Interaction of shear and normal stresses in multiaxial fatigue damage analysis

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ABSTRACT. Due to the abundance of engineering components subjected to complex multiaxial loading histories, being able to accurately estimate fatigue damage under multiaxial stress states is a fundamental step in many fatigue life analyses. In this respect, the Fatemi-Socie (FS) critical plane damage parameter has been shown to provide excellent fatigue life correlations for a variety of materials and loading conditions. In this parameter shear strain amplitude has a primary influence on fatigue damage and the maximum normal stress on the maximum shear plane has a secondary, but important, influence. In this parameter, the maximum normal stress is normalized by the material yield strength in order to preserve the unitless feature of strain. However, in examining some literature data it was found that in certain situations the FS parameter can result in better fatigue life predictions if the maximum normal stress is normalized by shear stress range instead. These data include uniaxial loadings with large tensile mean stress, and some non-proportional axial-torsion load paths with different normal-shear stress interactions. This modification to the FS parameter was investigated by using fatigue data from literature for 7075-T651 aluminum alloy, as well as additional data from 2024-T3 aluminum alloy fatigue tests performed in this study.

KEYWORDS. Multiaxial Fatigue; Critical Plane; Fatemi-Socie; Mean Stress; Load Path Interaction.

INTRODUCTION

Although there are many different steps in the fatigue life estimation process, relating the variation of stresses and strains to the fatigue damage that occurs within a material is the most fundamental part of any fatigue life analysis. Due to the abundance of engineering components subjected to complex multiaxial loading histories, being able to accurately estimate fatigue damage under multiaxial stress states is especially important.

To this end, significant effort in the last few decades has been put into developing damage parameters which reflect the actual damage mechanisms of the fatigue failure process [1,2]. Chief among these are critical plane approaches, which build on the observation that fatigue cracks tend to initiate on preferred planes within a material. Critical plane approaches are typically based on the idea of crack initiation occurring on or around either the maximum principal plane or maximum shear plane. As a result, these approaches have the added benefit of being able to predict failure plane orientation, which is useful if a subsequent crack growth analysis is to be performed.

A popular critical plane-based parameter for computing multiaxial fatigue damage in materials exhibiting shear failure mechanisms is the Fatemi-Socie (FS) parameter [3]. This parameter was formulated based on the idea that while alternating shear strain is the primary driving force behind fatigue crack initiation, the maximum normal stress on the

crack plane also affects the nucleation and growth of small cracks by influencing the amount of friction and interlocking between opposing crack faces. Therefore, the inclusion of a maximum normal stress term is able to account for the effects of mean stress in a manner that holds physical significance.

The FS parameter predicts fatigue life in terms of shear fatigue properties based on the following equation:

$$\frac{\Delta\gamma_{\max}}{2}\left(1+k\frac{\sigma_{n,\max}}{\sigma_y}\right)=\frac{\tau_f'}{G}\left(2N_f\right)^{b_o}+\gamma_f'\left(2N_f\right)^{c_o} \tag{1}$$

where $\Delta\gamma_{max}$ is the maximum range of shear strain experienced on any plane, $\sigma_{n,max}$ is the maximum normal stress occurring on the same plane for the cycle of interest, σ_y is the material yield strength, and k is a material dependent parameter reflecting the influence of normal stress on fatigue damage. The maximum normal stress is normalized by yield strength as a means of preserving the unitless feature of strain.

The right hand side of Eq. 1 represents the shear strain-life curve for the material under consideration. In the event that shear fatigue properties are not available for damage calculation, the right side of Eq. 1 may alternatively be expressed in terms of uniaxial fatigue properties as follows:

$$\frac{\Delta\gamma_{\max}}{2}\left(1+k\frac{\sigma_{n,\max}}{\sigma_y}\right)=\left[\left(1+\nu_e\right)\frac{\sigma_f'}{E}\left(2N_f\right)^b+\left(1+\nu_p\right)\varepsilon_f'\left(2N_f\right)^c\right]\left[1+k\frac{\sigma_f'}{2\sigma_y}\left(2N_f\right)^b\right] \tag{2}$$

where ν_e is elastic Poisson's ratio, ν_p is Poisson's ratio for fully plastic conditions, usually taken to be 0.5, and all other fatigue properties correspond to the fully-reversed uniaxial strain-life equation.

