# Simple criterion for predicting fatigue life under combined bending and torsion loading

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**ABSTRACT.** Multiaxial fatigue is a challenging problem and, consequently, a number of methods has been developed to aid in design of components and assemblies. Following the complexity of the problem, these approaches are often elaborate and it is difficult to use them for simple loading cases. In this paper, an empirical approach for constant amplitude, proportional axial and torsion loading is introduced to serve as a basic engineering tool for estimating fatigue life of rotational structural parts. The criterion relies on a quadratic equivalent-stress formula and requires one constant parameter to be determined from experiments. The comparison with similar classical stress-based approaches using data on diverse materials (several steels, aluminium alloy, and nickel base superalloy) reveals very good agreement with experimental data.

**KEYWORDS.** Multiaxial fatigue; Life prediction; Equivalent stress.

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## INTRODUCTION

Fatigue failure under multiaxial cyclic loading is undoubtedly one of the most common concerns among engineers. The multiaxial fatigue process itself is rather complex and a tremendous effort has been invested in developing methods capable of capturing some of its most relevant aspects, such as the importance of shear stresses for the fatigue crack initiation stage, the short crack problem, crack closure, or non-proportional hardening observed in some materials [1, 2]. Consequently, these methods are often quite elaborate and their employment may require a skilled person using a specialized software. On the other hand, the first-approximation estimation of multiaxial fatigue failure is frequently based on the von Mises stress, the Tresca criterion, or some other static hypothesis, e.g. [3, 4], which apparently are among a few simple formulas qualified for a widespread usage. As these simple methods are known to be generally not acceptable, the purpose of this work was to introduce a similarly simple empirical approach which is intended to serve as a basic engineering tool for initial estimation of fatigue life under combine proportional axial and torsion loading, being of a special interest for rotational structural parts operating under such conditions. The method relies on fitting the experimental data for the two loading channels and formulating the equivalent loading based on a single constant parameter.

## THE NEW S-N CURVE AND THE BENCHMARK CRITERIA

In the following, the combined cyclic loading is considered to be proportional, i.e., it consists of well-defined loading cycles and no additional non-proportionality effects, need to be taken into account. Furthermore, we assume that the relationship between loading and fatigue life, $N_f$, over the studied life range is reasonably linear in log-log coordinates. This relationship is expressed in terms of stresses using the formalism in ASTM E 739 standard [5]. For symmetric loading, the new equivalent S-N curve (*the middle-curve criterion*) is constructed as

$$\log N_{\mathrm{f}} = A_{\sigma,\tau} + m_{\sigma,\tau} \log \sigma_{\mathrm{eq,a}} = A_{\sigma,\tau} + \frac{1}{2} m_{\sigma,\tau} \log\left(\sigma_{\mathrm{a}}^2 + k_0 \tau_{\mathrm{a}}^2\right), \tag{1}$$

where $A_{\sigma,\tau}$ and $m_{\sigma,\tau}$ are the intercept and the (negative) slope and $\sigma_{eq,a}$ is the amplitude of the equivalent stress expressed as a quadratic combination of bending and torsion amplitudes, $\sigma_a$ and $\tau_a$. The constant $k_0$ which describes the relation between bending and torsion loading is obtained as

$$k_0 = \frac{\sigma_{\mathrm{a0}}^2}{\tau_{\mathrm{a0}}^2}, \tag{2}$$

where $\sigma_{a0}$ and $\tau_{a0}$ are bending and torsion fatigue strengths corresponding to fatigue life $N_f = N_{f0}$ being in the middle between the axial and torsion midpoints ($\log N_{f0} = ½ (\log N_{f0}^a + \log N_{f0}^t)$), see Fig. 1, where superscripts *a* and *t* reference, respectively, to the axial and torsion S-N curves and $N_{f0}^a$ and $N_{f0}^t$ correspond to the middles of the curves. The middle curve intersects the axial S-N curve at $\sigma_{a0}$, $N_{f0}$ and bisects the angle between the axial and torsion S-N curves. Its intercept and slope can be obtained from the intercepts and the slopes of bending ($A_\sigma$, $m_\sigma$) and torsion ($A_\tau$, $m_\tau$) curves:

