Multiaxial fatigue criterion based on parameters from torsion and axial S-N curve

M. Margetin
Slovak university of technology, Faculty of mechanical engineering, Slovakia
matus.margetin@stuba.sk

R. Ďurka
VAKUUMTECHNIK s.r.o., Slovakia
durka@vakuumtechnik.sk

V. Chmelko
Slovak university of technology, Faculty of mechanical engineering, Slovakia
vladimir.chmelko@stuba.sk

ABSTRACT. Multiaxial high cycle fatigue is a topic that concerns nearly all industrial domains. In recent years, a great deal of recommendations how to address problems with multiaxial fatigue life time estimation have been made and a huge progress in the field has been achieved. Until now, however, no universal criterion for multiaxial fatigue has been proposed. Addressing this situation, this paper offers a design of a new multiaxial criterion for high cycle fatigue. This criterion is based on critical plane search. Damage parameter consists of a combination of normal and shear stresses on a critical plane (which is a plane with maximal shear stress amplitude). Material parameters used in proposed criterion are obtained from torsion and axial S-N curves. Proposed criterion correctly calculates life time for boundary loading condition (pure torsion and pure axial loading). Application of proposed model is demonstrated on biaxial loading and the results are verified with testing program using specimens made from S355 steel. Fatigue material parameters for proposed criterion and multiple sets of data for different combination of axial and torsional loading have been obtained during the experiment.

KEYWORDS. Fatigue; Multiaxial; S355, Criterion, Material properties.

INTRODUCTION

Fatigue life time prediction plays an important part in mechanical equipment design, regarding operating safety as well as equipment's reliability and economical design. The ongoing increase of machines' operating parameters and the pursuit of both effective material use and operating reliability make the analysis of fatigue process significant in the area of constructions' mechanical endurance calculation.

The critical point of a construction that determines the life time of the whole equipment is often localized on a component that is exposed to a complex loading of external forces. Whether it's high-pressure piping systems [1] or mobile working machines [2], components of both are subjected to combined loading dependent on external conditions. Such a loading causes a stresses in the critical point, and this stress state is nearly always multiaxual.

In the process of life time estimation in multiaxial stress state, it is not sufficient to transform this state into uniaxial stress state according to static strength hypotheses, as they especially don't consider the cyclical properties of materials and the different effects of normal and shear stresses on the fatigue life time. Therefore, it's necessary to use a mathematical model that is both able to reduce the multiaxial stress state to uniaxial stress state and that respects the mentioned problems at the same time.

The methodology of transformation into uniaxial stress state then needs to be able to include also the change in the direction of damage, hence to respect the directional characteristic of the fatigue process.

Nearly a century passed since first attempts to tackle the problem of multiaxial fatigue have been made, and as for the situation today, there are plenty of criteria that consider component's multiaxial stress state. According to the methodology of assessment of loading process in the critical point, these criteria can be divided into stress-based criteria [3,4,5,6], strain-based criteria [7,8,9] and criteria based on fracture mechanics [10,11,12].

This text presents a new stress-based criterion that transforms the multiaxial stress state of a cyclic loading into an equivalent uniaxial stress. This criterion is based on the critical plane approach. After presentation in the text, the criterion is subsequently verified using proportional tension/compression and torsion loading in an experiment.

MULTIAXIAL FATIGUE CRITERION

Today's most used stress-based criteria that transform multiaxial stress state into equivalent stress amplitude in critical plane are in the form the following linear or non-linear combination:

$$\sqrt[a]{\left(b\sigma^{c}+d\tau^{e}\right)}=f\left(N_{f}\right) \tag{1}$$

Findley [3], McDiarmid [4] and Matake [5] have derived criteria for the calculation of the equivalent amplitude of shear stress as a linear combination of amplitude of shear stress and normal stress in the critical plane in the following form

$$\tau_{eq}=\sigma+k\tau=f\left(N_{f}\right) \tag{2}$$

On the other hand, Carpintieri with Spagnoli [6] and Papuga with Ruzicka [13] have derived criteria for the calculation of the equivalent amplitude of normal stress in the critical plane as a non-linear combination in the following form

$$\sigma_{eq}=\sqrt{k_{1}\sigma+k_{2}\tau}=f\left(N_{f}\right) \tag{3}$$

The difference between the respective criteria is in the definition of the critical plane and in the form of material parameters k that consider the effect of normal and shear stresses. The resultant amplitude of the equivalent stress is then compared with the adequate fatigue life time curve in order to determine the finite fatigue life time or with fatigue limit in order to determine the infinite fatigue life time.

