Focussed on multiaxial fatigue

Random accumulated damage evaluation under multiaxial fatigue loading conditions

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ABSTRACT. Multiaxial fatigue is a very important physical phenomenon to take into account in several mechanical components; its study is of utmost importance to avoid unexpected failure of equipment, vehicles or structures. Among several fatigue characterization tools, a correct definition of a damage parameter and a load cycle counting method under multiaxial loading conditions show to be crucial to estimate multiaxial fatigue life. In this paper, the SSF equivalent stress and the virtual cycle counting method are presented and discussed, regarding their physical foundations and their capability to characterize multiaxial fatigue damage under complex loading blocks. Moreover, it is presented their applicability to evaluate random fatigue damage.

KEYWORDS. Fatigue; Multiaxial random loading; Cycle counting method; Fatigue life; Damage accumulation.

INTRODUCTION

Usually, structures and mechanical components are subjected to random fatigue loadings. Car suspensions, wind towers for energy harvest or offshore structures are three examples, among others, that are subjected to this type of loading during service. Random loadings are quite different from the ones usually used in lab to evaluate fatigue strength, because they have unknown time histories. Therefore, damage assessment of these loadings is quite difficult, because exists an aleatory behaviour that is difficult to simulate. Nevertheless, in service conditions nothing is totally random; for instance, when a car suspension travels in the same path every day, the random forces of the car suspension will have similar patterns each day, but they do not have the same pattern. Thus, typical loading spectra are used in design stages of components and structures for reliability, where it is required reliable fatigue criteria.

Multiaxial random fatigue is the ultimate stage in fatigue life assessment; it takes into account several multiaxial fields that usually/traditionally are studied separately. In this stage, it is needed to cover a broad number of concepts related to multi-axial fatigue. In fact, it is not possible to deal with random accumulated damage without considering all of them. Fig. 1 shows the fatigue pyramid concept for random fatigue characterization. In this pyramid, each level depends of all levels below and each level deals with a wide number of material science concepts.

This is a very complex issue that has been handled in a segmented manner, i.e. each level of the fatigue pyramid has been studied and analysed without an evident interconnection between them. In literature, it can be gathered information related to each subject inherent to each fatigue level, but they are usually studied and presented separately.

Therefore, issues like damage accumulation, damage parameters, cyclic properties, multi-axial cycle counting among others, are concepts that capture partially fatigue phenomena when considered separately, however they have not been synergistically presented in literature on a regular basis. Nevertheless, to analyse and estimate random multi-axial fatigue life it is necessary to account with all of them.

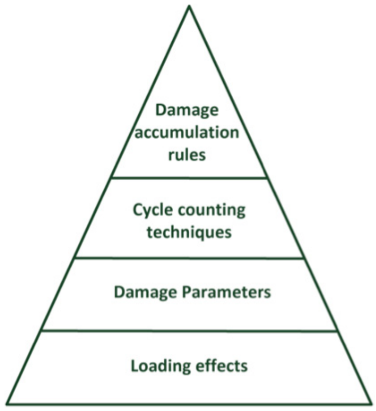


Figure 1: Fatigue levels of random fatigue characterization.

At the base of the fatigue pyramid, shown in Fig. 1, one can find level 1, here it is studied the loading effects on the material cyclic response, such as cyclic hardening, cyclic softening, non-proportional hardening, ratcheting, among others. Essentially, level 1 is focused on elastic-plastic cyclic models that cyclically update stress-strain states accordingly to the material cyclic properties and loading type, the outcomes of level 1 are the input of level 2.

Level 2 takes into account the damage parameter concepts. There are mainly four types of criteria to evaluate multiaxial fatigue; they are the critical plane criteria (which are stress-based, strain-based, and energy-based)[1–4], the equivalent stress criteria (mostly invariant-based)[5], the integral criteria, [6] and the spectral criteria [7].

The objective of these criteria is to capture multi-axial fatigue damage resulting from stress levels and several loading path types such as: sequential, proportional, non-proportional, stress gradient effects, mean stress effects, among others.

These criteria are used to obtain multiaxial fatigue estimates based on reference damage curves such as uniaxial shear/axial S_N curves. Among these criteria, the most appreciated one is the equivalent stress, because they reduce complex stress states (stress tensors) to a scalar (equivalent stress), which is a very suitable feature widespread in finite element packages. Although, their success in static mechanical design, they have huge limitations regarding fatigue damage assessment.

