Focussed on: Fracture and Structural Integrity related Issues

Static assessment of brittle/ductile notched materials: an engineering approach based on the Theory of Critical Distances

R. Louks, H. Askes, L. Susmel
Department of Civil and Structural Engineering, The University of Sheffield, Sheffield S1 3JD, United Kingdom
r.louks@sheffield.ac.uk, h.askes@sheffield.ac.uk, l.susmel@sheffield.ac.uk

ABSTRACT. Engineering components often contain notches, keyways or other stress concentration features. These features raise the stress state in the vicinity of their apex which can lead to unexpected failure of the component. The Theory of Critical Distances has been proven to predict accurate results, but, conventionally, requires two key ingredients to be implemented: the first is a stress-distance curve which can be obtained relatively easily by means of any finite element software, the second is two additional material parameters which are determined by running appropriate experiments. In this novel reformulation, one of these additional parameters, namely the critical distance, can be determined a priori, allowing design engineers to assess components whilst reducing the time and cost of the design process.

This paper investigates reformulating the Theory of Critical Distances to be based on two readily available material parameters, i.e., the Ultimate Tensile Strength and the Fracture Toughness. An experimental data base was compiled from the technical literature. The investigated samples had a range of stress concentration features including sharp V-notches to blunt U-notches, and a range of materials that exhibit brittle, quasi-brittle and ductile mechanical behaviour. Each data set was assessed and the prediction error was calculated. The failure predictions were on average 30% conservative, whilst the non-conservative predictions account for less than 10% of the tested data and less than 2% of the non-conservative error results exceed -20%. It is therefore recommended that a safety factor of at least 1.2 is used in the implementation of this version of the Theory of Critical Distances.

KEYWORDS. Theory of Critical Distances; Static Fracture; Notches; Design.

INTRODUCTION

Structural and mechanical engineering components will often contain notches or keyways that act as stress concentrators, raising the local stress ahead of the stress raiser apex. Predicting the failure of engineering components accurately has been the goal of many engineers in the last century, as improved accuracy leads to less unexpected failure and more efficient usage of materials. Examination of the state of the art suggests that the TCD produces very accurate predictions of failure in components that contain stress concentration features, typically predicting within ±20% error [1]. The TCD is a group of theories which use a common critical distance, L, to assess the local stresses ahead of a stress concentrator apex. The TCD have been formalised into four methods which include the Point Method (PM), the Line Method (LM), the Area Method (AM), and the Volume Method (VM)[2].

Historically, critical distance analysis was first proposed in the 1930s and by the 1950s Neuber [3] had developed and published a method which is equivalent to the TCD LM. Neuber's method averages the elastic stress ahead of the stress concentrator over a material dependent length. A few years later, Peterson [4] developed a simpler method which is the equivalent to the TCD PM. Peterson's approach assumes that the component would fail when the elastic stress at a material dependent distance from the stress concentrator reaches a critical value. Neuber and Peterson were investigating the fatigue life of notched metallic components, but since these early works, critical distance analysis has also been adapted to predict static fracture. Neuber and Peterson faced two problems with implementing their critical distance methods, namely:

1. What value of L should be ascribed to each material? Peterson hypothesised that the critical distance was related to grain size, however, this posed some measuring difficulties. Both Neuber and Peterson determined the critical distance empirically, fitting predictions to data.
2. Obtaining accurate stress-distance curves in real components. Neuber suggested various elegant solutions for some standard notch geometries but they only offer approximations when applied to real components.

In 1974, Whitney and Nuismer [5] investigated the problem of monotonic failure of fibre composite materials containing stress raising features. With seemingly no knowledge of the early work, they developed identical theories to the LM and PM but with different names. In their research they made the useful link between Continuum Mechanics and Linear Elastic Fracture Mechanics (LEFM) allowing them to express the critical distance as a function of the fracture toughness, K_{IC}.

