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THE USE OF FRACTIONAL CALCULUS FOR THE OPTIMAL PLACEMENT OF ELECTRONIC COMPONENTS ON A LINEAR ARRAY

Gilbert De Mey[1], Mariusz Felczak[2] and Bogusław Więcek[2]
[1]University of Ghent, Sint Pietersnieuwstraat 41,
9000 Ghent, Belgium
[2]Insitute of Electronics, Technical University of
Łódź, ul. Wolczańska 211-215, 90-924 Łódź, Poland

Abstract: Cooling of heat dissipating components has become an important topic in the last decades. Sometimes a simple solution is possible, such as placing the critical component closer to the fan outlet. On the other hand this component will heat the air which has to cool the other components further away from the fan outlet. If a substrate bearing a one dimensional array of heat dissipating components, is cooled by forced convection only, an integral equation relating temperature and power is obtained. The forced convection will be modelled by a simple analytical wake function. It will be demonstrated that the integral equation can be solved analytically using fractional calculus.

Keywords: Heat transfer, placement, convective cooling, thermal wake, integral equation, fractional calculus.

1 Introduction

Heat transfer in electronics and microelectronics has become an important topic. The reason is quite simple: the heat dissipation in electronic components is increasing. Integrated circuits dissipating 100 Watts are no longer

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Corresponding author: Gilbert De Mey
Department of Electronics, University of Ghent, Sint Pietersnieuwstraat 41, 9000 Ghent, Belgium
(e-mail: demey@elis.ugent.be))

exceptional. If you buy a pentium processor, you will receive the processor already mounted on a printed circuit board with the cooling fin and the fan. Otherwise the company cannot guarantee that the device will work at all. Some textbooks on heat transfer even include a chapter on "electronics cooling" [1].

The most obvious way to cool electronic components is to mount them on a cooling fin which is cooled either by natural convection or by forced convection if a fan is blowing. Normally, at first an electronic design is made and once this has been finished the cooling problems have to be solved. A few years ago, designers seem to be convinced that one should take the cooling problem into account from the beginning, i.e. during the electronic design phase. Let us give a simple example: you are designing a printed circuit board and one component on this board is dissipating a lot of heat. The cooling fan is blowing from the left. Do you put this component on the left, on the right, or somewhere in the middle? If you put this component on the left the cooling with be quite efficient due to the close presence of the fan. But furtheron, the air behind this component, the so called wake, will be warmed up. So, the other components will be warmed up by the warm air blowing. This can give rise to malfunctioning of the circuit if temperature sensitive components are involved. Alternatively you may decide to put the heat dissipating component on the right side of the printed circuit board. But the cooling air coming from the fan has first to cross over the printed circuit board before reaching the heat source. This gives rise to friction and hence a reduction of the air speed in the surroundings of the PCB. So the component will not be cooled so effectively. If you put the heat source in the middle you have a mix of the problems just mentioned. There is no simple answer to that simple question. The only solution is to make an electro-thermal design from the very beginning. A network simulator like SPICE should not only calculate voltages and currents but also temperatures and power dissipations.

In this paper we will deal with a simple problem depicted in fig.1. It

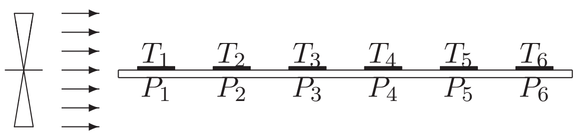

Fig. 1: Linear layout of integrated circuits.

shows a printed circuit board with 6 integrated circuit, in a linear array. The circuits dissipate powers P_1; P_2, ... giving rise to a temperature distribution T_1, T_2,... The fan is at the left side and provides a uniform flow over the circuit. If only the first circuit dissipates heat, not only will T_1 rise, but the downstream airflow (thermal wake) will be heated up so that the other components will be also heated even when $P_2 = ...P_6 = 0$. When only the rightmost component dissipates power ($P_6 \neq 0$), the 5 other components will not be warmed up as they are in an upstream position. By using a mathematical approximation for the wake function, i.e. the temperature rise caused by one heat dissipating component in all the other components located downstream, an integral equation will be set up for the temperature distribution. This equation will be solved by fractional calculus as will be outlined further on in this paper.