Since the normal stress term is multiplied by a strain term, the FS parameter is also able to account for changes in fatigue damage brought about by cyclic and/or non-proportional hardening. Damage parameters based on only stress or only strain terms, on the other hand, cannot reflect these changes in material constitutive behavior. Additionally, because the normal stress term is multiplied by the shear strain range, the FS parameter assumes that cyclic shear strain must be present in order for fatigue damage to occur. This is important because it prevents the prediction of fatigue damage in situations where only a static axial stress may exist on a plane.

Overall, the FS parameter, based on either shear or uniaxial fatigue properties, has been shown to correlate experimental and predicted fatigue lives well for a variety of materials and loading conditions. Despite this fact, when analyzing some recent literature data, e.g. [4,5], it was found that the parameter can result in non-conservative life predictions when significant tensile mean stress is present. Although increasing the k value in the FS parameter can improve mean stress correlation by increasing the influence of the maximum normal stress term, this has a detrimental effect on the correlation of fatigue data generated under other multiaxial loading paths. As a result, in the first part of this study, modifications to the FS parameter are investigated in an attempt to improve fatigue life predictions.

In addition to mean stress effects, Shamsaei [6] performed fatigue tests on 1050 steel for different multiaxial loading paths which produced the same fatigue damage predictions based on the FS parameter. From the results of these tests, however, it was found that the average experimental fatigue life for cyclic torsion with static tension loading was around half of the life predicted for in-phase axial-torsion loading at the same damage value. Conversely, the average life for cyclic torsion with pulsating tension loading was around a factor of 2.5 longer than for in-phase loading. Although a factor of ±2.5 error is still very reasonable for multiaxial fatigue life predictions, these results suggest that there is room for some improvement in fatigue damage calculation with respect to the quantification of normal-shear stress/strain interaction.

Considering the load path dependence of fatigue damage, the second part of this study focuses on a limited number of fatigue tests designed to differentiate between the original FS parameter and the modified parameter developed in the first part of the study. These tests were performed using tubular specimens of 2024-T3 aluminum alloy under loading conditions which result in the same damage value based on the original FS parameter, but different damage values for the modified parameter. Differences in experimental lives were then compared to predictions to determine which parameter more closely reflects the fatigue damage variation between loading paths.

MATERIAL AND TESTING PROCEDURES

The material chosen for the fatigue tests performed in this study was aluminum alloy 2024-T3. Mechanical properties for the material include a yield strength (0.2% offset) of 330 MPa, ultimate tensile strength of 495 MPa, and modulus of elasticity of 73.7 GPa. Additional deformation and fatigue properties can be found in [7]. All tests

were performed using un-notched specimens of a thin-walled tubular geometry. The specimens feature a 30 mm long gage section with an outside diameter of 29 mm and an inside diameter of 26 mm, resulting in a wall thickness of 1.5 mm. Additional details, including complete specimen geometry, can be found in [8].

While mean stress effects were not studied for the 2024-T3 aluminum alloy tested in the current study, several constant amplitude fully-reversed fatigue tests were performed under a variety of loading paths. These tests were performed in load control and include uniaxial (12 tests) pure torsion (15 tests), torsion with static axial stress (8 tests), in-phase axial-torsion (6 tests), and 90° OP axial-torsion (7 tests) loading conditions. Additional tests were also performed using triangular load paths (4 tests total) in order to study the effects of axial and shear stress interaction on fatigue damage.

All testing was carried out in a closed loop servo-hydraulic axial-torsion load frame with a dynamic rating of 100 kN axial load and 1 kN·m torsional load. Load train alignment was carefully maintained throughout testing to produce no more than 5% bending at 1000 microstrain. The definition of crack initiation was considered to be a 3% change in displacement or rotation amplitude when compared to a stable reference cycle. This generally corresponded to final crack lengths of approximately 10–15 mm, with growth from 1 mm to final length occurring very rapidly.