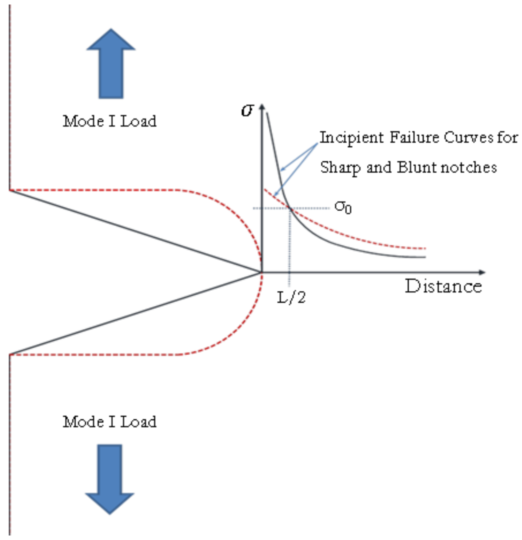

Figure 1: Critical local Stress-Distance curves ahead of two geometrically different stress concentrators.

The mathematical definition of L is shown by Eq. (1),

$$L = \frac{1}{\pi}\left(\frac{K_{IC}}{\sigma_0}\right)^2 \tag{1}$$

where σ_0 is the so-called materials inherent strength and K_{IC} is the plane strain fracture toughness. In the conventional application of the TCD for assessing components made from materials that exhibit some ductility prior to failure, the critical distance and inherent strength are not known *a priori,* meaning L and σ_0 have to be determined experimentally for each material. As shown in Fig. 1, by plotting, in the incipient failure condition, the critical stress-distance (S-D) curve for two like components containing different stress concentration features, the TCD parameters can be obtained. The material inherent strength, $\boldsymbol{\sigma_0}$, will often be greater than the material Ultimate Tensile Strength (UTS) [1]. If the inherent strength is greater than the UTS, the TCD cannot be used to design plain components as it would predict failures with large non-conservative errors.

In 2007, David Taylor published his book entitled *"The Theories of Critical Distances, a new perspective in fracture mechanics"* [2]. In the chapter on ceramics, which is based on his earlier work [6], it is said that some very brittle engineering ceramics take the inherent strength, σ_0, equal to the material UTS, σ_{UTS}. This finding is in agreement with Whitney and Nuismer's work [5]. In particular, they used the UTS as σ_0 for some quasi-brittle composite materials, achieving good results for both the PM and LM. On the contrary, it has been proven [7] that adopting $\sigma_0=\sigma_{UTS}$ to calculate the critical distance does not return accurate results in comparison to the conventional TCD when assessing materials that exhibit, prior to failure, limited plasticity in the vicinity of the stress raiser apex (such as, for instance, PMMA [3]).

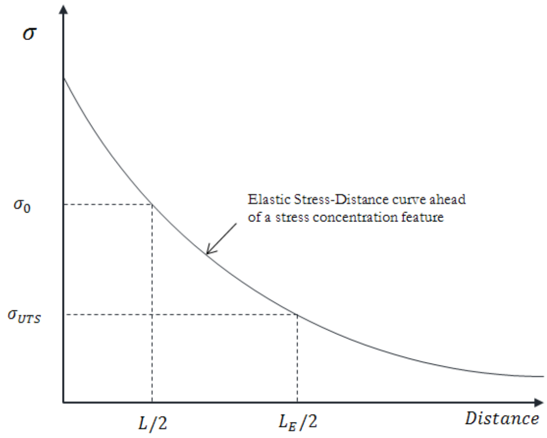

Figure 2: Failure stress and critical distance relationship.

