The research for mechanics stimulation method of nonlinear random vibration based on statistical linear theory

Wang Guodong
Chongqing water resources and electric engineering college Chongqing, 402160, China
673843103@qq.com

ABSTRACT. Earthquake disasters have brought great harm to people's life safety and economic property. Its effect on fabric mainly focus on random effects currently, the general pseudo excitation method could solve the inefficiency calculation problem of linear random earthquake. However it could not take the nonlinear problem factors into account for calculation. In this paper, we suggest that a nonlinear structural incentive method should be improved based on statistical linearity to calculate and solve absolute displacement value. Through the analysis and research for cases, we calculate the displacement, speed, random vibration spectrum of bridge's accelerated speed, as well as the influencing situation of axial force. The results indicate that such perfect incentive method could not only perform nonlinear structure analysis, but also to be very accurate and high effective. Such method could reasonably avoid the displacement decomposition and solution of the pseudo-static model, thus it will be widely applied in common software.

KEYWORDS. Structure; Incentive method; Nonlinear random vibration; Axial force.

INTRODUCTION

In recent years, earthquake disasters occurred frequently in our country, it has brought more and more loss to people. The prevention and control efforts that we have taken for earthquake disasters have been enlarged in China. However the seismic theory has been gradually advanced, and transformed from static force into dynamic force, and transferred from established pattern into random one. Now we should explore the transforming mechanism from linearity to nonlinearity. The effect of earthquake on structure mainly focuses on the random effects of seismic oscillation currently. The earthquake input process is generally assumed to be stationary stochastic process. When the actual earthquake occurs, its effect is small in early times, while it would be magnified in following period, afterwards it will be of attenuation gradually. Thus the earthquake input process should be considered as non-stationary random process .Accordingly, the effect of earthquake on structure could make construction be in linear elastic stage originally. As earthquake effect increases, the structure will enters into nonlinear phase gradually, and begins to have plastic accumulation. When plasticity accumulation reaches certain stage, the structure will be destroyed for it could not resist seismic action.

In the seismic process, how architectural structure operates under the load of earthquake is the standard of measuring anti-seismic effect. In the original seismic experience, applying proper method into designing building structure is an approach suitable to actual circumstance. Nowadays, the widely applied method in China includes response spectrum method, response spectrum method and vibration method. But these methods are relatively fixed, and could not be flexibly applied into the seismic construction of actual building, thus a set of more scientific and reasonable method was born. As it could adopt to the approach of earthquake's random excitation, it is called random vibration method. It takes power density as the core, and sufficiently considers the probability problem of earthquake occurring. It is very suitable for the building such as bridges, roads etc which have many structural support points with large span, and it could assist the more advanced analysis and be used as design tools. Random vibration method is taken as the design and calculation tool of structural seismic resistance by European structural seismic design code (Eurocode 8) in 1995.The highway bridge seismic design code in China also incorporates random vibration method into the calculating method of aseismic design currently. In recent years, Jia-Hao Lin [1] has proposed simple, convenient and accurate pseudo excitation method starting from computational mechanics; Jiwei Zhang etc[2]have proposed the simple calculation method of response value in structure peak under the non-stationary seismic excitation based on traditional incentive method; Yang Jiang etc [3]have proposed the pseudo-excitation algorithm with high precision which could only be obtained by a small amount of vibration mode, which is the modified absolute displacement method. So far, calculating working amount with pseudo excitation method could solve the response problem of structural linear stochastic earthquake. There is still no in-depth research for analyzing nonlinear seismic buildings. As architectural design has such response requirement to non- linear systematic random earthquake, such research method is urgently needed. Through referring to the solution of FAP numberical value by computer, this paper is to suggest simple and efficient approach of structural nonlinear pseudo excitation method, which could be applied into analyzing displacement, speed, random vibration spectrum of accelerated velocity as well as the influencing situation of axial force, thus we could realize the high accuracy and efficiency in mechanics stimulation method of nonlinear random vibration on the basis of statistical linear theory.