$$m_{\sigma,\tau} = \tan\left(\frac{\arctan m_\sigma + \arctan m_\tau}{2}\right), \tag{3}$$

$$A_{\sigma,\tau} = \log N_{\mathrm{f0}} - m_{\sigma,\tau} \log \sigma_{\mathrm{0a}} = A_\sigma + \left(m_\sigma - m_{\sigma,\tau}\right) \log \sigma_{\mathrm{0a}}. \tag{4}$$

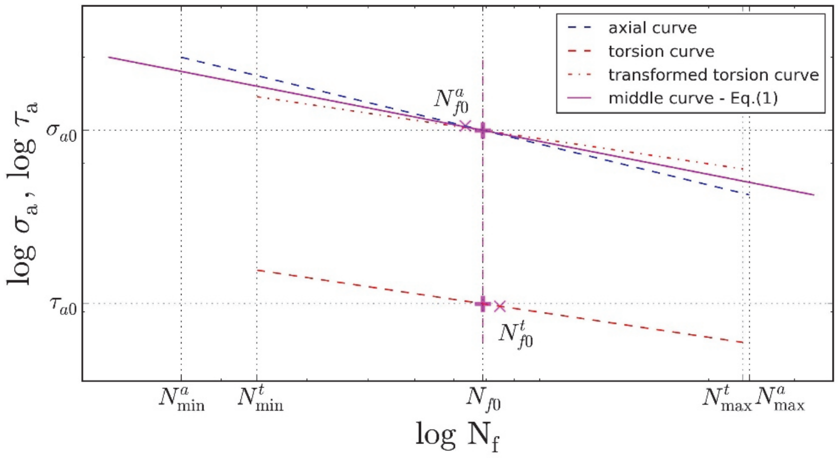

Figure 1: Definition of $N_{f0}$, $\sigma_{a0}$, and $\tau_{a0}$ using data of S355J2G3 alloy steel reported by Karolczuk et al. [4,5].

This method (Eqs. 1-4) was compared to the von Mises (Eq. 5) and Tresca (Eq. 6) criteria and to the Gough-Pollard criterion for ductile metals (Eq. 7), which are similar quadratic formulas:

$$\log N_\mathrm{f} = A_\sigma + m_\sigma \log \sigma_\mathrm{eq,a} = A_\sigma + \frac{1}{2} m_\sigma \log\left(\sigma_\mathrm{a}^2 + 3\tau_\mathrm{a}^2\right), \tag{5}$$

$$\log N_\mathrm{f} = A_\tau + m_\tau \log \tau_\mathrm{eq,a} = A_\tau + \frac{1}{2} m_\tau \log\left(\frac{1}{4}\sigma_\mathrm{a}^2 + \tau_\mathrm{a}^2\right), \tag{6}$$

$$\left(\frac{\sigma_\mathrm{a}}{\sigma_\mathrm{c}}\right)^2 + \left(\frac{\tau_\mathrm{a}}{\tau_\mathrm{c}}\right)^2 = 1\,. \tag{7}$$

In Eq. (7), $\sigma_\mathrm{c}$ and $\tau_\mathrm{c}$ are the axial and torsion fatigue strengths that, conceptually, correspond to the same fatigue life. Therefore, this criterion reflects the non-parallelism of the S-N curves (variation of the $\sigma_\mathrm{c}/\tau_\mathrm{c}$ ratio), as the middle curve criterion does, but is more difficult to apply, e.g. [7]. The middle-curve criterion is more general than the von Mises and Tresca criteria. Indeed, when $\sigma_\mathrm{c}/\tau_\mathrm{c} = \sqrt{3}$ (or $\sigma_\mathrm{c}/\tau_\mathrm{c} = 2$) holds in the whole range of fatigue life, the middle-curve criterion becomes equivalent to the von Mises (or Tresca) criterion. Furthermore, the middle-curve becomes equal to the Gough-Pollard criterion for materials with parallel S-N curves but gives better results for materials with non-parallel S-N curves (see hereafter).

