



Dynamic additional loads influencing the fatigue life of gears in an electric vehicle transmission

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ABSTRACT. In recent years the implementation of the electric engine in the automotive industries has been increasingly marked. The speed of the electric motors is much higher than the combustion engine ones, bringing transmission gears to be subjected to high dynamic loads.

For this reason the dynamic effects on fatigue life of these components have been taken into account in a more careful way respect to what is done with the usual gears.

In the present work the overload effects due to both speed and meshing in a gear couple of an electric vehicle transmission have been analyzed. The electric vehicle is designed for urban people mobility and presents all the requirements to be certified as M1 vehicle (a weight less than 600 kg and a maximum speed more than 90 Km/h).

To investigate the overload effects of teeth in contact, the reference gear design Standards (ISO 6336) introduce a specific multiplicative factor to the applied load called Internal Dynamic Factor (K_v).

Aim of this work is to evaluate how dynamic overloads may influence the fatigue life of the above quoted gears in term of durability.

To this goal, K_v values have been calculated by means of the analytical equations (ISO 6336 Methods B and C) and then they have been compared with the results coming from multibody simulations, involving full rigid and rigid-flexible models.

KEYWORDS. Multibody simulations; Electric vehicle; Gear design.

INTRODUCTION

In recent years, greater emphasis has been placed on the design of high-speed, lightweight, precision systems. Both performance analysis and design of such systems can be greatly enhanced through transient dynamic simulations. The need for a better design has made necessary the insertion of many factors that have not been considered in the past. Systems such as engines, robotics, machine tools and space structures may operate at high speeds and in very high temperature environments.

In particular, the implementation of the electric engine in the automotive industries has been increasingly marked. The speed of the electric motors is much higher than the combustion engine ones, bringing transmission gears to be subjected to high dynamic loads.

For this reason the dynamic effects on fatigue life of these components have been taken into account in a more careful way respect to what is done with the usual gears.

So, the behaviour of new automotive transmissions may be simulated with specific multibody programs, being these systems suitable to be described as a collection of subsystems called bodies and the motion of them is kinematically constrained. Basic to any analysis of multibody mechanics is the understanding of the motion of subsystems (bodies or components).

Referring to this environment, a gear is a complex body with a specific geometry, as an example the modulus, the number of teeth, the face width and the contact ratio.

Once a gear is designed as a function of the power to be transmitted, it is necessary to analyse it from both kinematic and dynamic point of view [1].

Some papers have been published about the numerical analysis of the dynamic meshing gears. Most of them are based on lumped parameter models [2-5], where the teeth contact is represented by an equivalent mass, a stiffness and damping of the teeth.

Another way to address the contact analysis is related to FEM simulations; this approach is helpful to determine the gear contact stiffness and the corresponding strain and stress distributions in the teeth. In particular the multibody kinematic approach uses a contact stiffness which depends on parameters that are not yet well understood [6].

In the present work the overload effects due to both speed and meshing in a gear couple of an electric vehicle transmission have been analyzed. The electric vehicle is designed for urban people mobility and presents all the requirements to be certified as M1 vehicle (a weight less than 600 kg and a maximum speed more than 90 Km/h).

The first aim of this research is to evaluate how the dynamic overloads may influence the fatigue life of the above quoted gears in term of durability.

To this goal, a specific multiplicative factor to the applied load, called Internal Dynamic Factor (K_v), involved in ISO 6336 Standards [7-10] has been taken into account; K_v values have been calculated by means of the related analytical equations (ISO 6336 Methods B and C [7]). Then, the dynamic analysis of the above quoted two gears has been performed by means of the multibody software RecurDyn (Functionbay), in order to obtain the real response of the components. The model has been developed by tuning step by step the contact parameters of engaging pair of gears and by optimizing the integration parameters.

Forces values obtained by the simulations have been utilised to determine the dynamic factor K_v involved in ISO Standard 6336 [7] for the calculation of gears load capacity.

Analytical results in terms of K_v values have been compared with those coming from multibody simulations, involving full rigid and rigid-flexible models.

GEAR MODEL

Object of this paper consists of two specific gears of a differential architecture for an electric vehicle. Its design has been developed in order to keep as low as possible the transmission weight; in this work, a single speed stage gearbox has been chosen.