STATISTICAL LINEAR THEORY

Statistical linear theory originated from the 1960s, R. Isaacs etc in America and L.S. Pontryagin etc in former Soviet Union had performed the initial study to this [4]. The numerical method applied in nonlinear differential game began taking shape in the 1980s, and it has developed into the method which could deal with random response problems in nonlinear dynamics system so far. For non-linear mechanics with n degree of freedom, it could be expressed with differential mode [5-7]:

$$[M]\{\ddot{y}\}+[G\{y\},\{\dot{y}\}]=\{F(t)\} \tag{1}$$

where
[M] is the matrix of structural mass with n order;
$[G(\{y\},\{\dot{y}\})]$ is the equivalent matrix of nonlinear restoring force and damping force.
The equivalent linear equation of nonlinear system could be established according to this method:

$$[M]\{\ddot{y}\}+[C_e]\{\dot{y}\}+[K_e]\{y\}=\{F(t)\} \tag{2}$$

where $[C_e][K_e]$ are the equivalent viscous drag coefficient matrix and stiffness matrix respectively, the following could be obtained through direct subtract between (1) type and (2) type:

$$[\varepsilon]=[G(\{y\},\{\dot{y}\})]-[C_e]\{\dot{y}\}\text{-}[K_e]\{\dot{y}\} \tag{3}$$

where $\{y\}=\{y(c_{ij},k_{ij},t)\}$ the steady-state solution of type (2) is, c_{ij},k_{ij} is the respective element in $[C_e][K_e]$ matrix, statistical linear theory is to reasonably select the c_{ij},k_{ij} in error matrix $[\varepsilon]$:

$$E[[\varepsilon]^T[\varepsilon]]=\min \tag{4}$$

where E is the mathematical expectation, the solution of type (4) could be obtained through calculation, and the equivalent linear stiffness and viscous damp of nonlinear system is derived.

THE SOLUTION OF PSEUDO-EXCITATION METHOD

For stimulation method, we generally establish the equation of motion through the variable generated on the basis of particle's relative displacement. The equation of motion is established according to d'Alembert principle, the inertia force is added on particle as load to avoid the matrix containing support quality in motion process. For the pseudo-excitation algorithm of multiple-support excitation, we could see from above solving idea that the dynamic relative displacement should be taken as the power balance equation of basic variable, then the dynamic relative displacement is derived, as well as the pseudo-static displacement of internal node caused by support movement will be solved, the sum of the two is the absolute displacement, and the solution procedure also avoids the participating computing of support quality matrix. But actually seismic action process is the quality mass vibration of support node generated by seismic excitation, the vibration of internal node is generated by the vibration of support node, earthquake force is directly added on support node, the quality of support node is not easy to determine, thus we usually solve the equation through applying this approach.

Type (2) could be written into partitioned matrix model:

$$\begin{pmatrix} M_s & 0 \\ 0 & M_b \end{pmatrix}\begin{Bmatrix} \ddot{y}_s \\ \ddot{y}_b \end{Bmatrix}+\begin{pmatrix} C_s & C_{sb} \\ C_{bs} & C_b \end{pmatrix}\begin{Bmatrix} \dot{y}_s \\ \dot{y}_b \end{Bmatrix}+\begin{pmatrix} K_s & K_{sb} \\ K_{bs} & K_b \end{pmatrix}\begin{Bmatrix} y_s \\ y_b \end{Bmatrix}=\begin{Bmatrix} 0 \\ F_b \end{Bmatrix} \tag{5}$$

where

y_b denotes the force displacement on ground of N supports,

y_s denotes the displacement of all unsupported node in construction system,

F_b denotes the force of ground effect on N supports,

M, C, K denote mass matrix, damping matrix and stiffness matrix respectively,

the small sign s, k correspond to the freedom degree of the internal structure node and support node respectively.

We assume the damping force is in direct proportion to relative velocity in type (5), $\dot{u}_d$, $\{0\}$ is used to replace the freedom degree of internal nodes and support nodes in type(5), by which to derive the equation:

$$M_s\ddot{u}_d + C_s\dot{u}_d + K_s u_d = -M_s A\ddot{y}_b \tag{6}$$

Virtual acceleration excitation is constructed on Eq. (6):

$$\overline{x}_j = \sqrt{\lambda_j}\Psi_j^* e^{i\omega t} \tag{7}$$

where superscript“*”represents taking complex conjugate, λ, Ψ denote the complex characteristics pair of power spectrum matrix of input ground motion. The product of virtual acceleration excitation and M_b in Eq. (7) is taken as F_b , then it will be substituted into Eq. (5):

$$\begin{pmatrix} M_s & 0 \\ 0 & M_b \end{pmatrix}\begin{Bmatrix} \ddot{y}_s \\ \ddot{y}_b \end{Bmatrix}+\begin{pmatrix} C_s & C_{sb} \\ C_{bs} & C_b \end{pmatrix}\begin{Bmatrix} \dot{y}_s \\ \dot{y}_b \end{Bmatrix}+\begin{pmatrix} K_s & K_{sb} \\ K_{bs} & K_b \end{pmatrix}\begin{Bmatrix} y_s \\ y_b \end{Bmatrix}=\begin{Bmatrix} 0 \\ M_b\sqrt{\lambda_j}\Psi_j^* e^{i\omega t} \end{Bmatrix} \tag{8}$$

