Fatigue characterization of a crankshaft steel: Use and interaction of new models

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ABSTRACT. The peculiar geometrical shape and working conditions of crankshafts make fatigue becoming responsible for most of the failure cases in such components. Therefore, improvement of crankshaft performance requires enhancing its fatigue life. In this work, the fatigue behavior of a D38MSV5S steel, used for crankshafts in compact vehicles, is investigated according to two traditional ways of analysis, namely the stress based and the fracture mechanics based approaches, though using advanced design models: On the one side, a probabilistic Weibull regression S-N model is assessed for experimental results obtained from fatigue resonance tests. On the other side, the crack growth rate curve is calculated from crack growth tests, carried out on SENB specimens, using a normalizing procedure. Specific Matlab programs are developed to facilitate the evaluation process. The information gained from both models will contribute to provide a probabilistic interpretation to the Kitagawa-Takahashi diagram.

KEYWORDS. Fatigue of crankshafts; Crack growth rate curves; S-N diagram.

INTRODUCTION

Due to the service conditions and its peculiar shape design, fatigue appears to be the main failure reason of crankshafts used in aeronautics and automotive engines [1]. Consequently, big effort is devoted to investigate fatigue failures in crankshafts in order to enhance their fatigue life. Generally, lifetime fatigue analysis is performed in two different ways: assuming no initial damage in the component or, alternatively, accepting the unavoidable presence of cracks. In the first case, the total lifetime is identified as initiation phase, at least for working loads. This estimation is usually performed by means of an adequate definition S-N field for the component. On its turn, in cracked components, the fatigue lifetime is only assigned to the propagation phase so that the lifetime is assessed based on fracture mechanics premises by determining the crack propagation law of fatigue cracks. Although both approaches are often applied as being independent each other their interconnection is apparent so that their simultaneous consideration is advantageous from the point of view of reliability due to its complementary character. In former publications [2], the limitations of the Paris law were pointed out in the definition of the crack growth rate curve as a power law sustained by incomplete self-similarity assumption [3]. Its substitution by a crack growth rate law applicable even to ΔK values close to the ΔK_{th} is advisable. In any case, dimensional inconsistencies, similar to those exhibited by the original Paris equation, are a common feature in most of the models being presently applicable even with international acknowledgement, as those of Forman, NASGRO and many others [4].

In this work, the fatigue behavior of a steel D38MSV5S, used in the manufacture of crankshafts of compact vehicles, is investigated according to the two approaches mentioned above: on the one hand, by defining the S-N field from the experimental data obtained in resonance fatigue tests using a probabilistic model [5,6], and by the other hand, by deriving the crack growth rate curve from SENB specimens using the approach proposed in [7]. The latter represents an advance with respect to others models by providing an easy, normalized formulation of the crack growth rate curve as a cumulative distribution function by fulfilment of the dimensional requirements, whereby the value of the threshold stress intensity range appears as model parameter. Moreover, under certain conditions of the geometric factor, a unique reference crack growth curve a-N is obtained, which allows any other crack growth curve by simple analytical transformation for different values of the initial crack size and stress range to be derived.
Understanding the theoretical fundamentals of the crack growth model as proposed in [7] facilitates the development of subroutines and their assembling into a practical program for general and comprehensible application of the crack growth rate curve irrespective of the initial crack size and applied load. Further, a possible extension to the variable loading case is envisaged. Derivation of propagation S-N curves is possible and, therefore, also of initiation lifetime curves once the conventional S-N field, representing the total fatigue life, is known. This will favor a future advance in what concerns the variability of the crack growth rate curves as well as the probabilistic concept of the Kitagawa-Takahashi diagram, nowadays still missed.

