Controller Design based on Fractional Calculus for AUV Yaw Control

Rasheed Abdulkader
Department of Electrical Engineering, Imam Mohammad Ibn Saud Islamic University (IMSIU), Saudi Arabia
rmabdulkader@imamu.edu.sa
(corresponding author)

Received: 16 January 2023 | Revised: 8 February 2023 and 12 February 2023 | Accepted: 15 February 2023

ABSTRACT

This research presents a fractional order integral controller strategy, which improves the steering angle for Autonomous Underwater Vehicles (AUVs). The AUV mathematical modeling is presented. A Fractional Order Proportional Integral (FOPI) control scheme is implemented to ensure the yaw angle stability of the AUV steering under system uncertainty. The FOPI controller is validated with MATLAB/Simulink and is compared to the conventional Integer Order PI (IOPI) controller to track the yaw angle of the structure. The simulation results show that the proposed FOPI controller outperforms the IOPI controller and improves the AUV system steering and the overall transient response while ensuring the system's stability with and without external disturbances such as underwater current and different loading conditions.

Keywords-Autonomous Underwater Vehicle (AUV); Nelder Mean Simplex (NMS); fractional calculus; FOPI

I. INTRODUCTION

Water covers more than 70% of Earth's surface and the oceans holds nearly 96.5% of all the Earth's water. The scientific area of exploration of new resources in the ocean is in rapid increase. However, investigating under sea water levels can have some limitations with manned or human operated systems, while the use of Automatous Underwater Vehicles (AUV) can provide huge benefits in terms of reducing the risk of human lives, exploring deep sea levels, underwater surveillance, and cost saving [1]. AUVs are unmanned robots that have the capability to move under the water surface typically on a pre-defined mission, with satellite networks used for communication. In the modeling and design stage, an important aspect is the mission requirements and objectives. Additional details on the process of designing underwater vehicles can be found in [2]. Underwater vehicles can be categorized into two major types, Remotely Operated Vehicles (ROVs) or manned vehicles, which need a human to function and send control instructions in order to operate. The other type is the AUV or unnamed vehicles, which can completely function independently. Currently, AUV's are rapidly used in a wide range of applications, which include environmental monitoring, underwater surveillance, scientific research, anti-submarine warfare, oceanographic discovery, subsea structure inspection, oil and gas natural research exploration, etc. [1].

The dynamics of underwater vehicles are well-known to contain significantly nonlinear dynamics and are dependent on a variant number of the system parameters. These nonlinearities can generate system uncertainties, time-varying dynamic model and severe effect by external disturbances such as unpredicted under water current, waves and environmental disturbances [1]. Controlling the AUV system is a challenging problem, and high control accuracy is needed to keep the system safe and stable when threatened by unpredicted factors. In order to handle AUVs' uncertainty and disturbance and to enhance their tracking performance, numerous control methods have been applied to ensure the stability of AUV systems, including the LQR control [3], neural networks [4], fuzzy control [5], PI/PID control [6], and Sliding Mode Control (SMC) [7]. Research findings on non-integer controllers indicate better quality control than Integer Order (IO) controllers. Some studies showing the advantages of implementing this control technique to stabilize the steering system of AUV's are [8-12].

In this paper, a control method is proposed that is essentially found in fractional calculus theory, a non-integer or fractional order control. At present, fractional order controller based on Nelder-Mead Simlex (NMS) algorithm has not been utilized for controlling and stabilizing the yaw angle of AUV's. The main contribution of this research is the development of a Fractional Order Proportional Integral (FOPI) controller scheme applied to enhance the steering stability and yaw angle for AUV dynamics with disturbances being present. The performance of the AUV structure and transient response is improved by designing the coefficients of the FOPI controller using the NMS method. The proposed controller is examined and its effectiveness to maintain the system stable with uncertain conditions such as underwater currents and load variation is shown.

II. AUV MODELING

A. AUV Dynamics (Coordinate System)

To derive a mathematical model and the equations representing the AUV structure, analysis of the system dynamics and kinematics is demonstrated, as three subsystems are considered. Figure 1 shows the two coordinate systems describing the movement of the AUV in 6 Degrees Of Freedom (DOF). The O-xyz axis is the motion coordinates and is static to the underwater vehicle, which is denoted as the body-fixed refence system. The movement of the body-rigid system is demonstrated to the earth (E) fixed system (ϕ, θ, Ψ) [13,14]. Thus, a non-integer order PI with feedback control scheme would be implemented to the AUV structure to achieve robust yaw angle stability with/without disturbances. Table I indicates the used AUV parameter values.

