Focussed on: Fracture and Structural Integrity related Issues

Integrated assessment procedure for determining the fracture strength of glass components in CSP systems

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ABSTRACT. The structural integrity and reliability of glass components are key issues for concentrated solar power (CSP) systems. For example, the glass windows in a solar furnace may suffer catastrophic fracture due to thermal and structural loadings, including reaction chamber pressure cycling. Predicting design strength provides the basis for which the optical components and mounting assembly can be designed so that failure does not occur over the operational lifetime of a given CSP system.
The fracture strength of brittle materials is dependent on the size and distribution of cracks or surface flaws. Due to the inherent brittleness of glass resulting in catastrophic failure, conservative design approaches are currently used for the development of optical components made of glass, which generally neglect the specific glass composition as well as subcritical crack growth, surface area under stress, and nature of the load – either static or cyclic – phenomena.
In this paper, several methods to characterize the strength of glass are discussed to aid engineers in predicting a design strength for a given surface finish, glass type, and environment. Based on the Weibull statistical approach and experimental data available on testing silica glass rod specimens, a theoretical model is developed for estimating their fracture strength under typical loading conditions. Then, an integrated assessment procedure for structural glass elements is further developed based on fracture mechanics and the theory of probability, which is based on the probabilistic modelling of the complex behaviour of glass fracture but avoids the complexity for calculation in applications. As an example, the design strength of a glass window suitable for a solar furnace reaction chamber is highlighted.

KEYWORDS Probabilistic model; Fracture strength; Structural glass; Reliability; Concentrated solar power (CSP).

INTRODUCTION

Improvements in production and refining technologies such as tempering and the production of laminated glass enabled glass to carry more substantial superimposed loads and therefore achieve a more 'structural' role [1]. Especially, the glass components play important role in some new energy industries such as the concentrated solar power (CSP) systems, etc. This gives impetus to studies on the mechanical behaviour of these materials and, in particular on their ability to resist fracture.

Glass failure is the consequence of the growth of flaws, its behaviour strongly depends on the surface condition as well as on the environmental conditions and the thermal and mechanical loading history to which they are exposed to. Due to the much more scatter in the data of glass materials, very large safety factors are often used in glass element design, up to 8 or more. These large safety factors are somewhat arbitrary and not satisfactory, because it is not very clear what the true factor of safety really is.

In recent years, considerable research efforts have been paid to improve the understanding of the load-carrying behaviour of structural glass elements, and many new design approaches have been proposed to improve the safety and serviceability of the structural glasses [2-6].

In 1972, Brown [7] proposed the "Load Duration Theory" (LDT), which combined the static fatigue theory of Charles & Hillings [8] with the statistical failure probability function proposed by Weibull [9]. In 1974, Evans [10] developed the "Crack Growth Model" (CGM) on the basis of the principles of Linear Elastic Fracture Mechanics. This method makes use of the empirical description of the sub-critical propagation of cracks (deduced from the experimental relationship between crack growth rate and stress intensity factor K_I) together with the Weibull failure probability under the hypotheses that a sub-critical crack growth takes place in all surface micro-cracks. Fishercripps & Collins [11] proposed a modified crack growth model, which is able to predict failure probabilities for both short and long term stresses [11].

Fernandes & Rosa [12, 13] presented a review on the "ring-on-ring" and "piston-on-3-ball" equibiaxial tests for ceramics and glasses, stress distributions in the test pieces were analysed, the importance of the effect of friction at the contact zones was discussed. Based on the Weibull statistics and experimental data obtained from testing silica glass rod specimens with diameters between 0.5 and 1 mm [14], a theoretical model was developed for estimating their fracture strength under different loading conditions [15]. By this method, the test results of strength from one testing type can be extrapolated to other test types, such as the uniaxial tension, 3-point bending, 4-point bending, etc. Besides, Rosa et al [16] studied the subcritical crack growth in three engineering ceramics under biaxial conditions, the results from the ring-on-ring tests were compared with 4-point bending tests.

In 2001, Porter [6] proposed the Crack Size Design method (CSD); and in 2006 Haldimann [4] developed the Lifetime Prediction Model (LPM) where he calculated directly the failure probability of a glass element starting from the probability distribution of its defects and from the deterministic knowledge of loading time-history [4].