FORMED SIMPLIFYING HYPOTHESIS TO CALCULATE THE CRITICAL DISTANCE VALUE

The aim of the present work is to reformulate the TCD to use it to assess Mode I fracture in brittle, quasi-brittle and ductile notched materials by determining the required critical distance via standard mechanical properties. In more detail, independently from the level of ductility characterizing the material being assessed, an engineering value for the critical distance is proposed here to be calculated as:

$$L_E = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_{UTS}}\right)^2 \tag{2}$$

where σ_{UTS} is the conventional ultimate tensile strength and K_{IC} the plane strain fracture toughness. Therefore, according to the PM, the component is assumed to fail when the effective stress at $L_E/2$ along the notch bisector reaches the UTS. Since the required material properties are supplied by most manufactures, definition (2) makes it evident that critical distance L_E can be determined without the need for running complex and expensive experimental investigations. This obviously would make the TCD even more appealing to those structural engineers engaged in designing real components against static loading. However, definition (2) raises an obvious and unavoidable question: *does the TCD still work?* The schematic S-D curve plotted, in the incipient failure condition, in the chart of Fig. 2 shows the way the PM assesses notch static strength when the critical distance is calculated according to definition (1) as well as to definition (2). As briefly recalled earlier, when final breakage is preceded by localised plastic deformations, σ_0 is seen to be larger than σ_{UTS} [2, 3]. This implies that, K_{IC} being constant, the critical distance value becomes larger than the corresponding value which would be calculated by applying the TCD rigorously. Having pointed out this theoretical argument, it is evident that the only way to answer the above key question, and therefore the validity of adopting Eq. (2), is by checking the accuracy of the TCD against appropriate experimental data, the critical distance being directly calculated according to definition (2). This will be done in the next section.

METHODOLOGY

A review of technical literature containing experimental data on static fracture was compiled. From the built database, all Mode I fracture data of notched components made of brittle, quasi-brittle and ductile materials were assessed. Tab. 1 summarises the different materials which were investigated and, where necessary, the temperature at which the tests were conducted. This table lists also the type of stress concentration feature being considered (which include external and internal U and V-notches) and the type of tests used (i.e., three/four point bending tests, tension tests, and testing done using blunt and sharp notched Brazilian/half Brazilian disks). The sharpness of the stress concentration features are also provided in the form of the notch root radii (which varied in the range 0.01-7.07mm).
The geometry of each data was modelled using finite element software Ansys®. After applying appropriate boundary conditions to each model, including the application of a unitary load/stress, the solution for each model was calculated by

assuming that that the investigated materials were linear-elastic, isotropic and homogeneous. The mesh density in the vicinity of stress concentration features apex was refined until convergence occurred at the critical distance (i.e., at $L_E/2$). The typical mesh spacing for convergence was between 1-10μm. The local effective stress calculated according to the PM was extracted from along the focus path, the focus path being coincident with the notch bisector under Mode I loading. The required S-D curves were calculated by FEA in terms of maximum principle stress. It is worth observing here that, under Mode I loading, the first principal stress is coincident with the maximum opening stress. Further, for the quasi-brittle and ductile materials, the S-D curves were calculated and post-processed also in terms of Von Mises equivalent stress. The S-D curves for each investigated geometrical feature were post-processed according to the PM. Finally, the failure prediction was compared with the experimental results, the error being calculated according to definition (3),

$$Error\ \% = \frac{\sigma_{Validation} - \sigma_{UTS}}{\sigma_{UTS}} \cdot 100 \quad (3)$$

where $\sigma_{Validation}$ is either the maximum principal stress or the Von Mises stress obtained, at a distance from the notch tip equal to $L_E/2$, from the finite element results calculated for the failure stress of the data. The error calculation for each data will show if the proposed method predicts the failure conservatively or non-conservatively by assigning either positive or negative results, respectively.

RESULTS

Shown in Figs 3 and 4 are the error predictions against changes in the material characteristic behaviour (i.e., from brittle to ductile) using the maximum principal stress and Von Mises equivalent stress, respectively.

