The concept of the average stress in the fracture process zone for the search of the crack path

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ABSTRACT. The concept of the average stress has been employed to propose the maximum average tangential stress (MATS) criterion for predicting the direction of fracture angle. This criterion states that a crack grows when the maximum average tangential stress in the fracture process zone ahead of the crack tip reaches its critical value and the crack growth direction coincides with the direction of the maximum average tangential stress along a constant radius around the crack tip. The tangential stress is described by the singular and nonsingular (*T*-stress) terms in the Williams series solution. To demonstrate the validity of the proposed MATS criterion, this criterion is directly applied to experiments reported in the literature for the mixed mode I/II crack growth behavior of Guiting limestone. The predicted directions of fracture angle are consistent with the experimental data. The concept of the average stress has been also employed to predict the surface crack path under rolling-sliding contact loading. The proposed model considers the size and orientation of the initial crack, normal and tangential loading due to rolling–sliding contact as well as the influence of fluid trapped inside the crack by a hydraulic pressure mechanism. The MATS criterion is directly applied to equivalent contact model for surface crack growth on a gear tooth flank.

KEYWORDS. Crack path; Maximum average tangential stress; *T*-stress; Mixed mode; Rolling-sliding.

INTRODUCTION

General aspects of the concept based on calculation of the average stress over the length of the process zone are described in Refs. [1-6] for a solid with cracks or notches. These references demonstrate an advantage of the critical average stress criterion over the classical elastic and elastic-plastic fracture mechanics criteria because this criterion avoids the confusion of applying the unrealistic continuum mechanics stress singularity to the fracture process zone in the vicinity of the crack (or notch) tip. For example, the failure criterion of the average stress within an effective distance has been successfully employed to relate the apparent fracture toughness in specimens with notches to the fracture toughness obtained from deeply cracked specimens (e.g., [4]). In this case, the effective distance corresponds to the point with a minimum of the stress gradient which requires finite element analysis to obtain the effective distance [5].
It should be noted that the effect of the high stress gradient ahead of the crack/notch tip can be taken into account by means of the theory of critical distance using the stress at some critical distance [6]. The theory of critical distances was very successful in predicting the fracture strength of ceramic materials.
Thus, the concept based on averaging the stress over the fracture process zone length can be successfully applied to cracks or notches. The concept of the average stress has been employed to propose the maximum average tangential stress (MATS) criterion under mixed mode I/II loading for predicting the direction of fracture angle. The effect of the

constant term of the series of Williams has been expressed by the parameter T and it is incorporated into the basic criterion equation. To demonstrate the validity of the proposed MATS criterion, this criterion is directly applied to experiments reported in the literature for the mixed mode I/II crack growth behavior.

MAXIMUM AVERAGE TANGENTIAL STRESS (MATS) CRITERION

The tangential crack-tip stress under mixed mode I/II loading can be characterized by the singular (the first order term) and the nonsingular term (the T-stress) in polar coordinate following to Williams [7]

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(K_I \cos^2\frac{\theta}{2} - \frac{3}{2} K_{II} \sin\theta \right) + T\sin^2\theta \qquad (1)$$

Here, r and θ are the crack tip coordinates, K_I and K_{II} are mode I and mode II stress intensity factor, respectively.
The maximum average tangential stress (MATS) criterion is assumed to be applicable and can be expressed mathematically as follows [8]

$$\frac{\partial \bar{\sigma}_{\theta\theta}}{\partial\theta} = 0\,, \ \frac{\partial^2 \bar{\sigma}_{\theta\theta}}{\partial\theta^2} < 0\,. \qquad (2)$$

This criterion states that a crack grows when the maximum average tangential stress $\bar{\sigma}_{\theta\theta}$ in the fracture process zone ahead of the crack tip reaches its critical value and the crack growth direction coincides with the direction of the maximum average tangential stress along a constant radius around the crack tip.
Combining Eq. (1) with Eq. (2), the MATS criterion leads to predicting the direction of fracture angle θ_0 [8]

$$\frac{\partial \bar{\sigma}_{\theta\theta}}{\partial\theta} = K_I \sin\theta_0 + K_{II}\left(3\cos\theta_0 - 1\right) - \frac{8T}{3}\sqrt{2\pi d}\,\cos\theta_0 \sin\frac{\theta_0}{2} = 0 \qquad (3)$$

