Analysis of time-dependent reliability of degenerated reinforced concrete structure

Zhang Hongping, Zhang Lili, Xia Zhengbing
College of Architecture and Civil Engineering, Jiangsu City Vocational College Nantong Campus, Nantong, Jiangsu, 226006, China
zhanghpingzhp@163.com

ABSTRACT. Durability deterioration of structure is a highly random process. The maintenance of degenerated structure involves the calculation of the reliability of time-dependent structure. This study introduced reinforced concrete structure resistance decrease model and related statistical parameters of uncertainty, analyzed resistance decrease rules of corroded bending element of reinforced concrete structure, and finally calculated time-dependent reliability of the corroded bending element of reinforced concrete structure, aiming to provide a specific theoretical basis for the application of time-dependent reliability theory.

KEYWORDS. Durability of structure; Reinforced concrete; Degeneration.

INTRODUCTION

Among multiple structures, reinforced concrete structure fully integrates the characteristics of steel reinforcement and concrete and cost little; hence it has been extensively promoted in infrastructure construction of China [1]. Scholars in civil engineering field keep searching for the application of new materials and innovation of structural system and seeking breakthrough in structural disaster-resistance analysis and design [2]. However, durability and life-cycle performance degeneration of structure is an important issue concerned by civil engineering workers. But so far, research progress in this field is not considerable, which may be correlated to the involvement of multiple subjects and large difficulty [3]. Randomness of resistance of reinforced concrete structure in the initial stage of construction and attenuation of resistance under the effect of environment is important characteristics [4]. With the development of infrastructure construction in China, deep research on the problem is urgent. Material degeneration and structural aging would happen under the effects of human-made environment or natural environment during long-term application, which can weaken carrying capacity and durability of structure. However, durability of reinforced concrete structure has been ignored for a long time [5]. An investigation [6] suggested that, more than 70 billion dollars were caused because of corrosion of various structures in America in 1975, 40% of which is caused by corrosion of steel reinforcement. Besides, Britain spends about 200 thousand pounds on ocean environment corrosion every year. Japan even spends 40 billion yen on the maintenance of house structure every year. Reinforced concrete structure is frequently used in China and structural damage caused by durability has been quite serious. Steel reinforcement corrosion has been one of major reasons for the failure of reinforced concrete structure. As civil engineering in China has entered into the stage of aging, scientific detection and evaluation on such kind of structure is in urgent need. Therefore, it is of great significance to make safety evaluation of structure through predicting structural durability.

In recent years, scholars in China and abroad have made a large number of experiments on the factors influencing durability of structure and obtained many durability degeneration models of concrete structure [7]. Traditional structural reliability calculation model does not vary along with time and the characteristics of existing structure determines that the calculation of structural reliability using time-invariant model cannot consider the degeneration performance of structure. But this study considered the variation characteristics of structural resistance and load along with time and calculated the reliability based on time-dependent concept. In addition, we find that, most scholars focus on the construction of degeneration model and concern little about degeneration parameters. Actually, degeneration parameter is an important component of degeneration model. Analysis of durability degeneration model and time-dependent reliability of structure is of significance only when correct parameters are used. As to another innovation point, this study gave uncertainty statistical parameters of reinforced concrete structure resistance based on the detailed introduction of degenerated reinforced concrete resistance decrease model and made an analysis on the rules of resistance decrease of corroded bending element.

DEVELOPMENT AND STATUS OF STRUCTURAL RELIABILITY ANALYSIS METHOD

The concept of reliability originated from America. In 1939, National Advisory Committee for Aeronautics proposed the concept of airplane accident rate; then the concept was extensively applied in the field of civil engineering. In 1940s, Pugsley and Freudenthal published a paper titled Safety Degree of Structure. They integrated the theory of structural safety degree and the concept of mathematical statistics and proposed a reliability analysis calculation model. Then earliest reliability calculation model was extensively recognized and studied by other scholars [9]. In 1969, Cornell, an American expert, proposed the concept of reliability index, integrated it with structural failure rate and put forward the conversion formula with regard to reliability index and structural failure rate [10]. Then Cornell and Ang proposed first-order second-moment method for the calculation of structural reliability, which was extensively applied in engineering. Since then, many scholars kept innovating structural reliability analysis and calculation and proposed many structural reliability calculation methods.

