On the stress investigation at the edges of the fixed elastic semi-strip

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ABSTRACT. The stress state of the elastic fixed semi-strip with the regarding of the singularities at its edge is investigated in the article. The initial boundary problem is reduced to a vector boundary problem in the transformation's domain by the use of integral Fourier transformation. The one-dimensional vector boundary problem is solved exactly with the help of matrix differential calculations and Green's matrix apparatus. The problem's solving was focused at the solving of the singular integral equation (SIE) with the two fixed singularities at the ends of the integration's interval. The symbol of SIE was constructed and the generalized method of the SIE solving was applied. The stress' distributions of the semi-strip are investigated.

KEYWORDS. Semi-strip; Vector boundary problem; Singular integral equation; Fixed singularity.

INTRODUCTION

T he plain elasticity problems for a semi-strip are important as the model examples for the different engineering applications. So many authors have examined these problems in their works. A short review of the different approaches to the solving of the plane elasticity problems for an elastic semi-strip is given below.
The reducing of the problem to the Fredholm's integral equation of the first kind was used in [1] for estimation of the symmetrically loaded semi-strip fixed by the short edge. Another approach based on the construction of the stress function as the combination of the Fourier's integrals and the series was used in [2, 3]. The problems for a semi-strip considered in [4, 5] were reduced to the singular integral equations which were solved numerically, it leaded to the solving

of an infinity system of the algebraic equations. The variation method was used for the analogical problem's solving in [6-8]. The energetic method was applied to the problem of the semi-strip with the free lateral edges and loaded short edge in [9]. In [10] authors constructed a special system of byorthogonal functions, with the help of which they solved the problem on a semi-strip loading at it's the short edge. The problem for the semi-strip with the free longitudinal sides was solved with the help of the stress function in [11, 12]. The Laplace's integral transformation was used for the problem's solving in [13]. The approach based on the use of Fadle-Papkovich functions was applied in [14-16].

In this paper the method, which was worked out by G. Ya. Popov, was used [17]. Accordingly to it the integral transformations were applied directly to the equilibrium equations and boundary conditions of a problem. It leaded the initial problem to one-dimensional boundary problem in the transformation's domain. The last one was formulated as the vector boundary valued problem and solved exactly with the apparatuses of the matrix differential calculations and Green's matrix function [18]. The problem was reduced to the singular integral equation's solving. Investigation of the signature's nature of the singular integral equation's solving was under consideration of many famous scientists. Today the new theories are appeared, which describe the solution's behavior at the particular points [19]. The investigations of the singularities' nature for the complex medium are continued [20]. But in most studies the authors did not pay attention to the fixed singularities at the angular points of the semi-strip usually, although these singularities play a main role in the estimation of the stress state. One approach that allows to find and to take such singularities into account was proposed in widely known work [21]. It was used in this paper for the fixed singularities' consideration. The special generalized method, which was proposed in [22, 23], was applied to obtain the solution of the (SIE) with regarding of the solution's two fixed singularities at the end of the integration's interval.
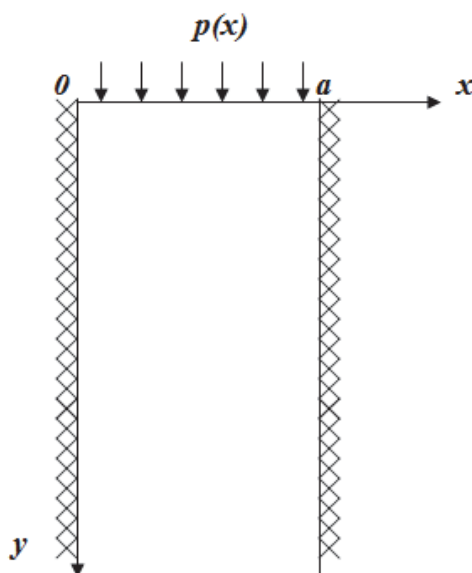

Figure 1: Geometry of the problem.

THE STATEMENT OF A PROBLEM

The elastic (G is a share module, μ is a Poison's coefficient) semi-strip, $0 < x < a, 0 < y < \infty$ is considered. At the edge $y = 0$, $0 < x < a$ the semi-strip is loaded

$$\sigma_y(x,0) = -p(x), \qquad \tau_{xy}(x,0) = 0, \qquad 0 < x < a \qquad (1)$$

where $p(x)$ is the known function.