In addition to the 2024-T3 tests data generated in this study, literature data for 7075-T651 (σ_y = 501 MPa, σ_u = 561 MPa), reported by Zhao and Jiang [4], were also analyzed. Loading conditions included were similar to those for the 2024-T3 tests: uniaxial (131 tests), torsion (17 tests), torsion with static axial stress (9 tests), in-phase (9 tests), and 90° OP axial-torsion (7 tests). Uniaxial fatigue tests were conducted at R ratios ranging from -∞ to 0.7, while tests for all other loading paths were performed under fully-reversed conditions.

MEAN STRESS EFFECTS ON DAMAGE CALCULATION

As mentioned in the introduction, although the Fatemi-Socie parameter has been shown to provide excellent fatigue life correlations under a variety of loading conditions, for 7075-T651 fatigue data reported in [4], it was found that the parameter resulted in non-conservative life predictions when significant tensile mean stress was present. Correlation of this test data using the FS parameter (k = 1) is shown in Fig. 1(a). Data for tests with experimental lives less than 50 cycles are excluded from the figure due to the possibility of unstable material behavior when maximum stresses are near the ultimate strength of the material. Additionally, data from runout tests are also excluded. It is clear from this figure that, despite reasonably good correlation between the different multiaxial load paths under fully-reversed conditions, life predictions are increasingly non-conservative for uniaxial loading conditions as the R ratio, and thus tensile mean stress, is increased. A similar trend of non-conservative fatigue life predictions in the presence of tensile mean stress was also observed for ductile cast iron data reported by Meyer [5].
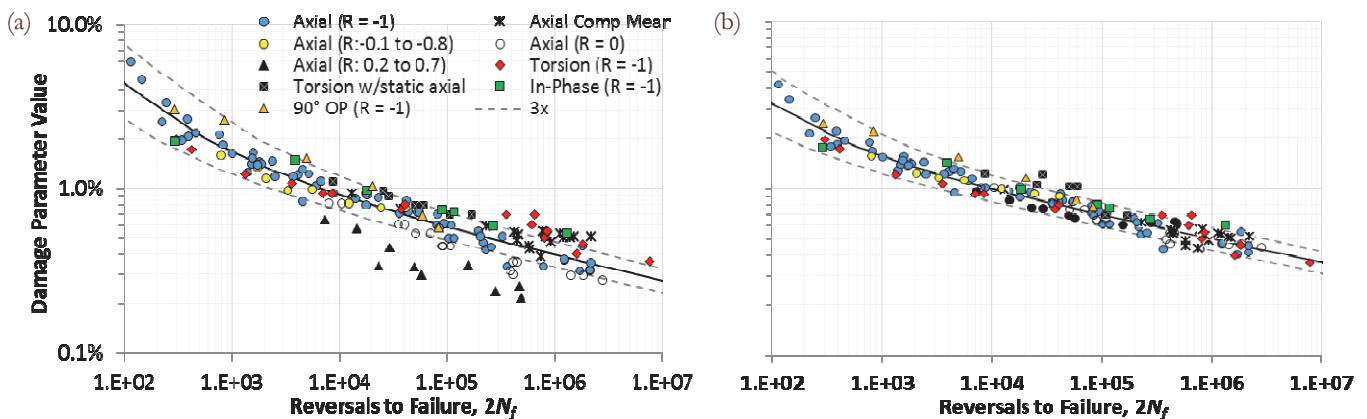

Figure 1: Fatigue life correlations for 7075-T651 aluminum alloy based on (a) FS and (b) modified FS parameters with uniaxial strain-life properties and k = 1.