Recently, Santarsiero and Froli [2] formulated a new semi-probabilistic failure prediction method, called "Design Crack Method" (DCM), defining a new quantity called Design Crack, which takes into account of the probability of failure and the surface damaging level.

Moreover, it is still a major concern to extrapolate the laboratory test results to applications for components under in-service conditions. A number of effects have to be considered, such as the size effect, the gradient effect or notch size effect, and multi-axial stress effect, etc. In Ref. [17], the extension of the weakest-link model to multiaxial stress states was verified by comparing fracture stress distributions obtained in four-point bending and in a concentric ring-on-ring test, and it was discussed about how the selected failure criterion influences the predicted distribution of the fracture stress of a component.

Danzer et al [18] presented a new method for biaxial strength testing of brittle materials, the so-called ball on three balls (B3B) test method. A detailed analysis of the stress field in the specimens and of possible measuring errors were studied. The B3B-testing method has several advantages compared to common three or four-point bending tests and the ring-on-ring tests.

From the above brief review of literature, it is shown that the mechanical behaviour of glass at breakage is very complex, more and more theoretical models as well as experimental methods have been developed. However, for engineering applications, the complexity of calculation procedures needs to be simplified reasonably. The motivation for this present work is to develop an integrated approach for analyzing the crack problem of the glass components in the CSP industry, to incorporate the probabilistic modelling, the principles of fracture mechanics and the details of the specific design in question.

MECHANICAL CHARACTERIZATION OF GLASSES FOR USE IN STRENGTH FORECASTING

The most commonly used mathematical representation of the relationship between applied stress and probability of survival for glasses is the two parameter Weibull distribution as defined [19]:

$$P_s = \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right] \tag{1}$$

where σ is the applied stress; P_s is the corresponding probability of survival; σ_0 is the characteristic strength at which 63.2% of the test specimen will break; and m is the Weibull modulus, which is a measure of the amount of scatter in the distribution (the shape parameter); small values of m imply wide variations in strength, whereas large values imply more consistent strength values.

Theoretical models were developed by Rosa et al [13-16] for estimating the fracture strength of brittle materials (such as ceramics and glasses, etc) under different typical loading conditions. The probability of survival P_s for glasses in a stressed volume V can be calculated as [12]:

$$P_s = \exp\left[-\int_V \left(\frac{\sigma}{\sigma_0}\right)^m dV\right] \tag{2}$$

The application of Eq. (2) to uni-axial tension testing stress, σ_t, yields:

$$P_s = \exp\left[-V\left(\frac{\sigma_t}{\sigma_0}\right)^m\right] \tag{3}$$

The above Eq. (3) can be expressed in the following linear equation, which facilitates to fit the Weibull parameters from test results:

$$\ln\left(\ln\left(1/P_s\right)\right) = \ln V + m\left(\ln\sigma_t - \ln\sigma_0\right) \tag{4}$$

Similarly, the application of Eq. (2) for 4-point bending testing stress, σ_{4p}, yields:

$$P_s = \exp\left[-\left(\frac{\sigma_{4p}}{\sigma_0}\right)^m \frac{V(m+2)}{4(m+1)^2}\right] \tag{5}$$

The above Eq. (5) can be expressed in the following linear equation, which facilitates to fit the Weibull parameters from test results:

$$\ln\left(\ln\left(1/P_s\right)\right) = \ln\left[\frac{V(m+2)}{4(m+1)^2\,\sigma_0^m}\right] + m\ln\sigma_{4p} \tag{6}$$

In the same way, the application of Eq. (2) for 3-point bending testing stress, σ_{3p}, yields:

$$P_s = \exp\left[-\left(\frac{\sigma_{3p}}{\sigma_0}\right)^m \frac{V}{2(m+1)^2}\right] \tag{7}$$

The above Eq. (7) can be expressed in the following linear equation, which facilitates to fit the Weibull parameters from experimental data:

$$\ln\left(\ln\left(1/P_s\right)\right) = \ln\left[\frac{V}{2(m+1)^2\,\sigma_0^m}\right] + m\ln\sigma_{3p} \tag{8}$$

