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IMPROVING TIME INTEGRATION SCHEME FOR FET ANALYSIS OF POWER SYSTEM ANGLE STABILITY

Marin Mandić, Ivica Jurić-Grgić, Nedjeljka Grulović-Plavljanić

University of Split, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, Split, Croatia

Abstract. *This paper presents improved algorithm for numerical analysis of power system angle stability achieved by improvement of the time integration when forming a local system of equations for power system finite elements (FE). Previously developed local system of equations of power system angle stability has been obtained using the generalized trapezoidal rule (ϑ - method). Improvement of accuracy was obtained by using Heun's method. Numerical solutions obtained using Heun's method and using the generalized trapezoidal rule are compared to Electromagnetic Transients Program (EMTP). It has been shown that Heun's method yields the results with much higher accuracy comparing to results obtained by generalized trapezoidal rule.*

Key words: *Heun's method, finite element technique, angle stability, time domain analysis.*

1. INTRODUCTION

The assessment of angle or transient stability of an electrical system plays a fundamental role, as it allows contingencies classification and provides indications for the design and planning of the power systems. Transient stability or also known as large-disturbance rotor angle stability concerns the ability of the system to withstand critical disturbances such as three-phase short circuits. As it is reported in [1-7], it is very difficult to analyze simultaneously the slow electromechanical and fast electromagnetic phenomena.

This paper presents improved algorithm for numerical analysis of power system angle stability. Complex numerical analysis is performed simultaneously by both the time-domain and the frequency-domain. The developed numerical model is based on a finite element technique (FET) procedure and time-varying phasors. In contrast to existing transient stability programs (TSP) that are limited to the first swing stability analysis, the developed numerical model can analyze large-disturbance rotor angle stability. The basis of developed numerical model for analysis of power system angle stability is application of FET to the power system so that the considered system is divided into smaller parts,

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Corresponding author: Marin Mandić
University of Split, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, Split, Croatia
(E-mail: mmandi00@fesb.hr)

which are treated as separated finite elements (FE) with a certain number of local nodes during entire time-domain simulation.

Fundamentals of FET analysis of power system angle stability have been described in [8] where the generalized trapezoidal rule (ϑ - method) is used for temporal integration when forming a FE equations system. The use of the generalized trapezoidal rule for the time integration when forming a FE equations system can sometimes cause numerical oscillations or numerical diffusion of results. Numerical oscillations and/or numerical damping of results are the consequences of the choice of the numerical integration parameter (ϑ). In principle, its choice is based on numerical experiments. The influence of ϑ on stability of one-step time integration methods for the first order initial value problems is discussed in [9-10], where the stability behaviour is investigated by examining the free response of the FE model. It was found that methods for which $0.5 \leq \vartheta \leq 1$ are unconditionally stable.

The aim of this article is to improve the algorithm for numerical analysis of power system angle stability by improving the time integration scheme when forming FE equations system.

Instead of using the generalized trapezoidal rule when forming a FE matrix, the accuracy of a numerical solution can be improved using one of the variants of the Runge-Kutta second-order method also known as Heun's method [11] for the time integration.

Sometimes Heun’s method is also referred as the improved Euler’s method. Heun’s method is suitable because, as a single-step method, it connects variables only at the beginning and at the end of the time interval and thus preserves the simplicity of numerical procedure. It is a single-step predictor-corrector method that improves the estimation of the slope for the time interval using an average value of slopes at the beginning and at the end of the time interval.

Desire for calculation speed must be balanced with the appropriate accuracy and stability required. We could get higher accuracy by multi-stage Runge-Kutta second-order method but it leads to time-consuming problem. Theoretically, one can develop infinite number of time schemes and their combinations, but the idea was to improve accuracy of FET model computation for power system angle stability analysis with one-step method that is not time-consuming.

Numerical solutions obtained using Heun’s method and using the generalized trapezoidal rule for time integration parameter $\vartheta = 0.66$ are compared to Electromagnetic Transients Program (EMTP). It has been shown that Heun’s method yields the results with much higher accuracy comparing to results obtained by generalized trapezoidal rule.

2. FET ANALYSIS - HEUN’S METHOD

Fundamentals of FET analysis of power system angle stability have been described in [8] where the generalized trapezoidal rule (also known ϑ - method) is used for temporal integration when forming a FE equations system. In the following text, it will be shown how to derive a local system of equations for power system finite elements by using Heun’s method for temporal integration instead of the generalized trapezoidal rule. Numerical model of turbine governor and excitation system have been included into synchronous generator FE model where numerical models have been improved using Heun's method [11] for the time integration. Finite elements of utility equivalent source, three-phase transformers and transmission lines are defined by the system of algebraic equations as it is already described in [8].