For instance, the equivalent stress concept under multi-axial loading conditions is independent from the loading path type, i.e. it can be reached the same equivalent stress in respect to different combinations of normal, and shear stresses, regarding the same stress level. However, experimental results show that the fatigue strength varies with different combinations of normal and shear stress amplitudes, even for the same equivalent stress amplitude. Thus, fatigue life estimates of equivalent stress criteria under multi-axial loading conditions give inconsistent results.

At level 3, cycle counting techniques are used to account for variable amplitude and loading blocks damage using the damage parameters of level 2. There are very few cycle counting methods for multiaxial loading conditions [8,9]. Most of them are very complex to implement and consumes too much computational resources, which can be a shortcoming in damage assessment in random loading conditions. Also multiaxial cycle counting methods based on Rainflow method do not show, so far, good correlations with experimental lab data even for well-defined loading paths. Thus, level 3 is the fatigue level that has less contribution in literature despite having equal importance. Finally, in level 4, it is focused in damage accumulation rules.

Damage accumulation rules translate an important procedure to estimate fatigue strength under complex loading paths [10]. Usually, they compute the unitary damage captured by the damage parameter in association with a cycle counting method. Thus, if a damage parameter and cycle counting method really capture the unitary damage of a loading block, thus the damage accumulation rule should be linear. It is found in literature some examples of non-linear damage accumulation rules, but in the present authors' opinion, those approaches aim to capture the damage accumulation without having into account the physical damage mechanisms within the material. Therefore, damage accumulation rules based in the Palmgren-Miner must be able to capture damage accumulation under random loading conditions.

This synergistic way to account with each pyramid level is an important feature because under random multi-axial loadings all of these pyramid levels may be activated in a sequential or simultaneous way. Fig. 2 summarizes the synergistic process of evaluating random multi-axial fatigue.

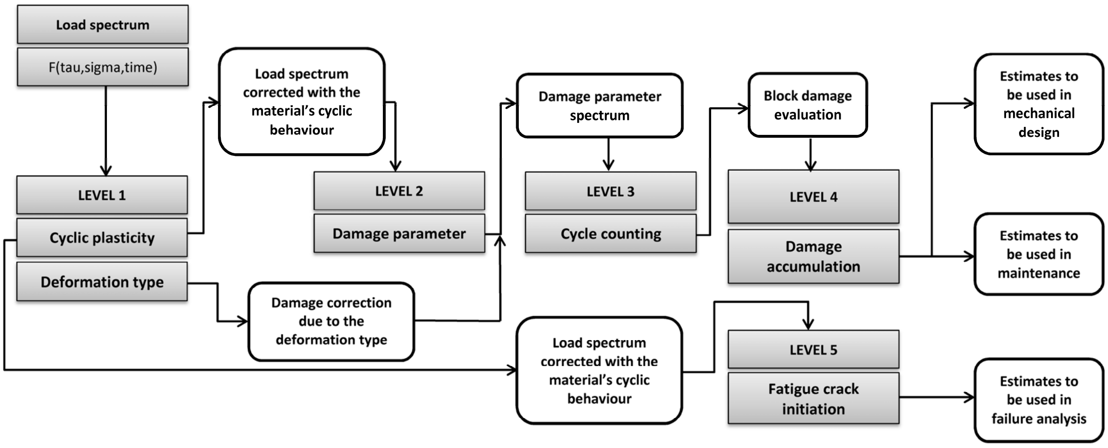


Figure 2: Fatigue estimates process under random loading conditions.

In this paper is presented by the present authors a contribution to levels 2, 3, and 4. For level 2, it was developed the SSF equivalent shear stress, and for level 3 it was developed the virtual cycle counting method, which is deeply dependent on the SSF equivalents stress time histories. Regarding level 4, it is presented the block extraction methodology, where loading blocks are extracted from loading spectra in order to be evaluated with the virtual cycle counting (vcc) technique and damage accumulation rules.

