A method for calculation of finite fatigue life under multiaxial loading in high-cycle domain

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ABSTRACT. A method for fatigue life assessment in high-cycle domain under multiaxial loading is presented in this paper. This approach allows fatigue assessment under any kind of load history, without limitations. The methodology lies on the construction - at a macroscopic level - of an "indicator" in the form of a set of cycles, representing plasticity that can arise at mesoscopic level throughout fatigue process. During the advancement of the loading history new cycles are created and a continuous evaluation of the damage is made.

KEYWORDS. High-cycle multiaxial fatigue; Metal fatigue; Multiaxial rainflow counting; Crack initiation; Non-proportional loading; Continuous damage.

INTRODUCTION

Numerous methods for multiaxial fatigue analysis exist in literature (see e.g. [1]) and they are widely applied in different industrial contexts. Nevertheless many of them are limited to specific loading conditions - typically proportional loading: see [2, 3] for a review - or intended only for an evaluation of the unlimited fatigue life (see [1, 2, 4]).

The motivation of the method presented in this paper comes from the need felt during years by the author in the fatigue assessment of industrial structures, to conceive a simple and easily applicable tool capable to estimate finite life crack initiation under any kind of loading, in the domain of High-Cycle Fatigue (HCF). Every type of variable amplitude multiaxial stress history can be treated without presenting incorrect filtering of significant cycling events.

GENERAL PRESENTATION OF THE METHOD

It is a widely recognized and well accepted phenomenon [5] that the fatigue damage in the HCF domain is related to the amount of plasticity created at least in some grains under a macroscopically elastic stress history.

The ref. [5] gives for instance a synthetic understanding: "…*the common principal mechanism responsible of the crack initiation is the plastic strains and the damage developed in the grains due to irreversible dislocations motion. The essential difference between HCF and LCF regimes is that the scale of the plastic localization in a material volume is mesoscopic and respectively macroscopic*".

Some criteria for the calculation of the fatigue life are actually based on the evaluation of the plasticity amount. They can be either direct - as described for instance in [6] and [7], where a computational crystal plasticity model is used at a refined microstructural scale - or indirect, like for example the work of Jabbado and Maitournam [8] where a macro-meso relationship is used.

Similarly, the methodology proposed in the present paper has its foundation on a plasticity approach, since usage is made of "yield surfaces" created in the stress space during the advancement of the stress history. But on the other side, the description remains here merely in the stress space: no explicit use of the mesoscopic plastic strain is made, but the equivalent fatigue stress is calculated directly on each created stress cycle. The yield surfaces used in the present approach just represent a way to describe at the macroscopic level the underlying phenomenon related to the plasticity arising at a mesoscopic level.

From a more general point of view, the present work follows the same scope of the initial work of Jabbado [3] to extend the endurance criterion of Dang Van [9] to a finite life assessment using a continuous damage evaluation. Nevertheless, differently from [3] a notion of stress cycle is used here, even though the cycles are not counted and extracted from the global sequence - as it is done in a rainflow method (see e.g. [1]) - but they are simply created and updated in a continuous way while the stress history advances. This continuous damage approach has the same viewpoint of the cited work of Jabbado [3] or for example of the methodology proposed by Stefanov [10]. On the other hand, the principle to adapt a multi-surface concept to fatigue assessment by creating the stress cycles during the stress history advancement has some similarities with the work of Herbland [11].

CONSTRUCTION OF THE STRESS CYCLES

The basic ingredient of the present approach is the geometrical creation of closed surfaces in the stress space while the stress point is moving on its path, in a way identical to the classical plasticity theory (see e.g. [12, 13]). Each created yield surface is afterward associated to an amount of fatigue damage, evaluated using the basic fatigue material properties. This usage of the yield surfaces is analogous to what is done for the stress cycles after their extraction by a classical cycle-counting, like the rainflow method. For this reason, the terms "yield surface" and "stress cycle" will be used hereafter as synonyms.

The rules described here below resume the stress cycles creation.