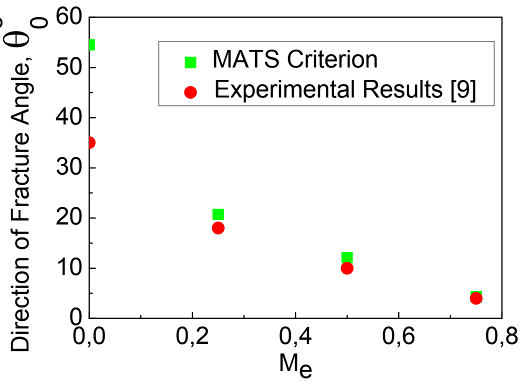

Figure 1: The predicted and experimental directions of fracture angle for the mixed mode I/II crack growth behavior of Guiting limestone.

VALIDATION OF THE MATS CRITERION

To validate the proposed MATS criterion, Eq. (3) is directly applied to the experimental results [9] for the mixed mode I/II crack growth behavior of Guiting limestone. Different combinations of modes I and II are characterized by a mixity parameter M_e

$$M_e = \frac{2}{\pi}\arctan\left(\frac{K_I}{K_{II}}\right) \tag{4}$$

The values of M_e were varied through 1 (pure mode I), 0.75, 0.5, 0.25, 0 (for pure mode II).
According to the MATS criterion (Eq. (3)), four parameters K_I, K_{II}, T and d at fracture need to be known for each mode mixity. The local strength of this brittle material and the fracture process zone length d are treated as a function of the tensile strength [8] which is equal to 2 MPa. The fracture toughness K_{mat} is $0.24\ \mathrm{MPa}\sqrt{\mathrm{m}}$. These values were assumed to be constant for all mixities.
The fracture trajectories predicted using the MATS criterion (Eq. (3)) are consistent with those observed in the broken specimens under various mode mixities (Fig. 1).

THE SURFACE CRACK PATH UNDER ROLLING-SLIDING CONTACT LOADING

Assuming the singular and non-singular (T-stress) terms are sufficient to characterize the crack tip stress under mode I/II loading in the case of rolling-sliding contact, the tangential stress $\sigma_{\theta\theta}$ is written in polar coordinate as follows [10]

$$\sigma_{\theta\theta}(\theta,r) = \frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(K_I\cos^2\frac{\theta}{2} - \frac{3}{2}K_{II}\sin\theta\right) + T\sin^2\theta - \sigma^c_{xy}\sin 2\theta + \sigma^c_{yy}\cos^2\theta\,. \tag{5}$$

Here, the tractions σ^c_{xy} and σ^c_{yy} are defined at the crack tip and their distribution is smooth enough along the crack surface. This equation is valid, if the crack surfaces are loaded with constant pressure $p_\alpha(\alpha = x, y)$. The stress component σ^c_{yy} is then equal to the $-p_y$ while $\sigma^c_{xy} = -p_x$.
Averaging the tangential stress over the fracture process zone, which is characterised by a critical distance d, the maximum average tangential stress (MATS) criterion leads to the following mathematical expression

$$\frac{\partial\overline{\sigma}_{\theta\theta}}{\partial\theta} = K_I\sin\theta_0 + K_{II}(3\cos\theta_0 - 1) + \frac{4}{3}\sqrt{2\pi d}\,\sigma^c_{xy}\frac{\cos 2\theta_0}{\cos\frac{\theta_0}{2}} - \frac{8}{3}(T - \sigma^c_{yy})\sqrt{2\pi d}\cos\theta_0\sin\frac{\theta_0}{2} = 0 \tag{6}$$

The fracture process zone size d in the above-mentioned equation can be determined for critical state $\overline{\sigma}_{\theta\theta} = \sigma_0$ as follows

$$T\cdot\sin^2(\theta_0) - \sigma^c_{xy}\cdot\sin(2\cdot\theta_0) + \sigma^c_{yy}\cdot\cos^2(\theta_0) + \cos\left(\frac{\theta_0}{2}\right)\cdot\left[K_I\cdot\cos^2\left(\frac{\theta_0}{2}\right) - \frac{3}{2}\cdot K_{II}\cdot\sin(\theta_0)\right]\cdot\sqrt{\frac{2}{\pi d}} = \sigma_0 \tag{7}$$

