quad (8)$$

More details can be found in [7].

III. Problem Statement

This section describes the process model and clarifies the assumptions about the data. The process starts from the physical framework, and the law of behavior and the conditions of contact should be specified. It was supposed that the body was electro-viscoelastic, rested on a rigid foundation by the part Γ_3 of its border, this contact was affected without friction, and the tangential movements were completely free. Additionally, as the quasistatic case was studied, the body Ω had a behavior law of the form of (9), see [8-10]. Under these considerations, the studied mechanical problem can be formulated as follows:

Problem 1. Find the displacement field $u : \Omega \times \mathbb{R}_+ \to \mathbb{R}^d$, the stress field $\sigma : \Omega \times \mathbb{R}_+ \to S^d$, the electric potential $\varphi : \Omega \times [0,T] \to \mathbb{R}$, and the electrical displacements field $D : \Omega \times [0,T] \to \mathbb{R}^d$, such that:

$$\sigma = \mathcal{A}\varepsilon(u) + \mathcal{F}\varepsilon(u) - \varepsilon^* E(\varphi) \quad in \ \Omega \times (0,T) \quad (9)$$

$$D = \mathcal{E}(u) + \beta E(\varphi) \quad in \ \Omega \times (0,T) \quad (10)$$

$$Div\sigma + f_0 = 0 \quad in \ \Omega \times (0,T) \quad (11)$$

$$div \ D = q_0 \ in \ \Omega \times (0,T) \quad (12)$$

$$u = 0 \quad on \ \Gamma_1 \times (0,T) \quad (13)$$

$$\sigma_\upsilon = f_2 \quad on \ \Gamma_2 \times (0,T) \quad (14)$$

$$\begin{cases} u_\upsilon \le 0, \ \sigma_\upsilon \le 0 \\ u_\upsilon\sigma_\upsilon = 0, \sigma_\tau = 0 \end{cases} on \ \Gamma_3 \times (0,T) \quad (15)$$

$$\varphi = 0 \quad on \ \Gamma_a \times (0,T) \quad (16)$$

$$D.\upsilon = q_2 \quad on \ \Gamma_b \times (0,T) \quad (17)$$

$$u = u_0 \quad in \ \Omega \quad (18)$$

Equations (9) and (10) represent the constitutive electro-viscoelastic law, (11) and (12) represent the equilibrium equations, (13) and (14) are the boundary conditions in displacement and traction respectively. The boundary condition (15) represents the conditions of contact without Signorini friction, (16) and (17) are the electrical boundary conditions, and (18) represents the initial condition. It should be noted that $\sigma = (\sigma_{ij})$ is the stress tensor, $\varepsilon(u)$ denotes the linearized strain tensor, $\mathcal{A}$ and $\mathcal{E} = (e_{ijk})$ represent the third-order piezoelectric tensor, $\mathcal{E}^*$ is its transpose, $\beta = (b_{ij})$ denote the electric permittivity tensor, and $D = (D_1, \ldots, D_d)$ is the electric displacement vector. The tensors $\mathcal{E}$ and $\mathcal{E}^*$ satisfy:

$$\mathcal{E}\sigma.v = \sigma.\mathcal{E}^* v; \ \forall(\sigma_{ij}) \in S^d, v \in \mathbb{R}^d$$

and Div and div denote the divergence operator for tensor and vector-valued function. For the electric displacement field, two Hilbert spaces were used.

$$\mathcal{W} = L^2(\Omega)^d; \ \mathcal{W}^1 = \{D \in \mathcal{W}: divD \in L^2(\Omega)\}$$

endowed with the inner products:

$$(D,E)_\mathcal{W} = \int_\Omega D_i E_i dx$$

$$(D,E)_{\mathcal{W}_1} = (D,E)_\mathcal{W} + (divD, divE)_{L^2(\Omega)}$$

The electric potential field was defined by:

$$W = \{\psi \in H^1(\Omega) : \psi = 0 \ on \ \Gamma_a\}$$

Since $meas\Gamma_a > 0$ the Friedrichs-Poincaré inequality holds, thus:

$$\|\nabla\psi\|_\mathcal{W} \ge c_F\|\psi\|_{H^1(\Omega)}; \ \forall\psi \in W.$$

