Focussed on: Fracture and Structural Integrity related Issues

The effect of thickness on out-of-plane constraint in terms of the T-stress

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ABSTRACT. The plastic zone is theoretically and numerically analyzed for various combinations of the non-singular terms T_{xx} to T_{zz} ratios in connection with specimen thickness under mode I loading. The plastic zone size distribution is based on the Mises yield criterion for mode I conditions and mapped in the specimen thickness direction. It is shown that the plastic zone size is affected by the T_{zz}-stress, i.e. there is strong effect of out-of-plane constraint (thickness) on crack tip plastic zones.
In addition, to study the effect of thickness and loading mode mixity (mode I and II) of the specimen on the singular (K_I, K_{II}) and the non-singular (T_{xx}, T_{zz}) terms along the 3D crack front, three-dimensional stress fields are analyzed by means of finite element analysis. The strong effect of thickness and mixed mode loading conditions on the T_{zz}-stress is observed. At the same time, there is not the effect of thickness on the non-singular T_{xx}-stress at the same loading conditions.

KEYWORDS. Non-singular T-Stresses; Thickness; Mixed Mode; In-plane and out-of-plane constraint; plastic zone;

INTRODUCTION

The results of analytical and numerical calculations show that stress fields in the vicinity of the crack tip, in many cases, are strongly dependent on constraint. The source of a change of out-of-plane constraint is thickness. In contrast to out-of-plane constraint, the different sources of a change in in-plane constraint at the crack tip are associated with crack size, geometry of specimens and type of loading. To describe in-plane and out-of-plane constraint effects in fracture analysis, the following parameters can be used, namely, T_z-parameter [1], local triaxiality parameter [2] and nonsingular components of the T-stresses (T_{xx} and T_{zz}) [3].

Not emphasize attention on merits and demerits of the above-mentioned parameters of constraint at the crack tip, this paper is concentrated on the nonsingular components of the T-stresses in the vicinity of the crack tip. It is well-known that the sign and value of the T_{xx}-stresses considerably effect on the shape and size of the plastic zone at the crack tip. At the same time, there is no information about influence of thickness on the T_{zz} component as well as the size of the plastic zone in literature. Moreover, an analysis of joint influence of the nonsingular T-stress (T_{zz} and T_{xx}) terms on the fracture mechanics parameters is not carried out yet.

Theoretical and numerical analysis of three-dimensional stress fields in the vicinity of through-thickness crack tip under mode I and mixed mode (I + II) loading is carried out to estimate the effect of thickness on the nonsingular T_{xx} and T_{zz}-stresses.

THE EFFECT OF OUT-OF-PLANE CONSTRAINT ON THE CRACK TIP PLASTIC ZONE

Basic equations

The components of the stress field, which take into consideration three-dimensionality of the stress state near mode I crack front in isotropic elastic body, can be represented in the manner of asymptotic formulas given in Ref. [3]

$$
\begin{aligned}
\sigma_{xx} &= \frac{1}{\sqrt{2\pi r}}\left[K_I\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)-K_{II}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)\right]+T_{xx}\\
\sigma_{yy} &= \frac{1}{\sqrt{2\pi r}}\left[K_I\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)+K_{II}\sin\frac{\theta}{2}\left(\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)\right]\\
\sigma_{xy} &= \frac{1}{\sqrt{2\pi r}}\left[K_I\sin\frac{\theta}{2}\left(\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)+K_{II}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\right]\\
\sigma_{zz} &= \frac{2\nu}{\sqrt{2\pi r}}\left[K_I\cos\frac{\theta}{2}-K_{II}\sin\frac{\theta}{2}\right]+T_{zz}\\
\tau_{yz} &= 0, \qquad \tau_{zx}=0\\
T_{zz} &= E\varepsilon_{zz}+\nu T_{xx}
\end{aligned}
\tag{1}
$$

Here, σ_{xx}, σ_{yy}, σ_{zz}, τ_{xy}, τ_{yz}, τ_{zx}, – components of the stress tensor which define stress state at the arbitrary point near the crack tip; r and θ – polar coordinates (Fig. 1); K_I and K_{II} are stress intensity factors, ν is Poisson's ratio.

