Development of a multiaxial fatigue damage parameter and life prediction methodology for non-proportional loading

Sandip Suman
Research Center, Environmental Solution Group, Dover Corporation, USA
ssuman@doveresg.com

Alan Kallmeyer
North Dakota State University, Fargo, ND, USA,
alan.kallmeyer@ndsu.edu

John Smith
Research & Validation, Environmental Solution Group, Dover Corporation, USA,
josmith@doveresg.com

ABSTRACT. Most of the prior studies on the prediction of fatigue lives have been limited to uniaxial loading cases, whereas real world loading scenarios are often multiaxial, and the prediction of fatigue life based upon uniaxial fatigue properties may lead to inaccurate results. A detailed exploration of multiaxial fatigue under constant amplitude loading scenarios for a range of metal alloys has been performed in this study, and a new methodology for the accurate prediction of fatigue damage is proposed. A wide variety of uniaxial, torsional, proportional and non-proportional load-paths has been used to simulate complex, real-world loading scenarios. Test data have been analyzed and a critical-plane based fatigue damage parameter has been developed. This fatigue damage parameter contains stress and strain terms, as well as a term consisting of the maximum value of the product of normal and shear stresses on the critical plane. The shear-dominant crack initiation phenomenon and the combined effect of shear and tensile stresses on micro-crack propagation have been modeled in this work. The proposed formulation eliminates many of the shortcomings of the earlier developed critical-plane fatigue damage models. It is mathematically simple with substantially fewer material dependent constants, and provides design engineers with a tool to predict the fatigue life of machine parts with minimal computational effort. This life prediction methodology is intended for a wide variety of LCF and HCF loadings on machine parts made of metals including advanced alloys.

KEYWORDS. Multiaxial; Fatigue Damage Parameter; Non-proportional loading.

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INTRODUCTION

Fatigue is one of the most prolific phenomena responsible for the failure of machine parts. Very often, the fatigue related damage stays undiagnosed, and by the time a visible fatigue crack is detected, the structure is left with a very small number of cycles required to propagate this crack until catastrophic fracture. Therefore, it is very important to understand this phenomenon, and consider it during the very early stages of the design process. Although it is very convenient to assume uniaxial loading conditions and perfect machine parts with no defects or irregularities, in practice this is seldom the case, and many cyclic loading conditions create some multiaxial stress states. Thus, the modeling of fatigue phenomena and the development of suitable damage methodologies is very challenging for engineers and researchers in the fatigue and fracture community.

Extensive review of several multiaxial fatigue theories [1-18] leads to three broad categorizations of fatigue analysis methodologies, namely equivalent stress-based, energy-based and critical plane-based models. Efforts have been made by several researchers [1-4] to represent the multiaxial stress state by an equivalent uniaxial stress value using von Mises or Tresca type equations. However, this approach fails to adequately account for many complexities like LCF-HCF interactions and the effects of non-proportional loading. Equivalent stress-based parameters often result in highly conservative fatigue life predictions with large factors of safety. Unlike equivalent-stress based models, energy-based fatigue theories compute the damage by estimating the strain energy within each fatigue cycle [5-9].

Critical plane-based fatigue theories [10-17] support the observation that the fatigue damage computation should be based upon the estimation of location and orientation of the crack. This approach is more convincing because it allows designers to determine the exact orientation of the fatigue crack plane to accurately make any design changes in the parts. Findley [10] was one of the earliest researchers to put forth the concept of the critical plane approach; however; the Findley model [10] is stress-based and thus often produces significant errors within the LCF regime. Other researchers [7-9] have proposed energy based-critical plane fatigue models, but it is difficult to exactly estimate the total energy of fatigue cycles that have mean stresses [11]. Brown & Miller [15] developed a strain-based fatigue model, and Fatemi & Socie [16] developed a critical plane model that included a normal stress term for non-proportional loading. The extra hardening and softening caused by multiple normal stress subcycles on the critical plane was addressed by Erickson et al. [17].

The critical-plane concept for the prediction of fatigue life of steel and titanium alloys has further been explored in this paper through a series of additional uniaxial and multiaxial fatigue tests under a wide variety of load paths. Special attention has been given to the need of a strain term, and a parameter to model the interaction of shear stress and normal stress on the critical plane. The Erickson et al. [17], Findley [10] and Fatemi-Socie [16] damage parameters have also been assessed for their complexity and ability to accurately estimate the fatigue damage, and a comparatively simpler formulation has been proposed.