– The equation defining a yield surface in the stress space has the form

$$f_P(\boldsymbol{\sigma} - \mathbf{X}_C) = \sigma_y \tag{1}$$

where f_P is a scalar function that in accordance with the hypothesis used by Jabbado [3] and by many classical multiaxial criteria (see [1, 4]) takes the Von Mises equivalent stress:

$$f_P(\boldsymbol{\sigma}) = \sqrt{\frac{1}{2}[(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2]} \tag{2}$$

where $\boldsymbol{\sigma}_I$, $\boldsymbol{\sigma}_{II}$, $\boldsymbol{\sigma}_{III}$ are the principal stresses of $\boldsymbol{\sigma}$. Let us note explicitly that f_P is function of the only deviatoric part $\mathbf{s}$ of the stress tensor $\boldsymbol{\sigma}$:

$$f_P(\boldsymbol{\sigma}) = f_P(\mathbf{s}) \tag{3}$$

$$\mathbf{s} = \text{dev } \boldsymbol{\sigma} = \boldsymbol{\sigma} - p_H \mathbf{I} \qquad \text{where } p_H = \text{tr}(\boldsymbol{\sigma}) / 3 \tag{4}$$

Hence each yield surface is a Von Mises hypersphere fully described by its centre $\mathbf{X}_C$ (back-stress tensor) and its size σ_y.

– In the same way of a classical plasticity criterion, when the current stress state moves along the stress history a yield surface is hardened if the stress lies on the surface itself and is moving outwards. These conditions will be integrated in the Eqs. (5.a), (5.b) and (12.a), (12.b) written below. As a consequence the stress point will be always inside - or exactly on – a yield surface, but never outside.

– A multi-surface model is used, inspired to the one proposed by Mroz and described e.g. in [12]. In such a model, more than one yield surface can exist at the same time. However a significant dissimilarity with the concept of Mroz is that intersections between surfaces are allowed in the present approach. As a consequence the existing surfaces are not necessarily all nested inside each other: this issue in the frame of multi-surface plasticity models is discussed for instance in ref. [14].

– The following hardening rules are used.

When the deviatoric part $\mathbf{s}$ of the stress tensor $\boldsymbol{\sigma}$ has an increment $d\mathbf{s}$, among the surfaces that are hardened at a given instant the one having maximum σ_y is hardened by the following superposed isotropic / kinematic hardening law:

$$d\mathbf{X}_C = \frac{1}{2} H(\phi_P) <d\mathbf{s} : \mathbf{n}> \mathbf{n} \tag{5.a}$$

$$d\sigma_y = \frac{1}{2} H(\phi_P) \sqrt{\frac{3}{2}} <d\mathbf{s} : \mathbf{n}> \tag{5.b}$$

where:

$$H(x) = 1 \text{ if } x \geq 0, \; H(x) = 0 \text{ if } x < 0 \tag{6}$$

$$<x> = x\, H(x) \text{ i.e. } <x> = x \text{ if } x \geq 0, \; <x> = 0 \text{ if } x < 0 \tag{7}$$

$$\phi_P(\mathbf{s}; \mathbf{X}_C, \sigma_y) = f_P(\mathbf{s} - \mathbf{X}_C) - \sigma_y \tag{8}$$

and $\mathbf{n}$ is the local outward vector normal to the surface in the deviatoric stress space, having unit length in the following sense:

$$\mathbf{n} : \mathbf{n} = 1 \tag{9}$$

The symbol **:** in Eqs. (5.a), (5.b) and (9) designates the following scalar product in the stress space, between two tensors $\mathbf{s}_A$, $\mathbf{s}_B$:

$$\mathbf{s}_A : \mathbf{s}_B = \sum_{i,j=1}^{3} s_{Aij}\, s_{Bij} \tag{10}$$

Since Von Mises stress is used, it is possible - as done in [3] - to give to $\mathbf{n}$ the following explicit form:

$$\mathbf{n} = \sqrt{\frac{3}{2}} \; (\mathbf{s} - \mathbf{X}_C) \,/\, f_P(\mathbf{s} - \mathbf{X}_C) \tag{11}$$

If at a given instant the same maximum σ_y belongs to two or more yield surfaces contemporarily then the one to be hardened according to the Eqs. (5.a) and (5.b) is chosen conservatively as the one having the maximum value of the scalar product $d\mathbf{s} : \mathbf{n}$.