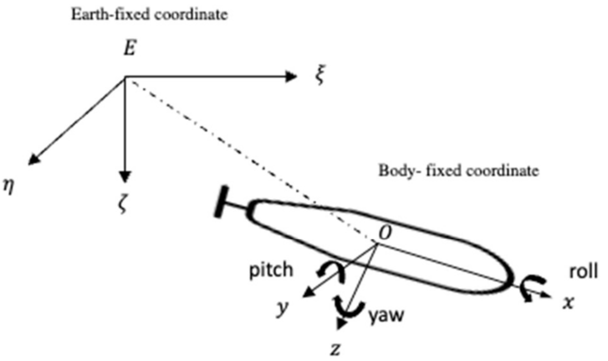

Fig. 1. AUV 6-DOF coordinate system.

B. AUV Kinematics

The physical demonstration of the AUV structure and the equation of motion follow fixed body dynamics. Consequently, it is valuable to utilize the physical system components by diminishing the number of coefficients needed to control the system. Thus, it is the main incentive for the development of the vertical description of the equations of motion, which are successful for computer processing [13]. The 6-DOF AUV equation of movement follows fixed body dynamics. $\dot{\eta} = [\eta_1 \quad \eta_2]^T$, $\eta_1 = [x \quad y \quad z]^T$ is the position vector and $\eta_2 = [\varphi \quad \theta \quad \psi]^T$ is the orientation vector of the body and earth fixed reference system. The linear and angular velocities are specified as $v_1 = [u \quad v \quad w]^T$, $v_2 = [p \quad q \quad r]^T$ where $v = [v_1 \quad v_2]^T$.

The two essential equations to model the AUV dynamical system are obtained from Newton's laws of motion. These equations can be defined as [13]:

$$\dot{\eta} = J(\eta)\, v \tag{1}$$

$$M\dot{v} + c(v)v + D(v)v + G(\eta) = \tau \tag{2}$$

where $J(\eta)$ is the transformation matrix, τ is the control input matrix, M, $C(v)$, $D(v)$, and $G(\eta)$ symbolize mass, Coriolis forces, damping matrix, and the gravitational matrix respectively.

By including in the AUV system non-linear equations of motion, the kinematics equation can be expressed as [13]:

$$\dot{v} = M^{-1}[C(v)v + D(v)v + G(\eta) + \tau_r] \tag{3}$$

TABLE I. AUV PARAMETERS

Parameter	Name	Value	Unit
M	AUV mass	50	kg
Y_v	Cross flow drag	-131	kg/m
Y_n	Cross flow drag	-0.632	kg.m/rad^2
N_r	Cross flow drag	-94	kg.m^2/rad
N_v	Cross flow drag	-3.2	kg
$X_{\dot{u}}$	Additional mass	-0.9	kg
$Y_{\dot{r}}$	Additional mass	1.93	kg.m /rad
$Y_{\dot{v}}$	Additional mass	-36	kg
$N_{\dot{r}}$	Additional mass	-4.9	kg.m^2/rad
I_z	Moment of inertia	3.45	kg.m^2/rad
u_0	AUV speed	10	m/s

C. AUV Model Decoupling

Due to the AUV system extreme nonlinear dynamics and coupling, a reduced system model is considered and linearized for the controller design, to ensure the stability and control of the AUV steering system. The vehicle model can be reduced and decoupled to study the yaw angle steering behavior. By considering the following three states, the sway v, yaw velocity r, and the yaw rate ψ. The steering movement can be acquired from the rudders and fins. Also, by disregarding the gravity forces, system damping and assuming an equilibrium point, the AUV system model can be decoupled as follows:

$$[m - Y_{\dot{v}}]\dot{v} = (X_{\dot{u}} - m)u_o r + Y_v v + Y_r r, \tag{4}$$

$$[I_z - N_{\dot{r}}]\dot{r} = (Y_{\dot{v}} - X_{\dot{u}})v\, r + Y_{\dot{r}} u_0 r + N_v v + N_r r + \tau_r \tag{5}$$