MATERIAL AND METHODS

Several material data sets were used to evaluate the accuracy of the new damage parameter, including a titanium alloy (Ti-6Al-4V) and nickel-based steel alloys (718 steel and Rene 104). These alloys have been used extensively in gas turbine engines for military and commercial applications, and also have many applications in the automotive and electronics industries. These data sets included both uniaxial and biaxial (axial/torsion) data subjected to a wide variety of cyclic load paths. Much of data were generated as part of a US Air Force program on High Cycle Fatigue, while other data sets were generated by industrial sources or taken from the literature [18]. Additional details about the materials can be found in Erickson et al [17].

Tests were conducted using both solid and tubular specimens with highly polished inner and outer surfaces. The majority of the tests were conducted in strain control on servo-hydraulic tension/torsion load frames. Some of the long-life tests were switched to load control after cyclic stabilization had occurred in order to accelerate the tests. An elastic-plastic stress strain analysis was performed using FEM to calculate the stresses on the outer surface of the specimens, using measured strains as input. This analysis utilized cyclically stabilized stress-strain curves for each material, which were generated from the half-life hysteresis loops recorded from the uniaxial fatigue tests. Separate curves were generated for both $R = -1$ and $R = 0$ loading conditions.

A wide variety of cyclic load paths were used in generating the fatigue data sets, including uniaxial, torsion, and combined axial/torsion. The biaxial tests included both proportional and non-proportional load paths, as illustrated schematically in Fig. 1. When both axial and shear stresses (strains) remain linearly proportional to one another throughout the cycle, the loading is considered proportional. In contrast, under non-proportional loading, the axial and shear stresses (strains) do not maintain a constant ratio during the load cycle. This causes the principle stress planes to rotate during the cycle, which has been observed to cause additional cyclic hardening or softening in some metals [16]. Such load paths also make the prediction of fatigue life more challenging.

The non-proportional load paths shown in Fig. 1 were designed to provide discriminating test conditions for evaluating the critical plane parameters. Specifically, the "check" path and "box" path simulated actual loading scenarios experienced in aircraft engine components, whereas the "triangle" path, "s" path, and "double check" path were designed to produce varying combinations of normal-stress "subcycles" on the shear-based critical plane. The critical-plane stresses are discussed in more detail in the following sections.

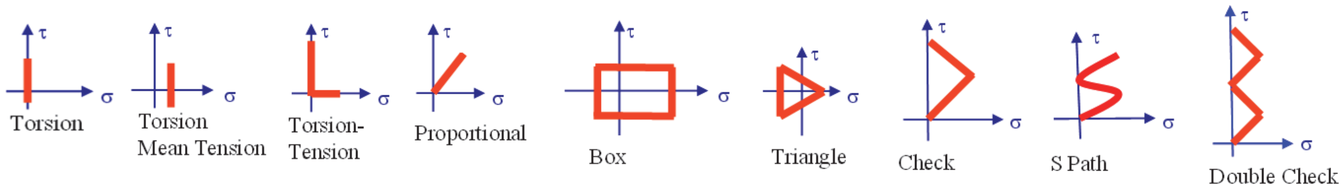


Figure 1: Multiaxial load paths considered in this study.

DAMAGE PARAMETER DEVELOPMENT FOR LIFE ESTIMATION

Critical Plane Analysis

As previously mentioned, the critical plane method allows designers to compute the fatigue damage on the crack plane; however, the identification of the critical plane is dictated by several factors including load level, load type, load path, and the behavior of the material. Similarly, the definition of the critical plane itself may vary among different researchers, and various proposals can be found throughout the literature. Many researchers define the critical plane as the plane possessing the maximum value of the damage parameter, whereas others have used the plane on which a particular stress or strain component (such as the normal or shear stress range) is maximized. In this work, the latter definition was adopted, with the maximum shear stress range used to identify the orientation of the critical plane. This definition was established after thorough analysis of a large amount of uniaxial and multiaxial fatigue data from high strength steel and titanium alloys. In comparing the correlation of the data sets from different parameters (described below) calculated on the maximum shear plane and the plane of maximum damage parameter, the differences were found to be very small. Additionally, using the maximum shear plane as the critical plane significantly reduces the mathematical computation (as it is easier to identify), and also makes optimization of the material constants needed by each parameter much simpler. It should also be noted that the critical plane definition proposed in this paper is for constant amplitude fatigue cycles, and may not be appropriate for variable amplitude loading cases because the orientation of the critical plane may change from cycle to cycle.