Transmission is constituted by an ordinary gear system and an epicyclical one named differential (see Fig. 1). The ordinary gear system consists of four helical gears, the differential of four bevel gears (two pinions and two suns).

From the dynamic point of view, only the ordinary gears system have to be taken into account; the transmitted torque passes directly to the wheel axes through the pinion and the vehicle has been analysed in a rectilinear motion condition.

Fig. 1 shows the complete transmission model that has been set up for the simulation.

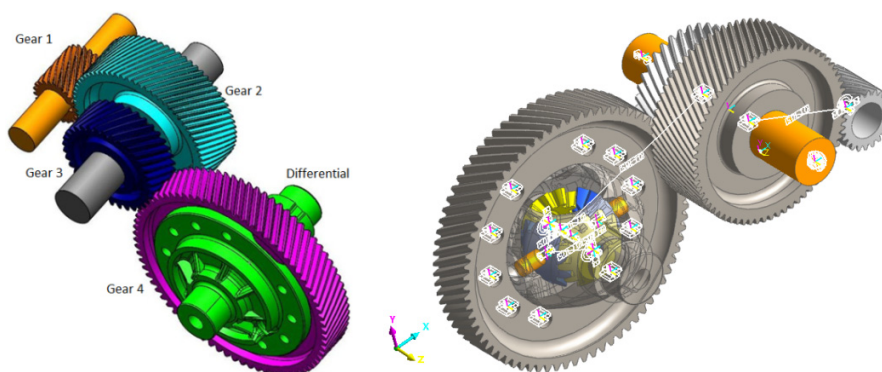


Figure 1: complete transmission model.



The main design data of the four helical gears are reported in Tab. 1 (engagement 1, Gears 1 and 2; engagement 2, Gears 3 and 4).

All gears have been calculated according to ISO 6336 Standard [7-10].

In this work the engagement 2 has been considered (gears 3 and 4).

		Gear 1	Gear 2	Gear 3	Gear 4
Pressure angle	α	20	20	20°	20°
Helix angle	β	30 (L.H.)	30 (R.H.)	30° (R.H.)	30° (L.H.)
Normal module	m_n	1.5	1.5	2 mm	2 mm
Number of teeth	Z	22	68	34	75
Face width	b	42	42	30 mm	30 mm

Table 1: Main design dimensions of gear's engagement 1 (input drive) and engagement 2 (final drive)

MULTIBODY MODEL

The simulation of meshing gears has been analysed (see Fig. 2). To obtain a correct simulation of this complex system, a lot of preliminary tests has been done in order to tune both kinematic and dynamic parameters. The final goal is to achieve a good matching between the multibody response and the corresponding theoretical one.

In this paper two different type of contact have been realized; the first (Fig. 2 on the left) consists of two rigid bodies in contact, the second (Fig. 2 on the right) consists of two rigid-flexible bodies, simulating the gears web as a rigid body and the rim as flexible.

Since the system is very complex, the computational time is very high. So, it was necessary to properly calibrate the simulation before its starting. The best compromise between parameter's calibration and the computational time has been searched.

Contact parameters have been changed from the default ones because the consequent increase in computation time is widely justified by the obtained improvement in the quality of the results.

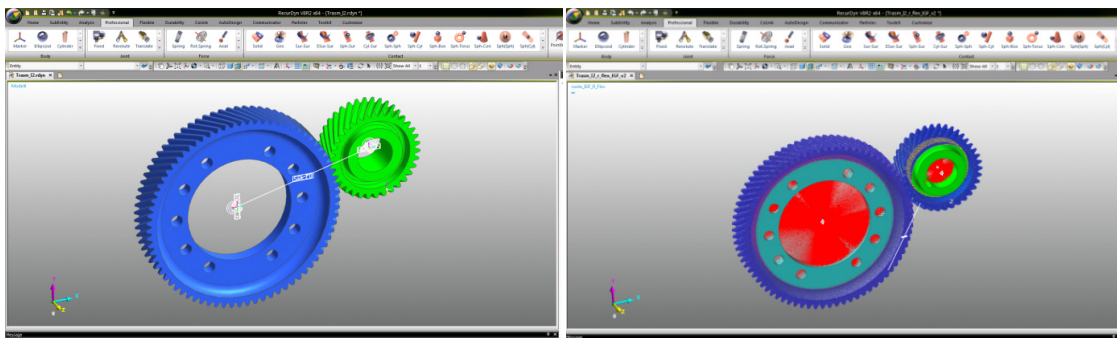


Figure 2: Multibody automotive transmission system, full-rigid on the left and rigid-flexible on the right.