2 A short introduction to fractional calculus

Fractional calculus is not so well known at the moment. Therefore a very short introduction will be given here. What is a semi derivative of a function. In simple words, it is a mathematical operator and if you apply twice a semiderivative, you get a well known classical derivative.

In the last decennia, it has been found that several physical phenomena can be described by differential equations involving fractional derivatives [2]. Also thermal diffusion problems can give rise to equations using fractional derivatives [3].

Mostly used is the so called semi derivative in the time domain defined by:

$$\left(\frac{d}{dt}\right)^{1/2} f(t) = {}_0D_t^{1/2} f(t) = \frac{1}{\sqrt{\pi}}\frac{d}{dt}\int_0^t \frac{f(t')dt'}{\sqrt{t-t'}}. \tag{1}$$

Applying two times the semiderivative (1), is nothing else than the classical derivative d/dt. The most straightforward way to interpret (1) is to transform (1) into the Laplace domain. One gets:

$$\mathcal{L}[\,{}_0D_t^{1/2} f(t)] = \sqrt{s}F(s). \tag{2}$$

where s is the Laplace variable and $F(s) = \mathcal{L}[f(t)]$. A semiderivative is just a multiplication by $\sqrt{s}$. Hence, two consecutive semiderivations are then represented by a multiplication by $\sqrt{s}\sqrt{s} = s$, which corresponds to a time derivative in the Laplace domain. A fractional derivative of order α $(0 < \alpha < 1)$ corresponds to a multiplication by s^α in the Laplace domain.

The second notation used in (1) is preferred because the subscript "0" in front of the operator D indicates that the integration should start from $t = 0$, which is common for the analysis of time dependent problems.

Generally, a fractional derivative of order α is then defined by:

$$\left(\frac{d}{dt}\right)^{\alpha} f(t) = {}_0D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{f(t')dt'}{(t-t')^{\alpha}}. \tag{3}$$

where Γ is the Euler gamma function. In the Laplace domain this corresponds to a multiplication by s^{α}. Integrating (3) with respect to time gives:

$$\left(\frac{d}{dt}\right)^{-1+\alpha} f(t) = {}_0D_t^{-1+\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{f(t')dt'}{(t-t')^{\alpha}}. \tag{4}$$

For $\alpha < 1$, (4) can be considered as a fractional integration of order $1 - \alpha$. In the Laplace domain this is equivalent to a multiplication by $1/s^{1-\alpha}$.

3 Integral equation for the thermal wake problem

In present day electronic and microelectronic components, the power density is such that the temperatures can attain quite high values, affecting seriously the overall circuit reliabilities [4, 5]. Designing a circuit is no longer possible if the heat removal from chip to the ambient is not taken into account. Not only the heat transfer by conduction from the semiconductor chip to the package, but also the convective heat transfer is modelled. The latter one is done by solving the Navier Stokes equations in order to model the flow around the packages and cooling fins [6–8].

On electronic substrates components are usually placed according to regular arrays. As a consequence, in case the substrate is cooled by forced convection caused by fan blowing, a component located in x' will heat the air used to cool all the remaining components located downstream $x > x'$ (fig.2). In case the components have different power dissipations, interchanging components can give rise to a more uniform temperature distribution without excessive hot spots. It should be mentioned here that a high operating temperature of just one single component will reduce the reliability of the whole circuit.

Such a problem can be attacked by computational fluid dynamics modelling. This requires the numerical solution of the Navier Stokes equations which is a difficult task from a numerical point of view. Any time some components interchange their positions a new CFD simulation has to be carried