Additionally, if two or more surfaces have not only the same maximum σ_y but also the same maximum $d\mathbf{s} : \mathbf{n}$, then the choice for the one to be hardened according to the Eqs. (5.a) and (5.b) becomes physically arbitrary: we simply choose the one which had been previously created firstly.

The other surfaces that are hardened at a given instant - because the stress point lies on the surface and is moving towards the outside - simply move rigidly with a pure kinematic hardening (σ_y remain constant) defined by:

$$d\mathbf{X}_C = H(\phi_P) <d\mathbf{s} : \mathbf{n}> \mathbf{n} \tag{12.a}$$

$$d\sigma_y = 0 \tag{12.b}$$

Actually only the yield surface selected by the criteria illustrated above is considered to be plastically active and it is called "active surface". The other hardened surfaces having a pure kinematic hardening expressed by the Eqs. (12.a) and (12.b) are not directly considered in the current global computation of the fatigue damage, but their updated position will have an influence on the subsequent response in fatigue. They are designated as "transported surfaces". A third category is given by those surfaces that are not moving at a current instant: they are denoted as "resting surfaces" since the stress point moves internally. The presence of the transported and resting yield surfaces represents a memory effect in the fatigue process, since they can be activated in the subsequent stress history. As already remarked above, all the existing surfaces do not form obligatorily a set of nested surfaces but they can intersect each other.

– An increment of plastic deformation at the grain scale can be by hypothesis created at each instant under a deviatoric stress variation. To take into account this, if the stress point is moving in such a way that none of the already existing yield surfaces are hardened then a new surface is created starting from the size $\sigma_y = 0$ and obeying to the hardening law for the active surface expressed by Eqs. (5.a) and (5.b). Of course a first surface is immediately created at the start of the stress history. A problem arises when a surface is created: its normal $\mathbf{n}$ is not defined since the surface is collapsed in a point ($\sigma_y = 0$). This problem is bypassed by defining, at the instant where a yield surface is created, a normal driven by the stress variation $d\mathbf{s}$:

$$\mathbf{n} = \frac{d\mathbf{s}}{\sqrt{d\mathbf{s}:d\mathbf{s}}} \qquad \text{for } \sigma_y = 0 \tag{13}$$

Once the rules for the creation of the stress cycles are defined, a "deviatoric fatigue equivalent stress" τ_y is calculated for the yield surfaces by means of the following equations:

$$d\tau_{EQ} = f_P(d\mathbf{X}_C) + d\sigma_y \qquad \text{for the active surface} \tag{14.a}$$