where $\dot{\psi} = r$. The linear AUV system can be expressed in state-space as:

$$\dot{x} = Ax + Bu\,,\ y = Cx + Du \tag{6}$$

where:

$$A = \begin{bmatrix} \frac{Y_v}{m - Y_{\dot{v}}} & \frac{(X_{\dot{u}} - m)u_0 + Y_r}{m - Y_{\dot{v}}} & 0 \\ \frac{N_v}{I_z - N_{\dot{r}}} & \frac{Y_{\dot{r}} u_0 + N_r}{I_Z - N_{\dot{r}}} & 0 \\ 0 & 1 & 0 \end{bmatrix}, \tag{7}$$

$$B = G(x) = \begin{bmatrix} 0 \\ \frac{1}{I_z - N_{\dot{r}}} \\ 0 \end{bmatrix},\ u = \tau_r. \tag{8}$$

From the state-space model in (7)-(8), the transfer function of the yaw angle control can be computed as follows:

$$G(s) = \frac{0.1198\, s + 0.2064}{s^3 + 10.67\, s^2 + 13.35\, s} \tag{9}$$

where a fractional controller is developed and applied to the AUV system dynamics in a feedback formation as shown in Figure 3.

III. FRACTIONAL ORDER PI CONTROL

The controller technique implemented in this study is based on fractional calculus, a non-integer or fractional order PI control. For AUV dynamics structure, it is crucial for the control scheme to track the required vehicle position and yaw angle in a precise manner to help the system obtain steering stability. By introducing fractional integral and differential operators, the perception of only IO operators can be stretched over a wider range, which results in better accuracy in system modeling or control. One critical advantage of a fractional system is the memory and hereditary annotation, which can be described with fractional order calculus theory. However, for classical IO plants the memory aspect and hereditary annotation are of a major concern [11]. Fractional calculus theory is a general notion to fractional order of differentiator and integrator simple operator ${}_aD_t^{\beta}$ defined in (10) [10]:

$$ {}_aD_t^{\beta} = \begin{cases} d^{\beta}/dt^{\beta} & \Re(\alpha) > 0, \\ 1 & \Re(\alpha) = 0, \\ \int_a^t (dt)^{-\beta} & \Re(\alpha) < 0. \end{cases} \quad (10) $$

where a and t are bounds of the process and β indicates the fractional component. β is assumed real, but it can be an imaginary component [10].

Fractional operators can be applied directly to systems or can be approximated as IO operators applied into the system. Further, a control system can be controlled by either an IO or non-integer order controller, hence the system plant can also possibly be derived as a fractional or classical IO system. Four distinct combinations can occur to implement a fractional order control: fractional order controller with a fractional order plant, fractional order plant with an IO controller, IO plant with a fractional order controller, and IO plant with an IO controller [14]. In this paper a linear fractional order controller scheme is developed and the controller would be implemented to an IO plant (AUV system).

Describing the control options by implementing a non-integer order controller in a graphic representation, Figure 2 indicates that all IO PID controllers are special cases of the non-integer order PID controller. It can be observed in Figure 2 that the IO PID controller only transforms at four rigid points, whereas any point on the graphical plane can be used with a fractional order PID controller, i.e. the control order can interchange continuously and smoothly throughout the entire shaded region, which makes the fractional order PID design more flexible more effective with system uncertainties than IOPID [17]. Thus, in this study, the value examined for the fractional order integral operator λ, lies in region [0-1] shown in Figure 2, since the system requires a lower order integral value to improve the transient response and maintain stability. The differential equation of the fractional order PI^{λ} control is provided by:

$$ u(t) = K_p\, e(t) + \frac{K_i}{D^{\lambda}}\, e(t) \quad (11) $$

By applying the Laplace transformation to (11) and with the assumption of zero initial conditions, the transfer function can be obtained as [16]:

$$ G_c(s) = K_p + \frac{K_i}{s^{\lambda}} \quad (12) $$

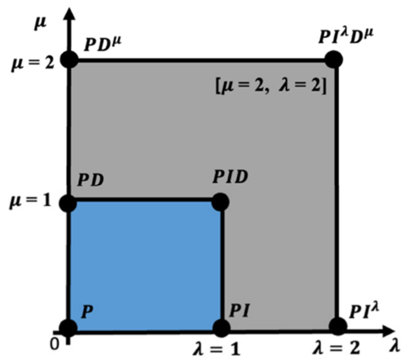

Fig. 2. Graphical representation of the range values of the FOPID controller.

