Duplex S-N fatigue curves: statistical distribution of the transition fatigue life

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ABSTRACT. In recent years, very-high-cycle fatigue (VHCF) behavior of metallic materials has become a major point of interest for researchers and industries. The needs of specific industrial fields (aerospace, mechanical and energy industry) for structural components with increasingly large fatigue lives, up to 10^{10} cycles (gigacycle fatigue), requested for a more detailed investigation on the experimental properties of materials in the VHCF regime.
Gigacycle fatigue tests are commonly performed using resonance fatigue testing machines with a loading frequency of 20 kHz (ultrasonic tests). Experimental results showed that failure is due to cracks which nucleate at the specimen surface if the stress amplitude is above the conventional fatigue limit (surface nucleation) and that failure is generally due to cracks which nucleate from inclusions or internal defects (internal nucleation) when specimens are subjected to stress amplitudes below the conventional fatigue limit. Following the experimental evidence, the Authors recently proposed a new statistical model for the complete description of S-N curves both in the high-cycle-fatigue (HCF) and in the VHCF fatigue regions (Duplex S-N curves). The model differentiates between the two failure modes (surface and internal nucleation), according to the estimated distribution of the random transition stress (corresponding to the conventional fatigue limit). No assumption is made about the statistical distribution of the number of cycles at which the transition between surface and internal nucleation occurs (i.e., the transition fatigue life).
In the present paper, the statistical distribution of the transition fatigue life is obtained, according to the statistical model proposed. The resulting distribution depends on the distance between the HCF and the VHCF regions and on the distribution of the random transition stress. The estimated distribution can be effectively used to predict, with a specified confidence level, the number of cycles for which an internal nucleation may probabilistically occur in a VHCF test and it is also informative for properly choosing the end of HCF tests in terms of number of cycles.
A numerical example, based on experimental datasets taken from the literature, is provided.

KEYWORDS. Ultra-high-cycle; Gigacycle; Random transition-stress; Random transition fatigue-life; Random fatigue limit.

INTRODUCTION

In recent years, Very-High-Cycle-Fatigue (VHCF) test results showed that specimens may also fail at stress amplitudes below the conventional fatigue limit and, therefore, drastically affected the way of modelling fatigue data and designing machine components under VHCF loading conditions [1].

Two distinct failure mechanisms are generally visible in VHCF data plots and, at a stress value near the conventional fatigue limit, plots show a plateau separating the two failure modes. For this reason, the conventional fatigue limit can be considered as a transition stress that differentiates between the two failure modes [2]. In particular, the plateau separating different failure mechanisms represent a transition stress, while the plateau separating finite lives from infinite lives can be considered as a real fatigue limit, if it exists [3, 4]. Following the experimental evidence, new fatigue life models [2, 5-7] were proposed in the literature for the description of S-N curves characterized by two failure modes.
A novel general statistical model, which can take into consideration the two failure modes (Duplex S-N curve) and the possible presence of a fatigue limit is described in [8]. The model differentiates between the two failure modes (surface and internal nucleation) according to the estimated distribution of the random transition stress (corresponding to the conventional fatigue limit). No assumption is made about the statistical distribution of the number of cycles at which the transition between surface and internal nucleation occurs (i.e., the transition fatigue life).
In the present paper, the statistical distribution of the transition fatigue life is obtained, according to the statistical model proposed in [8]. A numerical example, based on experimental datasets taken from the literature, is provided. The paper shows results obtained in case of a Duplex S-N curve with fatigue limit.

METHODS

In [8], a unified statistical model for various types of S-N curve was defined. In Subsection *Duplex S-N curves: statistical model*, the particular case of Duplex S-N curves is recalled. The model is able to take into account the possible presence of a fatigue limit. In Subsection *'Transition life: statistical distribution'* a procedure for the estimation of the statistical distribution of the transition life is presented.

Duplex S-N curves: statistical model

In case of Duplex S-N curve with fatigue limit, the cumulative distribution function (cdf) of the fatigue life Y (i.e., logarithm of the number of cycles to failure) can be expressed as [8]:

$$F_Y = F_{Y|surf} F_{X_t} + F_{Y|int} F_{X_l} \left(1 - F_{X_t}\right) \tag{1}$$

where

$F_{Y|surf}$ is the cdf of the fatigue life if crack nucleates superficially (i.e., of the random variable (rv) $Y|surf$),

$F_{Y|int}$ is the cdf of the fatigue life if crack nucleates internally (i.e., of the rv $Y|int$),

F_{X_t} is the cdf of the logarithm of the transition stress (i.e., of the rv X_t),

F_{X_l} is the cdf of the logarithm of the fatigue limit (i.e., of the rv X_l).