The model of the crack tip plastic zone

The von Mises yield criterion can be employed to estimate the influence of the nonsingular T_{xx} and T_{zz}-stresses in the vicinity of the crack tip on the shape and size of the plastic zone under mode I loading [4, 5]. In this case, the yield criterion can be written in the form

$$\left(\sigma_{xx}-\sigma_{yy}\right)^2+\left(\sigma_{yy}-\sigma_{zz}\right)^2+\left(\sigma_{zz}-\sigma_{xx}\right)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)=2\sigma_Y^2, \tag{2}$$

where σ_Y is the yield strength.

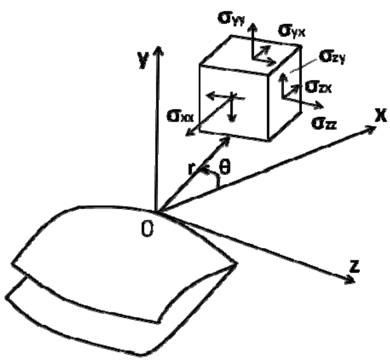

Figure 1: Stress components in the vicinity of the crack tip.

Substitution of asymptotic formulas (1) into the criterion (2) allows determining size r_p of the crack tip plastic zone

$$\frac{1}{2\pi \cdot r_p} \cdot \left(K_I^2 \cdot \left(A_I + \frac{D_I}{K_I} \cdot \sqrt{r_p} \right) \right) = 2\sigma_Y^2 \tag{3}$$

Here, some parameters are denoted as follows

$$D_I = \sqrt{\frac{\pi}{2}} \cdot \left(\left(\cos\frac{\theta}{2} + 3\cos\frac{5\theta}{2} \right) \cdot T_{xx} - 8\nu \cdot \cos\frac{\theta}{2} \cdot \left(T_{xx} + \frac{T_{zz}}{\nu} \right) + 16\nu \cdot \cos\frac{\theta}{2} \cdot (T_{zz}) \right) \tag{4}$$

$$A_I = (1-2\nu)^2 \cdot (1+\cos\theta) - \frac{3}{4} \cdot (\cos 2\theta - 1) \tag{5}$$

Plastic zone sizes can be estimated by the following equations

$$r(\theta)_{p1} = \frac{V + \sqrt{V^2 + 4U \cdot W}}{2U}, \quad r(\theta)_{p2} = \frac{V - \sqrt{V^2 + 4U \cdot W}}{2U} \tag{6}$$

As a result, the plastic crack zone is determined as follows

$$r(\theta) = \left[positive\left(r_{p1}, r_{p2} \right) \right]^2 \tag{7}$$

It should be noted that the value of r_{p1} is positive in the wide range of coefficients U, V, W. At the same time, the value of r_{p2} has a negative sign.

The effect of constraint on the crack tip plastic zone

The calculation results of the angular size distribution of the plastic zone around the crack tip in CT specimen with various relations between specimen thickness B and specimen width W are presented in Fig. 2.

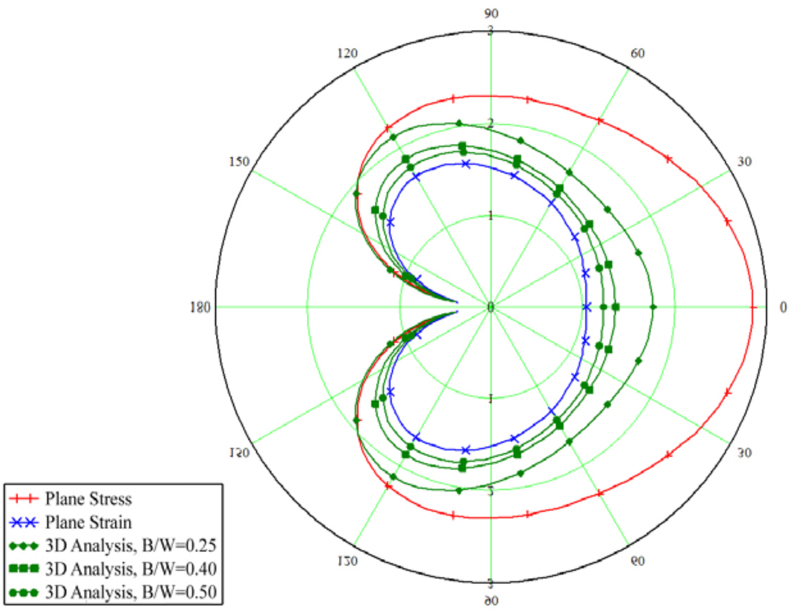

Figure 2: The angular distribution of crack tip plastic zone sizes for the middle plane in the CT specimen.