GEARS INTERNAL DYNAMIC FACTOR

Standard design procedures compare the gears strength with a calculated stress value, in both bending and pitting cases. The most common approach used in calculation of these stress values, as widely described in ISO Standard 6336 part 1 [7], involves the use of influence factors, derived from results of research and field service.

The influence factors may be distinguished between two categories, factors which are determined by gear geometry or which have been established by convention and factors which account for several influences and which are dealt with as independent of each other; in this last group is included the Internal Dynamic Factor K_v . As a matter of fact, even when the input torque and speed are constant, significant vibration of the gear masses and resultant dynamic tooth loads can exist. These loads result from the relative displacements between the mating gears as they vibrate in response to an excitation known as transmission error.



The internal dynamic factor makes allowance for the effects of gear tooth accuracy grade as related to speed and load. High accuracy gearing requires less derating than low accuracy gearing. It is generally accepted that the internal dynamic load on the gear teeth is influenced by both design and manufacturing.

“Perfect” gears are defined as having zero quasi-static transmission error at the nominal transmitted (design) mesh torque. They can only exist for a single load and, with proper modifications, have zero dynamic effects zero transmission error (perfect conjugate action), zero excitation, no fluctuation at tooth mesh frequency and no fluctuation at rotational frequencies. With zero excitation from the gears, there is zero response at any speed [7].

The Internal Dynamic Factor K_v takes into account the effects due to the rotating masses; ISO 6336 part 1 [7] suggests three methods for calculating this factor.

Method A (K_{v-A}) derives from the results of full scale load tests, precise measurements or comprehensive mathematical analysis of the transmission system and all gear and loading data shall be available, then this method, in this work, corresponds to the dynamic multibody analysis results. Method A generally results the most sophisticated.

Method B (K_{v-B}) is suited for all types of transmission, spur and helical gearing with any basic rack profile and any gear accuracy grade and, in principle, for all operating conditions.

Method C (K_{v-C}) supplies average values which can be used for industrial transmissions and gear systems with similar requirements, with restriction in the application field.

In the present paper Methods B (K_{v-B}) and C (K_{v-C}) have been taken into account as the resolution of the corresponding equations described in detail in [7].

K_{v-B} coefficient has been calculated on the basis of the different operating ranges (subcritical, main resonance, intermediate and supercritical ranges) related to the resonance ratio N of the mating gears [7].

Firstly, the resonance running speed of the gear pair n_{E1} has been determined as a function of the reduced gear pair mass per unit face width, of the mesh stiffness and of the pinion number of teeth, as indicated in detail in [7].

Then the resonance ratio N , where $N = n_1 / n_{E1}$, has been calculated, being n_1 is the rotational speed of the pinion in rpm.

Once the resonance ratio N has been obtained, the operating range has been determined as a function of the specific load $F_t \cdot K_A / b$, according to [7], where F_t is the tangential load, K_A the application factor, b the gear face width and N_s is the lower limit of the main resonance range.

The dynamic factor K_{v-B} has been computed for the different ranges, following the involved relationships indicated in [7]; non-dimensional parameters which take into account the effect of tooth deviations and profile modifications on the dynamic load are considered.

In particular, it has been calculated for: subcritical range $N \leq N_s$ (the majority of industrial gears operate in this range), main resonance range $N_s < N \leq 1.15$ (operation in this range should generally be avoided, especially for spur gears with unmodified tooth profile or helical gears of accuracy grade 6 or coarser, because the dynamic forces could be very high), supercritical range $N \geq 1.5$ (most high precision gears used in turbine and other high speed transmissions operate in this range), intermediate range $1.15 < N < 1.5$ (the dynamic factor is calculated by linear interpolation between K_v at $N = 1.15$ and K_v at $N = 1.5$).