Currently, researches on time-invariant theory have been relatively mature. However, time-dependent reliability theory involving multiple factors and more complex calculation remains to be further studied. On account of this, many scholars carried out a series of researches on time-dependent reliability of reinforced concrete structure. In 2008, Czarnecki et al. [11] proposed a time-dependent reliability model of reinforced concrete structure after finding the significant changes of reinforced concrete structure resistance along with the extension of time under corrosion environment and then applied the model for predicting the service time of structure. In 2012, Frangopol [2] applied structural health monitoring information obtained after integrating structure related priori knowledge and using Bayes updating probability to make a quantitative analysis on degeneration performance of structure. In 2014, Madsen [13] proposed a new structural time-dependent reliability and sensitivity analysis method. Similarly, many Chinese scholars are also dedicated to promoting the development of structural time-dependent reliability analysis. Through analyzing structural resistance decrease process induced by steel reinforcement corrosion, Chen Shoushan et al. [14] designed a method for evaluating and predicting reliability of structure. By investigating the existing resistance information of a structure and using a large number of measured data, Zhou Yan et al. [15] established a resistance time-independent reliability analysis model.

Resistance Decrease Model of Reinforced Concrete structure

The resistance of reinforced concrete structure depends on calculation model, geometrical parameters and material performance of structure. During the whole service process of structure, concrete carbonization and steel reinforcement corrosion can result in decline of strength of steel reinforcement and concrete, narrowing of cross section of steel reinforcement and deterioration of coordination between reinforced concrete and those adverse effects gradually accumulates as time goes on. Therefore, geometrical parameters and material performance of structure would both be degenerated as time goes on, which can weaken the resistance of structure. On account of this, when we consider the degeneration of resistance of reinforced concrete structure, the resistance decrease model can be expressed as:

$$P_r(t) = P[f_{aj}(t), d_j(t), k_{sj}(t)] \quad (1)$$

Where $P_r(t)$ stands for resistance of reinforced concrete structure, $P[X]$ stands for expression equation of reinforced concrete structure resistance, $f_{aj}(t)$ and $d_j(t)$ stands for performance and geometrical parameter of jth material and they are the functions of service time; and $k_{sj}(t)$ stands for coordination coefficient of jth steel reinforcement, and it is also the function of service time.

Uncertainty Statistical Parameters of Resistance

Uncertainty of reinforced concrete resistance includes uncertainty of calculation mode, material performance and geometrical parameters. Uncertainty of calculation mode refers to variability induced by basic assumption adopted by resistance analysis and evaluation, uncertainty of material performance refers to variability induced by durability degeneration, such as concrete carbonization, cross section loss induced by steel reinforcement corrosion, corrosion expansion, cracking and separation of concrete cover and mechanical property degeneration of corroded steel reinforcement.

Considering the uncertainty of calculation mode, the resistance of reinforcement concrete structure can be expressed using the following formula

$$P(t) = K_r P_r(t) \tag{2}$$

Where $P(t)$ stands for random process of resistance and K_r stands for random variable of uncertainty of calculation mode.

According to relevant methods of mathematical statistics and considering the uncertainty of model parameters, probability statistic characteristic of resistance can be obtained from the following formula.

$$\omega_P(t) = \omega_{K_r}\omega_{P_r}(t) \tag{3}$$

$$\sigma_P(t) = \sqrt{\sigma_{K_r}^2 + \sigma_{P_r}^2(t)} \tag{4}$$

$$\beta_P(t) = \sigma_P(t)\omega_P(t) \tag{5}$$

where ω_{K_r} and σ_{K_r} stand for the average value of random variable K_r and variable coefficient of calculation mode uncertainty respectively; $\omega_{P_r}(t)$ and $\sigma_{P_r}(t)$ stand for the function of average value and the function of variable coefficient respectively.