F_Y given in Eq. (1) depends on the cdfs of the continuous rvs X_l, X_t, $Y|int$ and $Y|surf$. According to what proposed in the literature [9-12] for the fatigue strength, both X_l and X_t can be assumed as Normal distributed (i.e., the fatigue limit and the transition stress are Log-Normal distributed). In particular, let X_l have mean value μ_{X_l} and standard deviation σ_{X_l}, and X_t have mean value μ_{X_t} and standard deviation σ_{X_t}, then:

$$F_{X_l} = \Phi\left[\frac{x - \mu_{X_l}}{\sigma_{X_l}}\right] \tag{2}$$

and

$$F_{X_t} = \Phi\left[\frac{x - \mu_{X_t}}{\sigma_{X_t}}\right] \tag{3}$$

where

Φ is the standardized Normal cdf,

x denotes the logarithm of the applied stress amplitude.

In the literature [9-11], different types of continuous distribution have been proposed for the number of cycles to failure. Usually, either a 2-parameter Weibull distribution or a Log-Normal distribution are used for the cycles to failure rv. Without loss of generality, the conditional fatigue life is supposed to be Normal distributed (i.e., the conditional number of cycles to failure is Log-Normal distributed). Therefore, suppose that the mean values of $Y|int$ and $Y|surf$ follow the Basquin's law and that the standard deviations are constant for any value of x, then:

$$F_{Y|int} = \Phi\left[\frac{y-\left(a_{Y|int}+x\cdot b_{Y|int}\right)}{\sigma_{Y|int}}\right] \quad (4)$$

and

$$F_{Y|surf} = \Phi\left[\frac{y-\left(a_{Y|surf}+x\cdot b_{Y|surf}\right)}{\sigma_{Y|surf}}\right] \quad (5)$$

where $a_{Y|int}$, $b_{Y|int}$, $a_{Y|surf}$ and $b_{Y|surf}$ are four constant coefficients related to the Basquin's law and $\sigma_{Y|int}$ and $\sigma_{Y|surf}$ denote the standard deviations of $Y|int$ and $Y|surf$, respectively.

Fig. 1 shows a schematic of a Duplex S-N curve together with the statistical distributions assumed in each characteristic region: the surface-nucleation and the internal-nucleation regions are described by a randomly variable fatigue life (Eqs. 4 and 5), while the transition and fatigue-limit regions are described by a randomly variable stress amplitude (Eqs. 2 and 3).

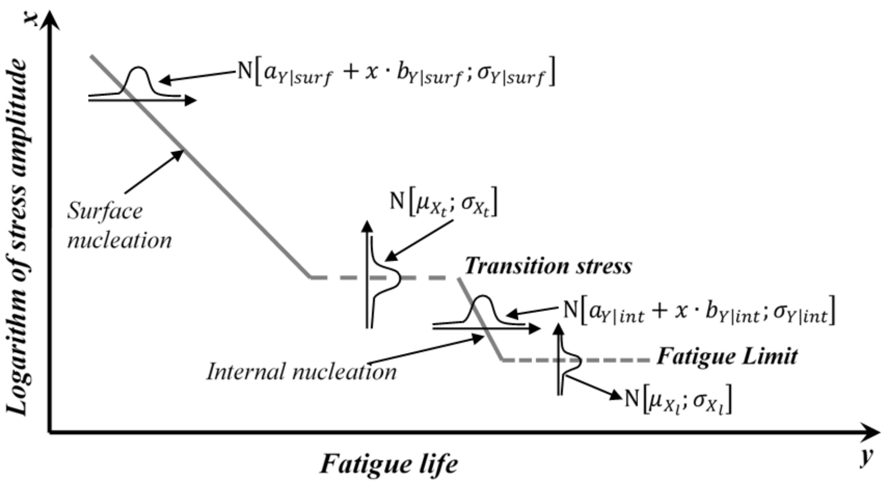

Figure 1: Schematic of a statistical Duplex S-N curve with fatigue limit.

By taking into account Eqs. 2-5, F_Y finally becomes:

$$F_Y = \Phi\left[\frac{y-\left(a_{Y|surf}+x\cdot b_{Y|surf}\right)}{\sigma_{Y|surf}}\right]\Phi\left[\frac{x-\mu_{X_t}}{\sigma_{X_t}}\right]+\Phi\left[\frac{y-\left(a_{Y|int}+x\cdot b_{Y|int}\right)}{\sigma_{Y|int}}\right]\Phi\left[\frac{x-\mu_{X_l}}{\sigma_{X_l}}\right]\left(1-\Phi\left[\frac{x-\mu_{X_t}}{\sigma_{X_t}}\right]\right) \quad (6)$$

with a number of parameters equal to 10.