According to method C [7], the dynamic factor K_{v-C} has also been obtained as a function of the specific load (as K_{v-B}), of the accuracy grades for spur and helical gears and of the tangential speed, without any attention to the operating ranges.

RESULT AND DISCUSSION

Obtained results are presented in this paper in terms of comparison between K_v values related to the different calculation methods.

For as concerns the Methods BISO 6336 [7], specific loads, resonance ratios, lower limits of main resonance range and corresponding K_{v-B} values for each electric motor speed are reported in Tab. 2; in particular, in column one is indicated the motor speed in rpm, in the second column is reported the specific loading of the gears, in the third one the resonance ration, in the fourth one the resonance ratio in the main resonance range and in the last one the internal dynamic factor of gear 4.

For as concerns Method C ISO 6336[8], since the specific load is always lower than 100 N/mm for each class, the dynamic factor is a function of the tangential speed only. So, for the maximum tangential speed $v = 12.02$ m/s, K_v is equal to 1.21.

Multibody Simulation allows to determine contact forces involved in the calculation of dynamic parameters.

The contact force components versus time of Gear 4, obtained by the rigid simulation, are shown in Fig. 3; the forces directions are referred to the ground system of the model.



The response of the elements (Fig. 3) emphasizes an initial perturbation of the signal, but very quickly the behaviour of the system becomes stable and leads to the theoretical values.

Calculated averaged values are: force in x direction $F_x=157.18$ N, force in y direction $F_y=-198.68$ N and force in z direction $F_z=-134.69$ N.

Electric motor rpm	$F_t \cdot K_A/b$	N	N_s	K_v
500	71.85	0.02	0.80	1.01
1000	71.85	0.05	0.80	1.02
2000	71.85	0.09	0.80	1.05
3000	71.85	0.14	0.80	1.07
4000	65.32	0.19	0.78	1.10
5000	58.79	0.24	0.77	1.14
6000	45.72	0.30	0.74	1.20
7000	39.19	0.35	0.72	1.27
8000	35.93	0.41	0.71	1.33
9000	29.39	0.47	0.69	1.44

Table 2: K_v values in function of the electric motor speed.