Material class Reference	Material	Notch Type	Test Type	σ_{UTS} (MPa)	K_{IC} (MPa.M$^{0.5}$)	L_E (mm)	ρ Range (mm)
B1 [8]	Soda-Lime Glass	V	BD	14	0.6	0.585	1 - 4
B2 [9]	Alumina-7%Zirconia	V	FPB	290	5.5	0.114	0.031-0.1
B3 [10]	Isostatic Graphite	Key U	Tension	46	1.06	0.169	0.25 - 4
B4 [11]	Polycrystalline Graphite	V	TPB & BD	46	1.06	0.169	1 - 4
B5 [12]	Isostatic Graphite	Internal Bean	Tension	46	1.06	0.169	0.25 - 4
B6 [13]	PMMA -60°C	U	Tension	128.4	1.7	0.056	0.04-7.07
QB1 [14]	PMMA 20°C	V	TPB	111.8	1.12	0.032	0.03-0.25
QB2 [15]	PMMA 20°C	U	TPB	71.95	2.03	0.253	0.01-2.5
QB3 [16]	PMMA 20°C	CVT	Tension	67	2.2	0.343	0.2 - 4
QB4 [17]	PMMA 20°C	V	TPB	75	1	0.057	0.08-0.08
QB5 [18]	PMMA 20°C	U	TPB	75	1	0.057	0.11 – 4
D1 [19]	High Strength Steel	U	TPB	1285	33	0.210	0.1 – 1
D2 [7]	En3B	U-V	TPB	638.5	97.4	7.407	0.1 - 5

Table 1: Summary of experimental data (B=Brittle, QB=Quasi-Brittle and D=Ductile, BD=Brazilian Disk, TPB=Three Point Bending, FPB=Four-Point Bending)

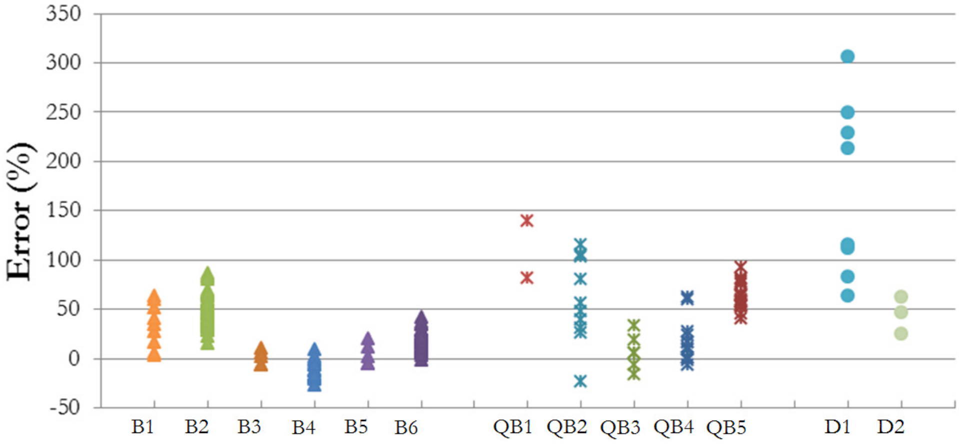

Figure 3: Error results using 1st Principal Stress as effective stress.

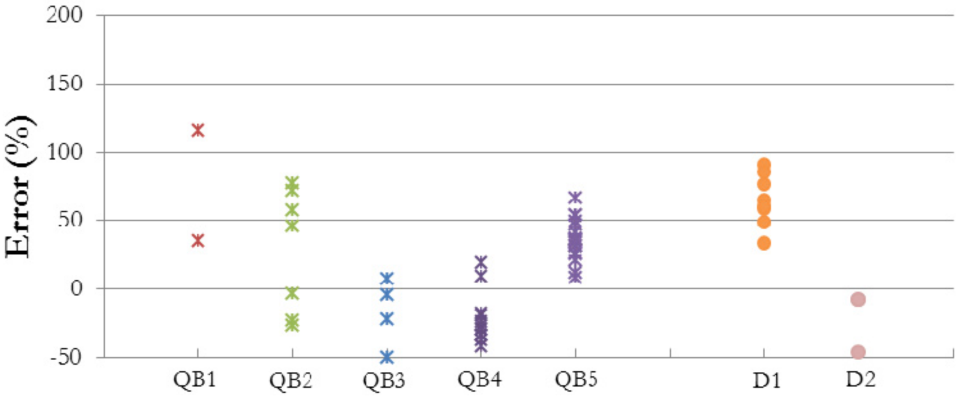

Figure 4: Error results using Von Mises Equivalent Stress as effective stress for Quasi-Brittle and Ductile materials.