Type (8) could be expanded as:

$$M_b\ddot{y}_b t + C_{bse}\dot{y}_s + C_{be}\dot{y}_b + K_{bse}y_s + K_{bse}y_b = M_b\sqrt{\lambda_j}\Psi_j^* e^{i\omega t} \tag{9}$$

Both sides of type (9)is multiplied by M_b^{-1}:

$$\ddot{y}_b t + M_b^{-1}C_{bse}\dot{y}_s + C_{be}\dot{y}_b + K_{bse}y_s + K_{bse}y_b = \sqrt{\lambda_j}\Psi_j^* e^{i\omega t} \tag{10}$$

When M_b mass is big, $M_b \to \infty$; when $M_b^{-1}(C_{bse}\dot{y}_s + C_{be}\dot{y}_b + K_{bse}y_s + K_{bse}y_b)$ is 0, type (10) could obtain virtual acceleration excitation of support:

$$\tilde{\ddot{y}}_b(t) = \sqrt{\lambda_j}\Psi_j^* e^{i\omega t} \tag{11}$$

In conclusion, as long as giving a larger mass on support and exerting stimulation, we could realize the load of virtual acceleration excitation. And support’s virtual velocity and virtual displacement incentives are:

$$\tilde{\dot{y}}_b(t) = \frac{1}{\omega}\sqrt{\lambda_j}\Psi_j^* e^{i\omega t}, \ \tilde{y}_b(t) = -\frac{1}{\omega^2}\sqrt{\lambda_j}\Psi_j^* e^{i\omega t} \tag{12}$$

Type (8) is expanded to be:

$$M_s\ddot{y}_s + C_{sc}\dot{y}_s + C_{bse}\dot{y}_b + K_{se}y_s + K_{bse}y_b = 0 \tag{13}$$

Through substituting (12) into (13), virtual absolute displacement $\tilde{y}_s$ is obtained. According to pseudo excitation method, the power spectrum matrix of absolute displacement is:

$$S_{y_s y_s}(\omega) = \sum_{j=1}^{m} \tilde{y}_s^* \tilde{y}_s^T \tag{14}$$

THE ANALYSIS AND RESEARCH FOR CASES

The Exposition of Main Information about Cases

We take a prestressed concrete cable-stayed bridge with two span, single tower and double cable planes as an example whose length is 130 m, span arrangement is 75 + 55 m. Its tower pier beam is semi-consolidated structural system. The girder section is the section of double solid girder cantilever, the center height of girder is 1.9 m, roof width is 38 m, cantilever length is 4.5 m, lateral solid beam of girder is 3 m, the width of lateral solid beam across back is 4 m, the thickness of roof between solid beam is 0.28 m. The girder section across back is added with baseplate as construction due to needing balance weight, thus box cross-section forms. The girder adopts two-way prestressed system, the king-tower is reinforced concrete leaning tower, the included angle between the center line of the tower and horizon is 75°, the vertical height above bridge is 50.7 m, the king-tower adopts the filled rectangle of variable cross-section, cross section height along the bridge changes from 3 m (tower top) into 8 m (the root of tower above bridge): The width across bridge is 2.5 m. The model is constructed through general finite element software ANSYS, main girder and main tower adopt C50 concrete, stayed-cable adopts high tensile steel wire PSEM7 - 241, cross section type of girder is simulated with finite element modeling, as it is shown in Fig. 1, calculation model performs dispersing to tower with space beam element -beam 4, the girder is simplified as fishbone shape through applying spatial beam- element beam 4 with rigid arm, stayed- cable adopts link 10 units, the left end of bridge is given vertical translational constraint of freedom degree, the right end of bridge is given transverse translational constraint of freedom degree, the bottom of cable support tower is completely constrained, cable element and beam element apply hinge constraint, as it is shown in Fig. 2.