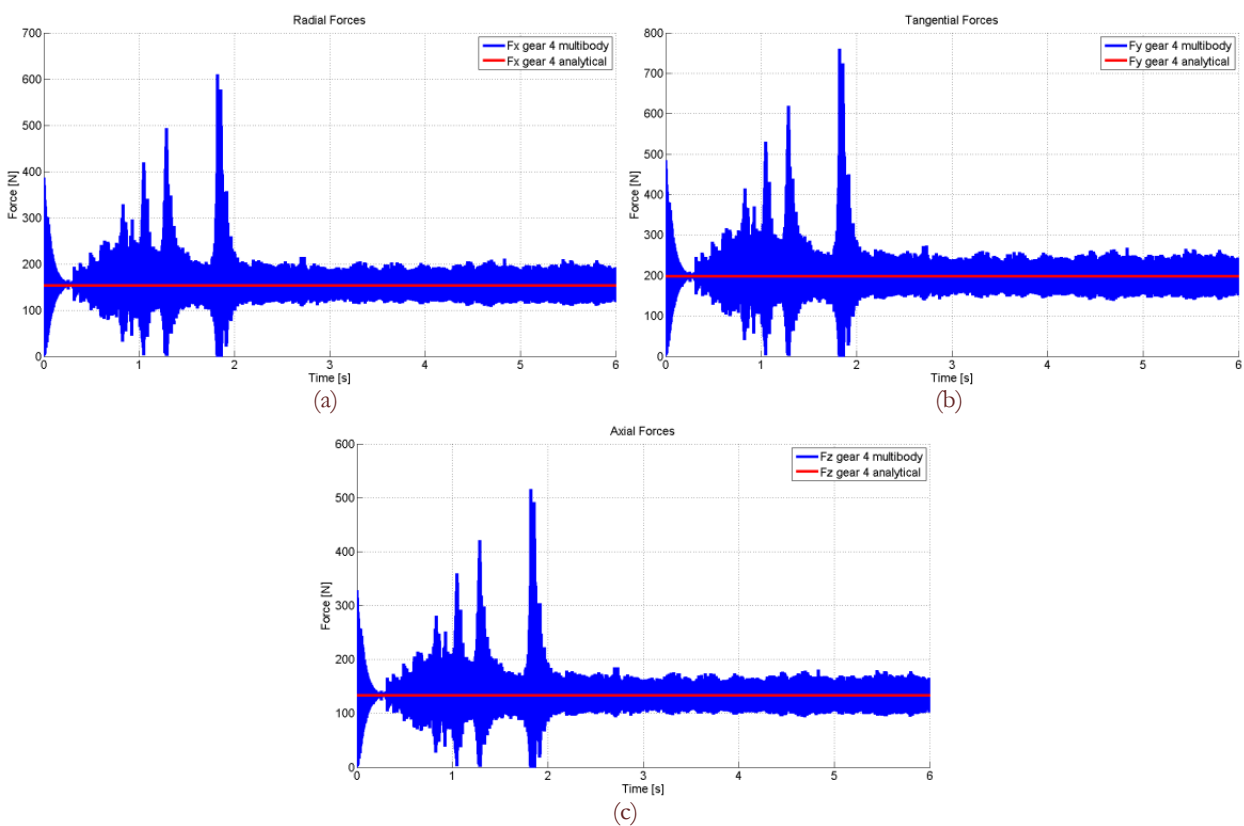


Figure 3: Forces of the Rigid simulation of Gear 4, (a) contact force in x direction (radial force), (b) contact force in y direction (tangential force), (c) contact force in z direction (axial force).



Theoretical values of the above quoted forces may be calculated by the analytical combination of tangential, radial and axial components; these equilibrium equations are the following:

$$F_x = F_t \cdot \sin \varphi + F_r \cdot \cos \varphi = 157.18 \text{ [N]} \quad (1)$$

$$F_y = -F_t \cdot \cos \varphi + F_r \cdot \sin \varphi = -198.68 \text{ [N]} \quad (2)$$

$$F_z = -F_a = -134.69 \text{ [N]} \quad (3)$$

where F_t , F_r , F_a are respectively tangential, radial, axial forces and $\varphi = 15^\circ$ is the misalignment angle of gears 3 and 4.

From the analysis of Fig. 3, it may be observed that numerical averaged forces well matches with the analytical ones.

Fig. 4 shows the Internal Dynamic Factor (K_v) trend of gear 4 during 6 second of the simulation.

To calculate K_v values, the following equation [7] has been used:

$$K_v = \frac{F_{\text{Total}}}{F_{\text{static}}} = \frac{F_{\text{dynamic}}}{F_{\text{static}}} + 1 = \frac{F_{\text{multibody}} - F_{\text{static}}}{F_{\text{static}}} + 1 = \frac{F_{\text{multibody}}}{F_{\text{static}}} \quad (4)$$

where F_{static} is the static theoretical force of the meshing gears obtained by means of above quoted equations for gears 3 and 4, while F_{dynamic} is the force trend obtained by means of the RecurDyn Simulation minus the Static one.

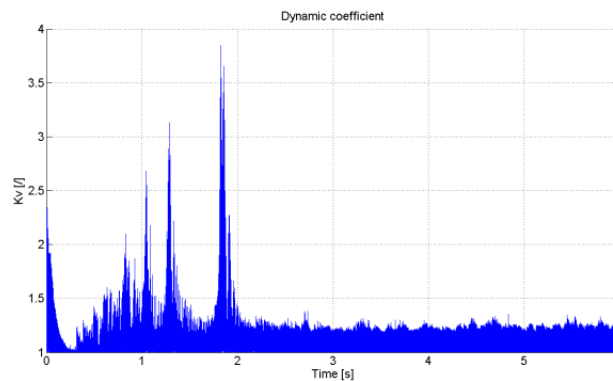


Figure 4: Dynamic Factor of Gear 4 for Rigid simulation.

By analysing the data reported in Fig. 4, it is possible also to calculate the mean value of the dynamic factor (Eq.(4)), that for gear 4 is equal to 1.125.

This dynamic factor is very important to be determined already in the design phase because the fatigue strength of gears depends on it, as indicated in ISO Standard [7].

Thanks to multibody simulations, however, it is possible to calculate a well defined value of K_v as in experimental tests, even if a high computation time is required for the simulation of the complete model of the transmission (38 to 47 hours).

