Technical note

A new formulation for estimating maximum stress intensity factor at the mid plane of a SENB specimen: Study based on 3D FEA

S. K. Kudari, K. G. Kodancha
Research Centre, Department of Mechanical Engineering, B. V. B. College of Engineering and Technology, Hubli 580 031, Karnataka, India.
s.kudari@rediffmail.com

ABSTRACT. In this investigation, three-dimensional elastic finite element analyses have been conducted to compute the stress intensity factor (K_I) in a SENB specimen with varied thickness (B) and crack-length to width ratio (a/W). The results indicate that the magnitude of K_I depends on B, and significantly varies along the crack-front from surface to the center of the specimen. The maximum value of K_I is found at mid plane of the specimen. Based on the present 3D finite element calculations an effort is made to propose the new analytical relationship between maximum K_I and specimen a/W, which helps to estimate maximum K_I by only knowing specimen geometry and applied load.

KEYWORDS. SENB specimen; Finite element analysis; Maximum stress intensity factor.

INTRODUCTION

In Fracture analysis, two-dimensional approach of LEFM is popular because of its simplicity in mathematical formulation and numerical analysis. In two-dimensional structural analyses, plane strain is governed by case where deformation is highly constrained for example plates with larger thickness, and plane stress is used for thin plates. Several methods have been developed in past, to compute the K_I for engineering problems with complex geometry and loading [1]. Fett [2] has used boundary allocation method to compute K_I solutions for components containing internal cracks. Chen and Lin [3] investigated the dependence of the K_I from the imposed boundary conditions in a rectangular cracked plate. Based on the asymptotic solution of the singular stress field, and the common numerical solution (stresses or displacements) obtained by finite element method, a simple and effective numerical method is developed by Yihua Liu *et al.* [4] to calculate stress intensity factors.

Due to complexity of the solution, the stress field around the crack-front in 3D continua is limited. Kwon and Sun [5] given the detailed literature review about the efforts in computating 3D stress intensity factor and presented 3D FE analyses to investigate the stress fields near the crack tip. The authors [5] suggested a simple technique to determine 3D K_I at the mid plane of a specimen by knowing 2D K_I and Poison's ratio (ν). Moreira *et al.* [6] studied the 3D effects in a central cracked plate by numerical determination of the stress field and K_I variation through the thickness. As Single edge notch bend (SENB) specimen is more preferred for fracture tests hence, it is essential to have such study on this specimen. The aim of this investigation is to study the variation of K_I along the crack-front considering a SENB specimen geometry having varied thickness (B) and crack length to width ratio (a/W). To calculate the maximum K_I at mid plane of any fracture specimen, it is essential to conduct complex 3D FEA. Hence, it is always very difficult for a structural engineer to conduct 3D FEA for estimating K_I at the mid plane of a specimen. Hence, based on the present 3D FE calculations, an attempt is made to propose a new analytical relationship between K_I at the mid plane, a/W and applied stress ratio (σ/σ_y) which shall be a great help in estimating maximum magnitude of K_I in a SENB specimen without complex 3D FE analysis.

FINITE ELEMENT ANALYSIS

A series of 3D finite element analyses have been made on SENB specimens using ABAQUS 6.5 [7] finite element software. The dimensions of SENB specimen has been computed according to ASTM standard E1290 [8] considering width of the specimen W=20mm. Figure1 shows the geometry of the SENB specimen used in this analysis. Finite element computations were carried- out considering only one-half of the specimen due to the geometric and loading symmetry. The analysis domain is descritized using 20-noded quadratic brick finite elements using reduced integration. This kind of elements were used in the work of Moreira *et al.* [6], Qu and Wang [9], Kim *et al.* [10], Courtin *et al.* [11], Kudari and Kodancha [12]. In this analysis, the number of elements in the analysis domain varied with the thickness (B) of the specimen. The specimen thickness modeled by considering 11 layers in the thickness direction. A typical mesh used in the analysis for specimen thickness B=4 mm is shown in Fig. 2. The stress distributions in the specimen and magnitudes of K_I have been extracted by using ABAQUS post processor. The variation of stress components and K_I along the crack-front has been studied for different specimen thickness and crack length to width ratio (a/W=0.45-0.70).