SSF EQUIVALENT STRESS (LEVEL 2)

SSF physical foundations

It is well-know that normal and shear stresses have different damage scales [11,12]. This is so because in experiments it is obtained different fatigue lives regarding the same stress amplitude in both uniaxial loading conditions (shear and axial). Usually, for structural materials, the uniaxial axial SN curve stands above the shear one. Thus, to obtain the same fatigue life in both loading conditions it is required higher normal stress amplitude comparatively to the shear stress amplitude.

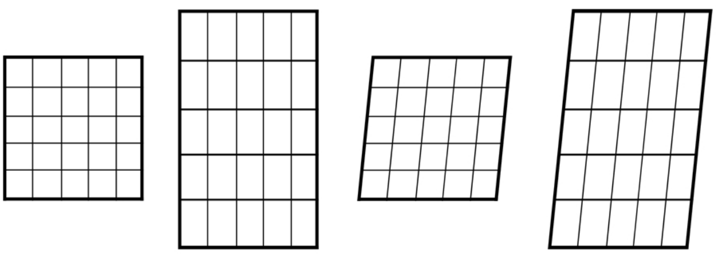


Figure 3: Illustration of different deformation patterns from different loading types. a) Without applied loads, b) axial loading, c) shear loading, and d) biaxial loading.

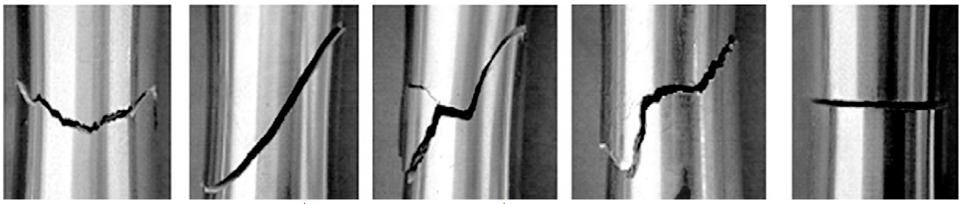
Under multiaxial loading conditions, fatigue damage results from the combination of both axial/normal and shear loading components of a multiaxial loading where it is necessary to account the axial and shear damage contribution to the overall damage. To do that, it is necessary to have both axial and shear damages in the same damage scale. For example, the von

Mises equivalent stress uses the $\sqrt{3}$ constant to reduce shear damage to the axial one. This procedure can be found in a wide range of multiaxial fatigue criteria being a common practice to account combined fatigue damage [13].

Fig. 3 illustrates these damage patterns. Fig. 3 a) depicts a grid at rest without any load (it can be imagined as a material grain). Now, consider Fig. 3 b), in this case the grid is loaded with an axial loading with an elongation pattern. Fig. 3 c) presents the same grid but now with a uniaxial shear loading causing a distortion deformation. As can be seen, the deformation pattern is quite different in both loading conditions.

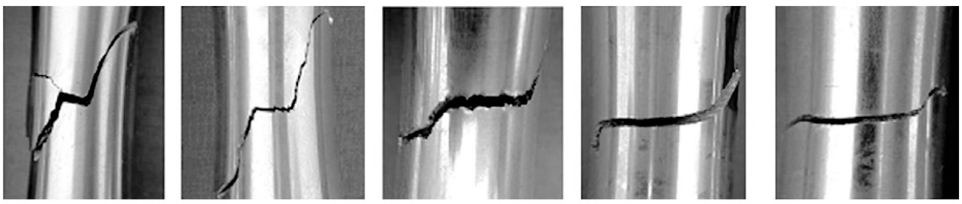
Consider now a material grain structure loaded with the loading conditions shown in Fig. 3. Under these loading conditions, the grain structure will have different deformation patterns like the ones depicted in Fig. 3, under axial, shear, and multiaxial loadings. Due to the different deformation patterns, the material strength will be different for each loading conditions. Now, consider a biaxial loading having simultaneously two types of deformation patterns (axial and shear) above represented in Fig. 4 d). In this case, the deformation pattern is a mixture of axial deformation and shear distortion. To account the contribution of each damage pattern from axial and shear deformations (different deformation mechanisms), it is required reduce both damages to the same scale, because they cause different types of deformation within the material grain structure.