At the lateral sides $x = 0$, $0 < y < \infty$ and $x = a$, $0 < y < \infty$ the boundary conditions of the fixed are given

$$u(0,y) = 0, v(0,y) = 0, u(a,y) = 0, v(a,y) = 0, \qquad 0 < y < \infty \qquad (2)$$

here $u(x, y) = u_x(x, y)$, $v(x, y) = u_y(x, y)$ are the displacements which satisfy the Lame's equations. The Lame's equations are written in the following form [24]

$$\begin{cases} \mu_* \dfrac{\partial^2 u(x, y)}{\partial x^2} + \dfrac{\partial^2 u(x, y)}{\partial y^2} + \mu_0 \dfrac{\partial^2 v(x, y)}{\partial x \partial y} = 0 \\[3mm] \dfrac{\partial^2 v(x, y)}{\partial x^2} + \mu_* \dfrac{\partial^2 v(x, y)}{\partial y^2} + \mu_0 \dfrac{\partial^2 u(x, y)}{\partial x \partial y} = 0 \end{cases} \tag{3}$$

where $\mu_0 = 1/(1 - 2\mu)$, $\mu_* = \mu_0 + 1$. After the expression of the constants μ_0, μ_* through the Muskchelishvili constant $\kappa = 3 - 4\mu$, one obtains the system (3) in the another form

$$\begin{cases} \dfrac{\partial^2 u(x, y)}{\partial x^2} + \dfrac{\kappa - 1}{\kappa + 1} \dfrac{\partial^2 u(x, y)}{\partial y^2} + \dfrac{2}{\kappa + 1} \dfrac{\partial^2 v(x, y)}{\partial x \partial y} = 0 \\[3mm] \dfrac{\partial^2 v(x, y)}{\partial x^2} + \dfrac{\kappa + 1}{\kappa - 1} \dfrac{\partial^2 v(x, y)}{\partial y^2} - \dfrac{2}{\kappa - 1} \dfrac{\partial^2 u(x, y)}{\partial x \partial y} = 0 \end{cases} \tag{4}$$

The boundary conditions on the semi-strip's edge are reformulated with the terms of the displacements

$$2G\mu_0 \left(\mu \frac{\partial u(x, 0)}{\partial x} + (1 - \mu) \frac{\partial v(x, 0)}{\partial y} \right) = -p(x), \quad 0 < x < a \tag{5}$$

$$\frac{\partial u(x, 0)}{\partial y} + \frac{\partial v(x, 0)}{\partial x} = 0, \quad 0 < x < a \tag{6}$$

One needs to solve the boundary value problem (2), (4)-(6) to estimate the stress state of the semi-strip.

THE GENERAL SOLVING SCHEME OF THE PROBLEMS ON THE SEMI-STRIP STRESS STATE ESTIMATION

The Fourier's transformation is applied to the system of Lame's equation and to the boundary conditions by the scheme

$$\begin{bmatrix} u_\beta(x) \\ v_\beta(x) \end{bmatrix} = \int_0^\infty \begin{bmatrix} u(x, y) \\ v(x, y) \end{bmatrix} \begin{bmatrix} \cos \beta y \\ \sin \beta y \end{bmatrix} dy \tag{7}$$

with the inverse formula

$$\begin{bmatrix} u(x, y) \\ v(x, y) \end{bmatrix} = \frac{2}{\pi} \int_0^\infty \begin{bmatrix} u_\beta(x) \\ v_\beta(x) \end{bmatrix} \begin{bmatrix} \cos \beta y \\ \sin \beta y \end{bmatrix} d\beta \tag{8}$$

The initial problem has the form after this

$$\begin{cases} u''_\beta(x) - \dfrac{\beta^2(\kappa-1)}{\kappa+1}u_\beta(x) + \dfrac{2\beta}{\kappa+1}v'_\beta(x) = \dfrac{3-\kappa}{\kappa+1}\chi'(x) \\ v''_\beta(x) - \dfrac{\beta^2(\kappa+1)}{\kappa-1}v_\beta(x) - \dfrac{2\beta}{\kappa-1}u'_\beta(x) = -\beta\dfrac{\kappa+1}{\kappa-1}\chi(x) \end{cases}$$

$$u_\beta(0) = 0, u_\beta(a) = 0$$
$$v_\beta(0) = 0, v_\beta(a) = 0$$

(9)

Here the new unknown function is inputted $\chi(x) = v(x,0), \chi'(x) = v'(x,0)$. As it is seen from the boundary condition (6), $\partial u/\partial y(x,0) = -\chi'(x)$, so the condition (6) is satisfied automatically.

With the aim to reduce the problem to the vector boundary problem one must input the vectors and the matrixes

$$\vec{y}_\beta(x) = \begin{pmatrix} u_\beta(x) \\ v_\beta(x) \end{pmatrix}, \quad \vec{f}(x) = \begin{pmatrix} \dfrac{3-\kappa}{\kappa+1}\chi'(x) \\ -\beta\dfrac{\kappa+1}{\kappa-1}\chi(x) \end{pmatrix}, \quad P = \begin{pmatrix} \dfrac{\kappa-1}{\kappa+1} & 0 \\ 0 & \dfrac{\kappa+1}{\kappa-1} \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & \dfrac{1}{\kappa+1} \\ -\dfrac{1}{\kappa-1} & 0 \end{pmatrix}.$$