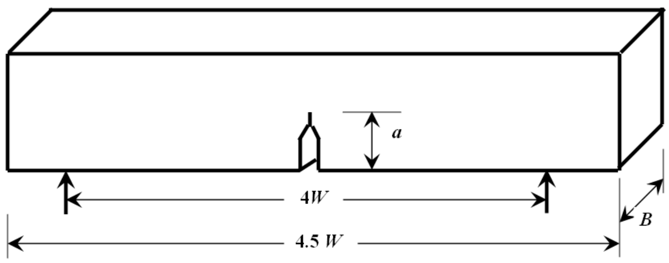

Figure 1: The geometry of the SENB specimen used in the analysis (W=20 mm).

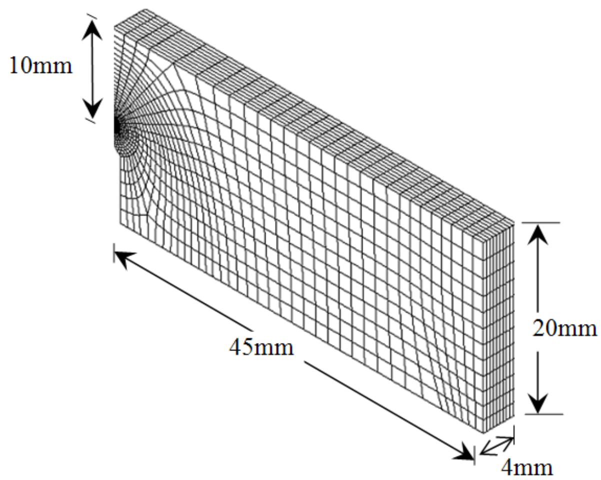

Figure 2: 3D Specimen FE mesh used for the analysis.

In this work the magnitude of applied stress (σ) is computed using the relation [13]:

$$\sigma = \frac{3PS}{2B(W)^2} \quad (1)$$

where, P is applied load, B is thickness of the specimen, W is width of the specimen, $S=4W$, the span of the specimen. The specimen thicknesses (B) considered in this analysis is 2 to 20 mm (B/W=0.1-1.0) in steps of 2 mm. In these finite element calculations, the material behaviour has been considered to be multi-linear kinematic hardening type pertaining to an interstitial free (IF) steel possessing yield strength of 155 MPa, elastic modulus of 197 GPa, Poison's ratio=0.30, and Ramberg-Osgood constants N=3.358 and α=19.22 [14].

RESULTS AND DISCUSSION

2D Stress Intensity Factor (K_I)

In this study, initially 2D elastic plane-stress FE analysis has been conducted on the SENB specimen with a/W=0.45-0.70 to extract the K_I. The magnitude of K_I computed by theoretical formulation [15] as given in Eq. (2) and present 2D FE analysis is plotted against a/W in Fig. 3.

$$K_I = \sigma\sqrt{\pi a}(Y) \tag{2}$$

where, σ is applied stress, a is crack length and Y is geometric factor. The Fig. 3 indicates that the present FE results of 2D K_I are in excellent agreement with the results obtained by theoretical Eq. (2). These analyses provide the validation of the FE computation of K_I in 2D.