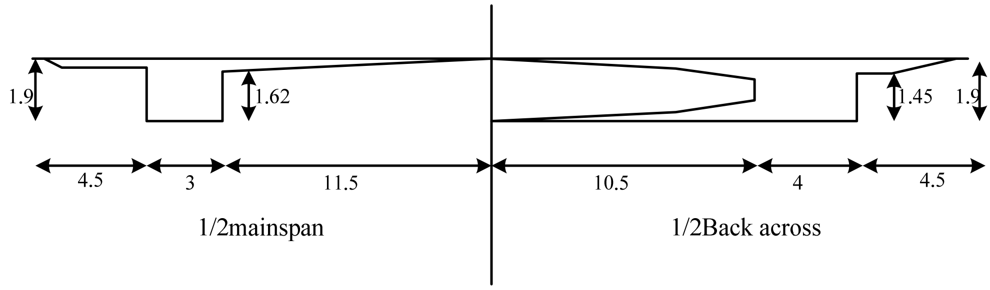


Figure 1: The girder's sectional view of cable-stayed bridge (unit: m).

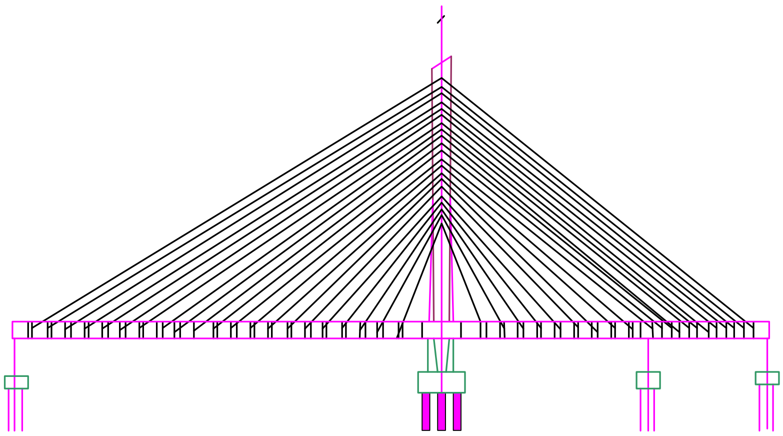


Figure 2 The diagrammatic figure of concrete deck cable stayed bridge.

Computational Analysis

In order to verify the validity of pseudo excitation method directly solved with absolute displacement, this paper is to linearize system through applying statistical linearization method, then analyze it through applying the random vibration module and harmonious response module in finite element software. Linearization procedure and its solution could refer to literature [8], which has well solved the response problem of nonlinear system with multi-degree of freedom under random excitation through taking full advantage of computer characteristics and combining FAP numerical method. FAP is the numerical method of statistical linearization method, and could compile corresponding computer program code with various algorithmic languages conveniently. This paper has compiled the procedure of FAP algorithm through applying FORTRAN 90, thus to realize the linearization of structure. For the pseudo excitation method directly solved with absolute displacement in this paper, the Full method in harmonic response analysis could be applied. Vibration isolation bearing is constructed through applying combin40 (it is the combination of mutually paralleled spring slider and damper, and cascaded with a clearance controller), its rigidity is k = 6×106N/, the former 150 order mode of vibration is taken during modal analysis, modal participation quality is as high as 97%, by which to guarantee the sufficient accuracy of algorithm. When applying pseudo excitation method directly solved with absolute displacement, mass 21 -big mass unit is added on 4 support nodes of model base, big mass unit is in rigid connection with support node, the mass takes 106 times of main bridges mass, then the constraint of freedom degree along x, y direction is released. According to the method proposed by above theory, every mass block is exerted with fictitious force, then harmonic response analysis is performed, the pseudo-excitation response after taking into account nonlinearity is obtained, and then the power spectral density matrix of absolute displacement is obtained according to type (14). The power spectrum inputted in calculation applies the practical seismic spectrum [10] obtained through GB50011 -2001 modifying response spectrum's model iteration proposed by literature [3]. The consistent ground motion parameters: Fortification of 7 degree, Ⅲ site, the first group designs earthquake grouping, the wave velocity takes 100m/s, coherence function adopts L－W model, the overall structure adopts Rayleigh damping, four nodes in base are taken out for performing consistent stimulus in three direction (the four nodes in base are: the constraint nodes at left end of bridge, the constraint nodes at right end of bridge as well as the constraint nodes on both sides of the centerline at the base of cable support tower along transverse direction of bridge).