EVALUATION OF THE S-N FIELD

First, as mentioned in the former Section, the probabilistic S-N field according to [5] was determined from the tests results carried out in a Rumul Testronic resonance machine using specimens taken out from the crankshaft axle, see Fig. 1. The tests were carried out at different constant stress ranges, $\Delta\sigma$ for a constant stress rate $R=\sigma_{mín}/\sigma_{max}= - 1$, for which the number of cycles until failure were registered. The test results pairs ($\Delta\sigma$, N) obtained are shown in Tab. 1. Considering the random character of the fatigue phenomenon a statistical analysis of the results is advisable to permit the definition of the S-N percentile curves representing the same probability of failure. This is achieved after estimation of the Weibull model parameters using the ProFatige software program [6] whereby the run-outs are taking into account. The Weibull model parameters fitted are included in Tab. 2 with which, in this case, the probabilities P=0, 0.05, 0.50 and 0.95 were considered (see Fig. 2).

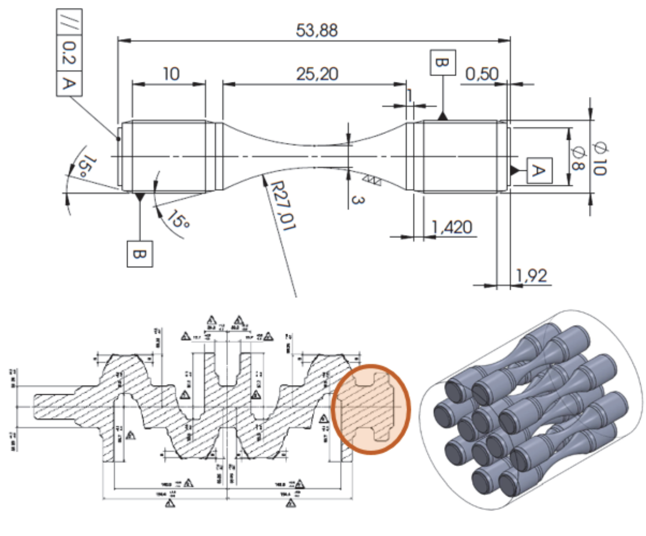

Figure 1: Specimen extraction and fatigue specimen geometry for resonance testing.

The parameters B and C represent, respectively, the limit number of cycles and the fatigue limit, in the sense of true endurance limit for $N\rightarrow\infty$, that is, the value of the stress range below which fatigue does not occur. In this way, the problem consists in estimating, for given stress range, the number of cycles to failure at which failure is expected for the certain probability.

Initial Lifetime (Cycles)	Stress (MPa)	Length (mm)	Runout	Expected Lifetime (Cycles)
106483	450	53.88		
518308	400	53.88		
10000002	350	53.88	R	2.876e+10
21479001	380	53.88	R	8.496e+07
1297002	400	53.88		
10000000	390	53.88	R	2.905e+07
133527	395	53.88		
359216	395	53.88		
10000000	395	53.88	R	2.449e+07
69554	425	53.88		
974659	425	53.88		
87004	395	53.88		
298661	395	53.88		
50698	437.5	54.88		
112488	437.5	55.88		
166587	412.5	56.88		

Table 1: Fatigue results from the resonance tests.

B	C	β	δ	λ
4.75 (115 cycles)	5.75 (312.7 MPa)	2.28	1.28	1.06

Table 2: Weibull model parameters from fitting of the S-N field.

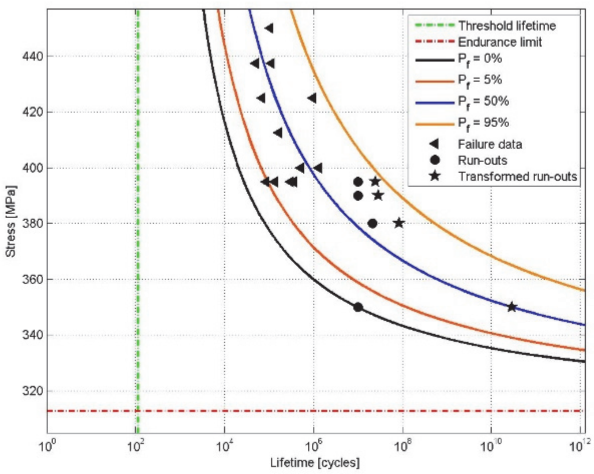

Figure 2: S-N field assessment using the Profatigue software [6].

CRACK GROWTH CURVES

The specimens used in the tests for the derivation of the crack growth rate curve of the material were cut out from the crankshaft axle as shown in Fig. 3.