A block diagram representation of the FOPI controller for a closed loop structure is shown in Figure 3, where the plant is the AUV system dynamics and $G_c(s)$ is the non-integer controller expressed as:

$$ G_c(s) = K_p + \frac{K_i}{s^{\lambda}}. \quad (13) $$

and λ is the non-integer operator.

The added tunable component λ with the FOPI control in comparison to the IOPI control generates more flexibility in controlling the structure and at the same time enhances the control quality implementation and helps minimizing the steady-state error and improves the transient response of the AUV system. Proper tunning of the FOPI controller is required to meet the system specifications. With a FOPI controller, reaching closed-loop zero steady-state error and diminishing the amplification of high-frequency disturbances can be achieved for the AUV system. Moreover, when using the IOPI control there would be a lagging phase of 90° for the system, while with the FOPI scheme, it delivers a reduced constant lagging phase to the system due to the fractional element λ which improves the transient response of the system dynamics as compared to the IOPI control.

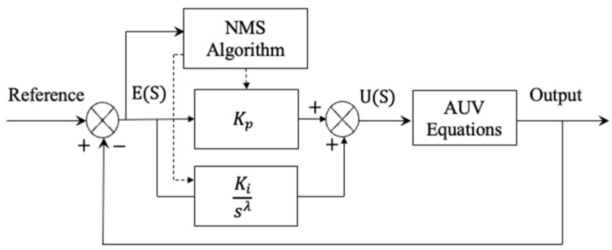

Fig. 3. FOPI structure applied for the AUV.

IV. THE NELDER-MEAD SIMPLEX METHOD

The simplex algorithm developed by Nelder and Mead in 1965 has been widely used to solve parameter estimation and optimization problems. The Nelder-Mead Simplex (NMS) has been applied by the United States army Corps Engineers for hydrologic engineering center-hydrologic modeling system (HEC-HMS) software as a search technique for optimizing the hydrologic parameters [18]. In this research, the value of the fractional operator λ is tunned using the NMS method. The NMS method can solve unconstrained optimization problems of the form $min_x\, f(x), x \in \mathbb{R}^n$. There are four basic operational steps of the NMS algorithm: reflection, expansion, contraction, and shrinking as demonstrated in Figure 4 [10, 18].

The five stages for implementing the NMS algorithm are [18]:

1. Order. Order $n+1$ vertices in order to meet:

   $f(x_1) \leq fx_2) \leq \cdots \leq f(x_{n+1})$

2. Reflect. Calculate the reflection point x_r:

$$x_r = \bar{x} + \rho(\bar{x} - x_{n+1}) \quad (14)$$

where:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad (15)$$

is the centroid of the n best points.

Then, $f_r = f(x_r)$. If $f_1 \leq f_r \leq f_n$, set the point x_r and terminate the iteration.

3. Expand. If $f_r < f_1$, evaluate the expansion point x_e by:

$$x_e = \bar{x} + \chi(x_r - \bar{x}) \quad (16)$$

Then compute $f_e = f(x_e)$. If $f_e < f_r$ then set $x_{n+1} = x_e$ and terminate the iteration. Otherwise, set $x_{n+1} = x_r$ and terminate the iteration.

4. Contract. If $f_r \geq f_n$, compute a contraction between $\bar{x}$ and the best of x_{n+1} and x_r:

- Outer contraction. If $f_n \leq f_r < f_{n+1}$, compute:

$$x_c = \bar{x} + \gamma(x_r - \bar{x}) \quad (17)$$

then determine $f_c = f(x_c)$. If $f_c \leq f_r$, set $x_{n+1} = x_c$, and terminae the process, else proceed to the next stage performing a shrinkage.