Critical Plane Damage Computations

Several well-accepted damage parameters were initially used to model the fatigue damage in the data sets mentioned above; however, the Findley parameter [10], Fatemi & Socie parameter [16], and Erickson et al. parameter [17] were found to provide better correlations between experimental and predicted fatigue lives for all the data sets, and thus are presented in detail in this paper. These damage parameters are expressed in the equations below: Findley (Eq. 1), Fatemi & Socie (Eq. 2), and Erickson et al. (Eq. 3).

$$DP = \tau_a + k\sigma_{max} \tag{1}$$

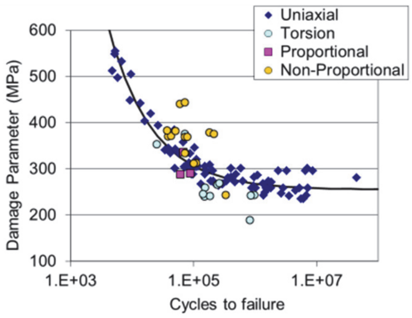
$$DP = \frac{\Delta\gamma_{max}}{2}\left(1 + k\frac{\sigma_n^{max}}{\sigma_y}\right) \tag{2}$$

$$DP = \tau_{max}\left(1-\frac{\tau_{min}}{\tau_{max}}\right)^{w1}\left[1+\frac{k^{+}\sigma_{max}+k^{-}\sigma_{min}}{\sigma_{y}}\right]^{w2}+k_{2}\sum\sigma_{max}\left(1-\frac{\sigma_{min}}{\sigma_{max}}\right)^{w3} \qquad (3)$$

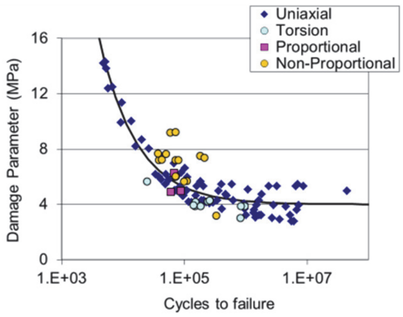
For each model, the stress and strain values on the surface of the specimen were taken from the finite element analysis and considered as the values on the zero degree plane. These values were rotated in 1-degree increments onto all possible planes to identify the critical (maximum shear) plane. The material constants required by each model were optimized using a least-squares process to minimize the error between predicted and experimental lives for each data set. A double power-law type of formulation was assumed to relate the DP value to the fatigue life, N, as shown in Eq. 4.

$$DP = AN^{b} + CN^{d} \qquad (4)$$

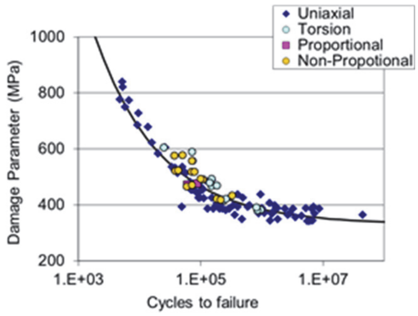
The initial model evaluations were performed using a large set of uniaxial and biaxial Ti-6Al-4V fatigue data. The resulting correlations of the data set using each model are shown in Fig. 2.



(a) Findley [10]



(b) Fatemi & Socie [16]



(c) Erickson et al. [17]

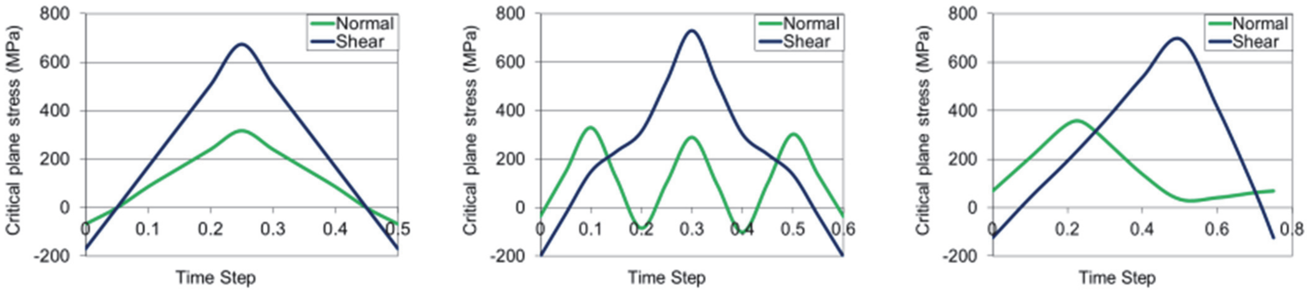
Figure 2: Correlation of Ti-6Al-4V fatigue data using three different models.