$$d\tau_{EQ} = 0 \qquad \text{for all the other surfaces} \tag{14.b}$$

The Eqs. (14.a) and (14.b) affirm that an increment of $d\tau_{EQ}$ is applied to the active surface only and not to the other surfaces: for both transported and resting surfaces the value of τ_{EQ} does not change at a given instant. At each instant of the stress history the result is thus the presence of a certain number of yield surfaces, each one with a related value of τ_{EQ}. The following figures show the concept illustrated above. They are intended only as a geometric plane representation of the stress cycles creation process, with pure explanation scope. The function f_P used here is the Cartesian distance in the plane, so that the yield surfaces are circles. During the stress advancement, the size and the center of the surfaces are determined by the same differential hardening laws defined above: Eqs. (5.a), (5.b) for the active surface and Eqs. (12.a), (12.b) for transported and resting surfaces. A closed stress sequence is given, defined by the points numerated from 1 to 10 and represented by thick continuous line. At four different instants, the stress cycles already created by the moving stress point are sketched with thin continuous lines. A dashed line represents the smallest circle enclosing all the points of the sequence, which is used in the Dang Van criterion for unlimited endurance [9].
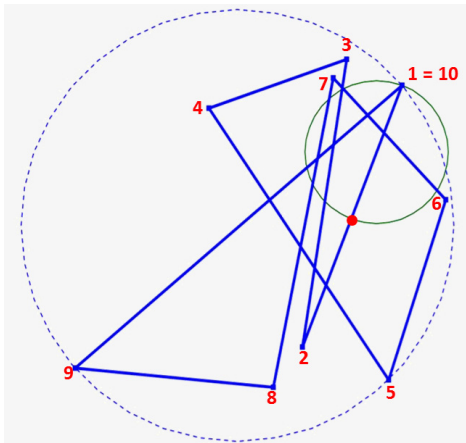

Figure 1: The stress point is moving between states '1' and '2'. Only one stress cycle has been created.
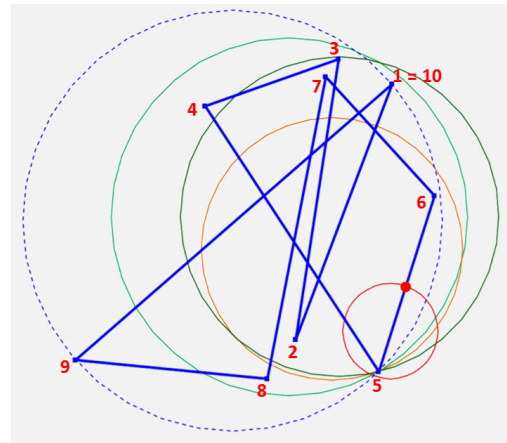

Figure 2: Created stress cycles when the stress point is moving between states '5' and '6'.
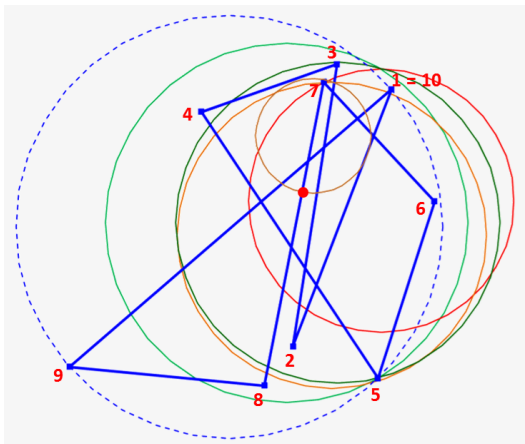

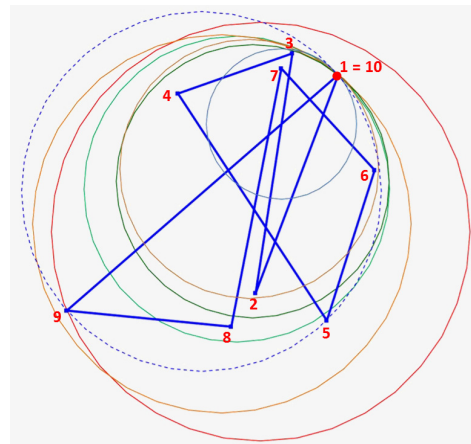Figure 3: Created stress cycles when the stress point is moving between states '7' and '8'.

Figure 4: Stress cycles at final instant of the sequence: the stress point has reached the instant '10'.

EFFECT OF THE HYDROSTATIC PRESSURE

I t is necessary to take into account the effect of the hydrostatic pressure, since the deviatoric fatigue equivalent stress τ_{EQ} calculated on the fatigue cycles as defined in the previous paragraph cannot entirely describe the fatigue damage process. This is due to the fact that the function f_P describing the yield surfaces depends on the deviatoric part only of the stress tensor.