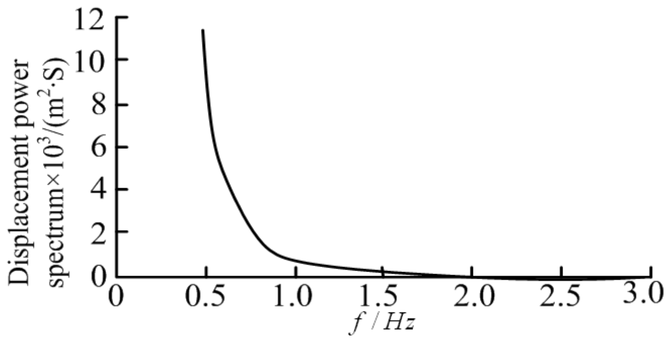
Contrastive Analysis

Through applying the traditional principle of random vibration solution, we perform solution through random vibration method, time-history method, spectra analysis respectively. Its projects are the respective effect of displacement, velocity and acceleration on power spectral density, by which to compare absolute displacement method.

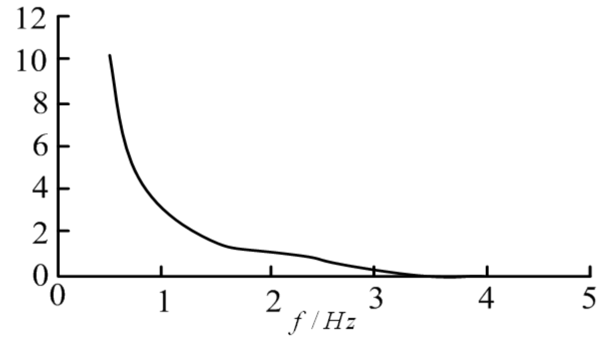
The Comparison between Absolute Displacement Method and Random Vibration Method

The charts in Fig. 3 are to respectively represent that the main span of cable-stayed bridge, the power spectral density of back span are influenced by several factors in the case, where the linear points denote random vibration method, the dashed points denote absolute displacement method. We could see from calculation results that after system linearization, the calculation result of two algorithms is basically consistent, and we find that there is also similar rule between the response power spectral density of other girders and main tower nodes through research, which demonstrates the validity of absolute displacement method. Furthermore, through observation and contrast, we find that the value of displacement, velocity and the density value of acceleration response power spectral obtained through calculating with absolute displacement method are smaller than the value obtained through calculating with random vibration method. We analyze from the area surrounded by curve and the abscissa that the mean square value of seismic random vibration wave obtained through calculation with absolute displacement method is on the smaller side, namely the energy acting on cable-stayed bridge is too small, and it demonstrates that random vibration method is a kind of seismic analysis method, which is conservative in relative speaking.

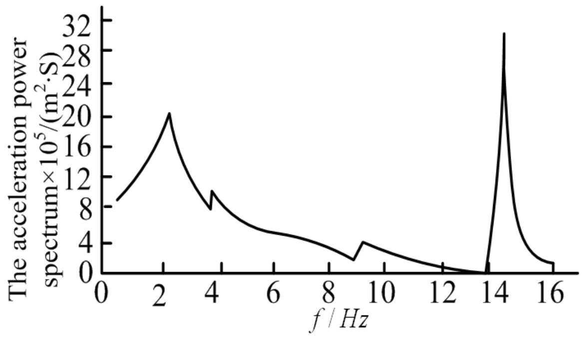
As the frequency range of inputted acceleration power spectrum density is 0.48-15.9 Hz, the figure does not represents the curved section whose frequency range is 0-0.48 Hz .Essentially, the inputted stimuli will make response power spectral density of structure have a sharp increase near the zero frequency. The average power spectral density model would reasonably reduce dynamic low frequency component of earthquake, when f =0, the power spectral density is generally 0 [9]. For displacement and velocity response power spectrum, the value is very small behind f ≥Hz, thus it is not represented in the figure. For acceleration response spectrum, its value represents a certain range of volatility after f ≥Hz, the changing curve is shown as Fig. 3 (c), (f).



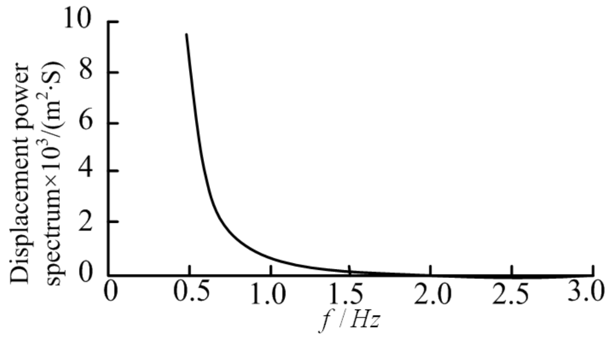
(a) The density of displacement response power spectral in main span.