2.1. FE Synchronous Generator – Heun's method

According to [12-13], salient pole synchronous generator subtransient phasor model is defined by the following system of algebraic and differential equations where time-domain swing equation (6) has been also included:

$$E''_q = U_q + R \cdot I_q + X''_d \cdot I_d \tag{1}$$

$$E''_d = U_d + R \cdot I_d + X''_q \cdot I_q \tag{2}$$

$$T'_{do} \cdot \frac{dE'_o}{dt} = E_f + (X'_d - X_d) \cdot I_d - E'_o \tag{3}$$

$$T''_{do} \cdot \frac{dE''_q}{dt} = E'_o + (X''_d - X'_d) \cdot I_d - E''_q \tag{4}$$

$$T''_{qo} \cdot \frac{dE_d{}''}{dt} = (X''_q - X_q) \cdot I_q - E_d{}'' \tag{5}$$

$$M_i \cdot \frac{d\Delta\omega}{dt} = P_m - P_e \tag{6}$$

$$\frac{d\delta}{dt} = \Delta\omega = \omega - \omega_n \tag{7}$$

$$P_e = \frac{E''_o \cdot U}{X''_d} \cdot \sin\delta_1 + \frac{U^2}{2} \cdot \left(\frac{X''_d - X''_q}{X''_d \cdot X''_q} \right) \cdot sin2\delta_1 \tag{8}$$

where E'_o, E''_q, E''_d are transient and subtransient generator voltages. Variable E_f represents generator field voltage while variables U_q, U_d, I_q and I_d are axial components of the terminal voltage $\bar{U}$ and stator current $\bar{I}$.

In order to perform the time integration according the Heun's method, it is necessary to put relations (3-7) into following form:

$$\frac{dE'_o}{dt} = f_1(E_f, I_d, E'_o) \tag{9}$$

$$\frac{dE''_q}{dt} = f_2(E'_o, I_d, E''_q) \tag{10}$$

$$\frac{dE''_d}{dt} = f_3(I_q, E''_d) \tag{11}$$

$$\frac{d(\Delta\omega)}{dt} = f_4(P_m, P_e) \tag{12}$$

$$\frac{d\delta}{dt} = f_5(\Delta\omega) \tag{13}$$

where:

$$f_1(E_f, I_d, E'_o) = \frac{E_f}{T'_{do}} + \frac{(X'_d - X_d)\cdot I_d}{T'_{do}} - \frac{E'_o}{T'_{do}} \tag{14}$$

$$f_2(E'_o, I_d, E''_q) = \frac{E'_o}{T''_{do}} + \frac{(X''_d - X'_d)\cdot I_d}{T''_{do}} - \frac{E''_q}{T''_{do}} \tag{15}$$

$$f_3(I_q, E''_d) = \frac{(X''_q - X_q)\cdot I_q}{T''_{qo}} - \frac{E''_d}{T''_{qo}} \tag{16}$$

$$f_4(P_m, P_e) = \frac{P_m}{M_i} - \frac{P_e}{M_i} \tag{17}$$

$$f_5(\Delta\omega) = \Delta\omega \tag{18}$$

According to Heun's method, the state of variables at the end of the time interval is defined by the following corrector equations:

$$E'^{+}_o = E'_o + \frac{\Delta t}{2}\cdot f_1(E_f, I_d, E'_o) + \frac{\Delta t}{2}\cdot f_1(E^+_f, I^+_d, E'^{+p}_o) \tag{19}$$

$$E''^{+}_q = E''_q + \frac{\Delta t}{2}\cdot f_2(E'_o, I_d, E''_q) + \frac{\Delta t}{2}\cdot f_2(E'^{+}_o, I^+_d, E''^{+p}_q) \tag{20}$$

$$E''^{+}_d = E''_d + \frac{\Delta t}{2}\cdot f_3(I_q, E''_d) + \frac{\Delta t}{2}\cdot f_3(I^+_q, E''^{+p}_d) \tag{21}$$

$$\Delta\omega^+ = \Delta\omega + \frac{\Delta t}{2}\cdot f_4(P_m, P_e) + \frac{\Delta t}{2}\cdot f_4(P^+_m, P^+_e) \tag{22}$$

$$\delta^+ = \delta + \frac{\Delta t}{2}\cdot f_5(\Delta\omega) + \frac{\Delta t}{2}\cdot f_5(\Delta\omega^+) \tag{23}$$

According to (14-18), the predicted slopes $f_1(E^+_f, I^+_d, E'^{+p}_o)$, $f_2(E'^{+}_o, I^+_d, E''^{+p}_q)$, $f_3(I^+_q, E''^{+p}_d)$, $f_4(P^+_m, P^+_e)$, $f_5(\Delta\omega^+)$ at the end of the time interval are:

$$f_1(E^+_f, I^+_d, E'^{+p}_o) = \frac{E^+_f}{T'_{do}} + \frac{(X'_d - X_d)\cdot I^+_d}{T'_{do}} - \frac{E'^{+p}_o}{T'_{do}} \tag{24}$$

$$f_2(E'^{+}_o, I^+_d, E''^{+p}_q) = \frac{E^+_o}{T''_{do}} + \frac{(X''_d - X'_d)\cdot I^+_d}{T''_{do}} - \frac{E''^{+p}_q}{T''_{do}} \tag{25}$$

$$f_3(I^+_q, E''^{+p}_d) = \frac{(X''_q - X_q)\cdot I^+_q}{T''_{qo}} - \frac{E''^{+p}_d}{T''_{qo}} \tag{26}$$