From experiments it was found that the contribution percentage of each axial and shear loading components of a multiaxial loading to the overall damage does influence the material fatigue strength results [11]. The predominance of an axial damage over the shear one or vice versa causes different cyclic damage rates and therefore different fatigue lives are obtained. For each loading direction depicted in a stress space, given by the stress amplitude ratio $\lambda = \tau_a / \sigma_a$, there will be a damage scale that reduces axial damages to the shear damage scale. Zhang et al. published an interesting work that correlates proportional loadings, with different stress amplitude ratios, with their crack initiation planes on a 2A12–T4 aluminium alloy [11]. Fig. 4 shows the crack opening paths obtained by Zhang et al. in respect to the stress amplitude ratios within the range $[0;\infty]$ where it can be found different damage mechanisms associated to each stress amplitude ratio. These results corroborate the premise in which different stress amplitude ratios have different damage scales being required their assessment to account combined axial and shear damages.



(a) $\lambda = 0$ (b) $\lambda = \sqrt{3}/2$ (c) $\lambda = \sqrt{3}$ (d) $\lambda = 3$ (e) $\lambda = \infty$

Figure 4: In-phase fracture appearance under different stress amplitude ratios [11].



(a) $\varphi = 0°$ (b) $\varphi = 24°$ (c) $\varphi = 46°$ (d) $\varphi = 68°$ (e) $\varphi = 90°$

Figure 5: Out-of-phase (φ) fracture appearance for a constant stress amplitude ratio, $\sqrt{3}$ [11].

Fig. 5 shows the phase angle variation effect on the crack opening, here the stress amplitude ratio was maintained equal to $\sqrt{3}$ in each phase-shift φ. Based on these results, it can be concluded that the phase-shift variation also induces different crack opening paths indicating different fatigue damage mechanisms. These mechanisms are related to the material sensitivity to non-proportional loadings, which is a level 1 issue. The present authors developed the Y non-proportional sensitivity factor to account with this phenomenon in the SSF multiaxial fatigue package, which can be found in [14].

The Stress Scale factor (SSF) concept

In the von Mises equivalent stress criterion, where the stress scale factor is a constant $\sqrt{3}$ and is defined under static loading conditions, it is not possible to capture the relation between axial, and shear damages for different stress amplitude ratios. It is assumed that the axial and shear damage relation given by the stress scale factor is constant for any stress amplitude ratios which has led to poor fatigue life estimates under multiaxial loading conditions. Fig. 6 depicts the reasoning that supports the damage scale concept used in the SSF method to reduce axial damage to the shear damage scale. Fig. 6 presents a new way to represents SN curves. Instead of using the usual practice where it is represented a damage parameter vs fatigue life (N_f) it is depicted the axial and shear stress amplitudes (of a multiaxial loading) vs their fatigue life. Thus, Fig. 6 has three SN curves, one is the uniaxial shear SN curve ($\tau_{uniaxial\_1}$), used here as reference damage across the fatigue life range, and the other two are the SN curves of the axial (σ_{load_2}) and shear (τ_{load_2}) loading components of a multiaxial loading. Now, suppose that a fatigue lifetime is selected, for example at 4E5 loading cycles, as shown in Fig. 6. The selected fatigue life can be achieved by using both loadings presented in Fig. 6 (the uniaxial shear loading (loading 1) and the multiaxial loading, loading 2). Thus, in the shear uniaxial loading it will be needed the τ_1 shear stress amplitude, and in the biaxial loading it will be needed the axial stress amplitude σ_2 and the shear stress amplitude τ_2. Note that, the τ_2 value is not enough to cause failure, it is required σ_2. Therefore, the damage difference between τ_1 and τ_2, is given by the damage caused by σ_2. Having this in mind, the damage from $\overline{BC}$ in the shear damage scale, is equal to the damage from $\overline{AD}$ in the axial damage scale.

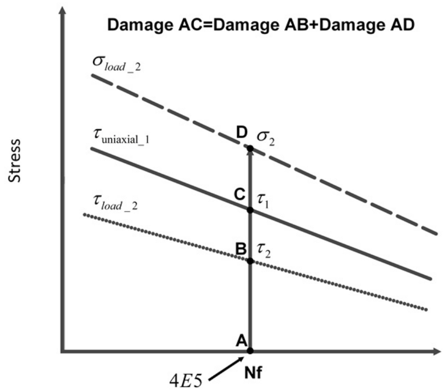


Figure 6: SFF damage scale concept.