The Weibull effective volume or surface can be used to scale ceramic and glass strengths from one component size to another, or from one loading state to another. Larger specimens or components are weaker, because of the bigger probability of containing larger and more critical flaws. The Weibull weakest-link model leads to a strength dependency on component size [12]:

$$\frac{\sigma_1}{\sigma_2} = \left(\frac{V_{E2}}{V_{E1}}\right)^{1/m} \tag{9}$$

where σ_1 and σ_2 are the mean strengths of specimens of type 1 and 2 (which may have different sizes and stress distributions), V_{E1} and V_{E2} are the effective volumes, and m is the Weibull modulus.
Similarly, the following relationships can be derived from Eq. (4), (6) and (8):

$$\frac{\sigma_{3p}}{\sigma_t} = \left[2(m+1)^2\right]^{1/m} \tag{10}$$

$$\frac{\sigma_{4p}}{\sigma_t} = \left[\frac{4(m+1)^2}{m+2}\right]^{1/m} \tag{11}$$

The above two Eq. (10) and (11) confirmed that the bending strength is higher than the tension strength. If the Weibull modulus m is equal to 10, the 3-point bending strength is 1.45 times the tension strength, and the 4-point bending strength is 1.73 times the tension strength.
In view of the fact that the Weibull modulus m is usually assumed to be a constant for a given material, only the characteristic strength σ_0 is needed to be extrapolated from laboratory specimen test data to components. For a component with a varying stress field σ, an effective surface area, A_{eff}, may be computed using the following relationship:

$$A_{eff} = \int \left(\frac{\sigma}{\sigma_{\max}}\right)^m dA \tag{12}$$

Then the characteristic strength σ_0 for the component can be calculated from the data of specimen as:

$$\frac{\sigma_0^{component}}{\sigma_0^{specimen}} = \left(\frac{A_{specimen}}{A_{component}}\right)^{1/m} \tag{13}$$

In service, the components are generally subjected to multiaxial loading conditions, hence, we need to analyze the effect of multi-axial tensile stresses on flaws and determining one equivalent stress based on the selected multiaxial criterion. Then, the equivalent stress can be assumed to be the applied stress σ in the above equations.
Glasses can demonstrate a loss of strength over time. This phenomenon is a kind of stress corrosion and it is known as static fatigue of glass. Chemical attack by water vapour (or other media) permits a pre-existing flaw to grow to critical dimensions and cause spontaneous crack propagation as shown in the following Fig. 1.

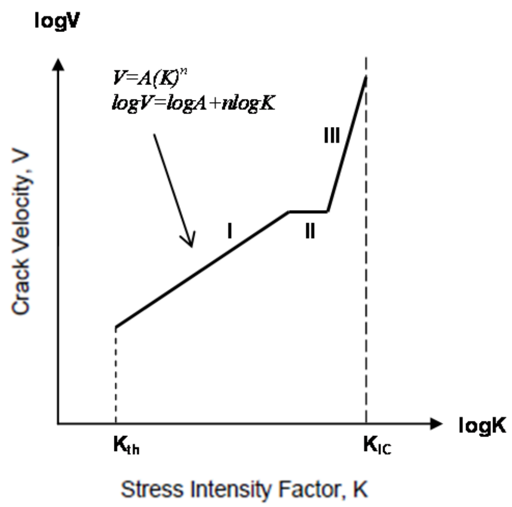


Figure 1: Regions of a typical log$V$ versus log$K$ plot.

Crack propagation velocity V is usually indicated in m/s. Region I is of primary interest since it represents the main duration of stable crack growth, which may be expressed as a power law:

$$V = A(K)^n \tag{14}$$

where A and n are parameters which depend on the material and the stress-corrosion conditions, they can be determined from experimental data.

INTEGRATED ASSESSMENT PROCEDURE FOR STRUCTURAL GLASS COMPONENT DESIGN

Following the above process of characterizing the mechanical properties of glasses, one integrated analysis procedure is developed in this section for the component design of glasses, which consists of two major steps:
Step1: analyse the maximum tensile stresses in the component by the finite element method, taking into account the multiaxial loading conditions and the contact stresses between the glass component and the parts for mounting the glass.
Step 2: transform the Weibull CDF (Cumulative Distribution Function that was adjusted to data obtained from specimens´ testing) for predicting the survival probabilities for the application conditions with different load type, load duration, and surface area.
The glass-mount contact can be approximated by Hertzian contact of a cylinder on a flat glass surface. The contact half-width b can be expressed as [20]:

$$b = \sqrt{\frac{4PR^*}{\pi E^*}} \tag{15}$$

where $E^* = \left(\frac{1-\nu_m^2}{E_m} + \frac{1-\nu_g^2}{E_g}\right)^{-1}$ is the effective modulus, $R^* = \left(\frac{1}{R_m} + \frac{1}{R_g}\right)^{-1}$ is the contact radius, Young's moduli E_m and E_g ,and Poisson ratios ν_m and ν_g are for the mounting part and the glass part, respectively.