- Inner contraction. If $f_r \geq f_{n+1}$, execute the inner contraction by computing:

$$x_{cc} = \bar{x} - \gamma(\bar{x} - x_{n+1}) \quad (18)$$

Then compute $f_{cc} = f(x_{cc})$. If $f_{cc} < f_{n+1}$, set $x_{n+1} = x_{cc}$, and terminate the iteration. Else proceed to the next stage performing a shrinkage.

5. Shrink. Calculate n new points by:

$$x_i = x_1 + \sigma(x_i - x_1), \quad (19)$$

where $i = 2, \dots, n+1$. Then compute f at those vertices.

The scalar coefficients ρ, χ, γ, and σ denote the coefficients, expansion, contraction, and shrink reflection, respectively. These cofficients should obey:

$\rho > 0, \chi > 1, 0 < \gamma < 1$, and $0 < \sigma < 1$.

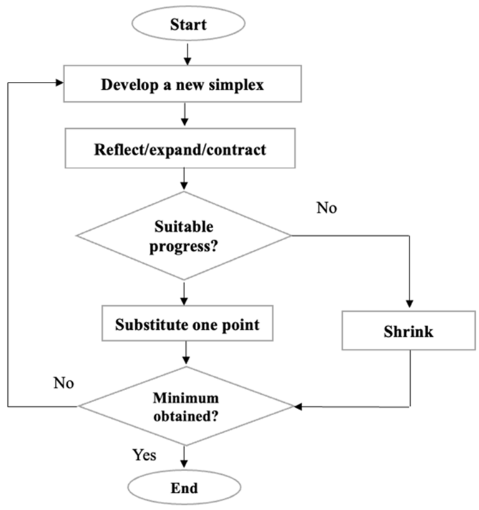

Fig. 4. NMS flowchart.

V. RESULTS AND DISCUSSION

The fractional order PI controller has been applied for AUV dynamics using MATLAB/Similink and the FOMCON toolbox. Figure 5 shows the Simulink model for the AUV system yaw angle response for the PI/FOPI controller, as a step disturbance is added to emulate an underwater current disturbance applied to the AUV system. The FOPI control is developed with the vehicle travelling at a speed of 10m/s, and maintains the yaw angle and steering stability for the AUV structure. The FOPI controller was designed using NMS with the minimum integral square error, as the bound of fractional integral inspected between the closed interval [0, 1]. Further, by applying the NMS technique the local minimization can be reached by eliminating the smallest needed vertex in a given function. The integral term λ of the fractional controller is optimized to be 0.1 with k_p = 200 and k_i = 79. Hence, the developed fractional controller is compared to the classical Integer-Order PI (IOPI) controller.

A. Simulation Results with the FOPI Controller for Different AUV Travelling Speeds

Figure 6 shows the simualtion results of the AUV operating at different speeds with the FOPI controller. It is demostarted that the vehicle reaches the desired steering angle of 40^o in 3s with a 50% increase and decrease in the AUV speed. As the speed is increased, the overshoot slightly increases, while the system still maintains its regulation to reach the desired yaw angle response.

B. Simulation of the Yaw Angle of the AUV with PI/FOPI Controllers

Figure 7 displays the response of yaw angle with the FOPI and IOPI controllers being applied for a desired yaw angle of 40^o. The response indicates improved performance of the FOPI over the IOPI controller, with a faster time response, settling time, lower overshoot, and increased damping. Table II shows the perfomrance comparison of the transient response of the controllers. We can see that the fractional controller has better performance in any feature.