SSF equivalent stress

Eq. (1) represents the SSF equivalent shear stress damage parameter [12], where the axial stress component of a multiaxial loading is reduced to the shear damage scale by a SSF damage function, the $ssf\left(\sigma_a,\lambda\right)$.

$$\tau_{eq.} = \tau + ssf\left(\sigma_a,\lambda\right)\cdot\sigma \tag{1}$$

The SSF damage function has as input the stress amplitude ratio and the axial stress amplitude where the stress amplitude ratios defines the axial and shear damage relation and the axial stress defines the stress level within a multiaxial loading; please see Eq. (2).

$$ssf\left(\sigma_a,\lambda\right) = a + b\cdot\sigma_a + c\cdot\sigma_a^{\ 2} + d\cdot\sigma_a^{\ 3} + f\cdot\lambda^2 + g\cdot\lambda^3 + h\cdot\lambda^4 + i\cdot\lambda^5 \tag{2}$$

Where, σ_a is the axial component of the biaxial loading and λ is the stress amplitude ratio (SAR), see Fig. 7. The constants from “a” to “i" are determined through the experimental tests discussed above. Therefore, the objective of the SSF damage function is to update the damage scale accordingly to each loading direction (SAR) under cyclic loading conditions. The SSF damage function is determined based in proportional SN curves for the loading paths presented in

Fig. 7 considering the damage concept depicted in Fig. 6. These loadings have stress amplitude ratios that cover different relations between axial and shear damages, starting from pure axial (0º) and ended with the pure shear loading condition (90º). The SSF damage function is a material cyclic property, and like many others should be determined by experimental tests.
There are some directions, between the ones depicted in Fig. 7, where it was not performed any experimental test. In these directions , the SSF damage function was established by using interpolation techniques between known experimental values of the SSF damage function. Eq. (3) shows the SSF criterion for fatigue life assessment.

$$\max\left(\tau + ssf\left(\sigma_a,\lambda\right)\cdot\sigma\right) = A\left(N_f\right)^b \quad (3)$$

The left side of Eq. (3), represents the maximum value of the SSF equivalent shear stress found within the loading period, where the shear component of a biaxial loading, τ, is added to the axial component previously reduced to the shear damage scale, $ssf\left(\sigma_a,\lambda\right)\cdot\sigma$. Then, the fatigue lifetime is estimated using the uniaxial shear SN curve, represented in the right side of Eq. (3).

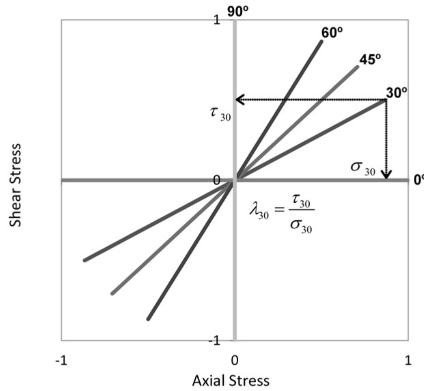


Figure 7: Loading paths used in experiments to calculate the SSF damage function.

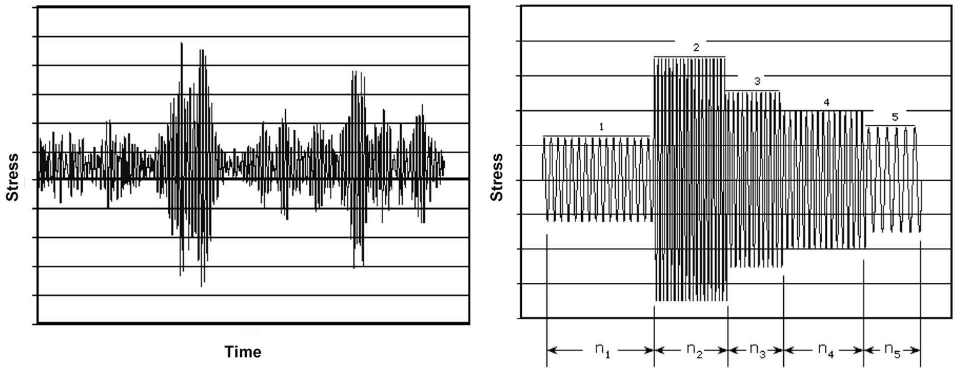
CYCLE COUNTING (LEVEL3)

The equivalent SSF shear stress only captures the damage inherent to the SSF maximum value found in a loading cycle, thus it is not captured the damage associated to all local SSF equivalent stress reversals within a loading history. However, the damage estimated for loading blocks considering only the maximum equivalent stress found in that same loading block is smaller than it should be leading to un-conservative fatigue life results. Thus, the SSF equivalent stress criterion requires a cycle counting technique to evaluate fatigue damage for multiaxial variable amplitude loadings.