Several possibilities arise. It is chosen here to give to the hydrostatic pressure effects a formulation inspired by the classical hypothesis of Sines (see e.g. [1, 4]) in which the average hydrostatic pressure during a load cycle is used.

But it is important to remark that this hypothesis is not the only possible in the frame of this methodology: by introducing it, we do not loose generality regarding the possibility to use other mechanisms of hydrostatic pressure influence.

Let us consider firstly that during the advancement of the stress history the deviatoric fatigue equivalent stress τ_{EQ} for a given stress cycle 'j' is a monotonic non-decreasing function going from 0 at the initial instant to its final maximum value $\tau_{EQ,j}$

Thus the evolution of the hydrostatic pressure p_H can be regarded as a function of $\tau_{EQ,j}$, so that it is possible to calculate for the considered stress cycle 'j' the average value $\overline{p}_{H,j}$ in the following way:

$$\overline{p}_{H,j} = \frac{1}{\tau_{EQ,j}} \int_0^{\tau_{EQ,j}} p_H(\tau'_{EQ,j}) d\tau'_{EQ,j} \tag{15}$$

Let us remark explicitly that $\overline{p}_{H,j}$ is different for each stress cycle. The quantity $\overline{p}_{H,j}$ is then used in combination with $\tau_{EQ,j}$ in order to evaluate the fatigue damage of the cycle itself.

To this scope in the frame of the present method it is required to have for a given material a relationship which provides, for an uniaxial tension/compression fatigue problem under Constant Amplitude (CA) cycles, the fatigue equivalent stress as a function of the stress amplitude S_a and of the average stress $\overline{S}$, as follows:

$$S_{EQ} = S_{EQ}(S_a, \overline{S}) \tag{16}$$

When such an equation is not available for a material, then it is necessary to do a hypothesis to describe the influence of the average stress for uniaxial CA cycles.

An "equivalent fatigue stress" $\sigma_{EQ,j}$ for each of the yield surfaces is then calculated by applying the same law of Eq. (16) on the calculated $\tau_{EQ,j}$ and $3\overline{p}_{H,j}$:

$$\sigma_{EQ,j} = S_{EQ}(\tau_{EQ,j}, 3\overline{p}_{H,j}) \tag{17}$$

This can be better illustrated by a simple example: in the "MMPDS" handbook [15] the expression of the equivalent stress for uniaxial CA cycles is given by the following formula:

$$S_{EQ} = S_{MAX}(1-R)^q \tag{18}$$

where R is the cycle ratio $R = S_{MIN}/S_{MAX}$.

Conformingly to the Eq. (16) this equation can be rewritten in terms of S_a and $\overline{S}$:

$$S_{EQ} = \left(\overline{S} + \frac{S_a}{2}\right)^{1-q} S_a^q \tag{19}$$

The application of Eq. (17) gives for a given stress cycle 'j':

$$\sigma_{EQ,j} = \left(3\overline{p}_{H,j} + \frac{\tau_{EQ,j}}{2}\right)^{1-q} \tau_{EQ,j}^{\ q} \tag{20}$$

The so-calculated equivalent fatigue stress $\sigma_{EQ,j}$ can be then entered directly in a S_{EQ}-N curve available for a material, in order to calculate the damage made by a single stress cycle.

However, an important remark must be made: in the evaluation of their damage, the yield surfaces obtained by the procedure described above must be considered as half-cycles (or stress-reversals) and not as entire cycles. This can be well

understood when thinking that in the present approach, when the stress point goes forth and back on a segment - from a point to a second point and then back to the initial point - then two identical yield surfaces are created.

Explicitly: if a S_{EQ}-N curve is available in which N has the meaning of number of complete cycles (from peak to peak), then the damage D_j associated to the yield surface 'j' calculated with the present method will be:

$$D_j = \frac{1}{2N_j} \qquad (21)$$

where N_j is the number of cycles on the S_{EQ}-N curve corresponding to $\sigma_{EQ,j}$.