$$f_4(P^+_m, P^+_e) = \frac{P^+_m}{M_i} - \frac{P^+_e}{M_i} \tag{27}$$

$$f_5(\Delta\omega^+) = \Delta\omega^+ \tag{28}$$

where $E_o'^{+p}$, $E_q''^{+p}$, $E_d''^{+p}$ are predicted transient and subtransient voltage values at the end of the time interval defined by the following predictor equations:

$$E_o'^{+p} = E_o' + \Delta t \cdot f_1(E_f, I_d, E_o') \tag{29}$$

$$E_q''^{+p} = E_q'' + \Delta t \cdot f_2(E_o', I_d, E_q'') \tag{30}$$

$$E_d''^{+p} = E_d'' + \Delta t \cdot f_3(I_q, E_d'') \tag{31}$$

where $f_1(E_f, I_d, E_o')$, $f_2(E_o, I_d, E_q'')$, $f_3(I_q, E_d'')$ are defined by equations (14-16).

The time integration of equations (3-7) using the generalized trapezoidal rule (ϑ-method) can be found in [8]. With application of the Heun's method [11] on Equations (3-7), which includes combining (19-31) and separating the variables at the end of time interval to the left-hand side and the variables at the beginning of time interval to the right-hand side, the following system of algebraic equations is obtained:

$$\begin{aligned} E_o'^{+} &= E_o' + \frac{E_f \cdot \Delta t}{2 \cdot T_{do}'} + \frac{\Delta t \cdot I_d \cdot (X_d' - X_d)}{2 \cdot T_{do}'} - \\ &- \frac{E_o' \cdot \Delta t}{2 \cdot T_{do}'} + \frac{E_f^{+} \cdot \Delta t}{2 \cdot T_{do}'} + \frac{\Delta t \cdot I_d^{+} \cdot (X_d' - X_d)}{2 \cdot T_{do}'} - \frac{E_o' \cdot \Delta t}{2 \cdot T_{do}'} - \\ &- \frac{E_f \cdot (\Delta t)^2}{2 \cdot (T_{do}')^2} - \frac{(\Delta t)^2 \cdot I_d \cdot (X_d' - X_d)}{2 \cdot (T_{do}')^2} + \frac{E_o' \cdot (\Delta t)^2}{2 \cdot (T_{do}')^2} \end{aligned} \tag{32}$$

$$\begin{aligned} E_q''^{+} &= E_q'' + \frac{E_o' \cdot \Delta t}{2 \cdot T_{do}''} + \frac{\Delta t \cdot I_d \cdot (X_d'' - X_d')}{2 \cdot T_{do}''} - \\ &- \frac{E_q'' \cdot \Delta t}{2 \cdot T_{do}''} + \frac{E_o'^{+} \cdot \Delta t}{2 \cdot T_{do}''} + \frac{\Delta t \cdot I_d^{+} \cdot (X_d'' - X_d')}{2 \cdot T_{do}''} - \frac{E_q'' \cdot \Delta t}{2 \cdot T_{do}''} - \\ &- \frac{E_o'' \cdot (\Delta t)^2}{2 \cdot (T_{do}'')^2} - \frac{(\Delta t)^2 \cdot I_d \cdot (X_d'' - X_d')}{2 \cdot (T_{do}'')^2} + \frac{E_q'' \cdot (\Delta t)^2}{2 \cdot (T_{do}'')^2} \end{aligned} \tag{33}$$

$$\begin{aligned} E_d''^{+} &= E_d'' + \frac{\Delta t \cdot I_q \cdot (X_q'' - X_q)}{2 \cdot T_{qo}''} - \frac{E_d'' \cdot \Delta t}{2 \cdot T_{qo}''} + \\ &+ \frac{\Delta t \cdot I_q^{+} \cdot (X_q'' - X_q)}{2 \cdot T_{qo}''} - \frac{E_d'' \cdot \Delta t}{2 \cdot T_{qo}''} + \frac{E_d'' \cdot (\Delta t)^2}{2 \cdot (T_{qo}'')^2} - \\ &- \frac{(\Delta t)^2 \cdot I_q \cdot (X_q'' - X_q)}{2 \cdot (T_{qo}'')^2} \end{aligned} \tag{34}$$

$$\Delta\omega^{+} = \Delta\omega + \frac{P_m \cdot \Delta t}{2 \cdot M_i} - \frac{P_e \cdot \Delta t}{2 \cdot M_i} + \frac{P_m^{+} \cdot \Delta t}{2 \cdot M_i} - \frac{P_e^{+} \cdot \Delta t}{2 \cdot M_i} \tag{35}$$

$$\delta^{+} = \delta + \frac{\Delta t}{2} \cdot \Delta\omega + \frac{\Delta t}{2} \cdot \Delta\omega^{+} \tag{36}$$

The variables in Equations (32-36) marked by "+" denote variables at the end of the time interval, while variables without mark denote variables at the beginning of the time interval. According to [8], synchronous generator can be defined as FE with three local nodes where FE local system has been defined by the system of algebraic Equation (8) and Equations (32-37).

$$\{\bar{I}_g\} = [\bar{Z}_e'']^{-1} \cdot \{\bar{\varphi}_g\} - [\bar{Z}_e'']^{-1} \cdot \{\bar{E}_o''\} \tag{37}$$