Limitations of Maximum equivalent stress concept

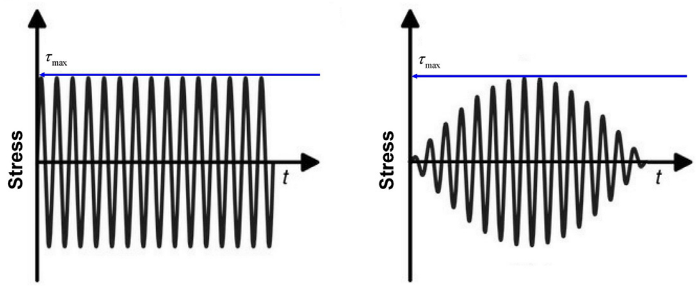
Usually variable amplitude time histories are transformed into an equivalent stress spectrum where each loading block is identified based on their stress amplitude level [15], please see Fig. 8.
The equivalent spectrum approach is suitable to be used in damage accumulation rules based in the Palmgren-Miner concept, however, the transformation from variable amplitude time histories to constant amplitude load spectrum (as one can see in Fig. 8 b), lead to lose the real damage monitoring found in the stress time histories. This means that the stress or damage parameter spectrum will hardly capture the fatigue damage, especially under multiaxial loading conditions. Fig. 9 shows two different loadings with the same loading period and maximum stress level. Despite, the same maximum stress during the load period is obtained in both loading cases; the fatigue damage inherent to each loading is quite different. However, this is the approximation performed when it is used an equivalent stress spectrum where the varying amplitude is reduced to a constant one. The loading depicted in Fig. 9 a) cannot be considered a loading block because the unitary damage is related to one load cycle which is repeated until reach time t, thus in damage accumulation rules the

unitary damage is computed for one loading cycle and then is multiplied by the number of loaded cycles. On the other hand, the load depicted in Fig. 9 b), can be considered as a loading block and the inherent damage unity is related to all load cycles performed during the loading period. None of these loading cycles have the same fatigue damage because they have different amplitudes and load sequence. Thus in order to quantify a unitary damage from a general loading it is necessary to account with all the loading cycles.



(a) (b)

Figure 8: Variable amplitude loadings, a) Time history, b) Equivalent stress spectrum.



(a) (b)

Figure 9: Two different loading paths with same maximum stress.

Unitary block damage concept

The main idea that supports the unitary block damage concept is based in the relation between the maximum equivalent stress fatigue life estimates (for a loading block) and the experimental results. This relation is given in Eq. (4).

$$\underset{Block}{Relative\_Damage} = \frac{N_{f_{estimated}}}{N_{f_{experimental}}} \tag{4}$$

The unitary damage concept aims to establish the loading block damage, which under cyclic loading conditions can be linearly added as if it was a simple sinusoidal loading cycle, like the ones used to obtain the uniaxial SN curves. It is expected that the experimental fatigue life be less than the estimated one; thus, the block unitary damage can be estimated as follows in Eq. (5).

$$N_{f_{experimental}} = N_{f_{estimated}} \frac{1}{\underset{Block}{Relative\_Damage}} \tag{5}$$

Where $1 / \underset{Block}{Relative\_Damage}$ is the block unitary damage. In order to estimate the relative damage without falling back on experimental tests, it was developed the virtual cycle counting method based in the SSF equivalent shear stress time histories.