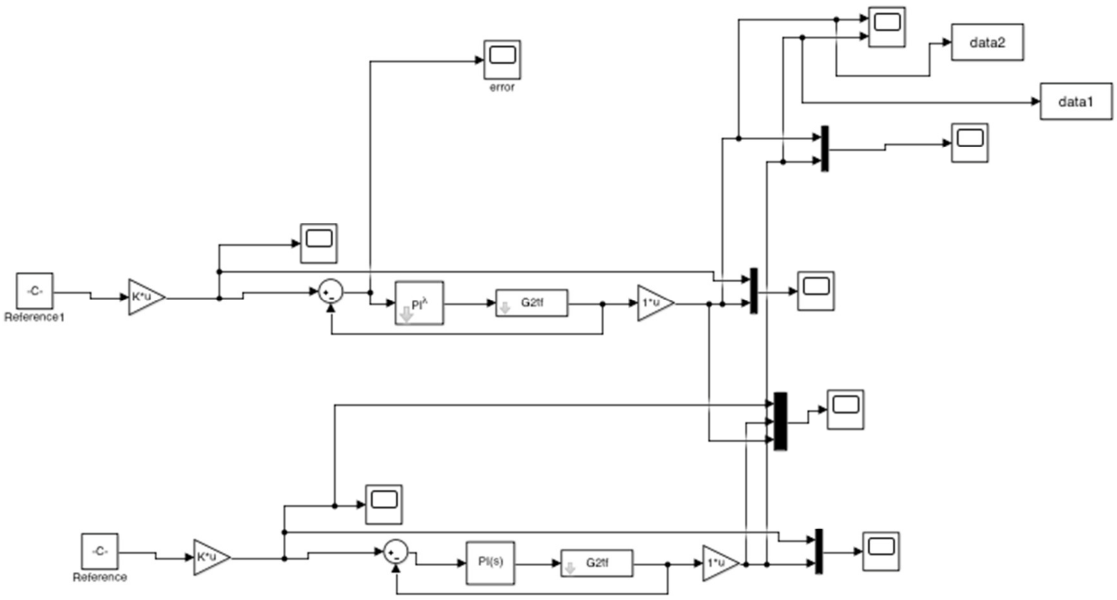

Fig. 5. Simulink model of the yaw angle for AUV using PI/FOPI.

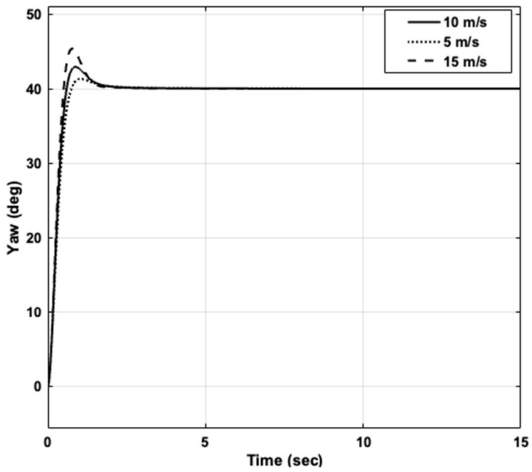

Fig. 6. Yaw angle for different AUV speeds with FOPI controller.

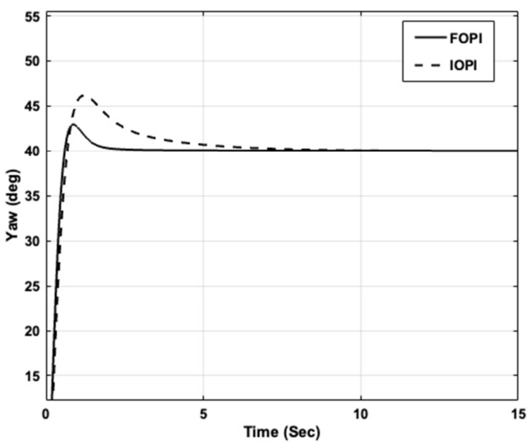

Fig. 7. Yaw angle of the AUV with FOPI/IOPI controller.

TABLE II. TRANSIENT RESPONSE

Response info	Controller	
	IOPI	FOPI
Settling time	4.75	1.48
Rise time	0.46	0.39
Overshoot	16.1	6.8
Undershoot	0	0
Peak	46.17	42.95

C. Simulation of the Yaw Angle of the AUV under the Presence of Disturbancet

Figure 8 presents the yaw angle reposne with an external disturbance, such as ocean curernt waves, being applied on the AUV system. It is seen that the FOPI controller outperforms the IOPI in terms of time response, overshoot, and settling time to reach the desired yaw angle with the presence of external disturance. Table III shows the respone comparizon between the controllers. It can be seen that the fractional controller improved the transisent response in all aspects.