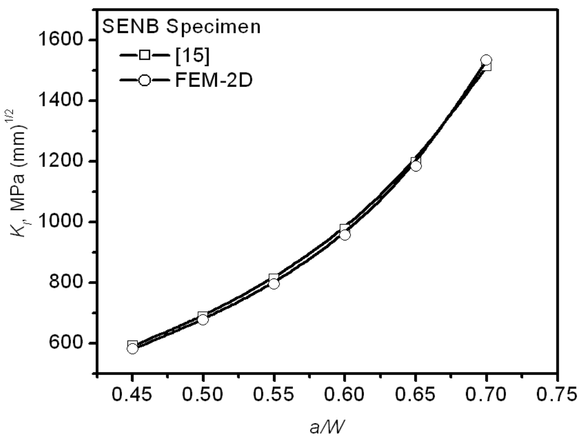

Figure 3: Variation of K_I vs. a/W obtained by Eq. (2) Ref [15] and 2D FEA.

3D Stress Intensity Factor (K_I)

A series of 3D FE stress analyses have been carried out on SENB specimen with varied B, a/W and normalized applied stress (σ/σ_y) to study the variation of K_I along the crack-front. The applied stress (σ) in this analysis is computed using the analytical formulation provided in the work of sherry *et al.* [13]. The details of extraction of magnitudes of K_I are discussed elsewhere [16].

A typical variation of K_I for B=10mm with a/W=0.55 for various loading (σ/σ_y=0.08 to 0.80) is shown in the Fig.4. This figure indicates that the magnitude of K_I is higher at the centre of the specimen than on the surface. This is due to variation of stress tri-axiality along the specimen thickness. The nature of variation of K_I shown in Fig.4 is in good agreement with the similar results presented by Fernandez *et al.* [17]. Further, the effect of a/W on variation of K_I is also studied. A typical variation of K_I along the crack-front for specimen having various a/W and B=10 mm for σ/σ_y=0.56 is shown in Fig. 5. This figure illustrates that for the similar applied load the magnitude of K_I at the centre and the surface of the specimen increases as a/W increases. This kind of variation of K_I along the crack front is not possible to study analytically. The 2D magnitude of K_I can be computed by well-known analytical formulation Eq. (2). The Eq. (2) do not consider the effect of specimen thickness, and provide the magnitude of K_I less than that at the center of the specimen as shown in Fig. 5 (Typically for a/W=0.60). From this result, one can infer that the analytical estimates of K_I (estimated by Eq. (2)) are not suitable for the analysis of fracture in case of specimens with higher thickness (plane strain). Earlier, Nakamura and Parks [18] have used degree of plane strain to demonstrate the plane strain behavior in 3D cracked plate. The degree of plane strain, $D_{p\varepsilon}$, a parameter that measures the variation of the constraint factors needs the computed values of stress variation in all three directions as [18]:

$$D_{p\varepsilon} = \frac{\sigma_{33}}{\upsilon(\sigma_{11}+\sigma_{22})} \tag{3}$$

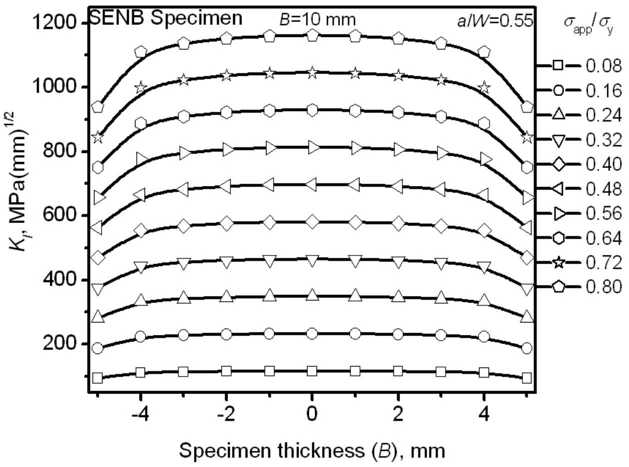

Figure 4: Variation of K_I along the crack-front for various loading.