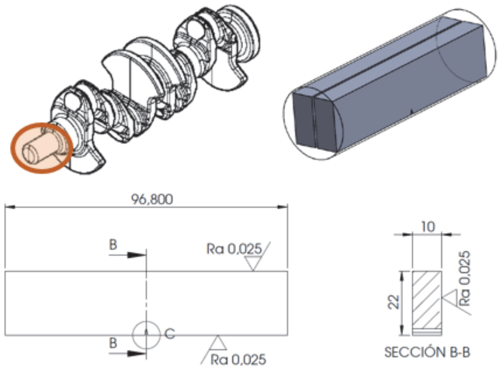

Figure 3: Extraction location and geometry of the specimens used in the crack growth tests.

Tests were carried out in accordance to the requirements of the ASTM-E1820 standard [8] and the *compliance method* was applied for determining the crack growth. Fig. 4 represents the crack growth rate da/dN as a function of the stress intensity factor range ΔK. The test was performed under constant ΔF and stress rate R= -1.

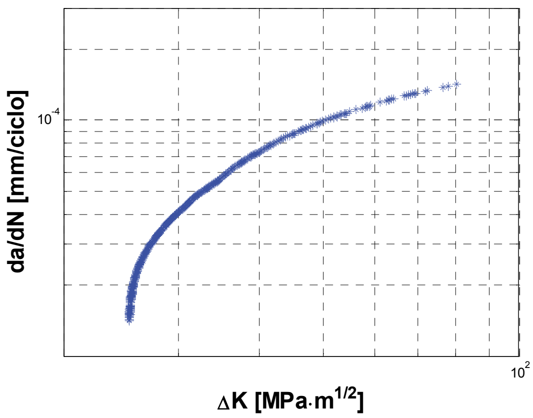

Figure 4: Crack growth rate curve from experimental data.

MODEL FOR CRACK GROWTH RATE CURVE

The fracture mechanics based approach, based on the application of crack growth rate curves, can be applied as an alternative to the approach based on stresses, i.e., that being related to the S-N field, due to its more general applicability to lifetime prediction of mechanical and structural components. With the aim of interrelating both models in the study of propagation of macrocracks or even of physical microcracks, Castillo et al. proposed to determine the crack growth rate curve based on a model [7], which considers the non-dimensional normalization of the stress intensity range factor according to the expression:

$$\Delta K^{*+} = \frac{\log \Delta K^* - \log \Delta K^*_{th}}{\log \Delta K^*_{up} - \log \Delta K^*_{th}} \tag{1}$$

in which ΔK_{th} represents the threshold stress intensity factor range and ΔK_{up}, is the upper bound of the stress intensity factor range, not necessarily identifiable with the failure stress intensity factor ΔK_f , see Fig. 5. This proposal offers the advantage that the sigmoidal shape of the crack growth rate curve *da/dN-ΔK* may be identified, analytically over its full existence range, as a cumulative distribution function, since ΔK^+ is a function growing up monotonically in the interval [0,1] (see Fig. 5) taking so advantage of the statistical experience gained about this family of curves. The fact that the threshold stress intensity factor ΔK_{th} is estimated as one of the model parameter ensures higher reliability in the curve estimation and makes easier a further variability analysis, still pending. A possible option consists in assuming an extreme distribution either for maxima or for minima, taking into account the characteristics of the phenomenon. In this case, a Gumbel distribution for minima [9] was searched so that a reliable fitting of the normalized crack growth rate curve is achieved from the experimental results. An important question lies in the interpretation of the fatigue life resulting from the integration of the crack growth rate curve that corresponds rather to the propagation life than to the total fatigue life.
The identification of the crack growth rate curve as a cumulative distribution function in which ΔK^+ is identified as the normalizing variable defined in the interval [0,1] leads to the consideration of *log (da/dN)* as being the random variable. Accordingly, the following equation must be used to fit the experimental results:

$$\frac{\log \Delta K^* - \log \Delta K^*_{th}}{\log \Delta K^*_{up} - \log \Delta K^*_{th}} = F\left(\log \frac{da^*}{dN^*}\right) = 1 - exp\left[-\exp\left(\frac{\log\left(\frac{da^*}{dN^*}\right) - \alpha}{\gamma}\right)\right] \tag{2}$$