Although increasing the k value in the FS parameter can improve the correlation of mean stress data by increasing the influence of the maximum normal stress term, this also has a detrimental effect on the correlation of fatigue data generated under other multiaxial loading paths. This is especially true for pure torsion loading where the maximum normal stress on the maximum shear plane is zero. It should also be noted that the accuracy of life predictions based on shear

strain-life properties is more sensitive to changes in k value. This is because the γ-N life prediction curve (Eq. 1) does not change with the k value in the same manner that the uniaxial life prediction curve (Eq. 2) does.

Given these results, modifications to the FS parameter were investigated in an attempt to improve life predictions. Because life predictions were found to be non-conservative in the presence of significant tensile mean stress, it was apparent that the effect of maximum normal stress should be increased when such conditions exist. However, in order to maintain good fatigue life correlations for fully-reversed and multiaxial loading conditions, the influence of the normal stress term should not change in these cases. It was determined that substituting the yield strength in the FS equation for a quantity based on stress amplitude/range could achieve this effect. In addition, it can also be observed in Fig. 1(a) that the correlation between fully-reversed axial and torsion fatigue data begins to degrade in the high cycle fatigue regime. Therefore, substituting yield strength for a value based on shear stress was thought to be beneficial. This was based on the idea that the ratio of normal stress to shear stress could allow for better consideration of interaction effects between the two types of stresses.

Through some trial and error, it was found that replacing σ_y in Eq. 1 with $G\Delta\gamma$, where $\Delta\gamma$ is the shear strain range on the maximum shear strain plane, resulted in improved fatigue life correlations in the presence of mean stress. This yields the following equation for multiaxial fatigue damage calculation:

$$\frac{\Delta\gamma_{\max}}{2}\left(1 + k\frac{\sigma_{n,\max}}{G\Delta\gamma}\right) = \frac{\tau_f'}{G}\left(2N_f\right)^{b_o} + \gamma_f'\left(2N_f\right)^{c_o} \tag{3}$$

Shear stress range was expressed using $G\Delta\gamma$, as opposed to $\Delta\tau$, in order to account for the effect that changes in material constitutive behavior can have on fatigue damage. Although $G\Delta\gamma$ is equal to $\Delta\tau$ at longer lives, where deformation is elastic, normalizing $\sigma_{n,max}$ by a quantity based on strain predicts an increase in damage when cyclic and/or non-proportional hardening occur. Normalizing $\sigma_{n,max}$ by $\Delta\tau$, on the other hand, would result in the same damage value in situations where the two stress components change proportionally as a material hardens.

This modified damage parameter maintains all of the advantages and physical interpretations of the original FS parameter, as discussed in the introduction, without introducing any additional empirical fitting constants. Additionally, from a physical standpoint, the reduction of the normal stress term with increasing shear stress/strain range may reflect the idea that as shear stress/strain increases, larger local shear deformations are able to overcome some of the resistance caused by friction and interlocking between opposing crack faces. Although there is a possibility for unrealistically large damage values to be computed as the shear strain range approaches zero, this mathematical issue can be overcome by imposing appropriate limits on the shear strain range required to produce fatigue damage. For example, in situations where the shear strain range is below its fatigue limit value, the damage parameter can be assumed to take a value of zero.

The right-hand side of Eq. 3, which relates the value of the damage parameter to fatigue life based on shear strain-life properties, can alternatively be expressed in terms of uniaxial fatigue life properties as follows:

$$\frac{\Delta\gamma_{\max}}{2}\left(1 + k\frac{\sigma_{n,\max}}{G\Delta\gamma}\right) = \left[\left(1 + \nu_e\right)\frac{\sigma_f'}{E}\left(2N_f\right)^b + \left(1 + \nu_p\right)\varepsilon_f'\left(2N_f\right)^c\right] + k\frac{\sigma_f'}{4G}\left(2N_f\right)^b \tag{4}$$

By recalculating fatigue damage for the 7075-T651 test data, improvements offered by the modified FS parameter become evident in Fig. 1(b). A k value of 1 was used in all modified parameter damage calculations. Fatigue life correlations are not only qualitatively improved, with a tighter grouping of test data from all loadings conditions, but the overall accuracy of predictions is also increased. Additionally, comparisons between modified parameter calculations based on the inclusion of $\Delta\tau$ versus $G\Delta\gamma$ revealed that differences in life predictions were negligible above 1000 reversals, and within a factor of around 1.5 at lives on the order of 50 reversals.