Transition life: statistical distribution

Statistical estimation of the parameters permits to compute the S-N curves corresponding to different probabilities of failure (quantile S-N curves). Eq. (6) can be exploited for the estimation of the α-th quantile S-N curve: if $F_Y = \alpha$ and

the 10 parameters are substituted with their estimates, Eq. (6) provides the relationship between x and y when the probability of failure equals α, which is the definition of the α-th quantile S-N curve.
Transition stress may vary from one specimen to another and, in a statistical framework, each specimen can be considered as representative of a particular quantile of the transition stress distribution. Similarly, a particular quantile S-N curve hides out each specimen. Therefore, for a given specimen, both the quantile S-N curve and the quantile of the transition stress distribution are uniquely determined. In particular, let the specimen be representative of the α-th quantile S-N curve (i.e., $F_Y = \alpha$) and of the α-th quantile of the transition stress distribution (i.e., $X_t = x_{t,\alpha}$). If the stress amplitude equals the transition stress of the specimen (i.e., if $x = x_{t,\alpha}$), then the fatigue life of the specimen corresponds to the transition life of the specimen (i.e., then $y = y_{t,\alpha}$). Thus, Eq. (6) becomes:

$$\alpha = \Phi\left[\frac{y_{t,\alpha} - \left(\tilde{a}_{Y|surf} + x_{t,\alpha}\cdot\tilde{b}_{Y|surf}\right)}{\tilde{\sigma}_{Y|surf}}\right]\alpha + \Phi\left[\frac{y_{t,\alpha} - \left(\tilde{a}_{Y|int} + x_{t,\alpha}\cdot\tilde{b}_{Y|int}\right)}{\tilde{\sigma}_{Y|int}}\right]\Phi\left[\frac{x_{t,\alpha} - \tilde{\mu}_{X_l}}{\tilde{\sigma}_{X_l}}\right](1-\alpha) \tag{7}$$

where
$\tilde{\cdot}$ denotes a parameter estimate,
$y_{t,\alpha}$ is the α-th quantile of the transition life distribution,
$x_{t,\alpha}$ is the α-th quantile of the transition stress distribution.

It must be noted that the term $\Phi\left[\frac{x_{t,\alpha} - \mu_{X_l}}{\sigma_{X_l}}\right]$ in Eq. (7) is almost equal to one since, according to the hypotheses stated in the definition of the unified statistical model [8], X_t must be larger than X_l (i.e., $x_{t,\alpha} \gg \mu_{X_l}$). By taking into account that $\Phi\left[\frac{x_{t,\alpha} - \mu_{X_l}}{\sigma_{X_l}}\right] \to 1$, Eq. (7) can be reformulated as follows:

$$\alpha = \frac{\Phi\left[\frac{y_{t,\alpha} - \left(\tilde{a}_{Y|int} + x_{t,\alpha}\cdot\tilde{b}_{Y|int}\right)}{\tilde{\sigma}_{Y|int}}\right]}{1 + \Phi\left[\frac{y_{t,\alpha} - \left(\tilde{a}_{Y|int} + x_{t,\alpha}\cdot\tilde{b}_{Y|int}\right)}{\tilde{\sigma}_{Y|int}}\right] - \Phi\left[\frac{y_{t,\alpha} - \left(\tilde{a}_{Y|surf} + x_{t,\alpha}\cdot\tilde{b}_{Y|surf}\right)}{\tilde{\sigma}_{Y|surf}}\right]} \tag{8}$$

Eq. (8) provides an implicit relationship between $y_{t,\alpha}$ and α and, consequently, permits the numerical computation of the statistical distribution of the transition life.