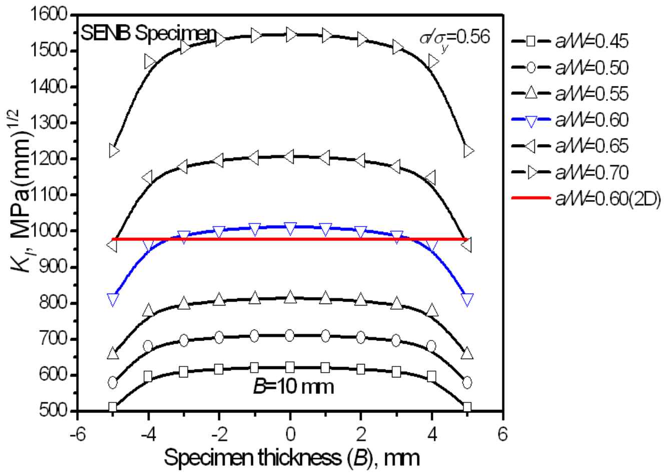

Figure 5: Variation of K_I along the crack-front for B=10 mm and various a/W.

This $D_{p\varepsilon}$ is quite complex to compute and analysis. The above limitations in fracture analysis demand the 3D analysis of fracture specimen to find the K_I at the center (maximum) of the specimen.

One can observe from Fig.5 that there exists a considerable difference in the magnitudes of K_I between center and surface of the specimen. A typical variation of difference between the magnitude of K_I at the center and on the surface (ΔK_I) for a specimen having B=10mm for σ/σ_y=0.56 and various a/W is shown in the Fig. 6. This figure indicates that ΔK_I of the specimen increases as a/W increases, which infers that higher magnitude of specimen a/W ratio provides maximum out-of-plane constraint. The magnitude of K_I are also extracted for B=2 to 20 mm (B/W=0.1 to 1) and a/W=0.45 to 0.70 for loading σ/σ_y=0.08 to 0.80. A typical variation of K_I for thicknesses B=2mm, 10mm and 20mm for a/W=0.50 and σ/σ_y=0.56 is shown in Fig 7. This figure clearly demonstrates an interesting finding that the magnitude of K_I at the center of the specimen is independent of specimen thickness (B).

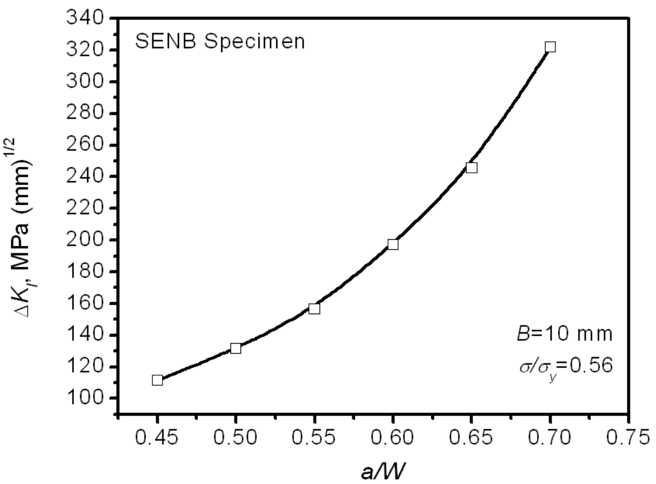

Figure 6: Variation of ΔK_I for various a/W ratios.

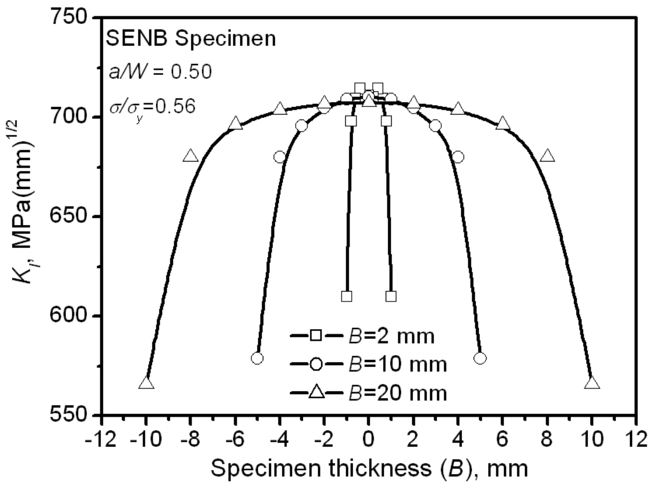

Figure 7: Variation of K_I along the crack-front for B=2 mm, 10 mm and 20 mm with a/W=0.50