In general case, the local strength σ_0 is dependent on the model of a solid and can be treated for plane strain according to von Mises yield criterion as a property of both the yield stress σ_Y and the T-stress which quantifies constraint in different geometries and type of loading [11, 12]

$$\sigma_0 = -\frac{T}{2} + \sigma_Y\sqrt{\frac{1}{4}\left(\frac{T}{\sigma_Y}\right)^2 - \frac{(1+\nu^2-\nu)\ (T/\sigma_Y)^2 - 1}{(1-2\nu)^2}}\,, \tag{8}$$

where .ν .is Poisson's ratio.
According to Zafošnik et al. [10], the real contact geometry (e.g. gear tooth flanks) can be transformed into a pair of equivalent contacting cylinders with the radii corresponding to curvature radii of analyzed mechanical elements. For small

coefficients of friction the distribution of tangential contact loading $q(x)$ due to relative sliding can be estimated with a simple Coulomb friction law within the Hertzian model [13]

$$q(x) = p_x = \mu \cdot p(x)\,, \tag{9}$$

where μ is the coefficient of friction between contacting elements.
Following loading configurations have been considered to investigate the effect of a moving contact on the crack propagation angle (Fig. 2). All configurations have the same normal $p(x)$ and tangential $q(x)$ contact loading distributions which are applied at different positions with reference to the crack mouth. The pressure on the crack faces is considered to be equal to that at the crack mouth, and is equal to

$$p(x) = p_y = p_0\sqrt{1-\left(\frac{x_0}{b}\right)^2}\,, \tag{10}$$

where x_0 / b is moving contact load position (Fig. 2).

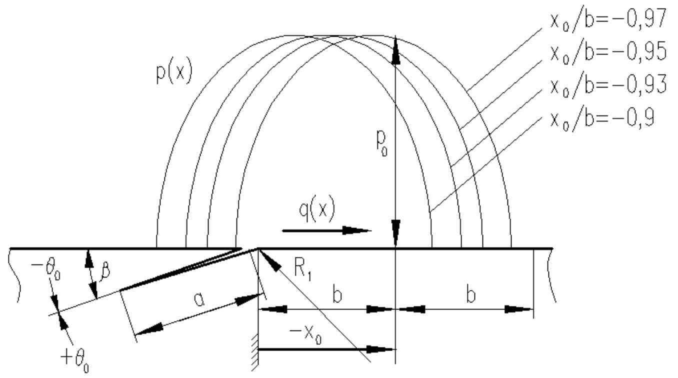

Figure 2: Simulation of the moving contact.

The following mechanical properties of carburized steel 16MnCr5 are used in the computational model: Young's modulus $E = 206 \cdot GPa$, Poisson's ratio $\nu = 0.3$, yield strength $\sigma_Y = 2200 \cdot MPa$ and fracture toughness $K_{mat} = 521 MPa\sqrt{m}$ [10]. The Hertzian contact pressure distribution $p(x)$ has the following parameters, namely, maximum value of $p_0 = 1779 \cdot MPa$ and the half-length of the contact area $b = 0.228\ mm$. The influence of different contact sliding is simulated with three different coefficients of friction (μ=0.04, 0.065 and 0.1). Initial length of the crack is equal to $a_0 = 20 \cdot \mu m$ with the initial inclination angle towards the contact surface $\beta = 20^o$. The computational data published by Zafošnik et al. [10] is employed in calculation to illustrate the variation of observed parameters in the region of maximum stress intensity factors $\overline{K}_I = K_I / p_0 \cdot \sqrt{b}$, $\overline{K}_{II} = K_{II} / p_0 \cdot \sqrt{b}$ and $\overline{T} = T / p_0$ for moving contact load over the crack mouth.
The computational results (Fig. 3) determined by the MATS criterion illustrate the variation of crack propagation angle θ_0 in the region of maximum stress intensity factors K_I, K_{II} and T-stress for moving contact load over the crack mouth, which is given in terms of a relative position of the loading case with respect to the half-contact width b. It can be seen that the influence of coefficient of friction on crack propagation angle is negligible.
To evaluate the analytical results obtained by the proposed maximum average tangential stress (MATS) criterion, the maximum tangential stress (MTS) criterion [10] is also employed. The obtained results illustrate the following. The MATS criterion gives slightly largest crack propagation angles as compared with the MTS criterion for load position corresponding to $x_0 / b < -0.94$ (Fig. 4). At the same time, the predicted crack propagation angles are smaller for contact load position corresponding to $x_0 / b > -0.94$.