## EXPERIMENTAL DATA

The studied methods were evaluated using plane-bending/torsion data on 2017A-T4 aluminium alloy, S355J2WP and S355J2G3 alloy steels, 30CrNiMo8 medium alloy steels, and Inconel 713LC nickel-base superalloy, see Refs. [6-9] and references therein for more detailed information on these fatigue experiments. Tab. 1 summarizes the ultimate strength $\sigma_\mathrm{u}$ and the yield strength $\sigma_\mathrm{y}$ of all materials and the parameters of bending and torsion S-N curves. The angle $\theta$ is the angle between the two curves and its positive value means that the axial curve is steeper. Except for 2017A-T4 aluminum alloy, the S-N curves are clearly not parallel.

| Material | $\sigma_\mathrm{u}$ (MPa) | $\sigma_\mathrm{y}$ (MPa) | $\sigma_\mathrm{y}/\sigma_\mathrm{u}$ | | $A_\sigma$ | $m_\tau$ | $A_\tau$ | $\theta$ (°) |
|---|---|---|---|---|---|---|---|---|
| 2017A-T4 [8] | 545 | 395 | 0.72 | -7.0 | 21.8 | -7.1 | 20.3 | 0.1 |
| S355J2WP [6] | 556 | 414 | 0.74 | -12.5 | 37.6 | -5.8 | 18.6 | -5.3 |
| S355J2G3 [6,7] | 611 | 394 | 0.64 | -7.2 | 23.9 | -11.7 | 32.8 | 3.0 |
| Inc713LC [9] | 982 | 801 | 0.82 | -4.5 | 17.4 | -7.5 | 23.9 | 4.8 |
| 30CrNiMo8 [6] | 1014 | 812 | 0.80 | -8.1 | 27.6 | -24.7 | 69.7 | 4.7 |

Table 1: Basic material properties and the parameters related to the bending and torsion S-N curves.

## RESULTS AND DISCUSSION

Tab. 2 summarizes parameters related to the middle-curve criterion. Note that the constant $k_0$ lies in the range from 2.22 to 4.29. Since $k_0$ = 3 for the von Mises criterion, this criterion can be expected to provide a plausible prediction for all materials except for S355J2WP steel, which should comply with the predictions obtained from the Tresca criterion.

Fig. 2 compares the experimental and calculated fatigue lives, $N_\mathrm{f,exp}$ and $N_\mathrm{f,calc}$, in the log-log space. A full diagonal line signifies a perfect agreement between predicted and observed values. The dashed and dash-dot lines constitute factors of two and three bandwidths. Fig. 3 plots the distribution of deviations from the perfect-agreement line for all materials using the box charts. Additional information is shown in Tab. 3 that summarizes the mean values obtained for each material.

| Material | $N_0$ | $\sigma_{0,a}$ (MPa) | $\tau_{0,a}$ (MPa) | $k_0$ | $m_{\sigma,\tau}$ | $A_{\sigma,\tau}$ |
|---|---|---|---|---|---|---|
| 2017A-T4 | $6.4\times10^5$ | 189 | 111 | 2.89 | -7.1 | 21.9 |
| S355J2WP | $5.7\times10^5$ | 343 | 166 | 4.29 | -7.9 | 25.9 |
| S355J2G3 | $6.9\times10^5$ | 328 | 204 | 2.57 | -8.9 | 28.2 |
| Inc713LC | $5.7\times10^4$ | 606 | 364 | 2.78 | -5.7 | 20.5 |
| 30CrNiMo8 | $1.1\times10^5$ | 623 | 418 | 2.22 | -12.2 | 39.1 |

Table 2: Parameters related to the middle-curve criterion.