LOAD PATH EFFECTS ON DAMAGE CALCULATION

I n addition to improved mean stress consideration, multiaxial fatigue life correlations and predicted failure planes under fully-reversed loading conditions remain very similar to the already accurate predictions based on the original FS parameter. In fact, fully-reversed data correlations were even found to improve slightly using the modified version

of the parameter. This is especially evident in the comparison between fully-reversed axial and torsion loading conditions in Figs. 1(a) and 1(b).

In order to further evaluate the characteristics of the proposed modified FS parameter, a limited number of fatigue tests were also performed in this study using specialized load paths meant to differentiate between the two different versions of the parameter. These tests featured loading conditions which result in the same damage value based on the original FS parameter, but different damage values based on the modified parameter. Differences in experimental lives were compared to predictions to determine which parameter more closely reflects the fatigue damage variation between loading paths. These tests were conducted using a triangular shaped load path in shear versus axial stress space. This path, along with its corresponding stress-time history, is shown in Fig. 2(a) based on normalized axial and shear stress variations.
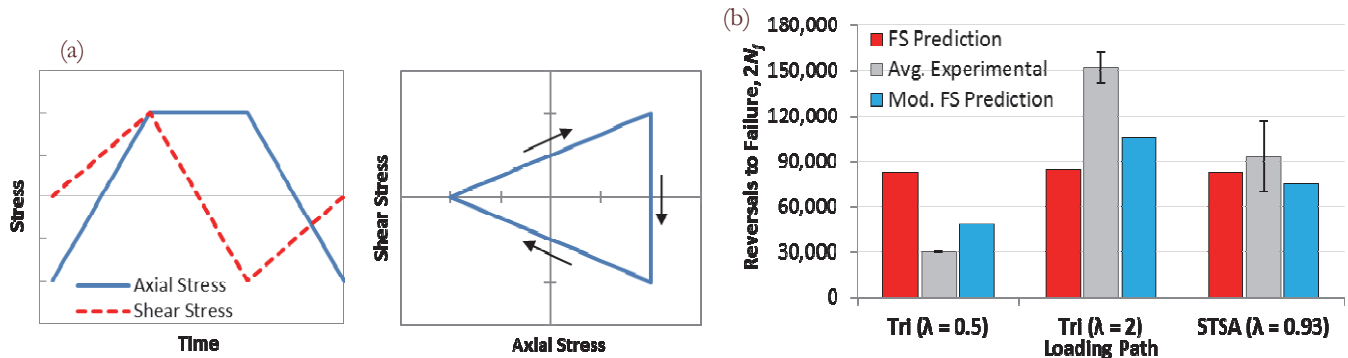

Figure 2: (a) Triangular load path in terms of stress-time history and shear vs. axial stress path, and (b) experimental and predicted fatigue life for discriminating load path tests.

In analyzing potential load paths, it was found that by changing the ratio of applied shear to axial stress, relatively large differences in predicted fatigue damage could be obtained between the original and modified FS parameters. Therefore, the same loading path was used in these tests, but with two different ratios of applied shear to axial stress, $\lambda = \tau/\sigma = 2$ and $\lambda = 0.5$. Stress values were carefully selected so that each path would result in the same fatigue damage value according to the FS parameter. Additionally, experimental results from fully-reversed torsion tests with static tensile stress were also available for comparison at the same damage value. This additional loading path allows for the evaluation of damage predictions for cycles containing different load-time interaction between shear and normal stress components.

The results of each of these tests are shown in Fig. 2(b) along with life predictions from both the FS and modified FS damage parameters. Results from the triangular load path tests (Tri), at both nominal stress ratios, along with results from torsion with static tension loading (STSA), are included. A k value of 1 and uniaxial fatigue life properties were used for all analyses. In this figure, gray columns represent the average experimental fatigue life from two duplicate tests, while error bars indicate the individual life of each test. From these results, it is clear that variations in experimental fatigue damage exist between the different loading conditions. While the FS parameter in its original form does not account for these differences, the modified form of the parameter reflects the observed differences in fatigue life relatively well for each loading path considered.