The proposed model provides an analytical expression to the normalized crack growth rate curve by fitting the experimental results referred to crack size vs. number of cycles using a minimum square error method, see [7]. Fitting of the curve succeeds by minimizing the function $Q\ (a, \gamma, \Delta K_{th}^*, \Delta K_{up}^*)$:

$$Q = \sum_{i=1}^{n}\left\{\log \Delta K^*_i - \log \Delta K^*_{ith} - \left(\log \Delta K^*_{up} - \log \Delta K^*_{th}\right)\cdot\left[1 - exp\left[-\exp\left(\frac{\log\left(\frac{da^*}{dN^*}\right) - \alpha}{\gamma}\right)\right]\right]\right\}^2 \tag{3}$$

with respect to those parameters, where *a* and *γ* are the location and the scale Gumbel parameters, and ΔK_{th}^*, ΔK_{up}^* the normalized values of ΔK_{th}, ΔK_{up} , respectively.
The formulation of a transcendent theorem proves that assuming certain premises concerning the function representing the geometric crack factor Y(a), a reference crack growth curve a-N may be obtained using a unique integration allowing any other a-N curve, corresponding to a given pair of values for the initial crack size ao and the remote stress range applied Δσ, to be derived.
Once the parameters are found from the experimental results obtained, the curves defining the normalized crack growth, *a*(N*)* can be determined by solving the differential Eq. (4) that provides a particular solution for a given initial crack size a_0* and a particular stress range, Δσ*.

$$\frac{da^*\left(N^*\right)}{dN^*} = exp\left(F^{-1}\left(\frac{\log u - \log \Delta K^*_{th}}{\log \Delta K^*_{up} - \log \Delta K^*_{th}}\right)\right) \tag{4}$$

where $u = \Delta\sigma^* Z(a^*(N^*))$. The *Z* function represents a transformation of the function defining the geometric crack factor being given by:

$$Z\left(a^*\right) = Y^*\left(a^*\left(N^*\right)\right)\sqrt{\pi a^*\left(N^*\right)} \tag{5}$$

This function can adopt, alternatively one of the following forms:

$$Z\left(a^{*}\right)=\begin{cases}\left(Qa^{*}+T\right)^{\rho}\\ T\exp\left(Ua^{*}\right)\end{cases} \quad (6)$$

where T, U, Q and ϱ in former expression are constants to be determined by fitting the function Z.
After proceeding in this way, it is possible to determine the reference crack growth curve *a*-N** from which, in turn, any other crack growth curve can be obtained for the given initial crack size and stress range. Finally, the *S-N* field for crack propagation can be derived proving the relation existing between the stress based approach and that based on fracture mechanics on a probabilistic basis.

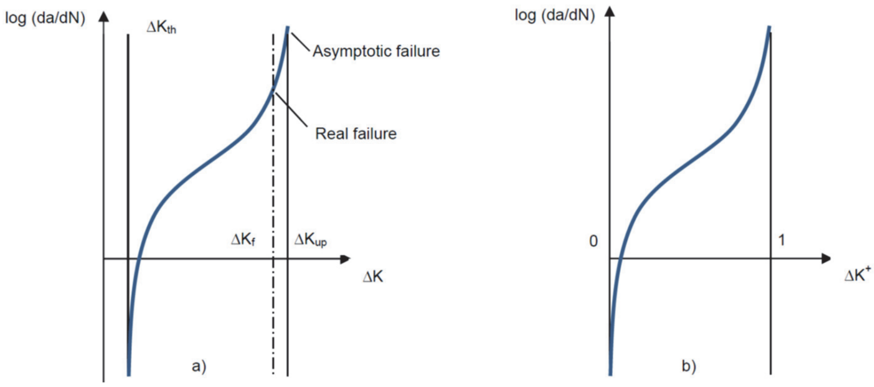

Figure 5: a) Original and b) normalized crack growth rate [7].