The assumptions on the problem data were listed for the study of problem $\mathcal{P}$. Then, the viscosity $\mathcal{A}$, the operator of elasticity $\mathcal{F}$, the piezoelectric tensor $\mathcal{E}$, and the electric permittivity tensor β satisfy the following properties:

$$\begin{cases} (a). \ \mathcal{A} = (a_{ijkl}) : \ \Omega \times S^d \to S^d \\ (b). \ a_{ijkl} = a_{lijk} \in L^2(\Omega) \\ (c). \ There \ exist \ m_A > 0 \ such \ that \\ a_{ijkh}\tau_{ijkh} \ge m_A\|\tau\|^2; \ \forall\tau \in S^d, a.e. x \in \Omega \end{cases} \quad (19)$$

$$\begin{cases} (a).\ \mathcal{F}:\ \Omega \times S^d \to S^d. \\ (b). There\ exist\ L_{\mathcal{F}} > 0\ such\ that \\ \|\mathcal{F}(x,\varepsilon_1) - \mathcal{F}(x,\varepsilon_2)\| \le L_{\mathcal{F}}\|\varepsilon_1 - \varepsilon_2\|, \\ \forall \varepsilon_1, \varepsilon_2 \in S^d, a.e. x \in \Omega. \\ (c). The\ mapping\ x \to \mathcal{F}(x,\varepsilon) \\ is\ measurable\ on\ \Omega, \forall \varepsilon \in S^d, \\ (d).\ The\ mapping\ \ x \to \mathcal{F}(x,0) \in \mathcal{H}. \end{cases} \quad (20)$$

$$\begin{cases} (a). \mathcal{E} = (e_{ijk}):\ \Omega \times S^d \to \mathbb{R}^d \\ e_{ijk} = e_{ikj} \in L^\infty(\Omega) \end{cases} \quad (21)$$

$$\begin{cases} (a). \beta = (\beta_{ij}) : \Omega \times \mathbb{R}^d \to \mathbb{R}^d. \\ \beta_{ij} = \beta_{ji} \in L^\infty(\Omega). \\ (b). There\ exist\ m_\beta > 0\ \ such\ that \\ \beta_{ij}E_iE_j \ge m_\beta\|E\|^2 \\ \forall E \in \mathbb{R}^d, p.p. x \in\ \Omega \end{cases} \quad (22)$$

The densities of body forces and surface tractions have regularity:

$$f_0 \in W^{1,1}(0,T;L^2(\Omega)^d),\ \ f_2 \in W^{1,1}(0,T;L^2(\Gamma_2)^d) \quad (23)$$

and surface free charge densities satisfy:

$$q_0 \in W^{1,1}(0,T;L^2(\Omega)), q^2 \in\in W^{1,1}(0,T;L^2(\Gamma_a)) \quad (24)$$

$$q_2(t) = 0\ on\ \Gamma_3\ ;\ \forall t \in [0,T] \quad (25)$$

Condition (19) allows providing the space V with the scalar product and the associated norm defined by:

$$(u,v)_V = (\mathcal{A}\varepsilon(u), \mathcal{A}\varepsilon(v))_{\mathcal{H}};\ \|u\|_V = \sqrt{(u,u)_V} \quad (26)$$

This norm on V is equivalent to that of $H^1(\Omega)^d$. Convex K which will be the space of admissible displacements, i.e. compatible with the connections (boundary conditions and unilateral conditions):

$$K = \{v \in V;\ \ vv \le 0\ \ on\ \Gamma_3\} \quad (27)$$

Finally, the following assumption was made:

$$u_0 \in K \quad (28)$$

For the rest, the following functional space was considered:

$$W = \{\psi \in H^{'}(\Omega) \mid \psi = 0\ on\ \Gamma_a\}$$

The operator A was said to be maximal monotone if it is monotone and if for all x and y in space X:

$$< y - A\zeta, x - \zeta > \ge 0\ \ for\ all\ \zeta \in dom(A) \Rightarrow y \in Ax$$

Now all the necessary ingredients are available to provide a weak formulation of problem $\mathcal{P}$ and present the main result of the existence and uniqueness of the weak solution.