Dynamic phasors of local matrix as well as dynamic phasors of vectors in Equation (37) are defined as follows:

$$[\bar{Z}_e''] = \begin{bmatrix} \bar{Z}_e'' & 0 & 0 \\ 0 & \bar{Z}_e'' & 0 \\ 0 & 0 & \bar{Z}_e'' \end{bmatrix}$$

$$\{\bar{E}_o''\} = \{\bar{E}_{01}'' \quad \bar{E}_{02}'' \quad \bar{E}_{03}''\}^T = \{E_0'' \cdot e^{j\delta_1} \quad E_0'' \cdot e^{j\delta_1 + 2\pi/3} \quad E_0'' \cdot e^{j\delta_1 - 2\pi/3}\}^T$$

$$\{\bar{\varphi}_g\} = \{\bar{\varphi}_1 \quad \bar{\varphi}_2 \quad \bar{\varphi}_3\}^T$$

$$\{\bar{I}_g\} = \{\bar{I}_1 \quad \bar{I}_2 \quad \bar{I}_3\}^T$$

where:

$$\bar{Z}_e'' = R + j[X_d'' + e^{j\psi} \cdot \cos\psi \cdot (X_q'' - X_d'')]$$

$$E_o'' = \sqrt{(\mathrm{E}_d'')^2 + (\mathrm{E}_q'')^2}$$

$$\mathrm{I} = \sqrt{(I_d)^2 + (I_q)^2}$$

$\psi = \delta + \varphi$ - angle between stator current phasor $\bar{I}$ and 'q' axis

φ- power angle

2.2. Model of Hydraulic Digital Turbine Governor – Heun's Method

According to [14], numerical model of digital turbine governor is defined by the following system of algebraic and differential equations:

$$P_s = P_{ref} - \Delta\omega_r - P_R \tag{38}$$

$$P_{s1} = P_{sa} + P_{sb} + P_{sc} \tag{39}$$

$$P_{sa} = k_p \cdot P_s \tag{40}$$

$$P_{g\sigma_1} = \sigma \cdot q \tag{41}$$

$$q_{v\_open} \leq q_v \leq q_{v\_close} \tag{42}$$

$$q_{\min} \leq q \leq q_{\max} \tag{43}$$

$$P_{sc} + T_d \frac{dP_{sc}}{dt} = k_d \cdot \frac{dP_s}{dt} \tag{44}$$

$$T_g \cdot q_v + T_g \cdot T_p \frac{dq_v}{dt} = P_{s1} \tag{45}$$

$$\frac{dq}{dt} = q_v \tag{46}$$

$$\frac{dP_{sb}}{dt} = k_i \cdot P_s \tag{47}$$

$$P_{g\sigma_2} + T_r \cdot \frac{dP_{g\sigma_2}}{dt} = \delta \cdot T_r \cdot \frac{dq}{dt} \tag{48}$$

$$P_m + a_{11} \cdot T_w \cdot \frac{dP_m}{dt} = a_{11} \cdot \delta + (a_{23} \cdot a_{11} - a_{13} \cdot a_{21}) \cdot T_w \cdot \frac{dq}{dt} \tag{49}$$

where:
ω - rotor angular frequency
ω_n - nominal angular frequency
P_m, P_e - mechanical and electrical power
k_p - PID controller proportional gain
k_i - PID controller integral gain
k_d - PID controller derivative gain
T_d - PID controller derivative time constant
P_{ref} - referent value of mechanical power
T_p - pilot time constant
T_r - transient droop time constant
T_w - water inertia time constant
T_g - gate time constant
q_{v_open} - gate max opening speed
q_{v_close} - gate min closing speed
$q_{\max}$ - maximal gate position
$q_{\min}$- minimal gate position
σ - permanent droop coefficient
a_{11}, a_{13}, a_{21} - turbine coefficients
a_{23} - turbine gain

The time integration of above equations using the generalized trapezoidal rule (ϑ - method) can be found in [8]. Using the same procedure shown in chapter 2.1. for time integration of Equations (44 - 49), that is application of Heun's method, the following system of algebraic equations is obtained:

$$\begin{aligned} P_{sc}^{\ +} = P_{sc} + \frac{\Delta t \cdot k_d \cdot P_{sn}}{2 \cdot T_d} - \frac{P_{sc} \cdot \Delta t}{2 \cdot T_d} + \frac{\Delta t \cdot k_d \cdot P_{sn}^{\ +}}{2 \cdot T_d} - \\ - \frac{P_{sc} \cdot \Delta t}{2 \cdot T_d} - \frac{k_d \cdot P_{sn} \cdot (\Delta t)^2}{2 \cdot (T_d)^2} + \frac{P_{sc} \cdot (\Delta t)^2}{2 \cdot (T_d)^2} \end{aligned} \tag{50}$$

$$P_s^{+} = P_s + \frac{\Delta t}{2} \cdot P_{sn} + \frac{\Delta t}{2} \cdot P_{sn}^{\ +} \tag{51}$$