The stress intensity factor K_I is assumed to be constant independently on specimen thickness and equal to 66 MPa·m$^{1/2}$. Corresponding values of the T-stress components are presented in Tab. 1. Sizes of the plastic zone around the crack tip with provision for the spatial stress state (3D Analysis) are surrounded by sizes of the zones corresponding to two limit conditions, namely, Plane Stress and Plane Strain. Moreover, when specimen thickness increases, the shape and size of the plastic zones tend to zones which are typical for plane strain conditions. Thus, the results clearly show that triaxiality of the stress state around the crack tip should be taken into account by means of both non-singular T_{xx}-stress and T_{zz}-stress according to Eq. (6).

The validity of analytical equations for calculation of the plastic zone around the crack tip is demonstrated by means of finite element analysis [4, 5]. The deviation between analytical and numerical results does not exceed 20% in the angular range (0°, 30…45°) and (90°…100°,135°…145°). It should be noted that the deviations between the results of the numerical analysis and the analytical calculation in the angular range (0°–145°) can be explained by the fact that the *T*-stress components around the plastic zone can be not constant and depend on angle θ as reported in Ref. [4].

Loading parameters	*B/W*=0.25	*B/W* =0.40	*B/W* =0.50
P, kN	6.0	9.6	12.0
K_I, $MPa{\cdot}m^{1/2}$	66.0	66.0	66.0
T_{xx}, *MPa*	186.59	182.36	176.28
T_{zz}, *MPa*	-159.47	-106.81	-84.97

Table 1: Loading conditions of the CT specimen.

THE EFFECT OF THICKNESS ON THE NON-SINGULAR T-STRESSES UNDER MIXED MODE LOADING

Loading conditions

Finite element analysis of 3D stress fields in the vicinity of the crack front is performed for the center cracked circular disc (CCCD-specimen) with the thorough-thickness crack of arbitrary space orientation. The specimen is loaded by 2 compressive forces acting in the vertical direction. This configuration of the specimen is very suitable to create different conditions of mixed mode (I + II) loading [6]. The orientation of the crack plane with respect to the disk is determined by the angle α. Changing the angle can provide almost any relationship between the magnitudes of stress intensity factors, namely, K_I and K_{II} . Loading mode mixity is characterized by the following parameter

$$M_e = \frac{2}{\pi} arctg(\frac{K_I}{K_{II}}) \tag{8}$$

Calculation of fracture mechanics parameters

The evaluation of the stress intensity factor is based on the well-known approach that includes the calculating this parameter in a number of points (at varied *r*) using relations from formulas (1) and extrapolation of the obtained values of the stress intensity factors to the point *r*=0

$$K_I = \sqrt{\frac{\pi r}{2}}\left(2\sigma_{xx}|_{\theta=0} - \sigma_{xx}|_{\theta=-\pi} - \sigma_{xx}|_{\theta=+\pi}\right) \tag{9}$$

$$K_{II} = \sqrt{\frac{\pi r}{8}}\left(\sigma_{xx}|_{\theta=-\pi} - \sigma_{xx}|_{\theta=+\pi}\right). \tag{10}$$

The computational procedure considers the nodes of the finite element mesh as the calculation points, but the nodes are located at some small distance from the crack front. To obtain the distribution of the stress intensity factor along the crack front, this procedure is used for a number of planes (x0y) orthogonal to the crack front [7- 9]. Their location is characterized by local coordinate *s* along the front and starts from the center of the crack front.

The calculation of the T_{xx}- and T_{zz}-stress is performed using the stresses in the points on the crack surface

$$T_{xx} = \frac{1}{2}\left[\sigma_{xx}\big|_{\theta=-\pi} + \sigma_{xx}\big|_{\theta=+\pi}\right] \tag{11}$$

$$T_{zz} = \frac{1}{2}\left[\sigma_{zz}\big|_{\theta=-\pi} + \sigma_{zz}\big|_{\theta=+\pi} \right] \tag{12}$$

The determination of T_{xx} and T_{zz} is similar to the procedure for the stress intensity factor including extrapolation to the point r=0.