The results achieved by presented hypotheses more or less correlate with the experimental results, however, there are some commonly known and well documented problems:

Parameters weighting the effect of normal and shear stress in hypotheses are independent on the loading level (i.e. number of cycles to failure), which is not true universally [14,15].

Material parameters are based on conventional values (yield stress, fatigue limit) that are strongly dependent on the methodology of determination and sometimes their existence itself is questionable (fatigue limit being the example - there's no agreement on whether there is an actual amplitude of stress that isn't damaging).

Neither one of the criteria provides correct results for both boundary loading conditions (pure torsion and pure tension/compression loading).

Based on the previous analysis of the problem, authors have decided to present their own criterion for transformation of multiaxial stress state into equivalent uniaxial stress state. Presented criterion results from the following theoretical premises:

The criterion is based on the critical plane approach and it assumes that the critical plane is the plane with the maximal shear stress amplitude. The premise that the plane with the maximal shear stress amplitude plays a key role in the process of the crack initiation is well documented in work [16].

The criterion reckons with non-linear combination of shear stress amplitude and normal stress amplitude in the critical plane.

$$\tau_{eq} = \sqrt{k\sigma_{a,cr}^2 + \tau_{a,cr}^2} \tag{4}$$

The criterion doesn't use conventional values of material parameters. Parameters that represent cyclical characteristics of the tested material are in the form of Baskin equation parameters for pure axial loading Eq. 5 and pure torsion loading Eq. 6.

$$\sigma_a = \sigma_f^{'}\left(2N_f\right)^{b_\sigma} \tag{5}$$

$$\tau_a = \tau_f^{'}\left(2N_f\right)^{b_\tau} \tag{6}$$

The criterion is derived so that it provides correct results for both boundary loading conditions (pure torsion and pure tension/compression loading represented by Eqs. 5,6).

The parameter weighting normal stress is then in Eq. 7 and the resulting form of the criterion is shown in Eq. 8.

$$k = \left(\frac{2\tau_f'}{\sigma_f'}\right)^2 \left(\frac{2\sigma_{max}}{\sigma_f'}\right)^{\frac{2\left(b_\tau - b_\sigma\right)}{b_\sigma}} - 1 \tag{7}$$

$$\tau_r = \sqrt{\left(\left(\frac{2\tau_f'}{\sigma_f'}\right)^2 \left(\frac{2\sigma_{max}}{\sigma_f'}\right)^{\frac{2\left(b_\tau - b_\sigma\right)}{b_\sigma}} -1\right) \cdot \sigma_{max}^2 + \tau_a^2} = \tau_f'\left(2N_f\right)^{b_\tau} \tag{8}$$

EXPERIMENTAL ASSESSMENT

To verify the function of the proposed criterion, an experiment was conducted in which experimental specimens were tested at different levels of proportional multiaxial loading. Experiment was carried out in the Strength and Elasticity Laboratory of the Faculty of Mechanical Engineering STU, using two experimental stands: Inova EDYZ testing system (tension/compression test) and MTS Bionix 370.02 Axial/Torsion testing system (torsion test and tension/torsion test). Two sets of experimental specimens were manufactured from steel S355J2+C (chemical composition is in Tab. 1 and specimens' geometry in Fig. 1.).

σ_{y02} [MPa]	σ_u [MPa]	A5 [%]
655	680	11.2

C [%]	P [%]	S [%]	Mn [%]	Si [%]	Cu [%]	Al [%]	Mo [%]	Ni [%]	Cr [%]
0.16	0.014	0.025	1.31	0.18	0.12	0.018	0.01	0.06	0.07