$$q_v^{\ +} = q_v + \frac{\Delta t \cdot P_{s1}}{2 \cdot T_g \cdot T_p} - \frac{q_v \cdot \Delta t}{2 \cdot T_p} + \frac{\Delta t \cdot P_{s1}^{\ +}}{2 \cdot T_g \cdot T_p} - \frac{q_v \cdot \Delta t}{2 \cdot T_p} - \frac{P_{s1} \cdot (\Delta t)^2}{2 \cdot T_g \cdot T_p^2} + \frac{q_v \cdot (\Delta t)^2}{2 \cdot (T_p)^2} \tag{52}$$

$$q^+ = q + \frac{\Delta t}{2} \cdot q_v + \frac{\Delta t}{2} \cdot q_v^{\ +} \tag{53}$$

$$P_{sb}^{\ +} = P_{sb} + \frac{\Delta t}{2} \cdot k_i \cdot P_s + \frac{\Delta t}{2} \cdot k_i \cdot P_s^+ \tag{54}$$

$$\begin{aligned} P_{g\sigma 2}^+ = P_{g\sigma 2} + \frac{\Delta t}{2} \cdot \delta \cdot q_v - \frac{P_{g\sigma 2} \cdot \Delta t}{2 \cdot T_r} + \frac{\Delta t}{2} \cdot \delta^+ \cdot q_v^+ - \\ - \frac{\Delta t \cdot P_{g\sigma 2}}{2 \cdot T_r} - \frac{\delta \cdot q_v \cdot (\Delta t)^2}{2 \cdot T_r} + \frac{P_{g\sigma 2} \cdot (\Delta t)^2}{2 \cdot T_r^{\ 2}} \end{aligned} \tag{55}$$

$$\begin{aligned} P_m^{\ +} = P_m + \frac{q \cdot \Delta t \cdot a_{23}}{2 \cdot a_{11} \cdot T_w} + \frac{\Delta t \cdot q_v \cdot (a_{23} \cdot a_{11} - a_{13} \cdot a_{21})}{2 \cdot a_{11}} - \frac{P_m \cdot \Delta t}{2 \cdot a_{11} \cdot T_w} + \\ + \frac{q^+ \cdot a_{23} \cdot \Delta t}{2 \cdot a_{11} \cdot T_w} + \frac{\Delta t \cdot (a_{23} \cdot a_{11} - a_{13} \cdot a_{21}) \cdot q_v^+}{2 \cdot a_{11}} - \frac{P_m \cdot \Delta t}{2 \cdot a_{11} \cdot T_w} - \\ - \frac{q \cdot a_{23} \cdot (\Delta t)^2}{2 \cdot (a_{11})^2 \cdot T_w^2} - \frac{(\Delta t)^2 \cdot q_v \cdot (a_{23} \cdot a_{11} - a_{13} \cdot a_{21})}{2 \cdot a_{11}^2 \cdot T_w} + \frac{P_m \cdot (\Delta t)^2}{2 \cdot (a_{11} \cdot T_w)^2} \end{aligned} \tag{56}$$

The numerical model of turbine governor is defined by the system of algebraic Equations (38-43) and Equations (50-56).

2.3. Model of Excitation System – Heun's Method

According to [15], numerical model of excitation system is defined by the following system of algebraic and differential equations:

$$v_C = v_{REF} + \frac{E_{f0}}{k_a} - v_T - v_f \tag{57}$$

$$v_{R\min} \le v_R \le v_{R\max} \tag{58}$$

$$\frac{dv_T}{dt} = -\frac{1}{T_r} \cdot v_T + \frac{k_r}{T_r} \cdot v_t \tag{59}$$

$$\frac{dv_R}{dt} = -\frac{1}{T_a} \cdot v_R + \frac{k_a}{T_a} \cdot v_C \tag{60}$$

$$\frac{dE_f}{dt} = -\frac{k_e}{T_e} \cdot u_f + \frac{1}{T_e} \cdot v_R \tag{61}$$

$$v_f + T_f \cdot \frac{dv_f}{dt} = k_f \cdot \frac{dE_f}{dt} \tag{62}$$

where:

k_a, k_r, k_f, k_e - regulator amplification factors
T_a, T_r, T_f, T_e - time constants
v_T, v_f, v_R , v_C - regulator signals
E_f - field voltage
$E_{f\,0}$ - initial field voltage
v_t - phase voltage
v_{REF} - referent phase voltage

The time integration of above equations using the generalized trapezoidal rule (ϑ - method) can be found in [8]. Using the same procedure shown in chapter 2.1 for time integration of Equations (59 - 62), that is application of Heun's method, the following system of algebraic equations is obtained:

$$v_T^{\,+} = v_T + \frac{\Delta t \cdot k_r \cdot v_t}{2 \cdot T_r} - \frac{v_T \cdot \Delta t}{2 \cdot T_r} + \frac{\Delta t \cdot k_r \cdot v_t^{\,+}}{2 \cdot T_r} - \frac{v_T \cdot \Delta t}{2 \cdot T_r} - \frac{k_r \cdot v_t \cdot (\Delta t)^2}{2 \cdot T_r^{\,2}} + \frac{v_T \cdot (\Delta t)^2}{2 \cdot T_r^{\,2}} \tag{63}$$