It can be known from formula (6), (7) and (8) that

$$\omega_{P_r}(t) = P[\omega_{faj}(t), \omega_{dj}(t), \omega_{K_{sj}}(t)] \tag{6}$$

$$\sigma_{P_r}(t) = \frac{\beta_{P_r}(t)}{\omega_{P_r}(t)} \tag{7}$$

$$\beta_{P_r}(t) = \sqrt{\sum_j \left[\frac{\partial \omega_P(t)}{\partial Z_j}|_\omega\right]^2 \beta_{Z_j}^2(t)} \tag{8}$$

where Z_j stands for relevant random variable influencing resistance and $\frac{\partial \omega_P(t)}{\partial Z_j}|_\omega$ stands for result of partial derivative when the average value was used.

It can be seen from the above deduction process that, some statistical characteristics of basic parameters of resistance needs to be determined at first before acquisition of statistical characteristics of final resistance. Depending on the current test means and research conditions, we can obtain some statistical characteristics of parameters easily. Using mathematical method, we can obtain uncertainty statistical rules of final model.

Statistical Parameters of Resistance of Corroded Bending Element

Bending element is the most common structural element in existing reinforced concrete structure. After years of research on corroded element, the calculation of bearing capacity of corroded bending element has been quite mature. Taking bending element of reinforced concrete as an example, resistance decrease rules of corroded bending element is analyzed in this section.

According to relevant knowledge of concrete structure design, bending capacity of normal section of rectangular section of corroded element can be obtained from the following formulas.

$$Y = \alpha_1 f_v lx(b - \frac{x}{2}) \tag{9}$$

$$x = \frac{B_{se} f_o}{\alpha_1 f_v l} \tag{10}$$

where Y stands for bearing capacity of normal section of corroded reinforcement (kN·m), f_v and f_0 stand for compressive strength of concrete axis and yielding strength of reinforcement (MPa); α_1 stands for coefficient (the value is determined according to relevant regulations in Code for Design of Concrete Structures); l and b stand for the width of section and effective height of section (mm); B_{se} refers to equivalent section area of tensile reinforcement (mm²) which is calculated using formula (11).

$$B_{se} = \sum_{i=1}^{n} K_{sj} \alpha_{sj} B_{sj} \tag{11}$$

where K_{sj} refers to coordination coefficient of ith reinforcement (decrease of cohesive force is considered), B_{sj} refers to section area of jth tensile reinforcement (mm²), and α_{sj} stands for reduction coefficient of yielding strength of jth reinforcement.

$$\omega_{P_r}(t) = \omega_C(t)\omega_b \left[1 - \frac{\lambda(t)}{2}\right] \tag{12}$$

$$\beta_{P_r}(t) = \sqrt{\omega_c^2(t)\omega b^2 \left\{ [1-\lambda(t)]^2 \sigma_C^2(t) + \frac{\lambda^2(t)}{4}\left[\sigma_l^2 + \sigma_{f_v}^2\right] + \sigma_b^2 \right\}} \tag{13}$$

$$\lambda(t) = \frac{\omega_C(t)}{\alpha_1 \omega_l \omega_b \omega_{f_v}} \tag{14}$$

where ω_{fv} and σ_{fv} stands for average value and variable coefficient of compressive strength of concrete axis, ω_l and σ_l refers to average value and variable coefficient of section width, ω_b and σ_b refer to average value and variable coefficient of effective height of section, and $\omega_C(t)$ and $\sigma_C(t)$ refer to average value and variable coefficient of yielding tensile force of corroded reinforcement.

Through analyzing parameters of mode, we can finally obtain relevant uncertainty statistical parameters of resistance of corroded bending element of reinforced concrete.

ANALYSIS METHOD OF STRUCTURAL TIME-DEPENDENT RELIABILITY

Basic Principle of Time Comprehensive Analysis method

Time comprehensive analysis method means to take the whole service period of a structure as a reference time period during analysis of structural reliability and obtain resistance and load model considering the changes of structural resistance and loading during the time period. The whole service period is regarded as a reference time period, and statistical characteristics of random variable relating to resistance and loading are all considered during the time period. Hence the maximum value of probability in the reference time period can be taken as the value of load. The practical conditions of P (t) and S (t) are shown in Fig. 1.