Table 1: Mechanical and chemical properties of S355J2+C steel

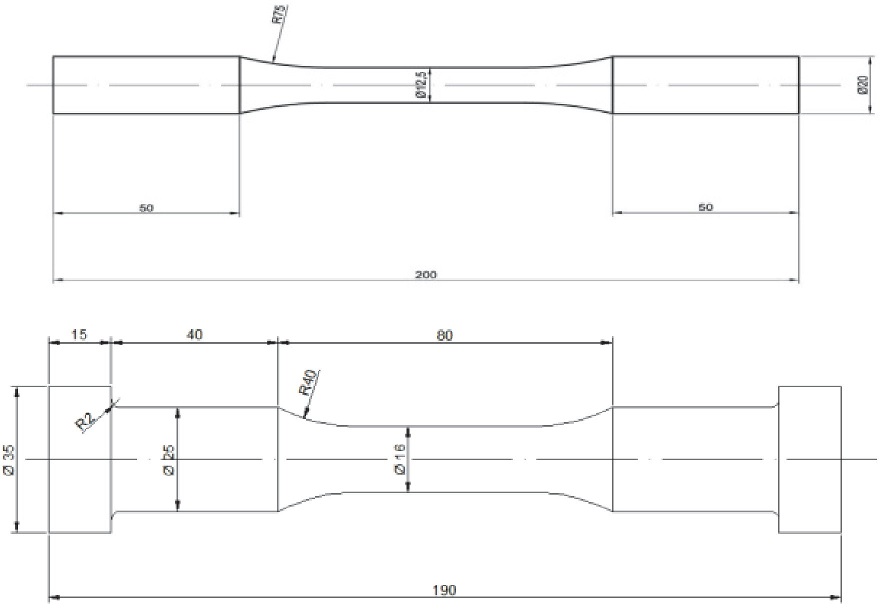

Figure 1: Geometry of the experimental specimens.

In the first part of the experiment, Baskin equation parameters for pure axial loading and pure torsion loading were acquired. Experimental specimens were loaded in the force control mode. Failure condition of the experimental specimen was defined by the moment of the so-called "technical initiation of fatigue crack" (0,5–1 mm). The number of cycles prior to the initiation of the fatigue crack was determined on the basis of a continuous measurement of the deformation response to the loading regime of the test specimen σ_a (or τ_a) = const.. Completion of the test was defined either by the increase of the deformation (or by the angle of the distortion) by 1% in reference to the mean value or by the achievement of the life time of 2.10^6 cycles. The values of material parameters for the regression line and for the upper and lower prediction intervals of reliability for pure axial loading and pure torsion loading are shown in Tab. 2.

P=50%		P=2.5%		P=97.5%		
τ_f [MPa]	b_τ [-]	τ_f [MPa]	b_τ [-]	τ_f [MPa]	b_τ [-]	R
550	-0.0736	565	-0.0739	536	-0.0732	-0.9876
σ_f [MPa]	b_σ [-]	σ_f [MPa]	b_σ [-]	σ_f [MPa]	b_σ [-]	R
636	-0.0531	655	-0.0531	619	-0.0531	-0.9779

Table 2: Fatigue properties.

To verify the validity of the proposed criterion, the experimental program was carried under multiaxial stress state. The experimental specimen was subjected to a proportional combination of axial and torsion loading using controlled loading force and torque. The test was completed by achieving the same conditions as in the uniaxial loading (see above).

CONCLUSIONS

Results of the experiment are tabularly summarized in Tab. 4. Life time estimated by the help of the presented hypothesis Eq. 4 is N_{f_com} and the actual measured life time is N_{f_exp}. For the purposes of comparison, Tab. 4 includes also estimated life times with the help of the well known hypotheses based on stresses in critical plane presented by Findley (N_{f_find}) [3] and McDiarmid (N_{f_mcd}) [4] Eqs 9 and 10. For the calculation of the fatigue life time, material parameters for regression line of fatigue curves were used (Tab. 2). Material parameters used in Eqs 9 and 10 are shown in Tabs. 1, 2 and 3.

$$\tau_{eq} = \left(\tau_a + k_{fin}\sigma_n\right)_{max} = \tau_f^{*}\left(2N_f\right)^{b_\tau} \tag{9}$$

$$\tau_{eq} = \tau_a + \left(\frac{k_{mcd}}{2\sigma_u}\right)\sigma_{n,max} = \tau_f^{'}\left(2N_f\right)^{b_\tau} \tag{10}$$

k_{fin} [-]	τ_{f*} [MPa]	k_{mcd} [MPa]
0.131	555	244

Table 3: Material parameters for Eqs 9 and 10

n.	τ_a [MPa]	σ_a [MPa]	N_{f_exp}	N_{f_com}	N_{f_find}	N_{f_mcd}
1	159	204	650800	643902	485504	291483
2	180	204	172700	202990	154283	94670
3	167	204	597300	415031	313706	189936
4	209	163	105300	96899	65261	42766
5	183	163	536120	449372	283251	181345
6	196	163	198810	205010	133788	86699
7	209	122	193800	165247	113010	77586
8	201	122	301140	267920	178905	122110
9	193	122	375630	441567	287341	194911
10	185	122	996040	740543	468583	315769
11	209	82	231490	226301	178946	129725
12	201	82	572500	375732	291264	210195
13	193	82	952600	635892	482358	346420
14	185	82	2000000*	1098588	813741	581393

Table 4: Experimental data.