Then the equations in the vector form will be written as the vector equation $L_2\vec{y}_\beta(x) = \vec{f}(x)$, where L_2 is a differential operator of the second order $L_2\vec{y}_\beta(x) = I\vec{y}_\beta''(x) + 2\beta Q\vec{y}_\beta'(x) - \beta^2 P\vec{y}_\beta(x)$, I is an identity matrix. The integral transformations also should be applied to the boundary conditions, with the aim to formulate the boundary functionals in the transformations' domain. As a result the vector boundary problem is constructed

$$L_2\vec{y}_\beta(x) = \vec{f}(x)$$
$$\vec{y}_\beta(0) = 0, \vec{y}_\beta(a) = 0$$

(10)

THE SOLVING OF THE VECTOR BOUNDARY VALUE PROBLEM

The solution of the vector boundary problem (10) will be searched as the superposition of a homogenous vector equation's general solution $\vec{y}_\beta^0(x)$ and a particular solution of the inhomogeneous one $\vec{y}_\beta^1(x)$

$$\vec{y}_\beta(x) = \vec{y}_\beta^0(x) + \vec{y}_\beta^1(x)$$

These solutions were constructed with the help of the matrix differential calculation apparatus earlier [18].

$$\vec{y}_\beta(x) = Y_1(x)\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + Y_2(x)\begin{pmatrix} c_3 \\ c_4 \end{pmatrix} + \vec{y}_\beta^1(x)$$

where $Y_i(x), i = 0,1$ are the matrix system of the fundamental matrix solutions [18]:

$$Y_1(x) = \dfrac{e^{\beta x}}{2}\begin{pmatrix} \dfrac{\kappa-\beta x}{\beta(\kappa-1)} & -\dfrac{x}{\kappa+1} \\ \dfrac{x}{\kappa-1} & \dfrac{\kappa+\beta x}{\beta(\kappa+1)} \end{pmatrix}, \quad Y_2(x) = \dfrac{e^{-\beta x}}{2}\begin{pmatrix} -\dfrac{\kappa+\beta x}{\beta(\kappa-1)} & \dfrac{x}{\kappa+1} \\ -\dfrac{x}{\kappa-1} & -\dfrac{\kappa-\beta x}{\beta(\kappa+1)} \end{pmatrix}$$

where constants $c_i, i = \overline{1,4}$ are founded from the boundary conditions [18].

For the obtaining of the vector boundary problem's particular solution $\vec{y}_\beta^1(x)$, was constructed the Green's matrix function [18]. Elements of matrix are shown in the Appendix A.

The inhomogeneous boundary problem's final form of the solution is constructed

$$\vec{y}_\beta(x) = Y_1(x)\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + Y_2(x)\begin{pmatrix} c_3 \\ c_4 \end{pmatrix} + \int_0^a G(x,\xi)\vec{f}(\xi)d\xi \tag{11}$$

The components of (11) can be written in the next form

$$u_\beta(x) = Y_1^{11}(x)c_1 + Y_1^{12}(x)c_2 + Y_2^{11}(x)c_3 + Y_2^{12}(x)c_4 + \frac{3-\kappa}{\kappa+1}\int_0^a G^{11}(x,\xi)\chi'(\xi)d\xi - \beta\frac{\kappa+1}{\kappa-1}\int_0^a G^{12}(x,\xi)\chi(\xi)d\xi$$

$$v_\beta(x) = Y_1^{21}(x)c_1 + Y_1^{22}(x)c_2 + Y_2^{21}(x)c_3 + Y_2^{22}(x)c_4 + \frac{3-\kappa}{\kappa+1}\int_0^a G^{21}(x,\xi)\chi'(\xi)d\xi - \beta\frac{\kappa+1}{\kappa-1}\int_0^a G^{22}(x,\xi)\chi(\xi)d\xi$$

where $G^{i,j}(x,\xi)$ is the Green's matrix function element in a i row and j column. The integrals with the function $\chi(\xi)$ are calculated by the parts and the inverse integral transformations' formulae were applied to the displacements' transformations.

$$u(x,y) = \int_0^a \chi'(\xi)\int_0^\infty f_1(x,\xi,\beta)\cos(\beta y)d\beta d\xi$$

$$v(x,y) = \int_0^a \chi'(\xi)\int_0^\infty f_2(x,\xi,\beta)\cos(\beta y)d\beta d\xi \tag{12}$$

where $f_i(x,\xi,\beta), g_i(x,\xi,\beta), i=1,2$ are known functions.

The formulae (12) would be the final ones if the unknown function $\chi'(\xi)$ is known. For its finding one must satisfy the boundary conditions (5) which are unsatisfied yet. It should be taken into consideration that integrals in these correspondences are conditionally convergent integrals. So, before the differentiating of the displacements' expressions, at first one must extract the weakly convergence parts at these integrals.