$$v_R^{\,+} = v_R + \frac{\Delta t \cdot k_a \cdot v_c}{2 \cdot T_a} - \frac{v_R \cdot \Delta t}{2 \cdot T_a} + \frac{\Delta t \cdot k_a \cdot v_c^{\,+}}{2 \cdot T_a} - \frac{v_R \cdot \Delta t}{2 \cdot T_a} - \frac{k_a \cdot v_c \cdot (\Delta t)^2}{2 \cdot T_a^{\,2}} + \frac{v_R \cdot (\Delta t)^2}{2 \cdot T_a^{\,2}} \tag{64}$$

$$E_f^{\,+} = E_f + \frac{\Delta t \cdot k_e \cdot v_R}{2 \cdot T_e} - \frac{E_f \cdot \Delta t}{2 \cdot T_e} + \frac{\Delta t \cdot k_e \cdot v_R^{\,+}}{2 \cdot T_e} - \frac{E_f \cdot \Delta t}{2 \cdot T_e} - \frac{k_e \cdot v_R \cdot (\Delta t)^2}{2 \cdot T_e^{\,2}} + \frac{E_f \cdot (\Delta t)^2}{2 \cdot T_e^{\,2}} \tag{65}$$

$$\begin{aligned} v_f^{\,+} = v_f &+ \frac{\Delta t \cdot k_f \cdot \left[-\frac{1}{T_e} \cdot E_f + \frac{k_e}{T_e} \cdot v_R \right]}{2 \cdot T_f} - \frac{v_f \cdot \Delta t}{2 \cdot T_f} + \frac{\Delta t \cdot k_f \cdot \left[-\frac{1}{T_e} \cdot E_f^{+} + \frac{k_e}{T_e} \cdot v_R^{+} \right]}{2 \cdot T_f} - \\ &- \frac{\Delta t \cdot v_f}{2 \cdot T_f} - \frac{k_f \cdot \left[-\frac{1}{T_e} \cdot E_f + \frac{k_e}{T_e} \cdot v_R \right] \cdot (\Delta t)^2}{2 \cdot T_f^2} + \frac{v_f \cdot (\Delta t)^2}{2 \cdot T_f^{\,2}} \end{aligned} \tag{66}$$

The numerical model of excitation system is defined by the system of algebraic Equations (57-58) and Equations (63-66).

3. TEST EXAMPLE

In order to verify a new numerical procedure, angle stability of the power system, shown in Figure 1, has been analysed. At the beginning of simulation, the generator 1 and generator 4 are in operating mode with P=108 MW (0.9 pu) and Q =0 Mvar (0 pu) while generator 2 and generator 3 are in operating mode with P=84 MW (0.7 pu) and Q =52.3 Mvar (0.436 pu). Global system of equations is obtained by assembling local system of equations of each FE according to the standard assembling procedure [9-10]. At the moment t=0.8 s, the three-phase fault with clearing fault time of 200 ms has been set at the busbar 'D' causing synchronous generators rotor angle (δ) oscillations and variation of mechanical powers (P_m) due to turbine governor control.

The generators data are: $S_n = 120$ MVA, $P_n = 108$ MW, $Q_n = 52.3$ Mvar, $U_n = 14.4$ kV, $I_n = 4811$ A, $I_{fo} = 690$ A, $I_{fn} = 1192$ A.
Available transient and subtransient generators time constants and reactances are: $x_d = 1.01$ pu, $x_q = 0.666$ pu, $x_q'' = 0.287$ pu, $x_d' = 0.421$ pu, $x_d'' = 0.268$ pu, $x_\sigma = 0.198$ pu, $r = 0.00236$ pu, $T_d' = 3.4$ s, $T_{d0}' = 7.5$ s, $T_d'' = 0.0554$ s, $T_q'' = 0.0876$ s, $T_{mG} = 8.63$ s.

The turbine governors time constants, permanent and temporary droop coefficients, maximal and minimal gate opening speed, maximal and minimal gate position, amplification factors and turbine coefficients are: T_p = 0.02 s, T_r = 4.5 s, T_w = 2.9 s, T_g = 0.5 s, $q_{v\_open} = 0.15$, $q_{\text{v\_close}} = 0.08$ $q_{\max} = 0.8$, $q_{\min} = 0.1$, $\sigma = 0.04$, $\delta = 0.378$, $k_p = 15$, $k_i = 0.8$, $k_d = 1.5$, $T_d = 5$ s, $a_{11} = 0.5$, $a_{13} = 1$, $a_{21} = 1.5$, $a_{23} = 1$.

The excitation systems time constants and amplification factors are: $T_r = 0.02$ s, $T_a = 0.001$ s, $T_e = 0.1$ s, $T_f = 0.1$ s, $k_r = 1$, $k_a = 50$, $k_e = 1$, $k_f = 0.001$.