The stress distributions at the glass-mount contact area can be analysed using the equations derived in [20], compressive stresses occur in the zone just beneath the contact area, and the maximum tensile stress, $\sigma_{t,max}$, occurs on the glass surface just outside the contact area, which can be derived as:

$$\sigma_{t,max} = \frac{2(1-2\nu_g)P}{3\pi b} \tag{16}$$

where P is the loading force, b is the contact half-width, ν_g is the Poisson ratio of glass.
Since the maximum tensile stress (the first principal stress) is critical for brittle materials, it is assumed as the applied stress in the following evaluation procedure.
From Eq. (1) the stress at fracture σ_{IC} can be calculated as:

$$\sigma_{IC} = \sigma_0 \left(\ln\left(\frac{1}{P_s}\right)\right)^{1/m} \tag{17}$$

The initial stress-intensity K_{Ii} at the crack tip may be estimated as:

$$K_{Ii} = K_{Ic}\left(\frac{\sigma}{\sigma_{Ic}}\right) \tag{18}$$

Substituting Eq. (17) into Eq. (18) yields:

$$K_{Ii} = K_{Ic}\left(\frac{\sigma}{\sigma_0}\right)\left(\ln\frac{1}{P_s}\right)^{1/m} \tag{19}$$

In order to take into account the strength reduction of the glass material over time due to subcritical crack growth, analytical expressions had been derived from the V versus K data. The total time-to-failure, t_s, of a component under a constant static stress with a known flaw was expressed as [5]:

$$t_s = \frac{2}{\sigma^2 Y^2}\int_{K_{Ii}}^{K_{IC}}\left(\frac{K_I}{V}\right)dK_I \tag{20}$$

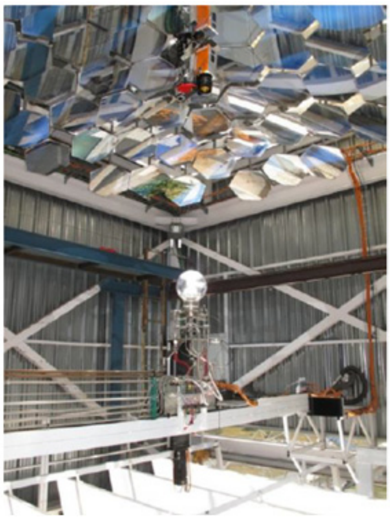
where K_{Ii} is the initial stress intensity factor, and V is the crack velocity. If a power law for the crack velocity (Region I of stable crack growth) is assumed, as shown in Eq. (14), then the Eq.(20) can be expressed as:

$$t_s = 2\left(K_{Ii}^{2-n} - K_{Ic}^{2-n}\right)/\left[(n-2)A\sigma^2 Y^2\right] \tag{21}$$

Therefore, one design strength diagram may be created from the above procedures and the time-to-failure versus equivalent stress curves as a function of the survivability probability (or reliability) can be shown in the diagram, which will be very helpful for the design processes as presented in the following section.

DESIGN STRENGTH EXAMPLE

Fig. 2 shows some examples of glass components of CSP systems used in the Plataforma Solar de Almeria (PSA). These glass windows are mounted inside a pressure vessel filled with argon gas at specified operating pressure, which may suffer catastrophic fracture due to thermal and structural loadings, including reaction chamber pressure cycling. We desire to accurately predict the state of stress created in the surface of the glass, and design the glass components for safety operation under severe thermal and mechanical loading conditions.



(a)

(b)

Figure 2: Examples of glass components of CSP systems at PSA-CIEMAT. a) Aspect of the solar concentrator and reaction chamber of SF-5 solar furnace; b) Aspect of the MiniVac chamber installed at the focal spot of the new SF-40 solar furnace; a 45º inclined mirror above the MiniVac allows the sunrays to reflect vertically.

During the operations, fracture problem occurred due to the severe thermal and mechanical loading conditions as shown in Fig. 3 for one example, crack initiated from the zone close to glass-mount contact area, where the tensile stresses were generated.