The contact force components versus time of Gear 4 obtained by the Rigid-Flexible simulation are shown in Fig. 5; the forces directions are referred to the ground system of the model.

Averaged values are:

force in x direction $F_x = 157.2 \text{ N}$,

force in y direction $F_y = -198.69 \text{ N}$

force in z direction $F_z = -134.79 \text{ N}$.

As it can be seen by Fig. 5, the trend of the forces in the three directions, respect to the rigid simulation (Fig. 3), is quite different, but the average values are practically the same. The difference is probably due to the flexibility of the teeth, as the Rigid-Flexible simulation results do not present the peaks emphasized in Fig. 3.

Finally, the trend of K_v factor (Fig. 6) is proposed: the average value for Gear 4 is 1.13, similar to that obtained by the rigid simulation.

The trends of forces and of K_v factor of the Rigid-Flexible simulation is slightly more accurate and do not present peaks respect to the rigid one, but a very high computational time has been required for this calculation (384 hours respect to a mean of 41 hours of the full rigid simulation).

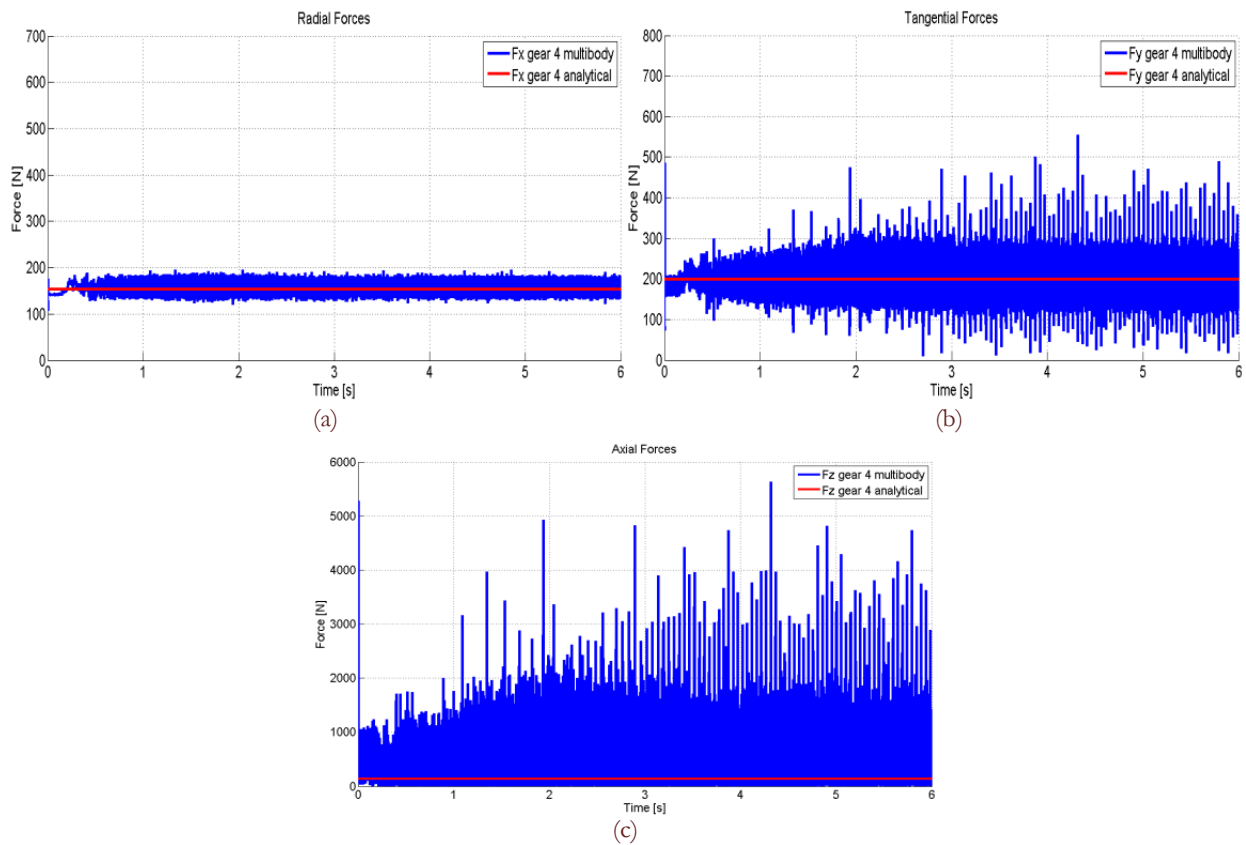


Figure 5: Forces of the Rigid-Flexible simulation of Gear 4, (a) contact force in x direction (radial force), (b) contact force in y direction (tangential force), (c) contact force in z direction (axial force).