Virtual cycle counting concept

The virtual cycle counting (vcc) [16] is a non-Rainflow approach to evaluate loading block damage and it is based on the SSF equivalent shear stress previously presented. In the present authors´ opinion, equivalent stress approaches are much more suitable to account block damages than other ones, because they condensed in one damage parameter the instantaneous values of axial and shear damages, and also it allows in an easy way to implement cycle counting techniques in finite element packages. The concept and use of the vcc approach is much easier than other methods usually adopted to quantify block damage accumulation. Basically, the vcc method estimates the relative damage described in the previous section and is presented in Eq. (6). In the numerator of Eq. (6), it is added all SSF equivalent stress absolute values at each peak and valley found between two consecutive zero stress points along the SSF equivalent shear stress time history of a loading block. In denominator hold two times the maximum SSF equivalent shear stress found in the entire SSF time history. Fig. 10 illustrates the vcc method and their variables.

$$vcc = \frac{\sum abs\left(\tau_{ssf}\right)_{peak,valley}}{2\cdot\tau_{\mathrm{ssf,max},Block}} \qquad (6)$$

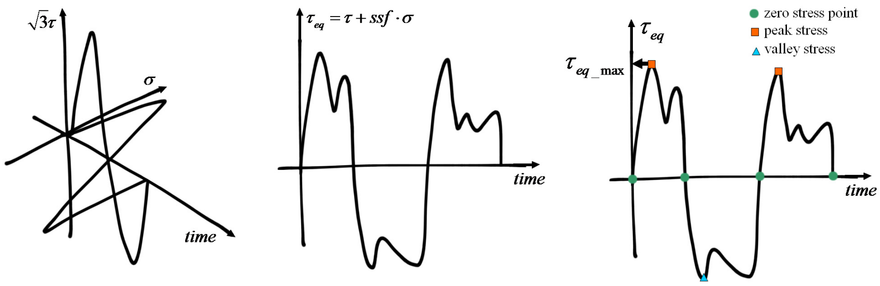


Figure 10: Virtual cycle counting variables identification.

The block fatigue life is estimated as follows in Eq. (7),

$$N_{f\_block} = \frac{\left(\dfrac{\tau_{ssf,\mathrm{max},block}}{A}\right)^{\frac{1}{b}}}{vcc} \qquad (7)$$

where A, and b are the power law regression variables obtained from pure shear fatigue life results from the considered material. $\tau_{ssf,\mathrm{max},block}$ it the maximum SSF equivalent stress within a loading block, and vcc is the virtual cycle counting.

RANDOM LOADINGS (LEVEL 4)

Damage assessment of loading spectra

One way to deal with random loadings is to extract loading blocks from the random loading time history in order to estimate multiaxial random accumulative damage. In order to do that, the SSF equivalent stress criterion in association with the virtual cycle counting method can be used.

The paradigm that supports the block extraction methodology presented here is based in the SSF equivalent stress time histories and is described in the following.

Fig. 11, shows a multiaxial loading represented with the SSF equivalent shear stress time evolution. Here the block extraction is based on the SSF maximum peak, the first maximum peak is settled as the reference damage on the SSF virtual cycle procedures, see Block 1 on Fig. 11, until that peak be exceeded, see Block 2 on Fig. 11, at this point the block 1 ends and starts the second block region with a new damage scale given by the SSF time evolution new peak, Block 3, in Fig. 11. The same methodology is used to extract the other blocks.

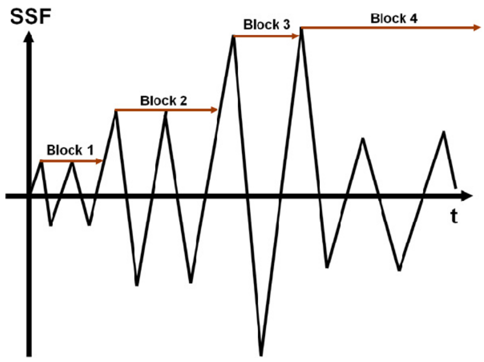


Figure 11: Block extraction methodology for random loadings.

During the block extraction, it can be established the inherent unitary damage of each block as follows in Eq. (8):

$$D_{unitary\_block} = \frac{1}{\dfrac{N_{f,\max,\text{ssf},\text{block}}}{vcc_{block}}} \tag{8}$$

Eq. (8) shows the methodology to estimate the unitary damage within a loading block. On the left, in numerator the number 1 is related to the number of loaded blocks, which in this context is 1. In the denominator, it is represented the material fatigue strength when subject to a loading block, where $N_{f,\max,\text{ssf},\text{block}}$ is the fatigue life estimate based in the maximum value of the SSF equivalent stress within a loading block, and vcc_{block} is the virtual cycle counting regarding each extracted block. In order to estimate the instantaneous accumulated damage, the Palmgren-Miner rule concept can be used as follows in Eq. (9).

$$D_{SSF.random} = \sum_{1}^{i} \frac{1}{\left[\dfrac{N_{f,\max,\text{ssf},\text{block}}}{vcc_{block}}\right]_i} \tag{9}$$

Eq. (9) represents the instantaneous accumulated damage value, which is updated block by block until $D_{SSF.random}$ reaches the value 1, which is the reference condition, considered here to estimate a high fatigue failure probability.