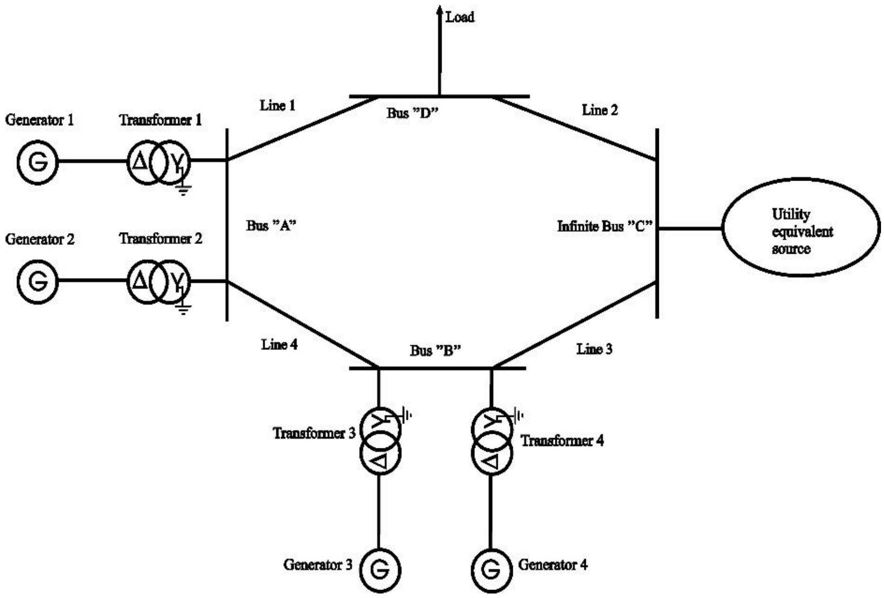

Fig. 1 Single line diagram of the electric power system

Step-up transformers as well as transmission lines reactance's have been taken 0.1 pu. To demonstrate that Heun's method is more accurate than ϑ - method, numerical solutions obtained using Heun's method and using the generalized trapezoidal rule for time integration parameter $\vartheta = 0.66$ are compared to Electromagnetic Transients Program (EMTP) as it shown in Figures (2-5). The below results, showing rotor angles (δ) and mechanical powers (P_m) of generators 1 and 2, clearly demonstrate that Heun's method yields the results with much higher accuracy comparing to results obtained by generalized trapezoidal rule.