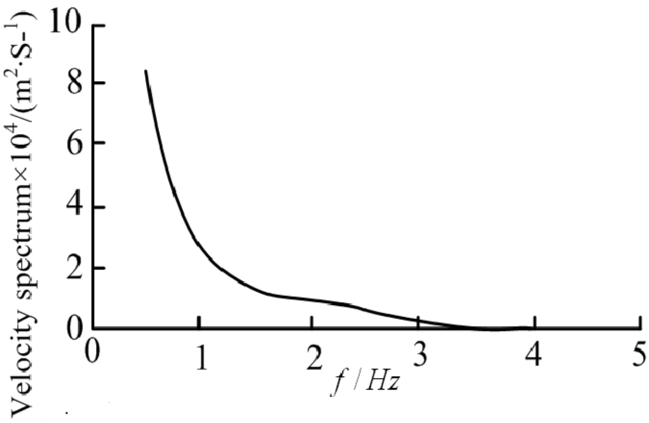
(b) The density of displacement response power spectral in main span.



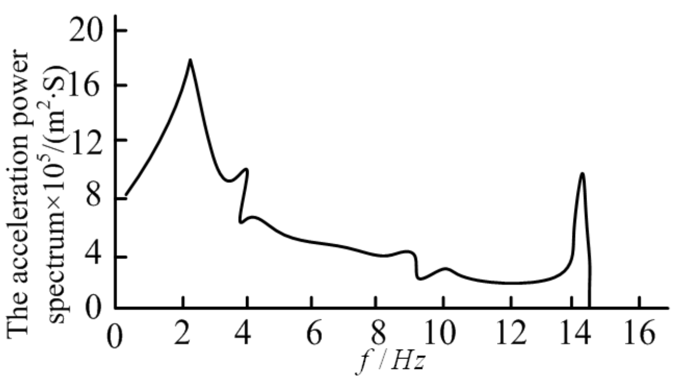
(c) The density of displacement response power spectral in main span.



(d) The density of displacement response power spectral in bridge's back span.



(e) The density of displacement response power spectral in bridge's back span.



(f) The density of displacement response power spectral in bridge's back span.

Figure 3 The response power spectral density of main span, mid-span across back in cable-stayed bridge.

When comparing the pseudo excitation method directly obtained with absolute displacement and traditional random vibration method, the relative error of its calculation results is shown as Tab. 1. We could derive the following conclusion from the data in table that:

(i) the calculation results of two algorithm are basically the same, it demonstrates that after linearization, the pseudo excitation method directly obtained with absolute displacement could replace traditional random vibration method in studying the seismic performance of structure. The excellence of this method has following points: the absolute displacement does not needs to be decomposed into quasi static displacement and dynamic relative displacement during calculating; quasi static modal matrix A does not needs calculation when constructing virtual stimuli for harmonic response analysis, thus it is simple and feasible; its essence does not changes the basic principle of traditional pseudo excitation method, its calculation efficiency is equivalent to traditional pseudo excitation method.

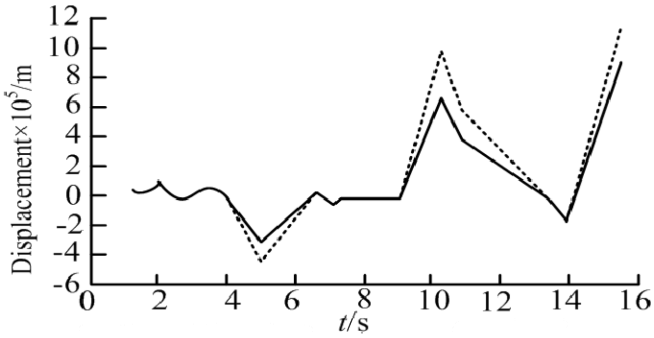
(ii) When comparing absolute displacement method with random vibration method, the error of acceleration response power spectral density is the smallest, the biggest error of main span is only 1.15%, the biggest error of back span is only 1.37%. The main reasons resulting into such phenomenon is that the inputted stimulus is the acceleration power spectrum density during the early analysis stage of model, thus after a series of calculation, its corresponding acceleration response power spectral density is closest to actual situation, while displacement and velocity response power spectral density have experienced more calculation steps than acceleration response power spectral density, then results into error accumulation, thus the maximum error of displacement power spectral density in main span reaches 11.1%, the maximum error of displacement power spectral density in back span reaches 9.14 %.