The substitution of (12) in the boundary conditions (5) leads to the singular integral equation

$$\int_0^a \chi'(\xi^*)f(\xi^*,x^*)d\xi^* = r(x^*), \quad 0 < x^* < a$$

here the function $f(\xi,x)$ contains Cauchy's type singularities and fixed singularities on the both ends of the integration interval.

THE SOLVING OF THE SINGULAR INTEGRAL EQUATION

The changing of the variables $\xi = \dfrac{2\xi^* - a}{a}, x = \dfrac{2x^* - a}{a}$ is done for the passing to the integration interval $[-1;1]$. As a result the integral equation is transformed to the form

$$c_1 \int_{-1}^{1} \frac{\tilde{\chi}(\xi)}{\xi - x} d\xi + \int_{-1}^{1} \tilde{\chi}(\xi) \left[\frac{c_2^-}{2 - \xi - x} + \frac{c_2^+}{\xi + x + 2} + \frac{c_{3,1}^-(1-\xi)}{(2-\xi-x)^2} + \frac{c_{3,2}^-(1-x)}{(2-\xi-x)^2} + \frac{c_{3,1}^+(1+\xi)}{(2+\xi+x)^2} + \frac{c_{3,2}^+(1+x)}{(2+\xi+x)^2} + \right.$$

$$\left. + \frac{c_4^+(1-\xi)(1-x)}{(2-\xi-x)^3} + \frac{c_4^-(1+\xi)(1+x)}{(\xi+x+2)^3} \right] d\xi + \int_{-1}^{1} \tilde{\chi}(\xi) K(x,\xi) d\xi = \tilde{r}(x), \qquad -1 < x < 1 \tag{13}$$

here $\tilde{\chi}(\xi) = \chi'\left(\frac{a(\xi+1)}{2} \right)$, $K(x,\xi), \tilde{r}(x)$ are the known regular functions, $c_i^\pm, i = \overline{1,4}$ are shown in the Application B.

The Eq. (13) is the partial case of the equation with two fixed singularities considered [21]

$$A\varphi(x) = c_0 \varphi(x) + \frac{c_1}{\pi i} \int_{-1}^{1} \frac{\varphi(y)}{y-x} dy + \sum_{k=0}^{n} \frac{1}{\pi i} \int_{-1}^{1} \frac{c_{k+2}(x,y)(1-x)^{m_k^+}(1+x)^{m_k^-} \varphi(y)}{(1-y)^{m_k^+ - k}(1+y)^{m_k^- - k}(1-xy)^{k+1}} dy + $$

$$+ \int_{-1}^{1} K(x,y)\varphi(y) dy = f(x), 0 \leq \mathrm{Re}\, m_k^+ \leq k$$

which can be rewritten as

$$A\varphi(x) = c_0 \varphi(x) + c_1 S_1 \varphi(x) + N_{-1}\varphi(x) + N_{+1}\varphi(x) + K_1\varphi(x) + \int_{-1}^{1} K(x,y)\varphi(y) dy = f(x)$$

where

$$N_{\pm 1}\varphi(x) = \sum_{k=0}^{n} \frac{c_{k+2}^\pm}{\pi i} \int_{-1}^{1} \frac{(1 \mp x)^{m_k^\pm} \varphi(y)}{(1 \mp y)^{m_k^\pm - k}\left[2 \pm (x+y)\right]^{k+1}} dy, c_k^\pm = c_k(\pm 1, \pm 1) = \lim_{\substack{x \to \pm 1 \\ \xi \to \pm 1}} c_k(x,\xi),$$

The symbol of the singular integral Eq. (13) was constructed, which has the following form, where all designations correspond to the designations in [21]

$$A^{(\beta,\delta)}(\lambda,\gamma) = \begin{cases} A_\beta(\lambda) = \begin{cases} c_1 S_\beta(\lambda) + \sum_{k=0}^{2} c_{k+2}^- n_{k,\beta}^{\left(m_k^-\right)}(\lambda), & \lambda \in R \\ -c_1 S_\delta(\gamma) + \sum_{k=0}^{2} c_{k+2}^+ n_{k,\delta}^{\left(m_k^+\right)}(\gamma), & \lambda = \infty, \gamma \notin R \end{cases} \end{cases} \tag{14}$$

$$\beta = \delta = (1+\alpha)/p, S_\varepsilon(z) = cth\,\pi(i\varepsilon + z), \varepsilon \in \{\beta,\delta\}, z \in \{\lambda,\gamma\},$$

$$n_{k,\varepsilon}^{\left(m_k^\pm\right)} = \frac{(-1)^k}{sh\,\pi(i\varepsilon_k + z)}(\varepsilon_k - iz - 1)_k, \varepsilon_k = \varepsilon + m_k^\pm,$$