Given the improvements in life predictions obtained when using the modified FS parameter to calculate fatigue damage, one final step was to analyze the remaining fully-reversed fatigue data generated for the 2024-T3 aluminum alloy tested in this study. These data, plotted against the original FS parameter in Fig. 3(a), are presented again in Fig. 3(b) with damage calculated using the modified form of the parameter. Similar to the 7075-T651 data, it can be seen from these figures that the modified FS parameter provides somewhat improved fatigue life correlations for the 2024-T3 data. This is, again, evidenced by the slightly tighter grouping of test data from all loadings conditions. Overall life prediction accuracy, however, is similar to that obtained using the original form of the parameter.

SUMMARY AND CONCLUSIONS

In this paper, the effects of mean stress and normal-shear stress interaction on multiaxial fatigue damage were investigated. Based on literature data for 7075-T651 aluminum alloy, the Fatemi-Socie critical plane damage parameter was found to produce non-conservative life predictions in cases where significant tensile mean stress was present. While increasing the k value can improve uniaxial mean stress data correlation, this results in worse predictions

for other multiaxial loading conditions. It was found that by changing σ_y to $G\Delta\gamma$ in the FS parameter, the correlation of high tensile mean stress data was improved while retaining all the same physical interpretations of the parameter and adding no new material constants. Additionally, a limited number of discriminating load path tests, performed using 2024-T3 aluminum alloy, suggest that the modified FS parameter may also be able to better account for some of the effects of axial and shear stress interaction on fatigue damage. Although the results of this study are encouraging, more experimental data and analysis are still needed, for different materials and loading conditions, to verify these findings.
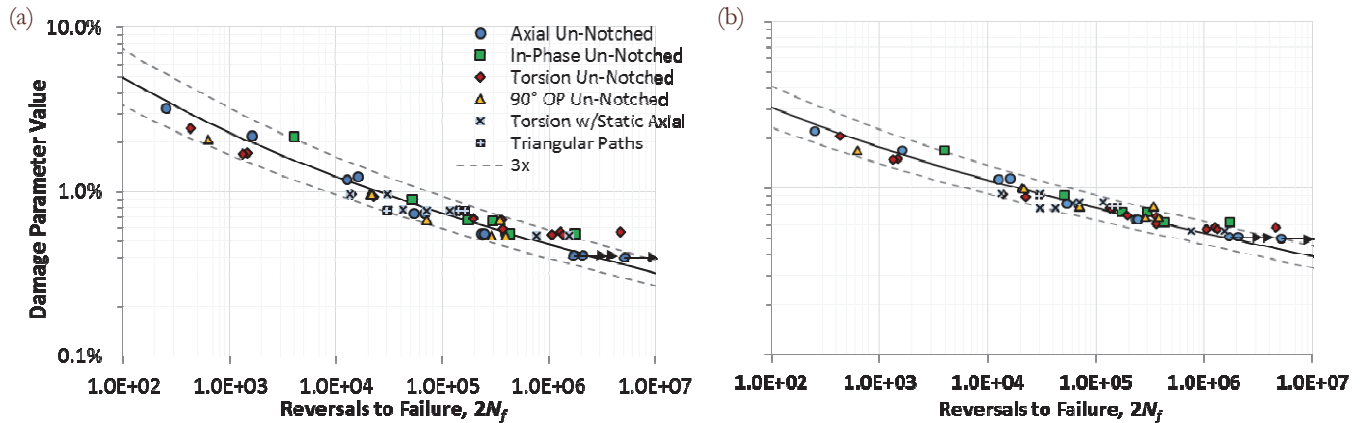

Figure 3: Fatigue life correlations for 2024-T3 aluminum alloy based on (a) FS and (b) modified FS parameters with uniaxial strain-life properties and $k = 1$.

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