Fig. 4 to 7 show that K_I strongly varies with distance along the crack-front and depends on the specimen thickness. Figure 4 to 7 show that the magnitude of K_I is higher at the centre of the specimen than on the surface. This nature of variation of K_I is in agreement with the result presented by Kwon and Sun [5]. Based on their FE results, Kwon and Sun [5] have proposed a formulation to obtain the highest magnitude of 3D K_I (at the centre of the specimen) based on magnitude of K_I obtained by 2D analysis as:

$$\frac{K_{3D}}{K_{2D}} = \sqrt{\frac{1}{1-\upsilon^2}} \qquad (4)$$

where ν is Poisson's ratio of the material.

The highest magnitude of the K_I (at the center of the specimen) extracted by the present 3D FE analysis for various a/W ratios is plotted in Fig. 8 along with the 3D K_I values estimated by the formulation, Eq. (4) using 2D results shown in Fig. 3. The 2D computation of K_I were made in order to verify the possible use of the formula (Eq. (4)) proposed by Kwon and Sun [5]. The Fig.8 shows that there is an acceptable agreement of 3D K_I at the center of the specimen and the computations made by the formulation given by Kwon and Sun [5] using present 2D FE results.

As magnitude of K_I is found to be maximum at the centre of the specimen, it becomes important to do the fracture analysis of material based on the value of K_I estimated at the center of the specimen. This demands 3D FE analysis of fracture specimen, which is difficult and time consuming for any fracture analyst. Hence, in this study an effort is made to propose a simple relation to estimate K_I at the center of the specimen, without any numerical computations. Further, the magnitude of K_I at the center of specimen having various B (B=2 to 20mm) and σ/σ_y=0.08 to 0.80 and a/W=0.45 to 0.70 are extracted from 3D FE analysis. A typical plot of K_I at center against σ/σ_y and B for a/W=0.50 is plotted in Fig.9.

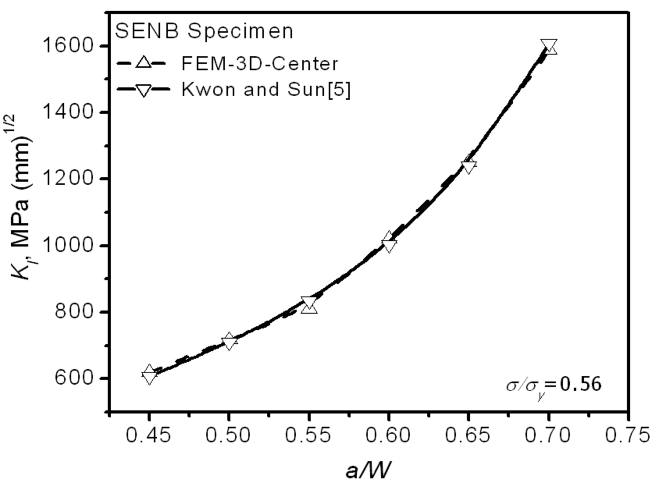

Figure 8: Variation of K_I for various a/W ratios obtained by present 3D FEA and Kwon and Sun [5]