$$A_\varepsilon(z) = -\frac{2G(2\mu - \kappa + \kappa\mu)i}{\kappa\sinh\left(\pi\left(z + \frac{\alpha+1}{p}i\right)\right)(2\mu-1)} - \frac{4G\left(zi - \frac{\alpha+1}{p} + 1\right)(2\mu - \kappa + \kappa\mu)i}{\kappa\sinh\left(\pi\left(z + \frac{\alpha+1}{p}i\right)\right)(2\mu-1)(\kappa+1)} -$$

$$-\frac{4G\left(zi - \frac{\alpha+1}{p}\right)(2\mu-1+\kappa\mu)i}{\kappa\sinh\left(\pi\left(z + \frac{\alpha+1}{p}i + i\right)\right)(2\mu-1)(\kappa+1)} - \frac{2G\cosh\left(z + \frac{\alpha+1}{p}i\right)(\mu-1)(\kappa+3)i}{\sinh\left(\pi\left(z + \frac{\alpha+1}{p}i\right)\right)(2\mu-1)(\kappa+1)} + \frac{16G\left(zi - \frac{\alpha+1}{p}\right)\left(zi - \frac{\alpha+1}{p} - 1\right)i}{\kappa\sinh\left(\pi\left(z + \frac{\alpha+1}{p}i + i\right)\right)(\kappa+1)}$$

here $p = 2$, $\alpha = -0.31$ is found from the known solution of the analogical problem for an edge with the angle of openness pi/2 [25].

According to [21] one needs to find the roots of the equation $A_\varepsilon(z) = 0$. The found roots of the kernel's symbol (14) have the next form: $\lambda_{1,2} = -0.5562 \pm 0.3690$, $\lambda_{3,4} = 1.2792 \pm 0.2380$, $\lambda_{5,6} = 3.2089 \pm 0.7127$, $\lambda_{7,8} = 5.2170 \pm 1.0251,...$, where $\lambda_k = \gamma_k$ because of the problem's statement.

The generalized method of SIE solving [22, 23] was applied for the solving of the Eq. (13). According to it the unknown function $\tilde{\chi}(\xi)$ is expanded by the series in each interval

$$\tilde{\chi}(\xi) = \begin{cases} \sum_{k=0}^{N-1} s_k \rho_k^-(\xi), & \xi \in [-1;0] \\ \sum_{k=N}^{2N-1} s_k \rho_k^+(\xi), & \xi \in (0;1] \end{cases} \tag{15}$$

where

$$\begin{aligned} \rho_{2k}^-(\xi) &= (1+\xi)^{\operatorname{Re}\lambda_k} \cdot \cos\left(\operatorname{Im}\lambda_k \ln(1+\xi)\right), \\ \rho_{2k+1}^-(\xi) &= (1+\xi)^{\operatorname{Re}\lambda_k} \cdot \sin\left(\operatorname{Im}\lambda_k \ln(1+\xi)\right), \end{aligned} \quad k = \overline{0, \frac{N}{2}-1},$$

$$\begin{aligned} \rho_{2k}^+(\xi) &= (1-\xi)^{\operatorname{Re}\lambda_{k-N}} \cdot \cos\left(\operatorname{Im}\lambda_{k-N} \ln(1-\xi)\right), \\ \rho_{2k+1}^+(\xi) &= (1-\xi)^{\operatorname{Re}\lambda_{k-N}} \cdot \sin\left(\operatorname{Im}\lambda_{k-N} \ln(1-\xi)\right), \end{aligned} \quad k = \overline{\frac{N}{2}, N-1}.$$

The segment $[-1;1]$ is divided on $2N$ equal segments with the length $h = \frac{1}{N}$. The Eq. (13) is considered when $x_i = -1 + ih + \frac{h}{2}, i = \overline{0, 2N-1}$.

After the substitution of the unknown function (15) into the singular integral Eq. (13) one obtains system of the linear algebraic equations relatively to the unknown constants $s_k, k = \overline{0, 2N-1}$ of the expansion (15).

$$\sum_{k=0}^{2N-1} s_k d_{ki} = f_i, i = \overline{0, 2N-1} \tag{16}$$

where $d_{ki}, f_i, i, k = \overline{0, 2N-1}$ are shown in the Application C.

The expression (16) presents the system of $2N$ equations with regard of $2N$ unknown constants s_k. The substitution of the founded constants in the formula (15) and following using of the formulae (12) completes the construction of the problem's solution.

THE RESULTS OF THE NUMERICAL ANALYSES

The calculations were done for the elastic semi-strip ($G = 61.2781955 \cdot 10^9$ Pa, $\mu = 0.33$). At Fig.2 one can admit that the values of the normal stress $\tilde{\sigma}_y = \sigma_y / P, \tilde{\sigma}_x = \sigma_x / P$ at the lateral side $x = 0$ decrease to zero with the increasing of the distance from the semi-strip's edge. When the semi-strip's side is $a = 10$. A similar situation is observed during the analyses of the stress $\tilde{\sigma}_y, \tilde{\sigma}_x$ when the semi-strip's side is $a = 50$ (Fig.4) and $a = 100$ (Fig.6).