The effect of thickness and loading mode mixity

To study the effect of thickness and loading mode mixity of the specimen on T_{zz}-stress, the compressive load is adjusted so that the T_{xx}-stress is remained approximately constant at variation of the specimen thickness (B= 10; 20; 040 and 80 mm) at constant crack length (for the corresponding values of mixity parameter M_e). To reflect mixed mode loading conditions, an effective stress intensity factor K_{eff} is introduced into consideration as follows

$$K_{eff} = \sqrt{K_I^2 + K_{II}^2} \tag{13}$$

The results of calculation of the effective stress intensity factor and the T_{zz}-stress in the middle plane of the crack front (s=0) for different mixed mode loading conditions and thickness of the specimen are summarized in Fig. 3. The increase of the specimen thickness is 8 times greatly reduces the T_{zz}-stress (up to 75% at M_e=0,25), whereas the value of T_{xx} is virtually unchanged [8]. Thus, magnitudes of the T_{zz}-stress have significant correlation with thickness of the specimen and allows taking into account the level of the out-of-constraint which should be included into assessment of the fracture toughness of full-scale structures.

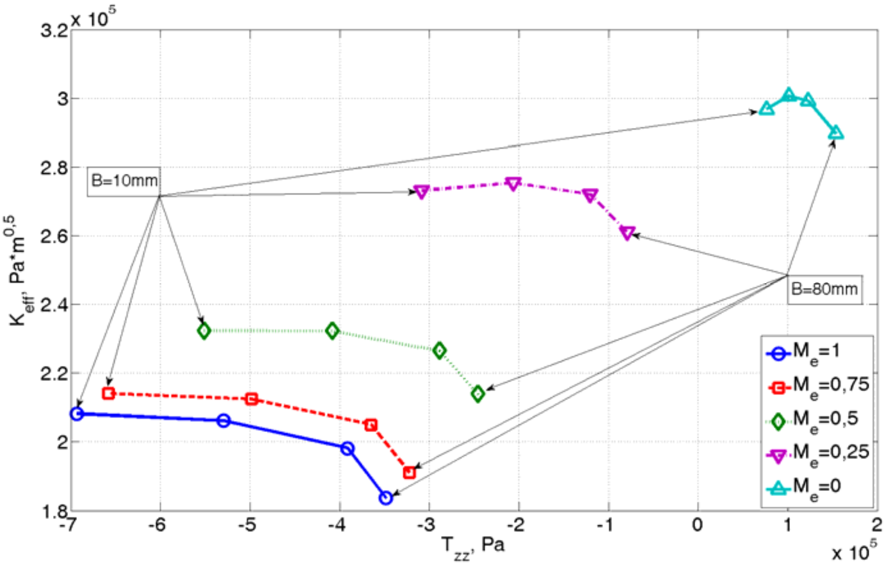

Figure 3: The effect of thickness and loading mode mixity on the effective stress intensity factor and out-of-plane constraint in the middle plane of the crack front.

CONCLUSIONS

The theoretical analysis of the joint influence of the nonsingular components of the T-stresses (T_{xx} and T_{zz}) on the plastic zone around the crack tip of mode I is carried out with attraction of asymptotic formulas, which take into account triaxiality of the stress state at the crack tip, and the von Mises yield criterion. The size of the plastic zone around the crack tip at the middle plane of the CT specimen decreases with the increase of specimen thickness. It is confirmed that the plastic zone is affected by the T_{zz}-stress, i.e. there is strong effect of out-of-plane constraint on crack tip plastic zones.

The three-dimensional stress field ahead of the through-thickness crack under mixed mode (I+II) loading conditions is also analyzed. The in-plane and out-of-plane constraint effect are discussed from viewpoint of the non-singular terms, namely, T_{xx}-stress and T_{zz}-stress, respectively. The significant effect of thickness and loading mode mixity of specimen on the T_{zz}-stress in the middle plane of the crack front has been observed at the same non-singular T_{xx}-stress.

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