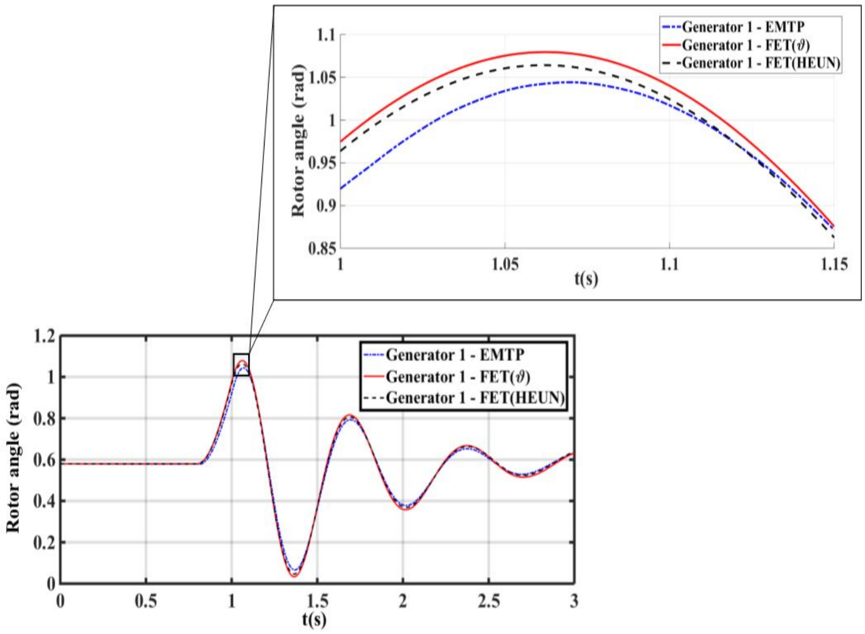

Fig. 2 Generator ‘1’ rotor angle (δ) during angle stability simulation

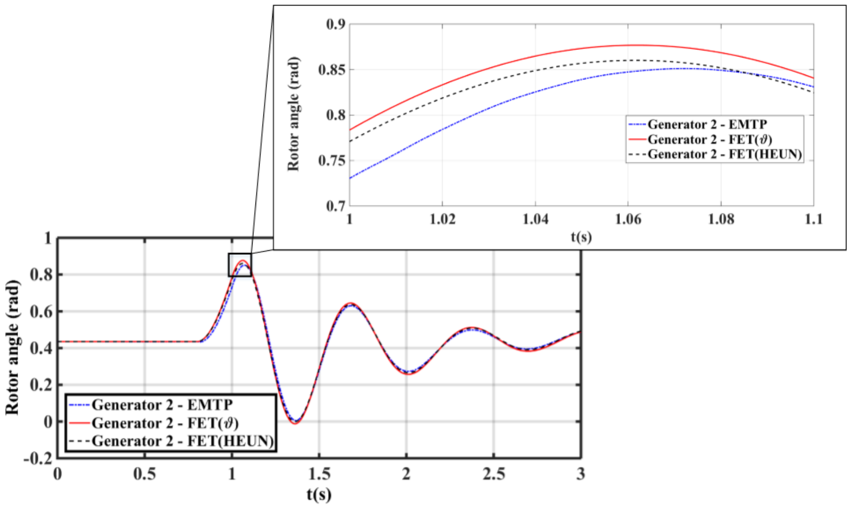

Fig. 3 Generator ‘2’ rotor angle (δ) during angle stability simulation

Since different values of ϑ yield different time-stepping schemes, all these schemes vary in accuracy. As it can be seen from Figures (2-5), numerical solutions obtained by Heun’s method yields highly accurate results while the generalized trapezoidal rule with $\vartheta = 0.66$ causes numerical damping of results. Furthermore, the generalized trapezoidal rule with $\vartheta = 0.5$ leads to numerical oscillations. Numerical oscillations and/or numerical damping of results are the consequences of the choice of the numerical integration parameter (ϑ). In principle, its choice is based on numerical experiments. The influence of ϑ on stability of one-step time integration methods for the first order initial value problems is discussed in [9-10], where the stability behaviour is investigated by examining the free response of the FE model. It was found that methods for which $0.5 \le \vartheta \le 1$ are unconditionally stable.

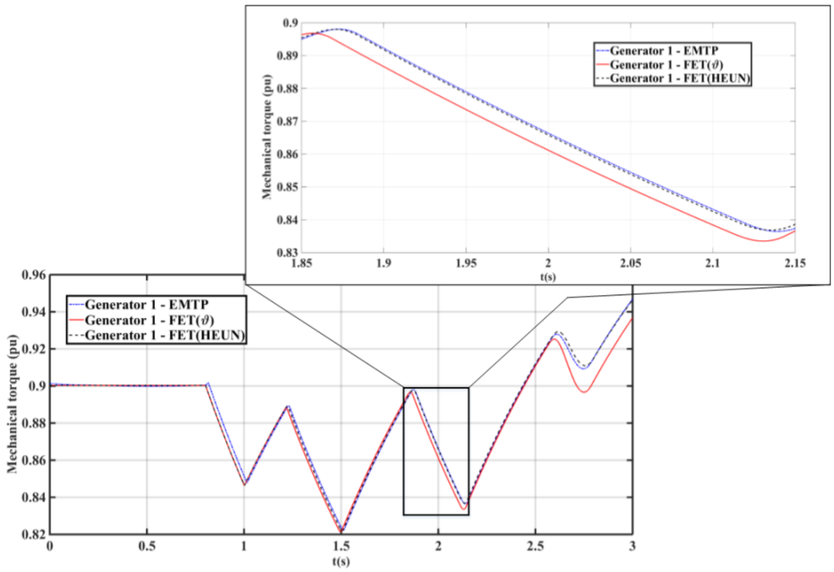

Fig. 4 Generator '1' mechanical power (P_m) during angle stability simulation

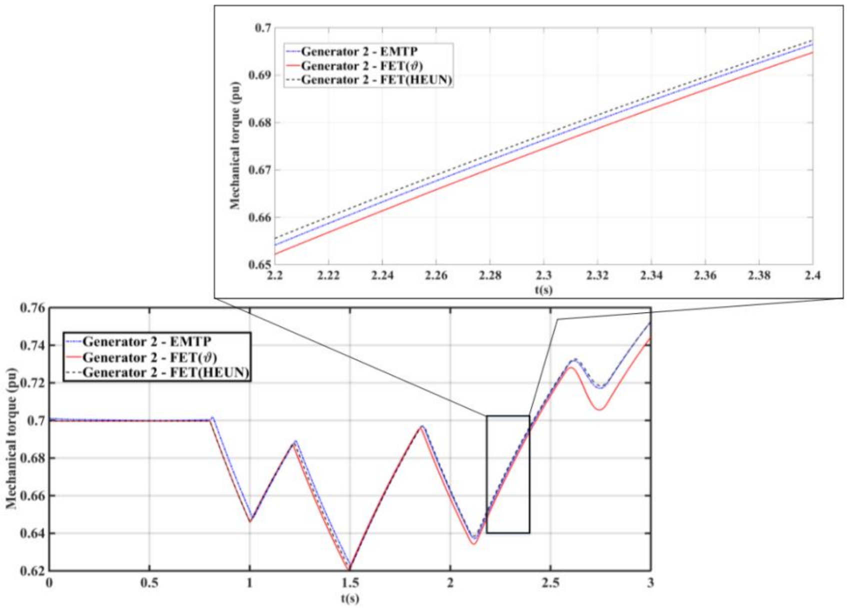

Fig. 5 Generator '2' mechanical power (P_m) during angle stability simulation

4. CONCLUSION

In this paper an improved time integration scheme for numerical analysis of power system angle stability using FET was presented. Improvement of accuracy was achieved by using Heun's method for the time integration scheme when forming a FE equations system. The proposed numerical method has been tested and compared to results obtained by ϑ - method and EMTP program. It has been shown that Heun’s method for time integration gives results with higher accuracy than the generalized trapezoidal rule (ϑ – method) previously used. In some cases where the generalized trapezoidal rule is used, the choice of the numerical integration parameter (ϑ) generates numerical oscillations and/or numerical damping of results. Using the proposed approach, these unwanted consequences are avoided.

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