FINAL REMARKS

In this paper was discussed and analysed several aspects related to random multiaxial fatigue. First, it was discussed the importance of having multiaxial fatigue packages in which is contained multiaxial fatigue models synergistically linked, in order to deal with all fatigue phenomena in multiaxial random fatigue. Second, it was presented the SSF equivalent shear stress damage parameter where it was focused the SSF physical foundations. Third, it was presented the virtual cycle counting method to evaluate a block damage and finally it was presented how the SSF equivalent stress in synergy with the virtual cycle counting method can be used to estimate random accumulated damage under multiaxial loading conditions.

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REFERENCES

[1] Fatemi, A., Socie, D.F., A Critical Plane Approach to Multiaxial Fatigue Damage Including out-of-Phase Loading, Fatigue & Fracture of Engineering Materials & Structures, 11 (1988) 149–165.
[2] Socie, D., Critical plane approaches for multiaxial fatigue damage assessment, ASTM Special Technical Publication, 1191 (1993) 7–7.
[3] Carpinteri, A., Spagnoli, A., Vantadori, S., A multiaxial fatigue criterion for random loading, Fatigue & Fracture of Engineering Materials & Structures, 26 (2003) 515–522.
[4] Carpinteri, A., Spagnoli, A., Multiaxial high-cycle fatigue criterion for hard metals, International Journal of Fatigue, 23 (2001) 135–145.
[5] Cristofori, A., Susmel, L., Tovo, R., A stress invariant based criterion to estimate fatigue damage under multiaxial loading, International Journal of Fatigue, 30 (2008) 1646–1658.
[6] Sonsino, C.M., Multiaxial fatigue assessment of welded joints–recommendations for design codes, International Journal of Fatigue, 31 (2009) 173–187.
[7] Lagoda, T., Macha, E., Nieslony, A., Fatigue life calculation by means of the cycle counting and spectral methods under multiaxial random loading, Fatigue & Fracture of Engineering Materials & Structures, 28 (2005) 409–420.
[8] Wang, C.H., Brown, M.W., A path-independent parameter for fatigue under proportional and non-proportional loading, Fatigue & Fracture of Engineering Materials & Structures. 16 (1993) 1285–1297.
[9] Langlais, T.E., Vogel, J.H., Chase, T.R., Multiaxial cycle counting for critical plane methods, International Journal of Fatigue, 25 (2003) 641–647.
[10] Fatemi, A., Yang, L., Cumulative fatigue damage and life prediction theories: a survey of the state of the art for homogeneous materials, International Journal of Fatigue, 20 (1998) 9–34.
[11] Zhang, J., Shi, X., Fei, B., High cycle fatigue and fracture mode analysis of 2A12–T4 aluminum alloy under out-of-phase axial–torsion constant amplitude loading, International Journal of Fatigue, 38 (2012) 144–154.
[12] Anes, V., Reis, L., Li, B., Fonte, M., de Freitas, M., New approach for analysis of complex multiaxial loading paths, International Journal of Fatigue, 62 (2014) 21–33.
[13] Pitoiset, X., Preumont, A., Spectral methods for multiaxial random fatigue analysis of metallic structures, International Journal of Fatigue, 22 (2000) 541–550.
[14] Anes, V., Reis, L., Li, B., Freitas, M., New approach to evaluate non-proportionality in multiaxial loading conditions, Fatigue & Fracture of Engineering Materials & Structures, 37 (2014) 1338–1354.
[15] Lee, Y.-L., Fatigue testing and analysis: theory and practice, Butterworth-Heinemann, (2005).
[16] Anes, V., Reis, L., Li, B., de Freitas, M., New cycle counting method for multiaxial fatigue, International Journal of Fatigue, 67 (2014) 78–94.