Fig. 4 shows the simulated deformation results of the glass component by ANSYS software, where the blue colour represents the deformed shape of the glass section due to the temperature and pressure loadings. It is shown that the zone near the glass-mount contact area had larger deformation, which explained why the crack initiated from there.
Based on the calculated maximum tensile stress in the glass component, the design strength diagram can be generated following the integrated assessment procedure as described in the above sections, which is shown in the Fig. 5. The time-to-failure versus the maximum tensile stress curves as a function of the probability of survivability (or reliability) is shown in the diagram, which will be very helpful for the design improvement of the glass component.

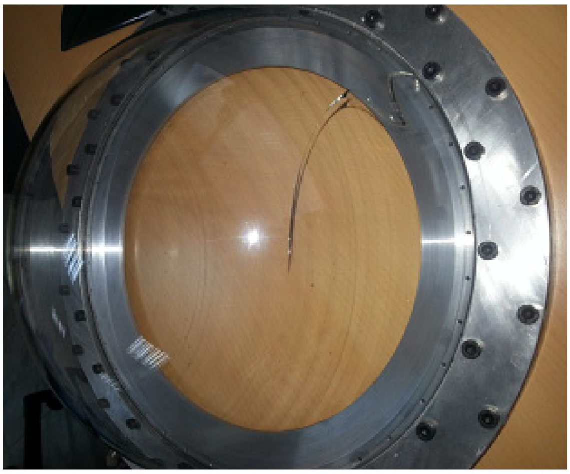


Figure 3: One example of the fracture of glass component; crack initiated from the zone close to glass-mount contact area.

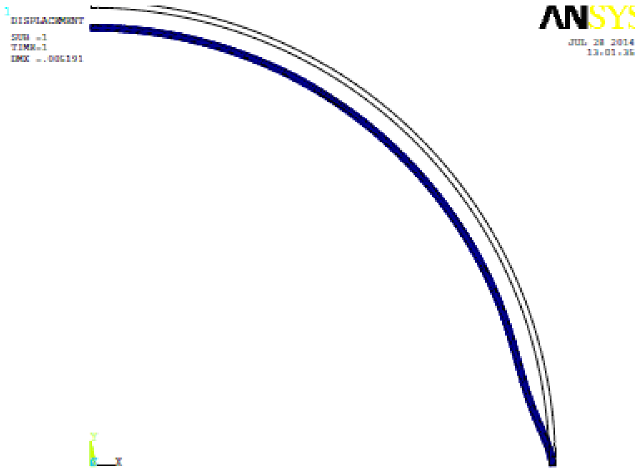


Figure 4: Glass section deformation simulated by FEM.

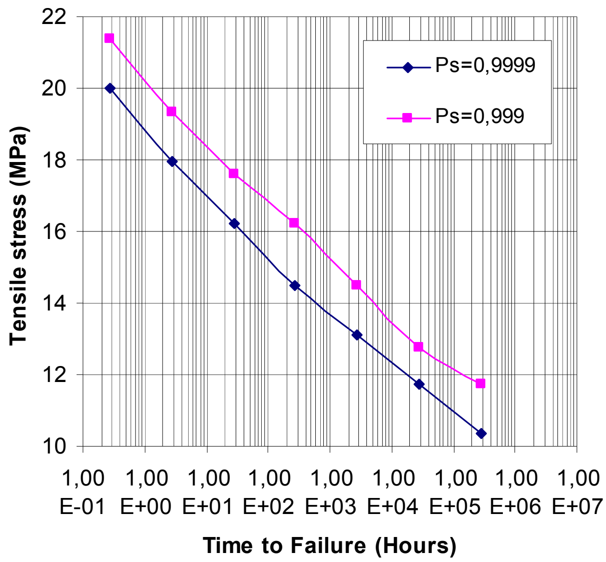


Figure 5: Design strength diagram, the time-to-failure versus the maximum tensile stress curves as a function of the probability of survivability (or reliability) P_s.

CONCLUSIONS

With the increasing applications of the glass and ceramic components in the new energy industries such as the CSP systems, it is imperative to develop advanced design methods for the safety and reliability of these components, which are quite different from the metal components. The integrated assessment procedure proposed in this paper is based on the probabilistic modelling of the complex behaviour of glass fracture but avoids the complexity for calculation in applications. With demonstrative examples, it is shown that the procedure is effective for analyzing the fracture problem of the glass components and helpful for design improvement.

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