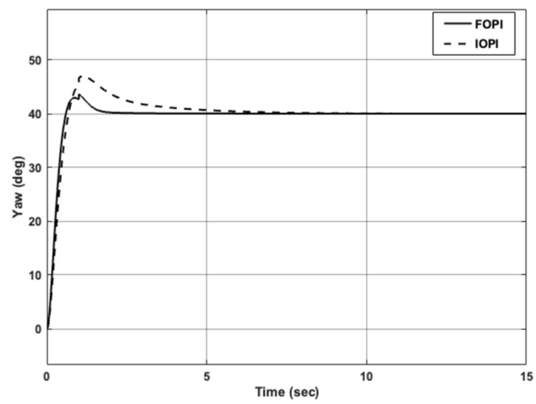

Fig. 8. Yaw angle with external disturbance applied to the AUV.

TABLE III. TRANSIENT RESPONSE WITH SYSTEM DISTURBANCE

Response info	Controller	
	IOPI	FOPI
Settling time	4.55	1.51
Rise time	0.47	0.38
Overshoot	9.3	17.1
Undershoot	0	0
Peak	43.66	46.97

D. Simulation of the Yaw Angle with Load Variation on the AUV System

Figure 10 demonstrates the response of the UAV with the load being increased and deceased by 50% along with the presence of disturbances. The FOPI controller shows its robustness in handling uncertain environmental disturbances with different loading conditions, in stabilizing the system, and in achieving the desired yaw angle with improved time response, stetting time, and minimized overshoot in comparison with the classical IOPI controller.

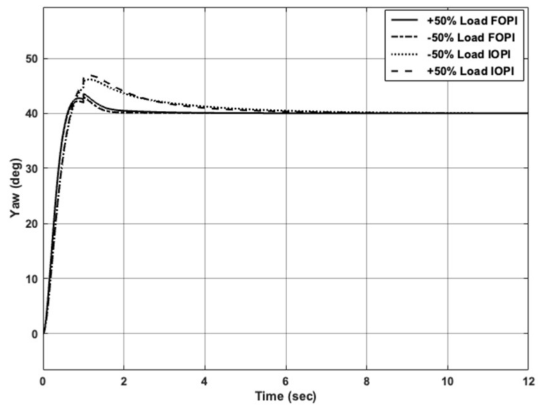

Fig. 9. FOPI/IOPI control of the yaw angle for AUV with varying loads.

VI. CONCLUSION

In this paper, a non-integer (fractional) order controller is implemented on the AUV dynamical system. It is demonstrated that the proposed FOPI control, which adds an additional tuning parameter, improves the system transient response, stability, and the yaw angle control of the AUV structure. The underwater vehicle yaw angle control is a very challenging problem and with the FOPI controller the system can preserve improved regulation and faster time response under system uncertainties and disturbances such as, underwater current waves and load variations. The FOPI controller was compared to the classical integer order controller, and the simulation results confirm that the fractional controller outperforms the IOPI controller and provides improved time response and reduced system overshoot and settling time to achieve the desired yaw angle. Regarding future work, the applied fractional order controller can be applied to an AUV system modeled as a fractional order system, which can provide more precision and accuracy to the system model dynamics and thus, improve the system overall performance.

REFERENCES

[1] M. T. Vu *et al.*, "Robust Position Control of an Over-actuated Underwater Vehicle under Model Uncertainties and Ocean Current Effects Using Dynamic Sliding Mode Surface and Optimal Allocation Control," *Sensors*, vol. 21, no. 3, Jan. 2021, Art. no. 747, https://doi.org/10.3390/s21030747.

[2] V. Upadhyay *et al.*, "Design and motion control of Autonomous Underwater Vehicle, Amogh," in *2015 IEEE Underwater Technology (UT)*, Chennai, India, Oct. 2015, https://doi.org/10.1109/UT.2015.7108287.

[3] L. Priyadarshini, S. Kundu, M. K. Maharana, and B. P. Ganthia, "Controller Design for the Pitch Control of an Autonomous Underwater Vehicle," *Engineering, Technology & Applied Science Research*, vol. 12, no. 4, pp. 8967–8971, Aug. 2022, https://doi.org/10.48084/etasr.5050.

[4] R. Hernández-Alvarado, L. G. García-Valdovinos, T. Salgado-Jiménez, A. Gómez-Espinosa, and F. Fonseca-Navarro, "Neural Network-Based Self-Tuning PID Control for Underwater Vehicles," *Sensors*, vol. 16, no. 9, Sep. 2016, Art. no. 1429, https://doi.org/10.3390/s16091429.