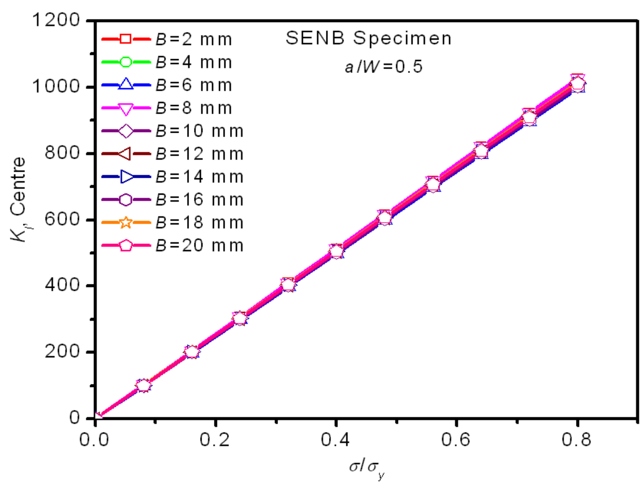

Figure 9: A typical plot of K_I at specimen centre against σ/σ_y for various B.

It is interesting to know from this figure that the variation of K_I *vs.* σ/σ_y is linear for various specimen thicknesses (B) and is independent of B. This nature of the variation of K_I *vs.* σ/σ_y allows us to obtain the relation between K_I and σ/σ_y for particular a/W. Next, the slope of K_I *vs.* σ/σ_y is evaluated by fitting a straight-line equation to all the values of K_I estimated at the centre of the specimen shown in Fig. 9. Now, it is required to find the variation of $K_I / (\sigma/\sigma_y)$ with the specimen thickness (B) for various a/W. $K_I / (\sigma/\sigma_y)$ against normalized thickness (B/W) for various a/W are plotted in Fig. 10. The plot indicates that the slopes are almost independent of normalized specimen thicknesses (B/W). The average slope values of B=2 to 20mm (B/W=0.1-1.0) are computed for a respective a/W. The plot between average slope value of $K_I / (\sigma/\sigma_y)$ and a/W is shown in Fig 11. This figure is used to obtain the relation between $K_I / (\sigma/\sigma_y)$ and a/W, in which K_I is the only unknown. As relation between $K_I / (\sigma/\sigma_y)$ and a/W is nonlinear, we have tried to fit the data with a suitable polynomial. In such exercise, we found a polynomial equation of third order fits the data with least error (Coefficient of determination, COD=0.99885), which is superimposed in Fig.11. From this third order polynomial fit, the relation between K_I, (a/W) and (σ/σ_y) can be expressed as:

$$\frac{K_I}{(\sigma/\sigma_Y)} = -2824.35 + 23133.54\left(\frac{a}{W}\right) - 50651.70\left(\frac{a}{W}\right)^2 + 41421.48\left(\frac{a}{W}\right)^3 \qquad (5)$$

Using Eq. (5) for various (a/W) and applied stress ratio (σ/σ_y), the value of K_I can be easily computed. To validate the formulation, Eq. (5), the computed values of K_I using Eq. (5) for typically, a/W=0.50 for B=10mm is compared with present 3D FE results and results of Kwon and Sun [5] in Fig.12. The figure shows that the results computed by Eq. (5) are in excellent agreement with the results of Kwon and Sun [5]. The maximum percentage of error estimated in use of Eq. (5) for B=2 to 20mm, a/W=0.45 to 0.70 and σ/σ_y=0.08 and 0.80 is 2.17%. Hence, the above proposed analytical formulation (Eq. (5)) is much capable and easy to use to compute maximum K_I in a specimen by knowing only specimen a/W and applied stress ratio (σ/σ_y). This formulation is much improved and simple to the one given by Kwon and Sun [5], which demands 2D K_I results.

Using the proposed new approximate analytical Eq. (5) one can estimate the magnitudes of maximum K_I at mid plane of the SENB specimen without complex numerical solutions. The proposed Eq. (5) can encourage the practicing engineers to estimate the realistic value of K_I as compared to Eq. (2).