At Fig.3 one can admit that the absolute values of the normal stress σ_y at the line $x = a / 2, y \in [0;10]$ are higher by its absolute value then normal stress σ_x when the semi-strip's side is $a = 10$. A similar situation is observed during the analyses of the stress σ_y, σ_x when the semi-strip's side is $a = 50$ (Fig.5) and $a = 100$ (Fig.7). As it is seen the stabilization of the stresses σ_y, σ_x is observed when the semi-strip's side is $a = 50$ (Fig.5) or $a = 100$ (Fig.7).
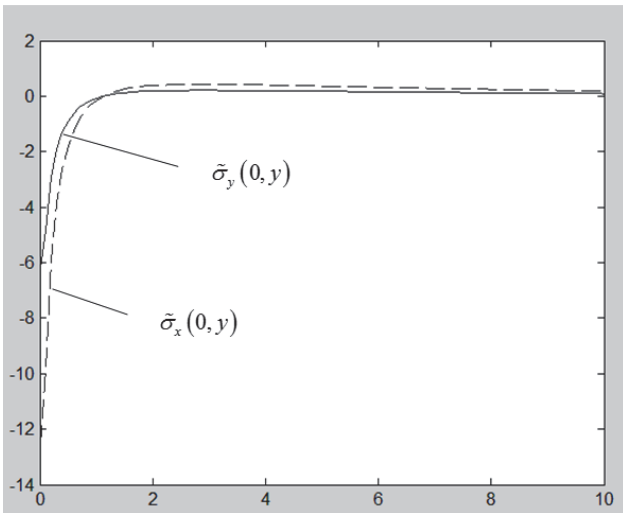

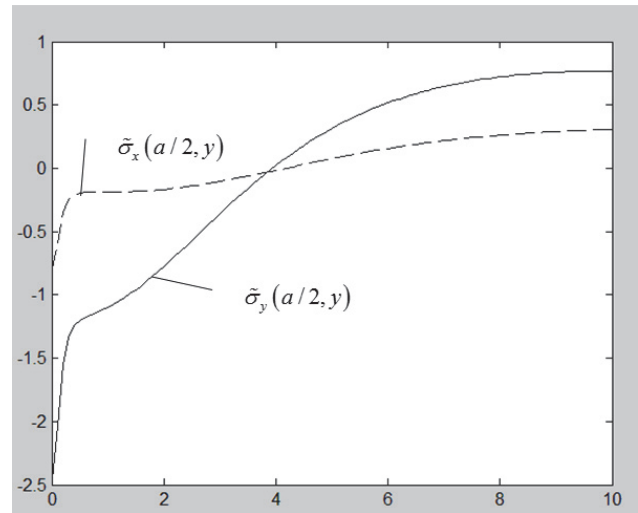Figure 2: Normal stresses $\tilde{\sigma}_y(0, y), \tilde{\sigma}_x(0, y), a = 10$.

Figure 3: Normal stresses $\tilde{\sigma}_y(a / 2, y), \tilde{\sigma}_x(a / 2, y), a = 10$.

Figure 4: Normal stresses $\tilde{\sigma}_y(0, y), \tilde{\sigma}_x(0, y), a = 50$.