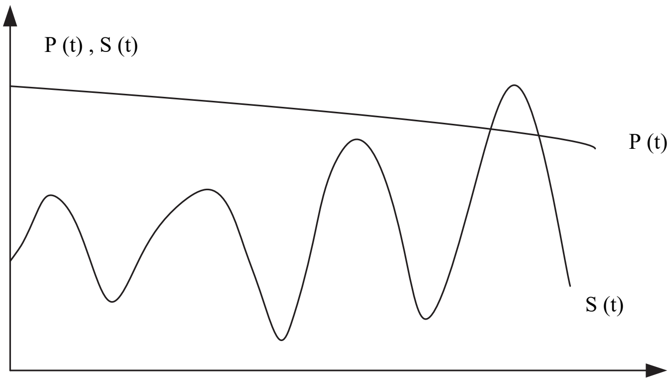

Figure 1. Changes of resistance and load.

From the perspective the concept of time-dependent reliability, P is a time-dependent function. But in time comprehensive analysis method, P is considered as a definite value and also the maximum value in the whole reference time period. It should be pointed out that, definite value herein does not refer to determined value, but random variable. We can obtain the practical value of P from probability density function fp.
In time comprehensive analysis method, failure probability can be obtained using the following formula.

$$D_f(t_A) = D\left[P_{\min} \le S_{\max}\right] \tag{15}$$

$S_{\max} = \max\limits_{0 \le t \le t_A} S(t)$ represents for the maximum load effect in the whole evaluation reference period [0, T] and $P_{\min} = \min\limits_{0 \le t \le t_A} P(t)$ represents for the maximum resistance in different stages in the whole evaluation reference period [0, T].
In practical application of time comprehensive analysis method, probability density function of $S(t)$ can be obtained through long-term observation of data. In this way, $S_{\max}$ can be obtained probability distribution function. But it is a pity that, long-term observation data are difficult to be obtained. Therefore, the extreme value distribution can be described by some short-term data. time-dependent reliability can be calculated by first order reliability method after analysis of load and resistance rules of structural element.

Analysis of time-dependent reliability of corroded bending element

A flexural simply supported beam component of reinforced concrete in someplace was taken as an example. Beam span was 6000 mm, spacing was 3,900 mm, and section size was 250 mm × 500 mm. The thickness of concrete cover was 25 mm, concrete strength was C20, and grade II rebar with a diameter of 16 mm was used, with a reinforcement ratio of 1.2%. As to the external environment, relative humidity was 71% and temperature was 13 $^{\circ}$C; besides, k_{co_2} was 1.2 and k_{ce} was 2.0. We know that, the reinforcement began to be corroded 13.7 years ago and the concrete cover began to crack due to corrosion expansion 26.5 years ago, after substituting relevant coefficients according the method stated in literature [16].
Other relevant resistance statistical parameters were as follows: ω_{K_r} = 1.0, σ_{K_r} = 0.04; ω_l = 1.0l, σ_l = 0.02; ω_b = 1.0b, σ_b = 0.03. The simply supported beam bears constant load and live load. The average value and standard error of constant load, i.e., ω_G and β_G, were 28.91 KN/m and 2.03 KN/m respectively. The average value and standard error of live load, i.e., ω_Q and β_Q, were 0.585 KN/m and 0.26 KN/m.
The relevant parameters were substituted into the resistance decrease model of corroded bending element of reinforced concrete, and then the curve for time-dependent variation of average value and variation coefficient of resistance could be obtained (Fig. 2 and 3).
According to Unified Standards for the Design of Reliability of Building Structure (GB50068-2001), the constant load followed normal distribution; the live load followed the I-type distribution of extreme value and the maximum value in the time interval [0, T] also followed I-type distribution of extreme value. The average value and standard error of live load can be obtained using the following formulas.