NUMERICAL EXAMPLE

An experimental dataset taken from the literature [13] is analyzed in order to show main characteristics of the statistical distribution of the transition life. The selected experimental data [13] are obtained by testing Ti-6Al-4V titanium alloy specimens and are shown in Fig. 2.
Estimates of the parameter involved in the model given in Eq. (6) can be computed by applying the Maximum Likelihood Principle to the experimental data. Results, obtained with a code developed in Matlab®, are given in the following list:

$$\begin{cases} \tilde{a}_{Y|surf} = 100.20 & \tilde{b}_{Y|surf} = -33.26 & \tilde{\sigma}_{Y|surf} = 0.4639 \\ \tilde{a}_{Y|int} = 40.36 & \tilde{b}_{Y|int} = -11.67 & \tilde{\sigma}_{Y|int} = 0.3280 \\ \tilde{\mu}_{X_t} = 2.8192 & - & \tilde{\sigma}_{X_t} = 0.0023 \\ \tilde{\mu}_{X_l} = 2.7200 & - & \tilde{\sigma}_{X_l} = 0.0059 \end{cases} \tag{9}$$

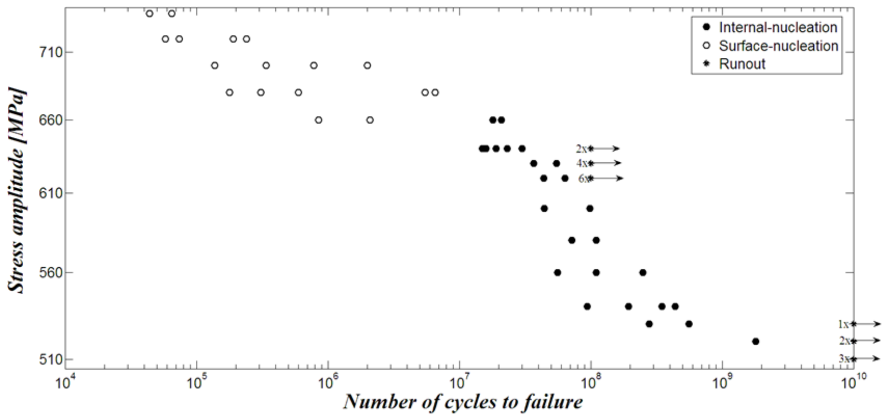

Figure 2: Experimental fatigue data plot [13].

Quantile S-N curves

Parameter estimates given in Eq. (9) can be used for computing quantile S-N curves. In particular, if the α-th quantile S-N curve is of interest, the following equation:

$$\alpha = \Phi\left[\frac{y-\left(\tilde{a}_{Y|surf}+x\cdot\tilde{b}_{Y|surf}\right)}{\tilde{\sigma}_{Y|surf}}\right]\Phi\left[\frac{x-\tilde{\mu}_{X_t}}{\tilde{\sigma}_{X_t}}\right]+\Phi\left[\frac{y-\left(\tilde{a}_{Y|int}+x\cdot\tilde{b}_{Y|int}\right)}{\tilde{\sigma}_{Y|int}}\right]\Phi\left[\frac{x-\tilde{\mu}_{X_t}}{\tilde{\sigma}_{X_t}}\right]\left(1-\Phi\left[\frac{x-\tilde{\mu}_{X_t}}{\tilde{\sigma}_{X_t}}\right]\right) \qquad (10)$$

must be solved with respect to y for different values of x. Fig. 3 shows the S-N plot together with the 10%, 50% and 90% quantile S-N curves.

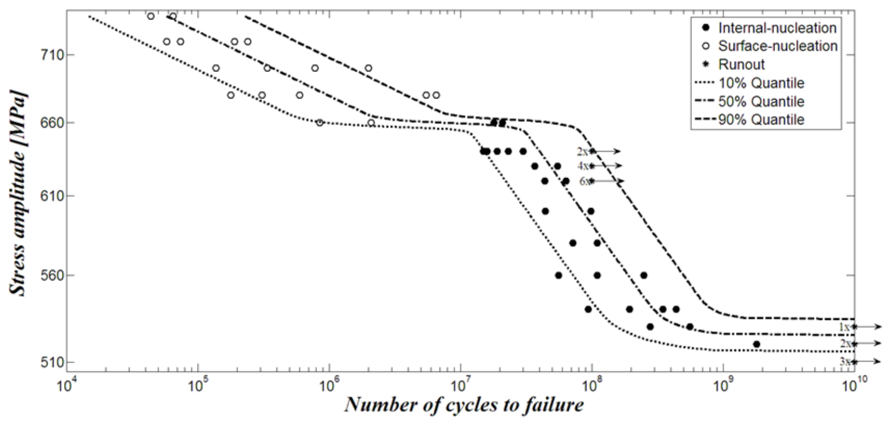

Figure 3: Quantile S-N curves.

As shown in Fig. 3 the region between the 10% and 90% quantile S-N curves includes about the 86% (which is close to the expected 80%) of the failure data; while the 50% quantile S-N curve is almost median between failure data at each stress amplitude.