It can be observed that the best correlation of all the uniaxial, proportional and non-proportional fatigue data has been achieved by computing the damage with the Erickson et al. model. The Findley parameter and Fatemi & Socie parameter provided good correlation for most of the uniaxial and proportional test data; however, both of these parameters failed to collapse some of the non-proportional fatigue data along the best-fit curve. It is to be noted that the Fatemi & Socie parameter has a shear strain term which allows it to better account for the plasticity in the short-life (LCF) regime, relative to the Findley parameter. Similarly, Erickson et al.'s model accounts for the strain-hardening effects, as well as the damage caused by multiple normal-stress "subcycles" on the critical plane, by summing the damage caused by these subcycles. This parameter was clearly superior in its ability to correlate the non-proportional test data.

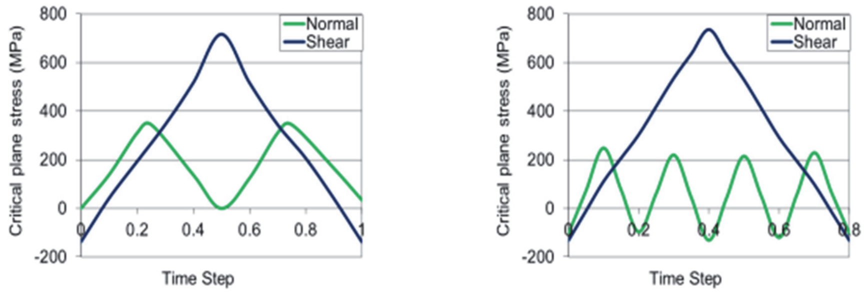
Development of New Damage Parameter

While all three damage parameters possess both normal and shear components to model the fatigue damage, the Erickson et al. [17] formulation (Eq. 3) contains extra terms in an effort to account for additional subtleties that may arise in non-proportional load paths. The first term, comprised of maximum and minimum values of shear stress, can account for the effect of mean shear stresses. The second term is designed to model the effect of the normal stress on the critical plane, while the third term has been introduced to capture the additional damage from multiple normal-stress subcycles on the critical plane. Despite the fact that this parameter captures many factors that contribute towards the nucleation of a fatigue crack, this model is very complex and has altogether six material dependent constants. The optimization of these constants requires extensive computations and a large volume of data.

In an effort to better understand the effects of normal-stress subcycles on the critical (maximum shear) plane, a series of specialized tests were designed to vary the number and timing of these subcycles, relative to the major shear cycle. The tests were designed in such a way that after rotating the stresses onto the critical plane, they produced identical shear stress cycles and one or more normal stress subcycles. These tests were performed on a different material, DA 718, for which a significant amount of uniaxial and proportional fatigue data was also available.



Proportional, N_f = 36,078 S-Path, N_f = 34,816 Triangle Path, N_f = 176,559



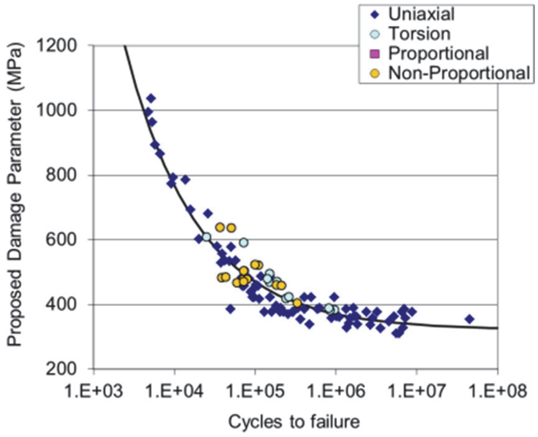
Check Path, N_f = 168,803 Double Check, N_f = 141,421

Figure 3: Critical plane stresses for the tests conducted on DA 718.