IV. VARIATIONAL FORMULATION

This section starts by giving a variational formulation in terms of displacement and electric potential. Once this weak formulation is established, a result of the existence and uniqueness of the weak solution emerges:

Let $f : [0,T] \to V\ et\ q : [0,T] \to W$ be the functional

$$(f(t),v)_V = \int_\Omega\ f_0(t).vdx + \int_{\Gamma_2} f_2(t).vda \quad (29)$$

$$(q(t),\psi)_W = \int_\Omega\ q_0(t).\psi dx - \int_{\Gamma_2} q_2(t).\psi da \quad (30)$$

for all $v \in V, \psi \in W, t \in [0,T]$. The conditions (23) and (24) imply:

$$f \in W^{1,1}(0,T;V)\ , q \in W^{1,1}(0,T;W) \quad (31)$$

Using Green's formula and assuming that (u,σ,ϕ,D) are regular functions satisfying (11) and (17):

$$u(t) \in K, (\ \sigma(t), \varepsilon(v) -\ \varepsilon(u(t)))_H \ge (f(t), u(t) - v)_V \quad (32)$$

$$(D(t), \nabla\psi)_{L^2(\Omega)^d} + (q(t),\psi)_W = 0 \quad (33)$$

for all $v \in K, \psi \in W$ and $t \in [0,T]$. Now, putting (9) in (32), (10) in (33), and keeping in mind that $E(\phi) = -\nabla\phi$ as well as the initial condition (18), the variational formulation of the mechanical problem $\mathcal{P}$ can be obtained in terms of displacement and electric potential following:

Problem 2: Find the displacement field $u: [0,T] \to V$ and electric potential $\phi :\ [0,T] \to W$ such that:

$$\begin{cases} u(t) \in K, \left(\mathcal{A}\varepsilon(\dot{u}(t)), \varepsilon(v) - \varepsilon(u(t))\right)_{\mathcal{H}} + \\ \left(\mathcal{F}\varepsilon(u(t)), \varepsilon(v) - \varepsilon(u(t))\right)_{\mathcal{H}} + \\ \left(E^*\nabla\phi(t), \varepsilon(v) - \varepsilon(u(t))\right)_{\mathcal{H}} \ge (f(t), v - u(t))_V \\ \forall v \in K, p.p. t \in [0,T], \end{cases} \quad (34)$$

$$\begin{cases} (\beta\nabla\phi(t), \nabla\psi)_{L^2(\Omega)^d} - \left(\mathcal{E}\varepsilon(u(t)) - \nabla\psi\right)_{L^2(\Omega)^d} = \\ (q(t),\psi)_W;\ \forall\psi \in W, \forall t \in [0,T], \end{cases} \quad (35)$$

$$u(0) = u_0 \quad (36)$$

The well-posedness of the above $\mathcal{P}^V$ problem is examined in the next section.

V. EXISTENCE RESULT

This section states and proves the existence and the uniqueness of the result.

Theorem 1: Assume that (19), (25), and (28) are verified. Then the variational problem $\mathcal{P}^V$ has a unique solution (u,ϕ), having the regularity:

$$u \in W^{1,\infty}(0,T;V)\ , \phi \in W^{1,\infty}(0,T;W) \quad (37)$$

A quadruplet (u,σ,ϕ,D) satisfying the (9) and (10) is called a weak solution to the mechanical problem $\mathcal{P}$. It can be concluded from Theorem 1 that problem $\mathcal{P}$ admits a unique solution. Regarding the regularity of the weak solution, it follows to refer to the regularity of the element $(u,\phi) \in W^{1,\infty}(0,T;V) \times W^{1,\infty}(0,T;W)$, to the constitutive laws (9) and (10), and also to the hypotheses (19) and (22). Then:

$$\sigma \in L^\infty(0,T;H), D \in L^\infty(0,T;L^{2}(\Omega)^d)$$

Now, taking $v = u(t) \pm z$, where $z \in C_0^\infty(\Omega)^d$ in (32) and $\psi\ \in C_0^\infty(\Omega)$ in (33), and using the notations (29) and (30) we get:

$$Div\sigma(t) + f_0(t) = 0\,, divD(t) = q_0(t)$$

for all $t \in [0,T]$. From (23) and (18), it follows that $Div\sigma \in L^\infty(0,T;L^2(\Omega)^d)$ and $divD \in L^\infty(0,T;L^2(\Omega))$, and therefore:

$$\sigma \in L^\infty(0,T;\mathcal{H}_1), D \in L^\infty(0,T;W) \quad (38)$$

So, it is concluded that the solution (u,σ,ϕ,D) of problem $\mathcal{P}$ will have regularity (37) and (38).

Proof: The abstract result will be used to obtain the existence and the uniqueness of the solution. For that, let's suppose in the continuation that the hypotheses (19), (25), and (28) are verified. Using the Riesz representation theorem, the operators $\mathcal{B}: W \to W$ and $C: V \to W$ are defined as follows:

$$(\mathcal{B}\varphi,\psi)_W = (\beta\nabla\phi,\nabla\psi)_{L^2(\Omega)^d} \quad (39)$$

$$(Cv,\varphi)_W = (\mathcal{E}\varepsilon(v),\nabla\varphi)_{L^2(\Omega)^d} \quad (40)$$

for all $\varphi,\psi \in W, v \in V$. From (22), it is deduced that ß is a linear, symmetric, and positive operator. Consequently, ß is a revertible and continuous operator on W. Now, using (26) and (21), it follows that C is a linear and continuous operator on V. Let $C^*: W \to V$ be the adjoint of C. So:

$$(C^*\varphi,v)_V = (E^*\nabla\phi,\varepsilon(v))_H; \forall v \in V, \varphi \in W \quad (41)$$

Let $t \in [0,T]$. By putting (39) and (40) in (35) we have:

$$(\varphi(t),\psi)_W = (Cu(t),\psi)_W + (q(t),\psi)_W, \forall \psi \in W$$

Consequently,

$$\varphi(t) = Cu(t) + q(t)$$

On the other hand, $B: W \to W$ is invertible. The previous equation then becomes:

$$\phi(t) = \mathcal{B}^{-1}Cu(t) + \mathcal{B}^{-1}q(t) \quad (42)$$

Using (42) in (34) and the definitions of operators ß, C, C^* given by (39-41) we get:

$$\begin{gathered} u(t) \in K, \left(\mathcal{A}\varepsilon(\dot{u}(t)), \varepsilon(v) - \varepsilon(u(t))\right)_{\mathcal{H}} + \\ \left(\mathcal{F}\varepsilon(u(t)), \varepsilon(v) - \varepsilon(u(t))\right)_{\mathcal{H}} + \\ \left(C^*\mathcal{B}^{-1}Cu(t), v - u(t)\right)_V \geq \left(f(t) - \mathcal{B}^{-1}q(t), v - u(t)\right)_V \\ \forall v \in V, p.p.t \in [0,T] \end{gathered} \quad (43)$$

Let the operator $L: V \to V$ be defined by:

$$L(v) = C^*\mathcal{B}^{-1}C(v); \forall v \in V \quad (44)$$

and keeping in mind the properties of the operators ß, C, and C^*, it is deduced that L is a linear operator, continuous on V.

$$\|Lu_1 - Lu_2\|_V \leq \|L\|\|u_1 - u_2\|_V; \forall u_1, u_2 \in V \quad (45)$$

$\mathcal{G}: V \to V$ denotes the operator based on the representation of Riesz given by:

$$(\mathcal{G}u,v)_V = (\mathcal{F}\varepsilon(u),\varepsilon(v))_{\mathcal{H}} + (Lu,v)_V : \forall u,v \in V \quad (46)$$

Now, taking into account (19), (20), (26), and (46) we have:

$$\|\mathcal{G}u^1 - \mathcal{G}u^2\|_V \leq \left(\frac{L_{\mathcal{F}}}{m_{\mathcal{A}}} + \|L\|\right)\|u^1 - u^2\|_V, \ \forall u_1, u_2 \in V \quad (47)$$