	Displacement power spectral density (main span)	Velocity power spectral density (main span)	Acceleration power spectral density (main span)	Displacement power spectral density (back span)	Velocity power spectral density (back span)	Acceleration power spectral density (back span)
The minimum error/%	6.14	2.58	0.50	3.11	5.50	0.43
The maximum error/%	9.99	7.50	1.04	8.23	8.77	1.23

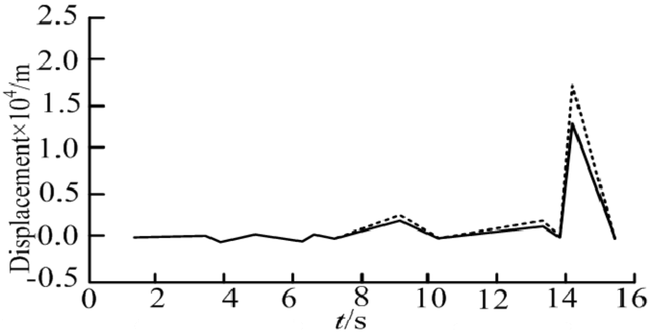
Table 1: The error comparison for two methods.

The Comparison for Absolute Displacement Method, Time History Method and Spectra Analysis

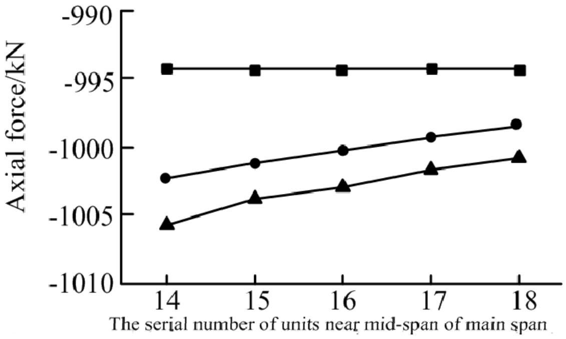
In current vibration resistance analysis, in addition to above mentioned random vibration method and harmony response analysis, spectrum analysis and time history method are also very important research method, the essence of dynamic time history analysis method is transient analysis Full method, various nonlinear factors could be considered, thus for the algorithm cases studied in this paper, the linearization process could be skipped to perform dynamic time history analysis directly. Vibration isolation bearing is considered during calculation, the structure is non-orthogonal damping system, the inputted artificial wave is generated according to the above adopted condition, and seismic action adopts both horizontal and consistent earthquake excitation. According to seismic code, earthquake acceleration at the second-rate direction is multiplied by the coefficient of 0.85. In order to compare above analyzed nonlinear and linear results, the method in last section is equally applied. As it is shown in Fig. 4, the straight line in (a) and (b) represents spectra analysis, point and line interval represents time history method, dotted line represents absolute displacement method; (c),(d) square lines represent spectra analysis, dot line represents time history method, triangle line represents absolute displacement method.
Through comparing the displacement time history diagram of nodes across main span and back span in Fig. 4(a) and (b), we could find out following rules: (i) as spectral analysis is to study the displacement response of cable-stayed bridge within linear elastic range, thus all the displacement response value of absolute displacement method are too small when compared with dynamic time history method and the absolute displacement method after linear processing, and the maximum deviation reaches more than 50% when compared with dynamic time history method which has the largest displacement response, it demonstrates that when studying the anti-seismic property of structure, the structural nonlinearity must be considered, otherwise more error will occur and the calculation results will be made deviate from true value. (ii)The calculation results of absolute displacement method and dynamic time history after taking account nonlinearity are very close, the maximum displacement error at main span X direction is only 7.8%, for back span Y direction ,its maximum error is 7.3%.And through research ,we find that there is similar regularity among the displacement response of main girder, main tower and stay-cables node, it demonstrates that the absolute displacement method could sufficiently consider structural nonlinearity after linearization, thus it could be taken as an effective method for aseismic design.
Through observing axial force response mean contrast figure at Z direction of typical unit under the action of consistent earthquake for three calculation method in figure 3.4, we could see that the unit axial force response value obtained through absolute displacement method after linearization is the biggest, and it is basically the same with unit axial force value obtained through calculating with dynamic time history method, its maximum error is only 0.23% (the units near the middle section of main span), for spectral analysis, all its axial force response values at Z direction are too small, when compared with dynamic time history method, the maximum error reaches 8%, thus it demonstrates that nonlinearity has certain effect on structural anti-seismic property, which should be considered in anti-earthquake analysis of bridge structure.