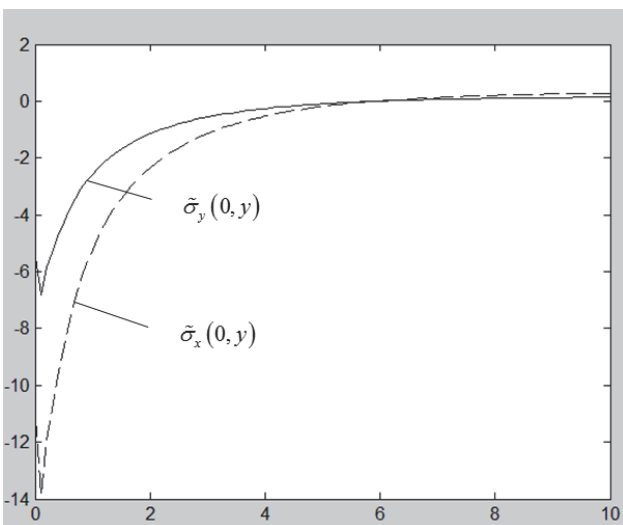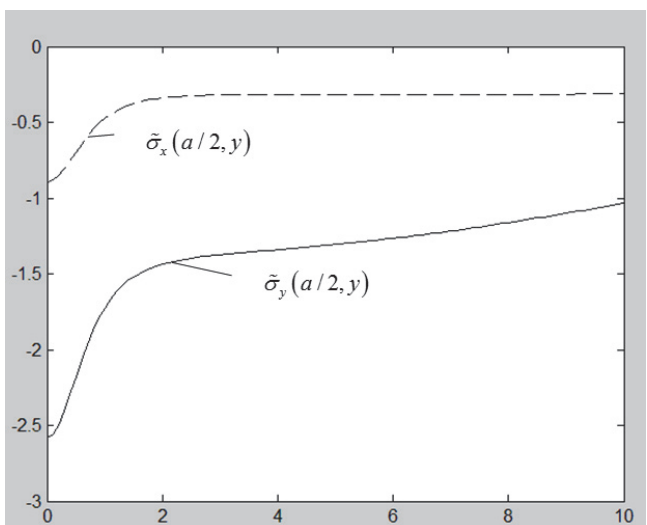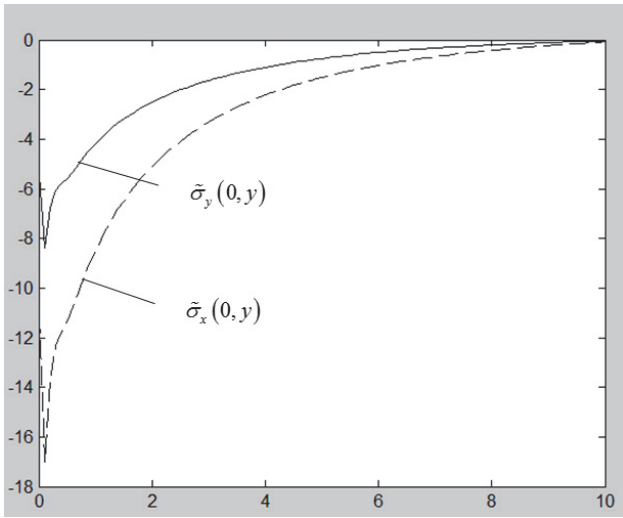Figure 5: Normal stresses $\tilde{\sigma}_y(a / 2, y), \tilde{\sigma}_x(a / 2, y), a = 50$.

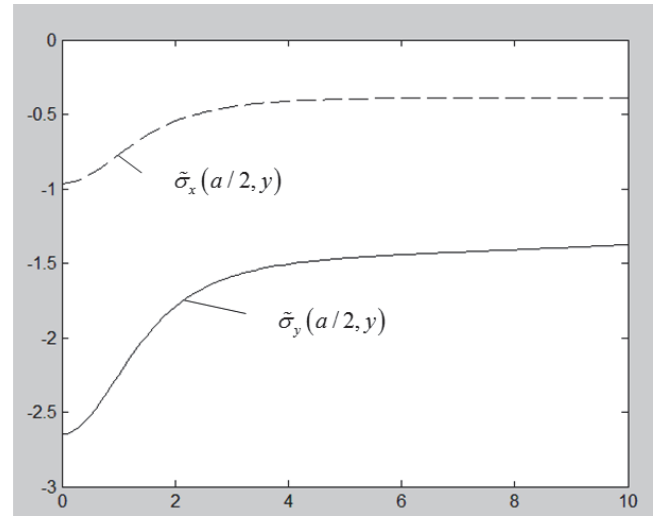Figure 6: Normal stresses $\tilde{\sigma}_y(0,y), \tilde{\sigma}_x(0,y), a = 100$.

Figure 7: Normal stresses $\tilde{\sigma}_y(a/2,y), \tilde{\sigma}_x(a/2,y), a = 100$.

CONCLUSIONS

1. The proposed solving method reduced the initial problem to the singular integral equation, which has the two fixed singularities at the end of the integration's interval. The special generalized scheme of SIE solving was applied with the aim to take these singularities into consideration.

2. The proposed approach may be applied to the solving of the elasticity mixed problem for the semi-strip with a crack.

3. The analyses of the numerical results show that the taking into consideration the existence of the two fixed singularities of the solution gives the possibility to obtain the numerical result on the distance less than a/1000 to the angular point of the semi-strip in comparison with the usual approach to the solving, allowing to get the stable results only on the distance to the angular points not less than a/10.

APPENDIX A

$$G_{11}(x,\xi) = \frac{\kappa ch(\beta(\xi - a + x)) - \kappa ch(\beta(a - |\xi - x|))}{2\beta sh(a\beta)(\kappa - 1)} -$$

$$-\frac{1}{2(sh(a\beta))^2(\kappa - 1)}\Big[sh(a\beta)\big(\xi sh(\beta(\xi - a + x)) + xsh(\beta(\xi - a + x)) -$$

$$-ash(\beta(\xi - a + x)) - (a - |\xi - x|)ash(\beta(a - |\xi - x|))\big) - ach(a\beta)\big(ch(\beta(\xi - a + x)) - ch(\beta(a - |\xi - x|))\big)\Big]$$

$$G_{12}(x,\xi) = \frac{1}{2sh(a\beta)(\kappa + 1)}\Bigg[-\frac{ach(a\beta)}{\big((ch(a\beta))^2 - 1\big)(\kappa + 1)}\big(sh(\beta(\xi - a + x)) -$$