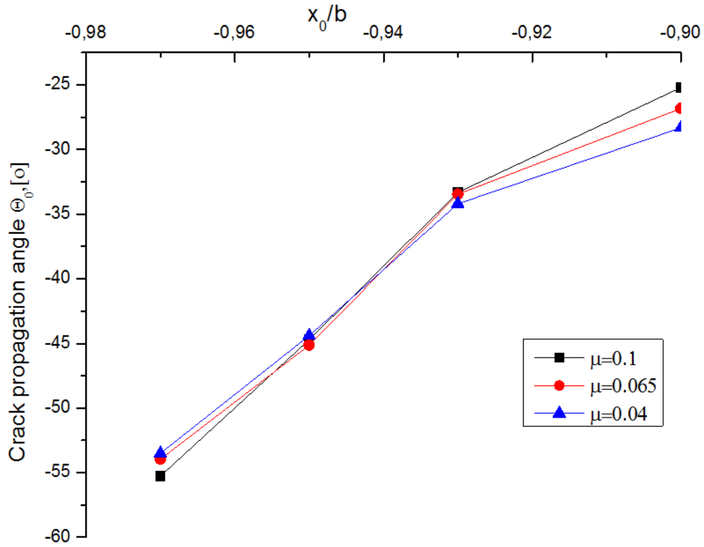

Figure 3: Crack propagation angle determined with the MATS criterion.

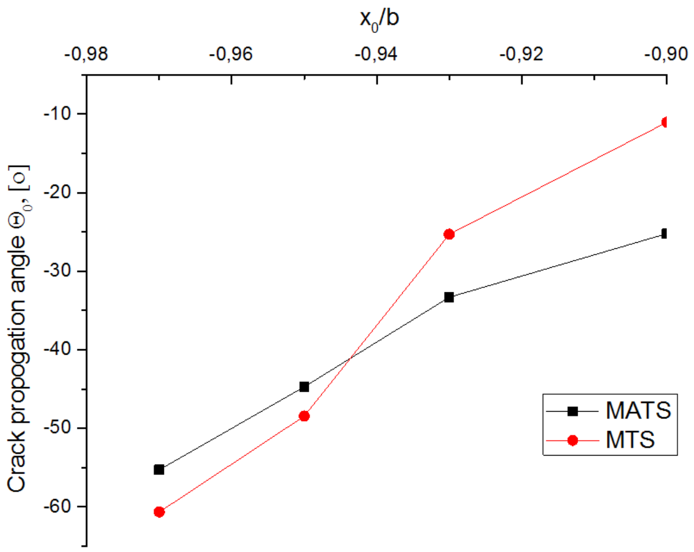

Figure 4: Crack propagation angle determined with MATS and MTS criteria (coefficient of friction $\mu = 0.1$).

Further investigation should be connected with numerical and experimental research to validate the proposed MATS criterion for estimation of crack propagation angles under the rolling-sliding contact loading.

CONCLUSIONS

The maximum average tangential stress (MATS) criterion, based on the concept of the average stress within the fracture process zone, has been proposed for predicting the direction of fracture angle. The effect of the constant term of the series of Williams has been expressed by the parameter T and it is incorporated into the MATS criterion. The predicted directions of fracture angle are good consistent with the experimental data for the mixed mode I/II crack growth behavior of Guiting limestone.

The criterion of the maximum average tangential stress, taking into account the non-singular T-stress and the lubricant hydraulic pressure on crack surfaces, is also spread to estimate the surface crack path under the rolling-sliding contact loading. The effect of coefficient of friction and load position on the crack path has been analysed. It is shown that the influence of coefficient of friction on crack propagation angle is negligible. At the same time, there is significant effect of contact load position on the crack propagation angle.

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