$$\omega_{Q_T} = \omega_Q + \frac{\beta_Q}{1.2826} \ln T \tag{16}$$

$$\omega_{Q_T} = \omega_Q \tag{17}$$

Where Q_T stands for the maximum random variable of live load in service period [0, T]. According to the existing conditions, the average value and standard error of constant load ω_{S_G} and β_{S_G} were 130.1 kN·m and 9.11 kN·m respectively. The average value of the maximum live load could be calculated using formula (18).

$$\omega_{S_{Q_T}} = 2.63 + \frac{1.17}{1.2826} \ln T \tag{18}$$

Standard error $\beta_{S_{Q_T}}$ was 1.17 kN·m.

According to the above conditions, we could obtain the changes of reliability index of corroded bending element of reinforced concrete along with service time of structure by applying time comprehensive analysis method. Results are shown in Fig. 4.

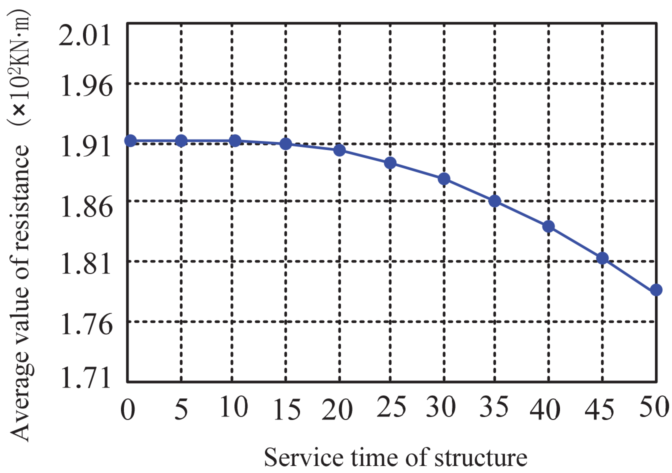

Figure 2. Time varying curve of average value of resistance.

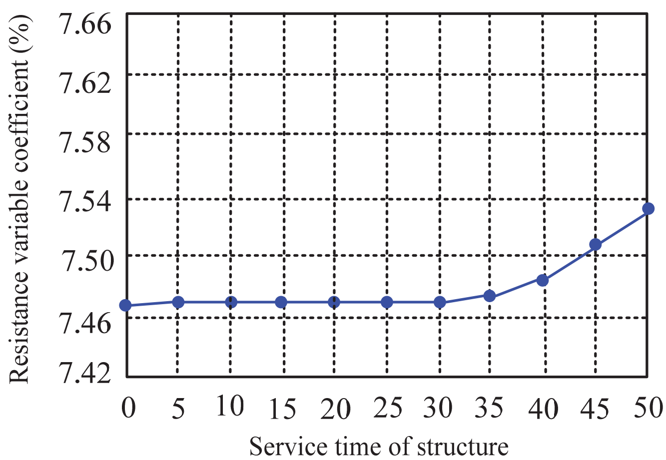

Figure 3. Time varying curve of variation coefficient of resistance.

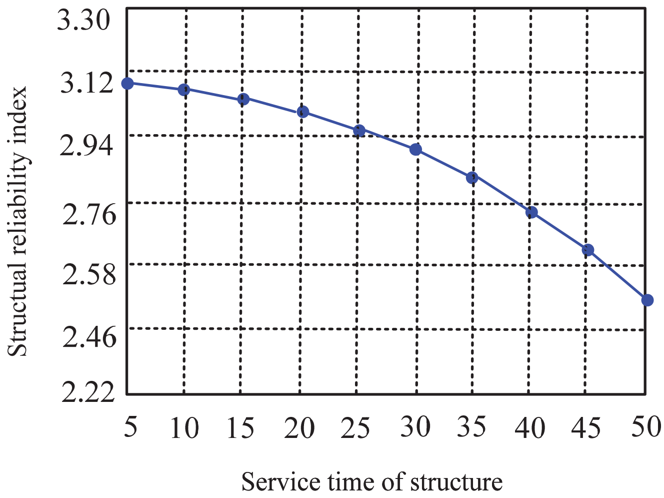

Figure 4. The changes of reliability index of corroded bending element of reinforced concrete along with service time of structure.