[5] X. Xiang, C. Yu, L. Lapierre, J. Zhang, and Q. Zhang, "Survey on Fuzzy-Logic-Based Guidance and Control of Marine Surface Vehicles and Underwater Vehicles," *International Journal of Fuzzy Systems*, vol. 20, no. 2, pp. 572–586, Feb. 2018, https://doi.org/10.1007/s40815-017-0401-3.

[6] F. Kong, Y. Guo, and W. Lyu, "Dynamics Modeling and Motion Control of an New Unmanned Underwater Vehicle," *IEEE Access*, vol. 8, pp. 30119–30126, 2020, https://doi.org/10.1109/ACCESS.2020.2972336.

[7] N. Wang, S. Lv, W. Zhang, Z. Liu, and M. J. Er, "Finite-time observer based accurate tracking control of a marine vehicle with complex unknowns," *Ocean Engineering*, vol. 145, pp. 406–415, Nov. 2017, https://doi.org/10.1016/j.oceaneng.2017.09.062.

[8] B. Bourouba and S. Ladaci, "A Novel Non-integer Indirect Adaptive Control for Non-integer Order Systems with Non-prior Knowledge," *Engineering, Technology & Applied Science Research*, vol. 10, no. 1, pp. 5186–5190, Feb. 2020, https://doi.org/10.48084/etasr.3236.

[9] A. W. Nasir, I. Kasireddy, and A. K. Singh, "Real Time Speed Control of a DC Motor Based on its Integer and Non-Integer Models Using PWM Signal," *Engineering, Technology & Applied Science Research*, vol. 7, no. 5, pp. 1974–1979, Oct. 2017, https://doi.org/10.48084/etasr.1292.

[10] A. Tepljakov, *Fractional-order Modeling and Control of Dynamic Systems*. Springer International Publishing, 2017, https://doi.org/10.1007/978-3-319-52950-9.

[11] S. Konar, M. D. Patil, and V. A. Vyawahare, "Design of a Fractional Order Sliding Mode Controller for Depth Control of AUV," in *2018 Second International Conference on Intelligent Computing and Control Systems (ICICCS)*, Madurai, India, Jun. 2018, pp. 1342–1345, https://doi.org/10.1109/ICCONS.2018.8662916.

[12] R. Abdulkader and R. McCann, "Fractional Controller for Improved Lateral Stability Control Using AFS," in *2020 6th International Conference on Control, Automation and Robotics (ICCAR)*, Singapore, Apr. 2020, pp. 242–246, https://doi.org/10.1109/ICCAR49639.2020.9108059.

[13] T. Fossen, *Hanbook of marine craft and motion contro*l. West Suusex, UK: Wiley and Sons, Ltd, 2011, https://doi.org/10.1002/9781119994138.

[14] R. Abdulkader, "Non-integer Controller for Autonomous Underwater Vehicle Steering Control," in *2021 IEEE International Conference on Robotics, Automation, Artificial-Intelligence and Internet-of-Things (RAAICON)*, Dhaka, Bangladesh, Sep. 2021, pp. 102–105, https://doi.org/10.1109/RAAICON54709.2021.9929838.

[15] I. Pan and S. Das, *Intelligent Fractional Order Systems and Control: An Introduction.* Berlin Heidelberg, Germany: Springer, 2013, https://doi.org/10.1007/978-3-642-31549-7.

[16] R. Abdulkader and R. McCann, "Fractional order state feedback control for improved lateral stability for tractor-trailers in platooning," *International. Journal of Heavy Vehicle Systems*, vol. 28, no.5, pp. 698-725 2020, https://doi.org/10.1504/IJHVS.2021.10037405.

[17] C. I. Pop, C. Ionescu, R. De Keyser, and E. H. Dulf, "Robustness evaluation of fractional order control for varying time delay processes," *Signal, Image and Video Processing*, vol. 6, no. 3, pp. 453–461, Sep. 2012, https://doi.org/10.1007/s11760-012-0322-4.

[18] R. Barati, "Parameter Estimation of Nonlinear Muskingum Models Using Nelder-Mead Simplex Algorithm," *Journal of Hydrologic Engineering*, vol. 16, no. 11, pp. 946–954, Nov. 2011, https://doi.org/10.1061/(ASCE)HE.1943-5584.0000379.