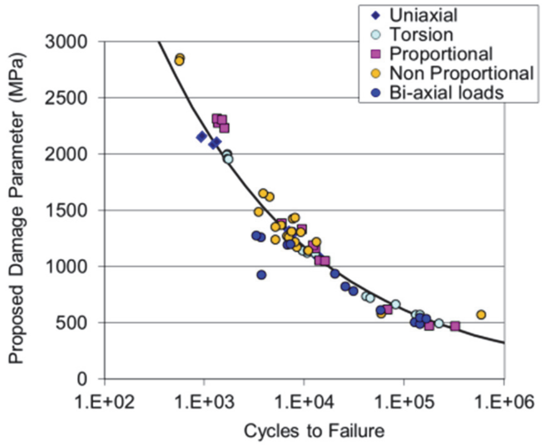
The critical-plane shear and normal stresses from these specialized tests are shown in Fig. 3, along with the resulting fatigue lives. Note that for all the tests, the shear stress cycles and normal stress cycles were nearly identical in magnitude; consequently, the differences in fatigue lives can be attributed to the number and positioning of the normal stress subcycles on the critical plane. Of particular interest is the fact that the proportional and S-path tests had similar lives, despite the fact the S-path contained three subcycles in comparison to one in the proportional test. This would indicate that the number of subcycles is of less importance than the magnitude. In comparing the triangle, check, and double-check tests, it is evident these tests also had similar lives (within expected scatter), but the lives were over four times longer than the proportional and S-path tests even though the normal stress cycles were similar in magnitude. The difference in the three latter tests was that the peak normal stress did not occur at the same time as the peak shear stress; i.e., the normal and shear cycles were offset. This again indicates a negligible effect from the number of subcycles, but a very large influence from the timing of a subcycle. In other words, when the normal stress cycle peaks simultaneously with the shear cycle, significantly more damage occurs, resulting in a shorter life.
Based on these observations, it was concluded that the Erickson parameter does not accurately model the fatigue damage in complex, multiaxial load paths. Specifically, the subcycle summation term is unnecessary. However, a different term is needed to model the interaction between the shear and normal stresses on the critical plane.
A new critical-plane parameter that eliminates many of the shortcomings of the previous models is shown in Eq. 5. This parameter makes use of the shear strain range ($\Delta\gamma$) multiplied by the maximum shear stress in order to capture the effects of strain hardening in the LCF regime and mean shear stresses in the HCF regime. The effect of the interaction between normal and shear stresses on the critical plane is accounted for by the product of these stresses in the secondary multiplicative term. The value of σ_o in this equation is arbitrary, and simply used to maintain unit consistency. Note also that the parameter contains only two material-dependent constants (k and w), relative to the six required by the Erickson model. Thus, this parameter requires substantially less computational effort to fully implement in comparison to the Erickson model.

$$DP = \left(G \times \Delta\gamma\right)^{w} \times \tau_{max}^{(1-w)} \left(1 + k \frac{\left(\sigma \times \tau\right)_{max}}{\sigma_o^2}\right) \quad (5)$$

The application of this new parameter to the Ti-6Al-4V data set referenced in Fig. 2 is shown below in Fig. 4(a). In comparing this plot with the one shown in Fig. 2(c), it is evident the new parameter provides a similarly excellent correlation of both the uniaxial and multiaxial fatigue data throughout the full LCF/HCF spectrum. A second set of fatigue data, taken from the thesis of Morrow [18], is shown in Fig. 4(b). This data set includes uniaxial, torsional, proportional and non-proportional multiaxial data from an IN 718 alloy. It can be seen that the new parameter also collapses this data set extremely well to a single curve, providing further evidence of its ability to predict fatigue lives in a variety of materials and load paths.



(a)



(b)

Figure 4: Proposed damage parameter applied to (a) Ti-6Al-4V and (b) IN 718 [18].

CONCLUSION

A new critical-plane damage parameter for the prediction of fatigue life under multiaxial loading has been presented. This new parameter has been evaluated using a significant amount of fatigue data from several high strength titanium and steel alloys, and found to provide excellent correlation of the data. The parameter can account for strain hardening in the LCF regime, and the effect of mean stresses in the HCF regime. The parameter is computationally inexpensive and requires determination of only two material constants.

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