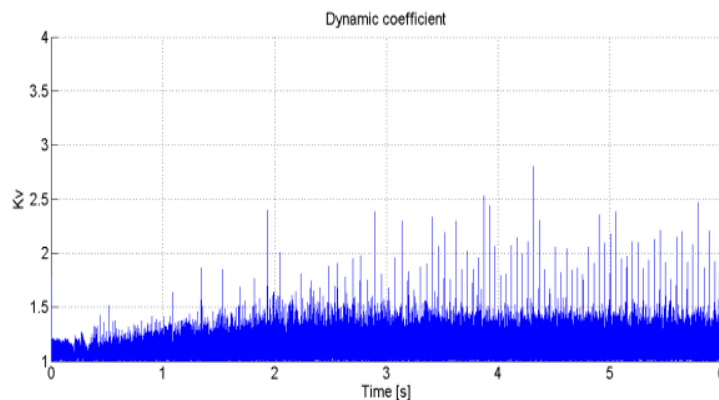


Figure 6: Dynamic Factor of Gear 4 for Rigid-flexible simulation

Finally, Tab. 3 resumes the mean values of the Internal Dynamic Factor calculated respect to the ISO Standard [7] and by the Multibody simulation respectively with rigid and Rigid-Flexible approaches:

Method	Internal Dynamic Factor (K_v)
ISO Standard Method B	1.163
ISO Standard Method C	1.210
Multibody Rigid Simulation	1.125
Multibody Flexible-Rigid Simulation	1.130

Table 3: K_v values for Gear 4.



CONCLUSIONS

Object of the present paper is the simulation of the dynamic behaviour of an automotive transmission for electric vehicle by means a Multibody approach, performed with the software RecurDyn, so as to obtain some necessary information to be involved in the evaluation of the Internal Dynamic Factor.

The results obtained are very helpful in the evaluation of the dynamic loads. Two different simulations have been done, the first has considered the two meshing gears as rigid bodies, the second one has been done with flexible rims and rigid webs.

Forces values have been compared to the corresponding theoretical ones and a very good agreement has been obtained. Then, by processing force trends calculated with the Multibody approach, the dynamic factor K_v , involved in a fatigue study following the ISO Standard 6336 [10], has been determined for the Gear 4.

All obtained K_v values match very well, emphasizing a similar dynamic behaviour of the system.

It may be concluded that the Multibody approach provides a satisfactory information about the dynamic response of the system in both instantaneous and averaged condition.

However, on the basis of the obtained results, it may be observed that the more efficiency in calculation doesn't justify the corresponding increasing in computational time by shifting from rigid (at maximum 47 hours) to flexible-rigid (384 hours) simulation, for as concerns the transmission considered in the present work.

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NOMENCLATURE

α = Pressure Angle, °;

β = Helix Angle, °;

m_n = Normal module, mm;

Z = Number of teeth;

b = Face width, mm;

K_v = Internal Dynamic Factor;

K_{v-A} = Internal Dynamic Factor calculating with Method A of ISO Standard 6336-1 [7];

K_{v-B} = Internal Dynamic Factor calculating with Method B of ISO Standard 6336-1 [7];

K_{v-C} = Internal Dynamic Factor calculating with Method C of ISO Standard 6336-1 [7];

N = Resonance ratio;

N_s = Lower limit of the main resonance range;

n_1 = Rotational speed of the pinion, rpm;

n_{E1} = Running speed of the gear pair, rpm;

F_t, F_r, F_a = Tangential, radial, axial forces, N;

K_A = Application factor;

$F_t \cdot K_A / b$ = Specific Load, N/mm;

F_x, F_y, F_z = Forces in the main directions, N;

v = Maximum tangential speed, m/s.

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