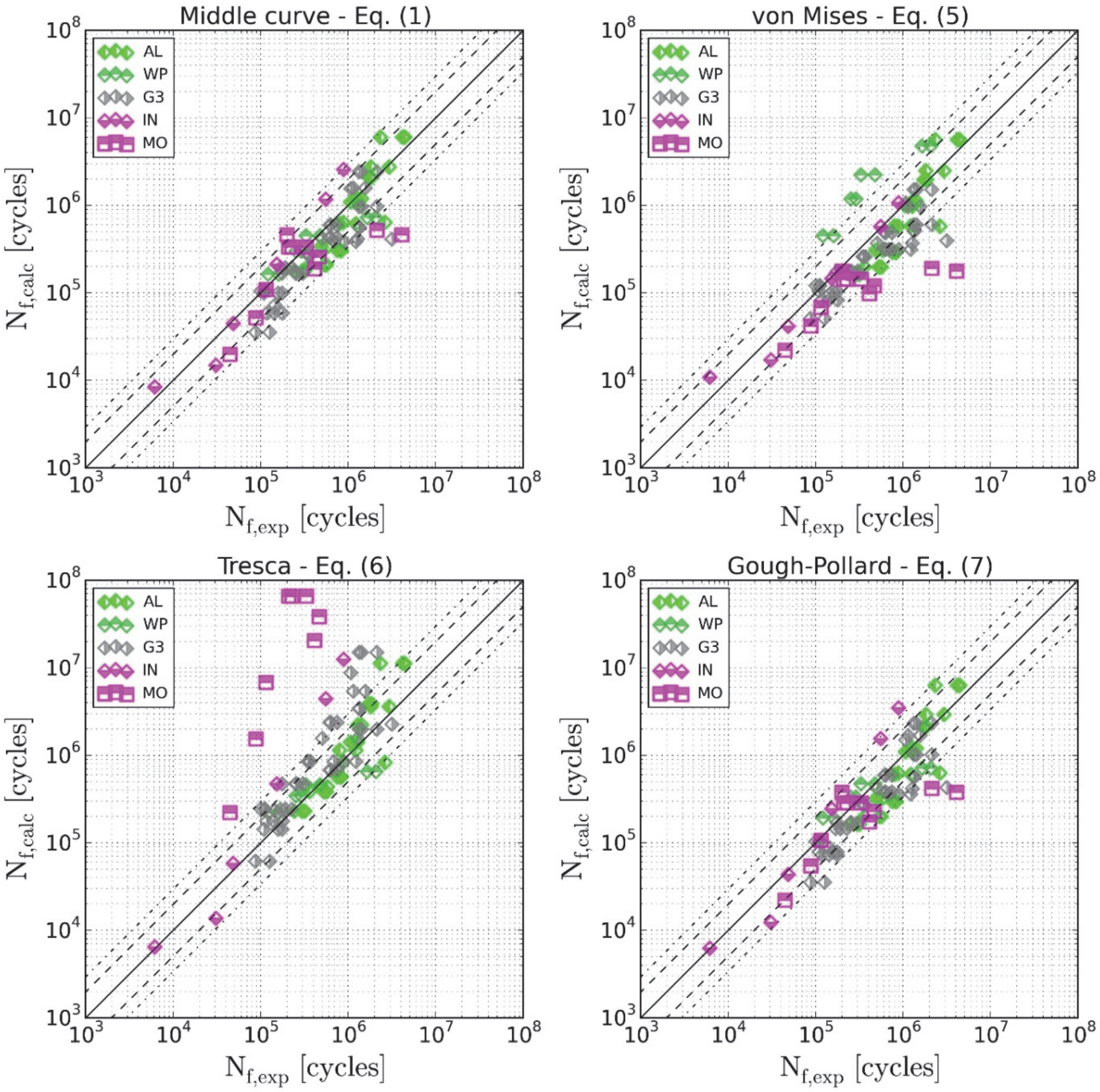

Figure 2: Comparison of experimental and predicted fatigue lives.