Statistical distribution of the transition life

If parameters are substituted by their estimates, Eq. (8) can be used for numerically computing the statistical distribution of the transition life. To this aim, Eq. (8) must be solved with respect to $y_{t,\alpha}$ for different values of α ranging from zero to one. It is worth noting that for a given value of α, $x_{t,\alpha}$ in Eq. (8) is a known quantity and is equal to $\tilde{\mu}_{X_t}+z_\alpha\tilde{\sigma}_{X_t}$, being z_α the α-th quantile of a standardized Normal distribution. Fig. 4 shows the computed statistical distribution of the transition life.

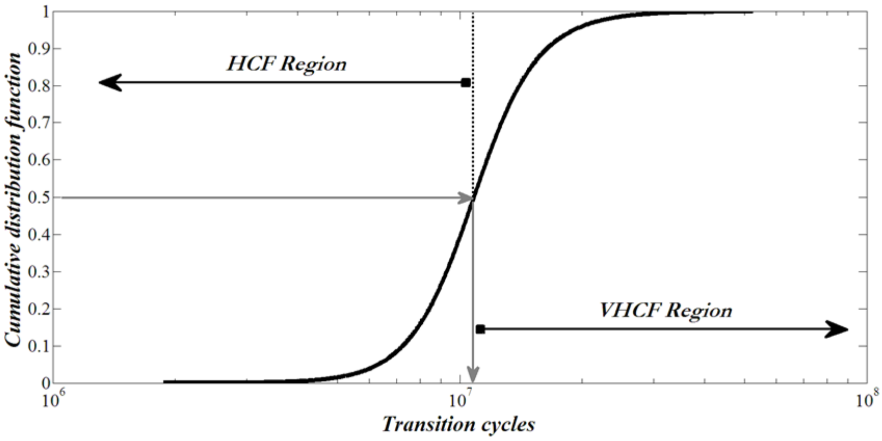

Figure 4: Statistical distribution of the transition life together with the probable fatigue regions.

As shown in Fig. 4, the median of the transition life can be usefully considered to discriminate between the two fatigue regions of HCF and VHCF: failures that occur at a number of cycles smaller than the median more probabilistically belong to the HCF region; while failures that occur at a number of cycles larger than the median more probabilistically belong to the VHCF region. For the analyzed case, the median of the transition life is equal to 7.035, which results in a median transition cycle equal to $1.08 \cdot 10^7$. As visible in Fig. 5, the median value properly differentiates between the two fatigue regions: each internally nucleated failure is above the median value, while each superficially nucleated failure is below the median value.

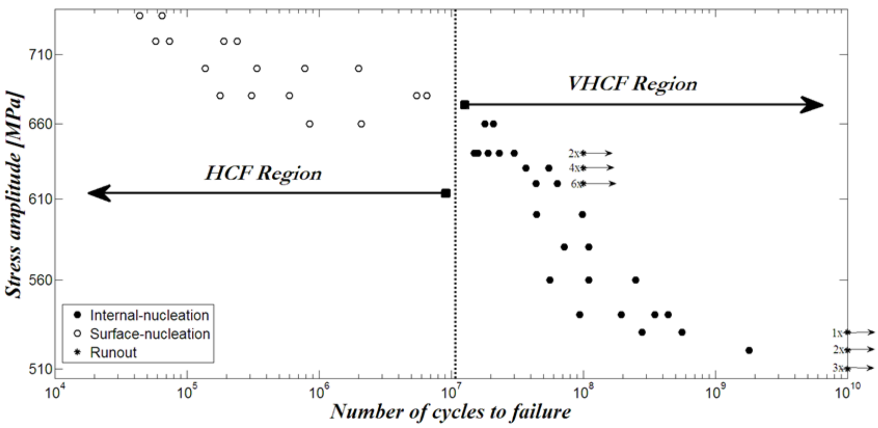

Figure 5: Experimental data and probable fatigue regions as discriminated by the median value of the transition life.

CONCLUSIONS

A procedure for the estimation of the statistical distribution of the transition life in a Duplex S-N curve was shown. The statistical distribution was estimated by numerically solving an equation which correlates the cumulative distribution function to the quantile of the distribution. As shown with a numerical example taken from the literature, the resulting distribution depends on the distance between the HCF and the VHCF regions and on the distribution of the random transition stress. The estimated distribution can be effectively used to predict, with a specified confidence level, the number of cycles for which an internal nucleation may probabilistically occur in a VHCF test and it is also informative for properly choosing the end of HCF tests in terms of number of cycles.

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