Looking at the table, it's evident that estimations using Findley's or McDiarmid's criteria are too conservative for the specimen material and the loading levels we used. At the same time, comparing these two criteria, Findley's criterion has smaller deviation from the experimentally acquired data.

Figure 2 shows comparison of calculated and experimentally acquired life times for the proposed hypothesis. For each specimen, N_{f_cal} are listed in the chart shown in a probabilistic form for the regression line and the lower and the upper prediction reliability intervals of the material parameters (Tab. 2). Chart shows that the proposed hypothesis correlates well with the experimentally acquired values of the fatigue life time. Majority of the experimentally acquired life times are placed within the reliability interval of the estimated life times. Experimentally acquired life times of specimens number 3,10,12, 13 and 14 were outside of the reliability interval. For these cases, the criterion provided conservative results. The specimen number 14 was put aside after going through 2 .10^6 loading cycles.

Based on the experimental verification of the proposed hypothesis (Figure 2) and on its comparison with the well known hypotheses (Tab. 4), following can be stated:

- The hypothesis correlates well with the experimentally acquired data - the majority of measured life times are placed within the prediction interval of the calculated life times.
- In case the hypothesis doesn't provide correct results (experimentally acquired life time is outside of the prediction interval of the calculated life time), the results are on the conservative side of the calculation.

- In comparison with Findley's and McDiarmid's hypotheses, the life times calculated for the particular experimental program are closer to the measured values. At the same time, both hypotheses provide significantly more conservative results then proposed criterion.

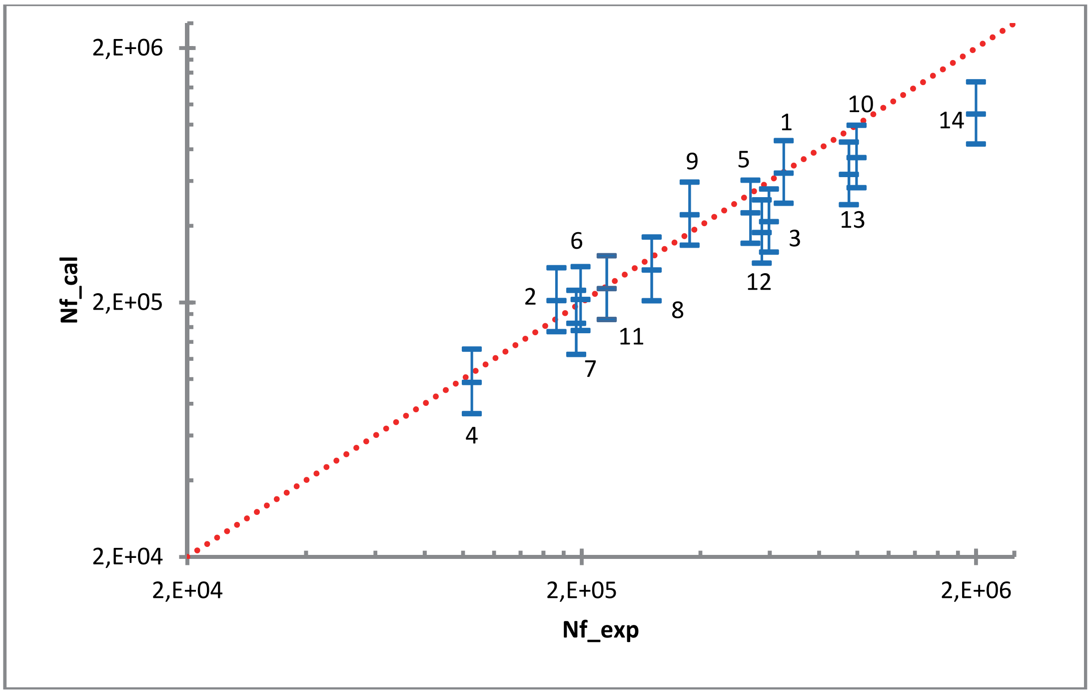

Figure 2: Comparison of experimentally acquired and calculated life times.

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NOMENCLATURE

σ, τ - normal and shear stress respectively
σ_a, τ_a, $\sigma_{a,eq}$ - normal, shear and equivalent stress respectively
$\sigma_{f'}$, $\tau_{f'}$ - normal and shear fatigue strength coeficient respectively
b_σ, b_τ - normal and shear fatigue strength exponent
σ_n - normal stress in computed plane
N_f - cycles to failure
σ_{y02} - yield strength
σ_u - ultimate strength
k_i - material parameters used to weight normal and shear stress respectively
R - correlation coefficient
P - probability of occurrence