The decision for selecting the initial crack size in order to proceed to the calculation of the fatigue life propagation can be adopted on the base of probabilistic considerations relative to the distribution of the cracks provided by the possible correspondence between the *S-N* curves and those referred to the crack growth *a-N*.
Indeed, some questions remain unsolved, among them the following can be referred to: a) The experimental verification of the model, which is, at least partially, one of the contributions of this work, b) the interpretation of the S-N field as composed by the initiation and propagation lives, and c) the possibility of considering a fictitious microcrack size, which allows the correspondence between both fatigue lives, i.e. between the one determined from the S-N field and that resulting from the integration of the crack growth rate curve, to be determined. On its turn, the crack growth rate curve provides a direct relation between the initial crack size and its progression till failure. In this way, ΔK presents the advantage, or perhaps disadvantage, of coupling crack size *a* and stress range $\Delta\sigma$ as a unique parameter apparently related to the crack micromechanism. Nevertheless, the interpretation of the results is less apparent since the same ΔK can be arise from different combinations of crack size and stress range whereas notable differences are observed in the behavior according to the characteristics of the crack size (macrocracks and microcracks in the two variants, physical and microstructural): Therefore the interest consists in relating crack size and crack growth rate curve to the *S-N* field where both parameters are uncoupled.

DERIVATION OF THE PROPAGATION S-N CURVES FROM THE CRACK GROWTH CURVE

In the development of this Section, the methodology proposed in [7] has been applied step by step.

a- Selection of the normalizing variables

First of all, the participating variables are normalized according with the values shown in Tab. 3.

W [mm]	K_{IC} [MPa m$^{1/2}$]	N_0 [cycles]
22	89.26	1000

Table 3: Values of the normalizing variables.

b- Minimization of function Q

Thereafter, the parameters fitting the crack growth rate curve are determined from the experimental data (see Tab. 4).

α	γ	$\log(\Delta K_{th}^{*})$	$\log(\Delta K_{up}^{*})$
-3.7462	1.9224	-1.7159	0.5721

Table 4: Parameter values found by fitting the crack growth rate curve as a cumulative distribution function.

Fig. 6 exhibits the normalized values for log(ΔK*) vs. log(da*/dN*) along with the curve fitting obtained after minimization.

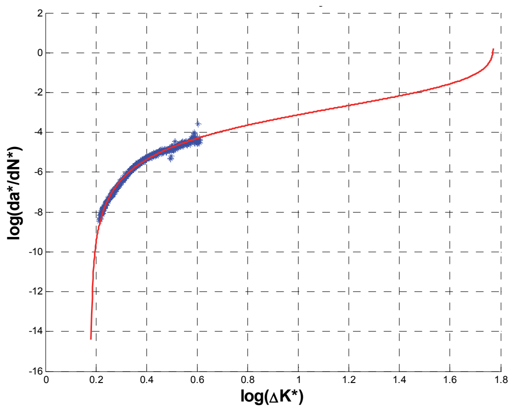

Figure 6: Representation of the normalized experimental data and fitting obtained.

c- Obtaining crack growth curves

Figs. 7 and 8 represent the result of the integration of Expr. (4), firstly for different initial crack sizes a_0* when maintaining a fixed value for the non-dimensional stress range, Δσ*.

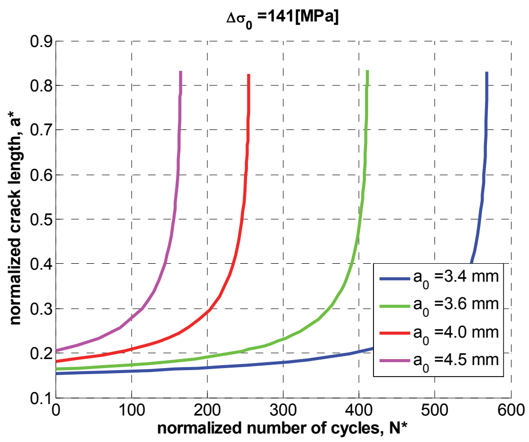

Figure 7: Curves a*-N* for different initial crack sizes a_0*, while maintaining constant Δσ*.