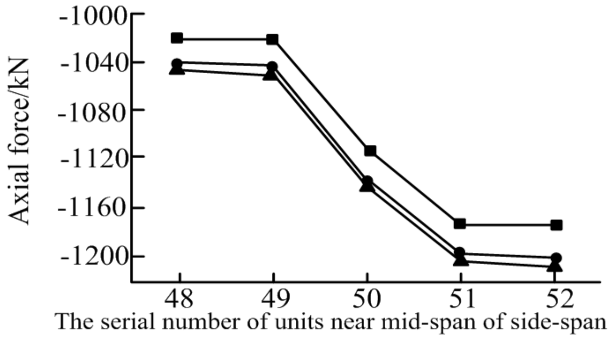
(a) The displacement course of main span at X direction.



(b) The axial force response of back span unit at Z direction.



(c) The axial force response of main span unit at Z direction.



(d) The axial force response of main span unit at Z direction.

Figure 4: The comparison diagram of axial force response

CONCLUSIONS

Compared with traditional pseudo excitation method, the new pseudo excitation method directly obtained through the absolute displacement based on statistical linearization could sufficiently consider structural nonlinearity and make its seismic analysis results be more close to actual situation; secondly, absolute displacement needs not to be discomposed into quasi static displacement and dynamic relative displacement during calculation, and quasi static modal matrix needs not to be calculated when constructing virtual stimuli for harmonic response analysis, it could be directly applied in general finite element analysis software and convenient for the application of pseudo excitation method in actual engineering. After system linearization, the node response power spectral density obtained through calculating with absolute displacement method and random vibration method is basically the same, and it demonstrates the correctness of the absolute displacement method, and the accuracy of response power spectral density outputted in calculation depends on the type of input excitation. Compared with dynamic time history method and the absolute displacement after linearization method, spectrum analysis does not take into account the nonlinear structure, the value of node displacement response and the unit axial force response are too small, it demonstrates that nonlinearity has great influence on seismic performance of structure, and seismic analysis of bridge structure must be taken into account. The response value of node displacement and the unit axial force calculated through the absolute displacement method and dynamic time history method after linearization are basically the same. It demonstrates that the pseudo-excitation method directly solved through absolute displacement based on statistical linearization is equivalent to the dynamic time history method, as they can effectively analyze and study the seismic behavior of the nonlinear system. One point needs to distinguish is that performing harmonic response analysis with absolute displacement method is the required stimuli when taking frequency as interval input, while the dynamic time history method is the seismic wave data established by taking time as interval, but their essence is the same.

REFERENCES

[1] Lin, J., Zhang, Y., Pseudo-excitation method of random vibration. Beijing: science press, (2004).
[2] Zhang, J., Consider large-span spatial structure seismic analysis and design method of seismic spatial effect. Tianjin: Tianjin University, (2008).
[3] Jiang, Y., The multi-point input earthquake response calculation of large-span structure and seismic design method research. Beijing: Tsinghua University, (2010).
[4] Bardi, M., Raghavan, T. E. S., Parthasarathy, T., Stochastic and Differential Games: Theory and Numerical Methods. Berlin: Birkhauser, (1999) 69−177.
[5] Huang, J. H., Li, X,. Wang, G. C., Near-optimal control problems for linear forward-backward stochastic systems, Automatica, 46(2) (2010) 397−404.
[6] Cheng. D. Z., Hu, X. M., Shen, T. L., Analysis and Design of Nonlinear Control Systems. Beijing: Science Press, (2010) 1−20.
[7] Huber, M. F., Adaptive Gaussian mixture filter based on statistical linearization, In: Proceedings of the 14th International Conference on Information Fusion (FUSION), Chicago, IL: IEEE, (2011) 1−8.
[8] Chemyshov, K. R., Statistical linearization based on the maximal correlation, In: Siberian Conference on Control and Communications (SIBCON). Moscow, Russia: IEEE, (2007) 29−36,
[9] Ernesto, H. Z., Vanmarcke, E. H., Seismic random-vibration analysis of multisupport-structural systems, Journal of Engineering Mechanics, 120(5) (1994) 1107 —1128.
[10] Shi, Y., Jiang, Y., Wang, Y., The application and improvement of direct solving method in earthquake response calculation for structural multi-point input, Engineering mechanics, 28(1) (2011) 75—81.