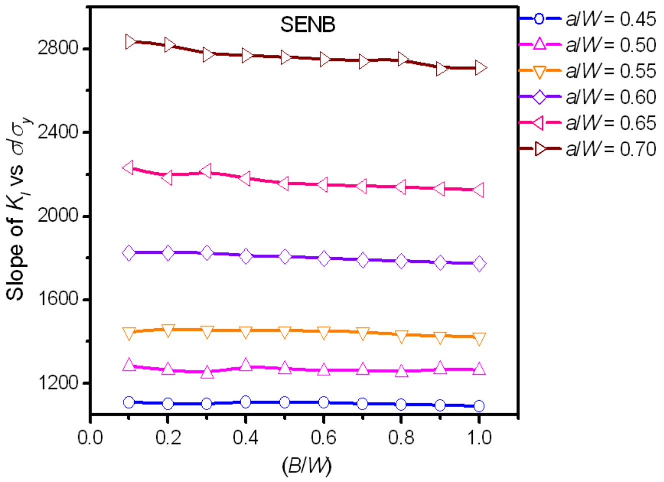

Figure 10: Variation of $K_I/\left(\sigma/\sigma_y\right)$ against normalized thickness (B/W) for various a/W

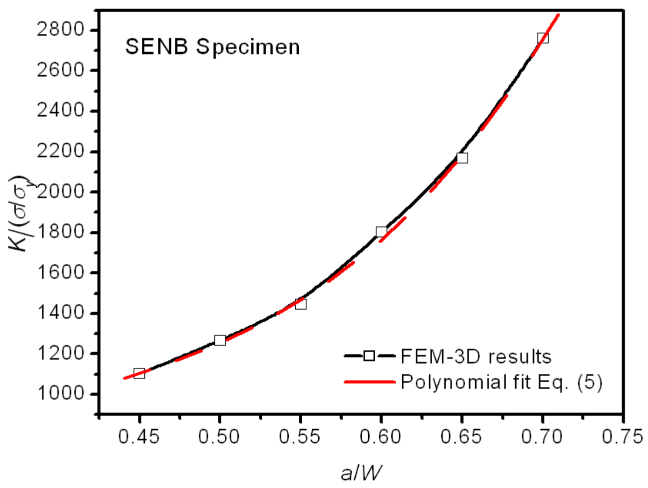

Figure 11: Variation of average slope value of $K_I/\left(\sigma/\sigma_y\right)$ vs. a/W

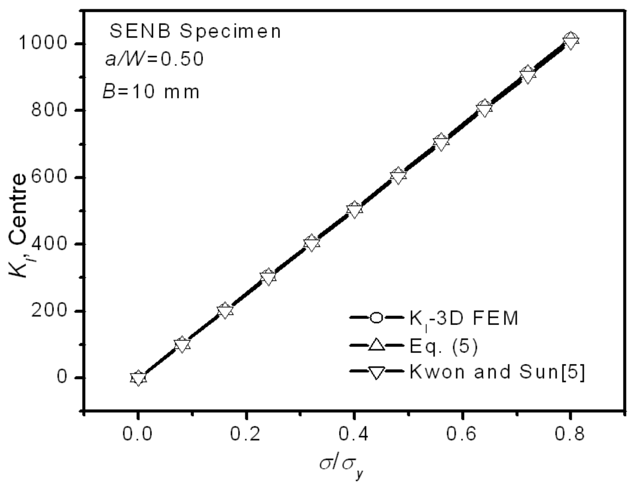

Figure 12: Comparison of K_I estimated using Eq.(5) for typically, a/W=0.50 and B=10mm is with present 3D FE results and results of Kwon and Sun [5].

CONCLUDING REMARKS

(i) The magnitude of K_I varies along the crack-front and is found to be maximum at the centre of the specimen
(ii) The magnitude of K_I is observed to be independent of B at the center of the specimen

(iii) The proposed formulation, Eq. (5) is more simple than the one proposed by Kwon and Sun [5] given in Eq. (4), which needs magnitude of K computed by 2D FE analysis. The proposed Equation can be readily used to evaluate the maximum K_I in a SENB specimen by knowing only applied stress, specimen thickness (*B*) and *a*/*W*

ACKNOWLEDGEMENT

Authors gratefully acknowledge the computational facilities provided by Research Center, B. V. B. College of Engineering & Technology, Hubli, and ProSim, Bangalore.