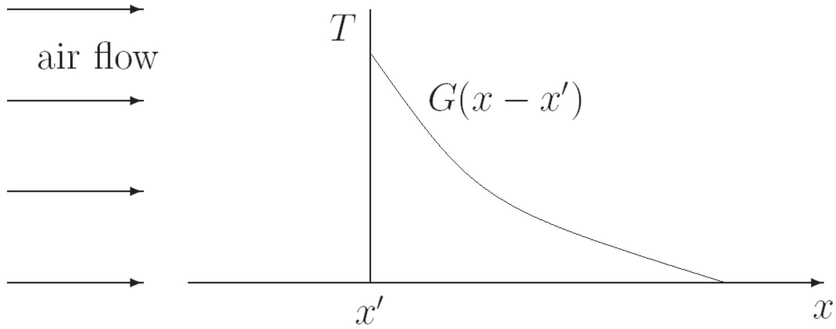

Fig. 2: Temperature profile downstream a heating component (thermal wake function)

out. This has to be repeated till the optimum layout has been obtained. It is quite obvious that this method requires a huge amount of computing time so that it is no longer of practical use during the design phase of a circuit.

In this contribution the thermal wake function approach will be presented. It gives rise to a one dimensional integral equation which can be solved with a minimum of computational effort. Finding the optimal position of the individual components can be quickly performed during the design phase of a circuit.

By definition, the thermal wake function G is the temperature distribution of the downstream components caused by single component having a unit heat dissipation. All downstream component should not have any heat production then. Several authors have studied the thermal wake function properties [9,10]. From experimental data and the own measurements of the authors [10], it was found that the thermal wake function G can be very well approximated by (fig.1):

$$G(x-x') = \frac{1}{(x-x')^p} \quad \text{if} \quad x > x', \qquad G = 0 \quad \text{if} \quad x < x' \tag{5}$$

If the heat production of a linear array of components can be described by a continuous function $q(x)$, one gets the following equation for the temperature distribution:

$$T(x) = \int_0^x q(x')G(x-x')dx' = \int_0^x \frac{q(x')dx'}{(x-x')^p} \tag{6}$$

Finding the optimal placement is now finding the optimal function $q(x)$.

Usually a uniform temperature $T(x) = T_0$ is considered optimal because all components will then have equal reliability if they are of course identical.

4 Solution with fractional calculus

The integral equation (6) has been solved analytically for a uniform temperature distrbution T_0. A rather artificial method was used based on a particular property of the Euler Beta function [10, 11]. For a non constant temperature this method cannot be used.

A uniform temperature distribution $T(x) = T_0$ is often considered as the optimal situation. If all components do not have the same degradation rate as a function of temperature a non uniform temperature distribution can then considered as the optimal situation. The problem is now to find the power $q(x)$ for the given $T(x)$. This problem will be solved now using fractional calculus.

The equations (3) and (4) being time dependent, the causality principle is then automatically taken into account. However, the fact that a heat source can only warm up the downstream part, can be interpreted as the causality principle in the space domain. Hence, by comparing (4) and (5), the equation (6) can be rewritten as:

$$T(x) = \Gamma(1-p)\ {}_0D_x^{-1+p} q(x) \tag{7}$$

If $T(x)$ is given the solution $q(x)$ is immediatly found to be:

$$q(x) = \frac{1}{\Gamma(1-p)}\ {}_0D_x^{1-p} T(x) = \frac{1}{\Gamma(1-p)\Gamma(p)} \frac{d}{dx} \int_0^x \frac{T(x')dx'}{(x-x')^{1-p}} \tag{8}$$

Taking into account that [10]:

$$\Gamma(1-p)\Gamma(p) = \frac{\pi}{\sin \pi p} \tag{9}$$

One obtains the general solution:

$$q(x) = \frac{\sin \pi p}{\pi} \frac{d}{dx} \int_0^x \frac{T(x')dx'}{(x-x')^{1-p}} \tag{10}$$

In case one wants to get a uniform temperature $T(x) = T_0$, the heat production $q(x)$ turns out to be:

$$q(x) = T_0 \frac{\sin \pi p}{\pi} \frac{d}{dx} \int_0^x \frac{dx'}{(x-x')^{1-p}} = T_0 \frac{\sin \pi p}{\pi} \frac{1}{x^{1-p}} \tag{11}$$

which is exactly the same solution found by a rather artificial method [9]. The use of fractional calculus offers us a general solution which can be used for any temperature function $T(x)$.

5 Conclusion

It has been shown that fractional calculus can be succesfully used for some particular problems in electronics. We treated the placement problem of components on a one dimensional array, using the prescribed temperature distribution as the optimisation criterion.

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