Using the Palmgren-Miner law, the total damage D at a given instant of the complete stress history is given by:

$$D = \sum_j D_j \qquad (22)$$

PRELIMINARY VALIDATION OF THE METHODOLOGY

The approach presented in this paper needs obviously at least firstly a robust numerical implementation and secondly a wide validation, by comparison with data coming from test or literature. This activity is in progress, in order to reach a comprehensive substantiation of the methodology. It is the intention of the author to present the complete results in a forthcoming publication.

At the present time, the main purpose of the paper is to submit, in a first time, this methodology to the judgment of the community working in the field of the fatigue analysis and dealing with the necessity of treating complicated spectra and non-proportional loading in a HCF multiaxial context.

Nevertheless, a first important element of the validation process is presented here: the scope is to prove that for an uniaxial closed sequence having Variable Amplitude (VA) cycles, the present methodology is equivalent to a classical 1-D rainflow counting (see e.g. [1]) in the sense that the size τ_{EQ} of the yield surfaces is equal to the amplitude of the 1-D stress cycles extracted by the rainflow.

This has been tested by comparison between the present procedure and the rainflow counting prescribed in [16]: namely, a set of 10 uniaxial stress sequences has been created, each one made of 50 stress points randomly generated with probability uniformly distributed between -100 and +100 MPa. The stress points have been preliminarily discretized using a step of 1 MPa.

The cycles extracted from both the rainflow procedure [16] and the present approach have been compared in terms of stress amplitude and they have been found to be strictly identical. The figure below gives for one of the sequences the stress points as well as the cycles extracted with the two methods.
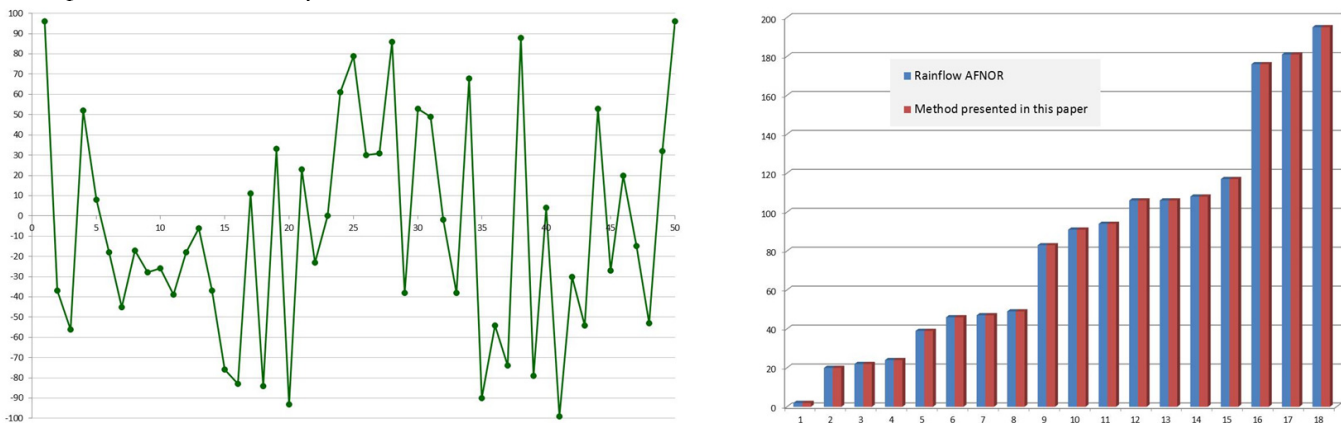

Figure 5: Stress points of a test-sequence and amplitude of extracted cycles.

CONCLUSIONS

In the previous paragraphs a general approach for multiaxial HCF crack initiation assessment has been presented. As mentioned in the *Preliminary validation of the methodology* paragraph, its validation by comparison with available experimental data is in progress and will be enclosed in a future publication.

Regarding the evolution of this work a first significant enhancement should be the extension to yield surfaces more general than the only Von Mises hypersphere, namely the possibility to use non-smooth surfaces based for example on the Tresca equivalent stress.

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