REFERENCES

[1] Murakami, Y., Stress Intensity Factors Hand Book, Pergamon Press, Oxford, (1987).
[2] Fett, T., Stress Intensity and T- stresses for internally cracked circular disks under various conditions, Engineering Fracture Mechanics, , 68 (2001) 1119-1136.
[3] Chen, Y.Z., Lin, X.Y., On dependence of the stress intensity factor and T-stress from imposed boundary conditions in a rectangular cracked plate, Computational Materials Science, 42 (2008) 149-55.
[4] Yihua, L., Zhigen, W., Yongcheng, L., Xiaomei, L., Numerical methods for determination of stress intensity factors of singular stress field, Engineering Fracture Mechanics, 75 (2008) 4793–4803.
[5] Kwon, S.W., Sun, C.T., Characteristics of three-dimensional stress fields in plates with a through –the-thickness crack, International Journal of Fracture, 104 (2000) 291-315.
[6] Moreira, P.M.G.P., Pastrama, S.D., Castro, P.M.S.T., Three-dimensional stress intensity factor calibration for a stiffened cracked plate, Engineering Fracture Mechanics, 76 (2009) 2298–2308.
[7] ABAQUS User's Manual. Version 6.5-1. Hibbitt, Karlsson & Sorensen, Inc. (2004).
[8] ASTM Standard E1820-13, Standard Test Method for Measurement of Fracture Toughness, American Society for Testing and Materials, Philadelphia, Pennsylvania (2013).
[9] Jie, Q., Xin, W., Solutions of T-stresses for quarter-elliptical corner cracks in finite thickness plates subject to tension and bending, International Journal of Pressure Vessels and Piping, 83 (2006) 593–606.
[10] Kim, Y., Zhu, X.K., Chao, Y.J., Quantification of constraint on elastic-plastic 3D crack front by the J-A_2 three-term solution, Engineering Fracture Mechanics, 68 (2001) 895-914.
[11] Coutin, S., Gardin, C., Bezine, G., Ben Hadj Hamouda, H., Advantages of the J-integral approach for calculating stress intensity factors when using the commercial finite element software ABAQUS, Engineering Fracture Mechanics, 72 (2005) 2174-2185.
[12] Kudari, S.K., Kodancha, K.G., Effect of Specimen Thickness on Plastic Zone, In: 17th European Conference on Fracture (ECF-17), (2008).
[13] Sherry, A.H., France, C.C., Edwards, L., Compendium of T-stress solutions for two and three dimensional geometries, Fatigue and Fracture of Engg Mat and Structures, 18 (1995) 141-155.
[14] Kudari, S. K., Maiti, B., Ray, K. K., The effect of specimen geometry on plastic zone size: a study using the J integral. Journal of Strain Analysis, 42 (2007) 125-136.
[15] Saxena, A., Nonlinear fracture mechanics for engineers, CRC Press, Boca Raton, Florida (1997), pp. 51–52
[16] Kodancha., K.G., Kudari., S. K., Variation of stress intensity factor and elastic T-stress along the crack-front in finite thickness plates. Frattura ed Integrità Strutturale, 8 (2009) 45-51.
[17] Fernandez, Z.D., Kalthoff, J.F., Fernandez, C.A., Canteli, A., Grasa , J., Doblare, M., Three dimensional finite element calculations of crack-tip plastic zones and K_{IC} specimen size requirements, In: 15th European Conference on Fracture (ECF-15), (2005).
[18] Nakamura, T., Parks, D.M., Three-dimensional stress field near the crack -front of a thin elastic plate, Journal of applied Mechanics, 55 (1988) 805-813.