This relation proves that the $\mathcal{G}$ is a Lipshitz operator. Now, let the function $f: [0,T] \to V$ given by:

$$\not{f}(t) = f(t) - C^*\mathcal{B}^{-1}q(t); \ \forall t \in [0,T] \quad (48)$$

Using (39) and the fact that $C^*\mathcal{B}^{-1}$ is linear and continuous, it comes by observing (48) that:

$$f \in W^{1,1}(0,T;V) \quad (49)$$

On the other hand, the operator:

$$\mathcal{G} + \left(\frac{L_{\mathcal{F}}}{m_{\mathcal{A}}} + \|L\|\right)I: V \to V \quad (50)$$

is a Lipshitz operator on V.

Now, the indicator function $\psi_k: V \to]-\infty,+\infty]$ of the set K is introduced as well as its sub-differential $\partial\psi_k$. Being the set K non-empty, closed, and convex, the sub-differential $\partial\psi_k$ is a strongly monotone operator on the space V. The domain of this sub-differential is thus: $D(\partial\psi_k) = K$. It can be said that the sum:

$$\partial\psi_k + \mathcal{G} + \left(\frac{L_{\mathcal{F}}}{m_{\mathcal{A}}} + \|L\|\right)I: K \subset V \to 2^V$$

is a strongly monotone operator. Being the hypotheses (49) and (28) satisfied, (8) can be applied with:

$$X = V\,; A = \partial\psi_k + \mathcal{G}: \ D(A) = K \subset V \to 2^V$$

and:

$$\omega = \frac{L_{\mathcal{F}}}{m_{\mathcal{A}}} + \|L\|$$

This result deduces that there is a unique element $u \in W^{1,\infty}(0,T;V)$ which verifies:

$$\dot{u}(t) + \partial\psi_K(u(t)) + \mathcal{G}u(t) \ni f(t)\, p.p.t \in (0,T) \quad (51)$$

and:

$$u(0) = u_0 \quad (52)$$

In addition, for all $g \in V$ there is the equivalence:

$$g \in \partial\psi_K(u) \Leftrightarrow u \in K\,, (g, v - u)_V \ ; \forall v \in K$$

The differential inclusion (51) is equivalent to the variational inequality:

$$\begin{gathered} u(t) \in K, (\dot{u}(t), v - u(t))_V + (\mathcal{G}u(t), v - u(t))_V \\ \geq (f(t), v - u(t))_V \forall v \in K, p.p.t \in (0,T). \end{gathered} \quad (53)$$

From (53), (46), and (26) comes that u satisfies the inequality:

$$\begin{gathered} u(t) \in K, (\mathcal{A}\varepsilon(\dot{u}(t)), \varepsilon(v) - \varepsilon(u(t)))_{\mathcal{H}} \\ +\left(\mathcal{F}u(t), v - u(t)\right)_V + (Lu(t), v - u(t))_V \\ \geq \left(f(t), v - u(t)\right)_V; \ \forall v \in K, p.p.t \in (0,T). \end{gathered} \quad (54)$$

From (53), (46), (44), and (48), it is concluded that u satisfies (43). Let φ be the function given by (42). Using (43), (52), and (42), it follows that (u,φ) is the solution to the variational problem $\mathcal{P}^V$. Regarding the regularity of the function φ, it follows to refer to the regularity of the element

$u \in W^{1,\infty}(0,T;V)$ and to the hypotheses (42) and (31). Then it is obtained that function φ has the regularity $\varphi \in W^{1,\infty}(0,T;W)$.

A solution (u,φ) of regularity $u \in W^{1,\infty}(0,T;V), \varphi \in W^{1,\infty}(0,T;W)$ was just shown. The uniqueness part of the function φ is deduced from the uniqueness of the function $u \in W^{1,\infty}(0,T;V)$ solution of (51) and (53) given by Lemma 1, thus taking into account (42).

VI. CONCLUSION

This paper investigated a mathematical model to describe the quasistatic contact between a piezoelectric body and a deformable foundation. The contact was frictionless and described with Signorini conditions. The proof of the existence and the uniqueness of the weak solution was presented using a classical result of elliptic variational inequalities and a maximal monotone operator.

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