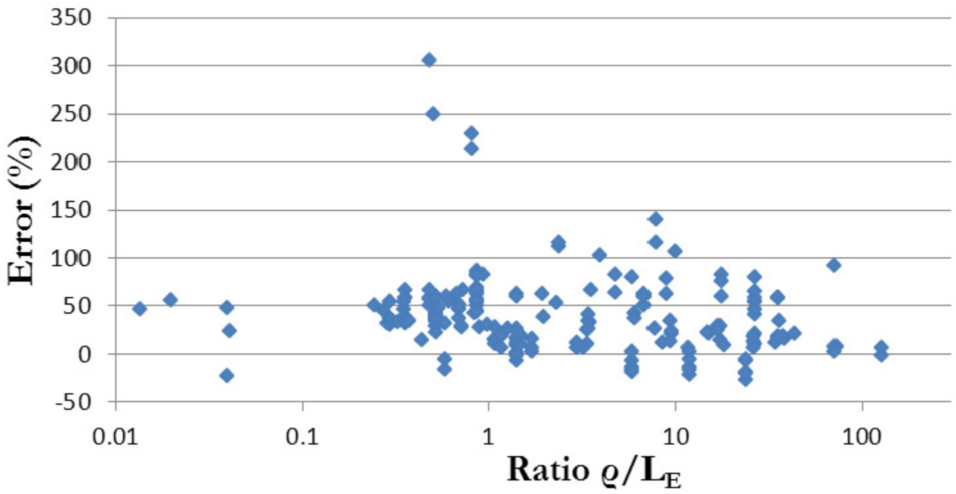

Figure 5: Error % against ratio of notch root radius, ϱ, and the critical distance, L_E (local stress fields post-processed in terms of maximum principal stress).

Initially the database was assessed by extracting the maximum principal stress S-D curve from the FEA for each of the data. The calculated error results are shown in Fig. 3. Using the maximum principal stress as the effective stress, the error scatter appears to increase with ductility. The results on the non-conservative side, negative error, do not exceed -27%. To counteract the apparent increasing conservatism with ductility, Von Mises effective stress S-D curves were extracted and post-processed. The results are presented in Fig. 4 for semi-ductile and ductile materials. The scatter and conservatism of the predictions is reduced for the ductile materials, however, the non-conservative error increases.

DISCUSSION

The predictions made for the data set assessed using the maximum Principal Stress as effective stress (Fig. 3) show a maximum non-conservative error of -27%, the majority of the data being on the conservative side. This suggests that the PM applied along with the maximum principal stress criterion can safely be used in situations of practical interest by directly estimating the required critical distance according to definition (2).

It has been proven [1] that the TCD applied along with Von Mises equivalent stress is successful in estimating the static strength of notched ductile materials subjected to both uniaxial and multiaxial static loading, provided that, both inherent strength σ_0 and critical distance L are determined experimentally. As shown in Fig. 4, the use of σ_{UTS} and L_E, Eq. (2), to assess notched ductile components results in a lower level of scattering, the maximum non-conservative error reaching -50%. This suggests that, as far as notched ductile materials subjected to Mode I loading are concerned, the use of the simplified methodology proposed here to calculate the critical distance values results in the largest level of accuracy when the TCD is applied along with the maximum principal stress criterion.