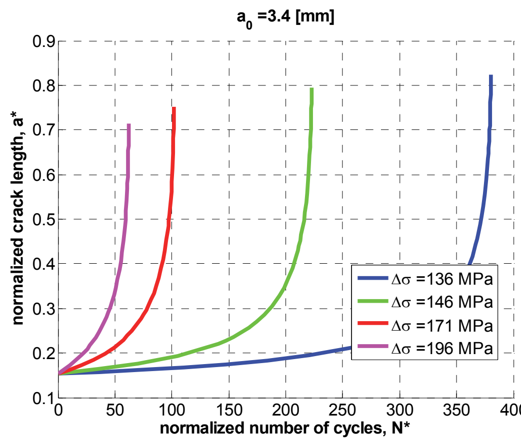

Figure 8: a*-N* curves for different stress ranges, Δσ*, while maintaining the initial crack size, a_0* constant

The integration follows after fitting the geometric factor to the first Expression in (6). Fig. 9 illustrates the fit obtained for the geometrical crack factor in the specimen

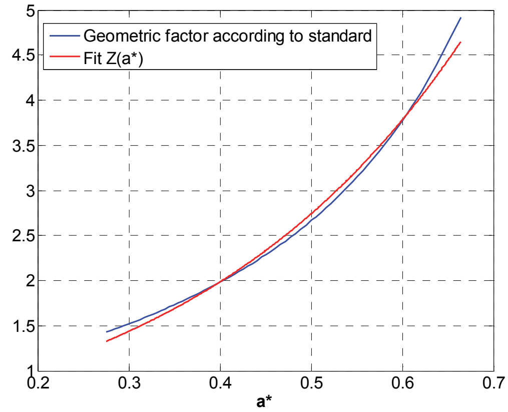

Figure 9: Fitting of function Z(a*).

d- Derivation of the S-N field

Finally, the S-N field in propagation can be derived from former information using Expr. (7). Fig. 10 shows the S-N field in which both axes representing non-dimensional variables.

The information gained from both models, that is that representing the probabilistic S-N field and that related to the crack growth rate curve as proposed here, provide a basis for the probabilistic interpretation to the Kitagawa-Takahashi diagram [10]. First, it seems possible to assign crack sizes to iso-probabilistic curves in the S-N field, making use of the El Haddad equation, and second, the asymptotic character of the S-N field allow a true endurance limit to be defined, thus extending the concept and application of the Kitagawa-Takahashi diagram for finite number of cycles (limited propagating cracks) and infinite number of cycles (non-propagating cracks) according to the asymptotic value of the fatigue limit for $N \rightarrow \infty$. Finally, the study of variability of the crack growth rate curve is facilitated by the normalizing concept that includes the consideration of the threshold stress intensity factor ΔK_{th} as a model parameter.

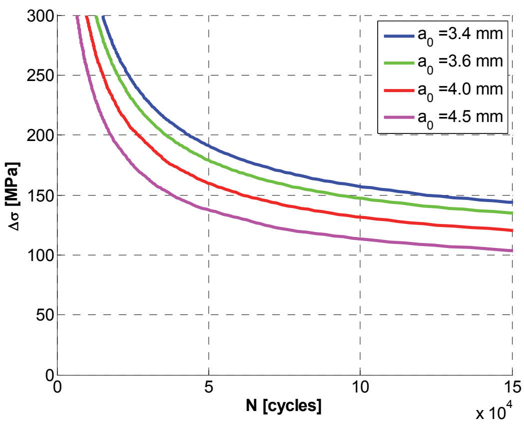

Figure 10: S-N field in propagation as derived from the crack growth curves.

CONCLUSIONS

The main conclusions derived from this work are:

- The validity of the methodology proposed is confirmed for fitting experimental data to the crack growth rate curve of one steel used for crankshafts. Also the crack growth curves a-N curves for different initial crack size values and load, or remote stress range.
- This opens new perspectives for the crack growth prediction under variable loading.
- The derivation of the S-N requires further study in order to solve scale problems observed.

- Matlab subroutines for any of the procedures used here were developed in order to facilitate the application to practical cases and will be offered as free program in a next future.

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