CONCLUSIONS

It is of great significance to study the prediction of structural durability. But there is no relatively mature method which can obtain correct prediction results. This study gave out uncertainty statistical parameters of reinforced concrete structure and calculated time-varying reliability of corroded bending element of reinforced concrete using time comprehensive analysis method. Finally, the variation rules of reliability index of corroded bending element along with the changes of service time of structure were obtained. It is no doubt that the method provides a new idea for the prediction of durability and the research results have great value for the prediction and evaluation of durability of existing structures.

As durability degeneration of reinforced concrete structure is a quite complex process, some researches concerning degeneration mechanism has not been broken through. Only few issues were considered for durability degeneration of reinforced concrete in this study, and many problems remain to be solved.

REFERENCES

[1] Li, J., Gao, X., Probability density evolution method and its application in life-cycle civil engineering. Structure and Infrastructure Engineering, 10(7) (2014) 921-927.

[2] Madsen, H.O., Tvedt, L., Methods for time-dependent reliability and sensitivity analysis. American Society of Civil Engineers, 116(10) (2014) 2118-2135.

[3] Shi, X., Xie, N., Fortune, K., et al., Durability of steel reinforced concrete in chloride environments: An overview. Construct Build Mater, 30(2012)125-138.

[4] Neves, R., Branco, F.A., De Brito. J., A method for the use of accelerated carbonation tests in durability design. Construct Build Mater, 36(2012)585-591.

[5] Yu, CL., Ye, P., Jin, R.H., Analyze of the depths of concrete carbonization experience predictive models. Ready-mixed Concrete, (3) (2009) 52-53.

[6] Kabir, G., Sadiq, R., Tesfamariam, S., A review of multi-criteria decision-making methods for infrastructure management. Structure and Infrastructure Engineering, 10(9) (2013)1176-1210.

[7] Biondini, F., Frangopol, D.M., Lifetime reliability-based optimization of reinforced concrete cross-sections under corrosion. Structural Safety, 31(6) (2009)483-489.

[8] Shodja, H.M., Kiani, K., Hashemian, A., A model for the evolution of concrete deterioration due to reinforcement corrosion. Mathematical and Computer Modelling, 52(9-10) (2010) 1403-1422.

[9] Chiu. C., Chi, K., Analysis of lifetime losses of low-rise reinforced concrete buildings attacked by corrosion and earthquakes using a novel method. Structure and Infrastructure Engineering, 9(12) (2013) 1225-1239.

[10] Cornell, C.A., A probability-based structural code. ACI Structural Journal, 100(3) (1969) 94-107.

[11] Czarnecki, A.A., Nowak, A.S., Time-variant reliability profiles for steel girder bridges. Structural Safety, 30(1) (2008) 49-64.

[12] Okasha, N.M., Frangopol, D.M., Integration of structural health monitoring in a system performance based life-cycle bridge management framework. Structure and Infrastructure Engineering, 8(11) (2012) 999-1016.

[13] Madsen, H.O., Tvedt, L., Methods for time-dependent reliability and sensitivity analysis. American Society of Civil Engineers, 116(10) (2014) 2118-2135.
[14] Chen, C.S., Zhang, J.Q., Li, W.H., Reliability analysis of deterioration of construction performance caused by corrosion of steel bar. Journal of Highway and Transportation Research and Development, 23(4) (2006) 33-36.
[15] Zhou, Y., Wang, G.H., Analysis of the existing reinforced concrete bridge's time-dependent and reliability. Journal of Lanzhou Jiaotong University, 26(1) (2007)75-77.
[16] Niu, D.T., Chen, Y.Q., Yu, S., Analysis of carbonization mode and life of concrete structure. Journal of Xi'an University of Architecture & Technology, 27(4) (1995) 365-369.