Fig. 5 plots the errors obtained by using the maximum principal stress against the ratio of notch root radius, ϱ, and the critical distance, L_E. As the notch root radius decreases, the stress concentrator is considered to be more like a crack; as the critical distance reduces, the material is considered to be more sensitive to defects and/or machined stress raisers. By using a dimensionless abscissa, the error predictions can be treated independently of the materials sensitivity to defects/notches and the stress concentration feature, which is considered to be mostly governed by the notch root radii, making this version of the TCD suitable to design components containing any geometrical feature and constructed of any material. This diagram confirms that the accuracy of the TCD applied by calculating critical distance via definition (2) is independent from ratio ϱ/L_E.

The error results are plotted as a cumulative probability distribution, displayed in Fig. 6. It can be seen that, as far as the maximum principal stress is concerned, less than 10% of the predictions fall on the non-conservative side and do exceed -27%. Less than 2% of the non-conservative error results exceed -20%, it is therefore recommended that a safety factor of at least 1.2 be implemented with the maximum principal stress assessment. The use of Von Mises stress to assess semi-ductile and ductile data is also shown in Fig. 6. The results show an increase of non-conservatism but a decrease in the scatter, therefore, using Von Mises stress in this assessment of components experiencing mode I loading should be incorporated with a safety factor of at least 1.5.

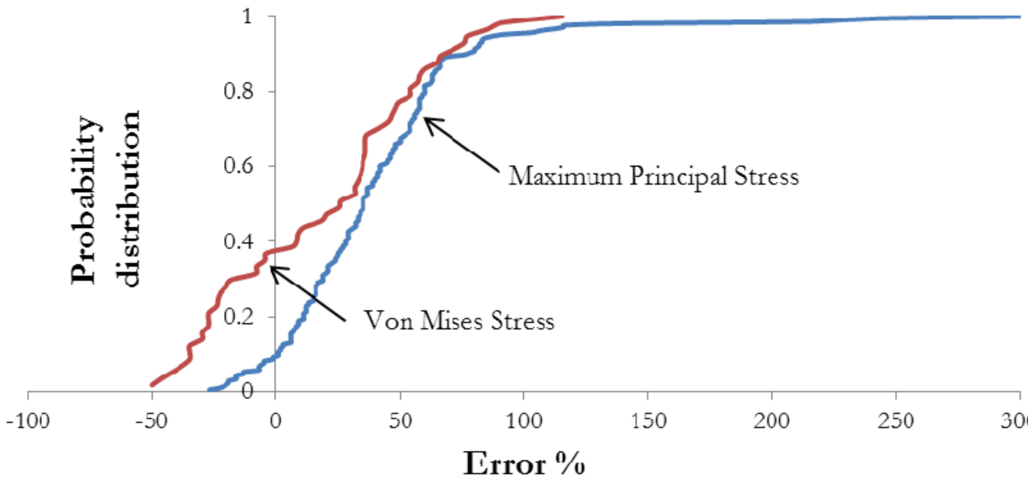

Figure 6: Probability distribution of the error predictions using the maximum principal stress for all data and Von Mises stress for semi-ductile and ductile data.

The simplified engineering method proposed in the present paper is suitable for the design of engineering components that experience Mode I loading, without the need for expensive testing. If the level of conservatism is an important factor such as high performance components where weight and/or size are crucial, it is recommended that the rigorous

application of the TCD be used, the conventional method has been proven to predict static failures with an accuracy of ±15% [7]. It should also be highlighted that the TCD has been reported to give similar levels of accuracy, typically ±20%, when used to assess other fracture and fatigue problems [2].
Finally, it is worth mentioning that predictions made in practical applications may have increased conservatism. This is due to engineering values supplied by manufacturers typically being given as minimum values compared to the average values typically reported in technical literature, from the design engineers point of view this should be seen as a positive factor in achieving a safe design.