$$-\text{sgn}(x - \xi)sh(\beta(a - |\xi - x|))\big) + \big(ch(\beta(\xi - a + x))(\xi - a + x) - \text{sgn}(x - \xi)ch(\beta(a - |\xi - x|))(a - |\xi - x|)\big)\Bigg]$$

$$G_{21}(x,\xi)=\frac{1}{2sh(a\beta)(\kappa-1)}\left[-\frac{ach(a\beta)}{\left((ch(a\beta))^2-1\right)(\kappa-1)}\left(sh(\beta(\xi-a+x))+\right.\right.$$

$$\left.+sgn(x-\xi)sh(\beta(a-|\xi-x|))\right)+\left(ch(\beta(\xi-a+x))(\xi-a+x)+sgn(x-\xi)ch(\beta(a-|\xi-x|))(a-|\xi-x|)\right)\right]$$

$$G_{22}(x,\xi)=-\frac{ch(\beta(\xi-a+x))+ch(\beta(a-|\xi-x|))}{2\beta sh(a\beta)}+$$

$$+\frac{1}{2\beta(sh(a\beta))^2(\kappa+1)}\left[sh(a\beta)\left(ch(\beta(\xi-a+x))+ch(\beta(a-|\xi-x|))+\right.\right.$$

$$\left.+\beta\left((a-x-\xi)sh(\beta(\xi-a+x))-(a-|\xi-x|)sh(\beta(a-|\xi-x|))\right)+a\beta ch(a\beta)\left(ch(\beta(\xi-a+x))+ch(\beta(a-|\xi-x|))\right)\right]$$

APPENDIX B

$$c_0=0,c_1=-\frac{2G(\kappa+3)(\mu-1)}{\pi(2\mu-1)(\kappa+1)},c_2^-=\frac{2G(\kappa+2\mu-\kappa\mu)}{\pi(2\mu-1)\kappa},c_2^+=-\frac{2G(\kappa+2\mu-\kappa\mu)}{\pi(2\mu-1)\kappa},c_{3,1}^-=-\frac{4G(2\mu+\kappa\mu-\kappa)}{\pi\kappa(2\mu-1)(\kappa+1)},$$

$$c_{3,2}^-=-\frac{4G(2\mu+\kappa\mu-1)}{\pi\kappa(2\mu-1)(\kappa+1)},c_{3,1}^+=\frac{4G(2\mu+\kappa\mu-\kappa)}{\pi\kappa(2\mu-1)(\kappa+1)},c_{3,2}^+=\frac{4G(2\mu+\kappa\mu-1)}{\pi\kappa(2\mu-1)(\kappa+1)},c_4^-=\frac{16G}{\pi(\kappa+1)\kappa},c_4^+=-\frac{16G}{\pi(\kappa+1)\kappa}$$

APPENDIX C

$$d_{ki}=\int_{-1}^{0}\rho_k^-(\xi)\left[\frac{c_1}{\xi-x_i}+\frac{c_2^-}{2-\xi-x_i}+\frac{c_2^+}{\xi+x_i+2}+\frac{c_{3,1}^-(1-\xi)}{(2-\xi-x_i)^2}+\frac{c_{3,2}^-(1-x_i)}{(2-\xi-x_i)^2}+\frac{c_{3,1}^+(1+\xi)}{(2+\xi+x_i)^2}+\frac{c_{3,2}^+(1+x_i)}{(2+\xi+x_i)^2}+\right.$$

$$\left.+\frac{c_4^+(1-\xi)(1-x_i)}{(2-\xi-x_i)^3}+\frac{c_4^-(1+\xi)(1+x_i)}{(\xi+x_i+2)^3}+K(x_i,\xi)\right]d\xi,k=\overline{0,N-1},i=\overline{0,2N-1}$$

$$d_{ki}=\int_{0}^{1}\rho_k^+(\xi)\left[\frac{c_1}{\xi-x_i}+\frac{c_2^-}{2-\xi-x_i}+\frac{c_2^+}{\xi+x_i+2}+\frac{c_{3,1}^-(1-\xi)}{(2-\xi-x_i)^2}+\frac{c_{3,2}^-(1-x_i)}{(2-\xi-x_i)^2}+\frac{c_{3,1}^+(1+\xi)}{(2+\xi+x_i)^2}+\frac{c_{3,2}^+(1+x_i)}{(2+\xi+x_i)^2}+\right.$$

$$\left.+\frac{c_4^+(1-\xi)(1-x_i)}{(2-\xi-x_i)^3}+\frac{c_4^-(1+\xi)(1+x_i)}{(\xi+x_i+2)^3}+K(x_i,\xi)\right]d\xi,k=\overline{0,N-1},i=\overline{0,2N-1}$$

$$f_i=\tilde{r}(x_i),i=\overline{0,2N-1}$$

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