CONCLUSIONS

- The proposed method was validated using over 200 Mode I test data, however, many test data were presented as an average of up to 5 tests.
- Using the maximum principal stress for the assessment of components subjected to mode I loading should be incorporated with a safety factor of at least 1.2.
- Using Von Mises stress for the assessment of components subjected to mode I loading should be incorporated with a safety factor of at least 1.5.
- Further work is required to extend this engineering approach to predict failure under multiaxial loading conditions.

REFERENCES

[1] Susmel, L., Taylor, D., The Theory of Critical Distances to estimate the static strength of notched samples of Al6082 loaded in combined tension and torsion. Part II: Multiaxial static assessment, Eng. Fract. Mech., 77 (3) (2010) 470–478.
[2] Taylor, D., The Theory of Critical Distances: A new perspective in fracture mechanics. Elsevier Ltd, (2007).
[3] Neuber, H., Theory of Notch Stresses, Second. Heidelberg: Springer-Verlag, (1958).
[4] Sines, G., Peterson, R. E., Metal Fatigue, New York; London; McGraw-Hill, (1959) 293–306.
[5] Whitney, J. M., Nuismer, R. J., Stress Fracture Criteria for Laminated Composites Containing Stress Concentrations, J. Compos. Mater., 8(3) (1974) 253–265.
[6] Taylor, D., Predicting the fracture strength of ceramic materials using the theory of critical distances, Eng. Fract. Mech., 71(16–17) (2004) 2407–2416.
[7] Susmel, L., Taylor, D., On the use of the Theory of Critical Distances to predict static failures in ductile metallic materials containing different geometrical features, Eng. Fract. Mech., 75(15) (2008) 4410–4421.
[8] Ayatollahi, M. R., Torabi, A.R., Experimental verification of RV-MTS model for fracture in soda-lime glass weakened by a V-notch, J. Mech. Sci. Technol., 25(10) (2011) 2529–2534.
[9] Yosibash, Z., Bussiba, A., Gilad, I., Failure criteria for brittle elastic materials, Int. J. Fract., 125(1957) (2004) 307–333.
[10] Lazzarin, P., Berto, F., Ayatollahi, M. R., Brittle failure of inclined key-hole notches in isostatic graphite under in-plane mixed mode loading, Fatigue Fract. Eng. Mater. Struct., 36(9) (2013) 942–955.
[11] Ayatollahi, M. R., Torabi, A. R., Tensile fracture in notched polycrystalline graphite specimens, Int. J. Carbon, 48(8) (2010) 2255–2265.
[12] Berto, F., Lazzarin, P., Marangon, C., Brittle fracture of U-notched graphite plates under mixed mode loading, Mater. Des., 41 (2012) 421–432.
[13] Gómez, F. J., Elices, M., Planas, J., The cohesive crack concept: application to PMMA at −60°C, Eng. Fract. Mech., 72(8) (2005) 1268–1285.
[14] Priel, E., Bussiba, A., Gilad, I., Yosibash, Z., Mixed mode failure criteria for brittle elastic V-notched structures, Int. J. Fract., 144(4) (2007) 247–265.
[15] Cicero, S., Madrazo, V., Carrascal, I. A., Analysis of notch effect in PMMA using the Theory of Critical Distances, Eng. Fract. Mech., 86 (2012) 56–72.
[16] Susmel, L., Taylor, D., The theory of critical distances to predict static strength of notched brittle components subjected to mixed-mode loading, Eng. Fract. Mech., 75(3–4)(2008) 534–550.
[17] Gómez, F. J., Elices, M., Fracture of components with V-shaped notches, Eng. Fract. Mech., 70(14) (2003) 1913–1927.

[18] Gomez, F. J., Elices, M., Valiente, A., Cracking in PMMA containing U-shaped notches, Fatigue Fract. Eng. Mater. Struct., 23(9) (2000) 795–803.
[19] Gómez, F. J., Elices, M., Valiente, A., Fracture of a high strength steel containing U-notches, In: ECF13, San Sebastian, Espana, (2000).