The results reveal that the middle-curve criterion provides, in general, the best estimates of fatigue life. The Gough-Pollard criterion also yields very reasonable predictions but it is computationally more complicated since it requires an iterative solution. As expected, the von Mises criterion provides good results for materials with the $k_0$-value close to 3 but much worse estimates for S355J2WP steel with $k_0$ = 4.29, for which the Tresca criterion is more suitable. The latter criterion is, however, generally inaccurate and nonconservative.

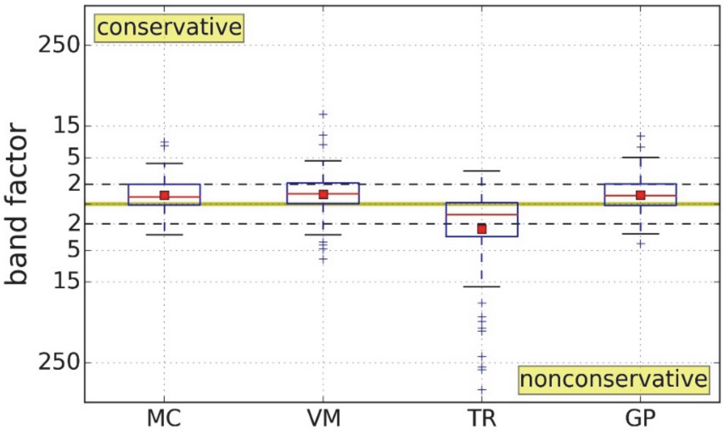

Figure 3: Box chart of prediction differences: MC – middle curve, VM – von Mises, TR – Tresca, GP – Gough-Pollard.

| Material | MC | VM | TR | GP |
|---|---|---|---|---|
| 2017A-T4 | 1.4 | 1.5 | -1.2 | 1.4 |
| S355J2WP | 1.2 | -3.8 | 1.1 | 1.1 |
| S355J2G3 | 1.5 | 1.6 | -1.9 | 1.5 |
| Inc713LC | -1.3 | 1.0 | -2.5 | -1.4 |
| 30CrNiMo8 | 1.5 | 3.0 | -78 | 1.7 |

Table 3: Average band factors.

## CONCLUSIONS AND PROSPECTS

This paper introduces a simple, computationally non-intensive empirical method, termed as the middle-curve criterion, to be used as a fast and efficient tool for the initial estimate of life of structural component subjected to combined bending and torsion proportional loading. The criterion requires one constant material parameter, $k_0$, to be determined from experiments based on a suitably defined reference number of cycles to failure $N_{f0}$. The new referential S-N curve is obtained as the curve that intersects the axial S-N curve at $N_{f0}$ and bisects the angle between the axial and torsion curves, thus reflecting the fact that the axial and torsion curves are generally divergent. The von Mises and Tresca criteria are special cases of the middle-curve criterion which is equal to the Gough-Pollard criterion for materials with parallel S-N curves. A comparison of all these criteria was made by using experimental data on five diverse metallic materials and the analysis clearly demonstrated advantages of the new criterion over the studied classical criteria. Therefore, the middle-curve criterion is very useful for a preliminary estimation of life of rotational structural parts under mixed axial and torsion loading. Despite the formulas presented in this paper do not account for the mean stress effect, it can be easily included by using experimental data obtained from non-zero mean stress tests. Transformation of the data based on the Goodman diagram or other similar methods [10] is also possible. Furthermore, our unpublished data show that the precision of prediction can be increased by using referential S-N curve obtained by fitting of joined axial data and torsion data expressed in terms of equivalent stress $\left(\sigma_a^2 + k_0\tau_a^2\right)^{1/2}$. The non-proportionality of loading can also be taken into account by a suitable definition of a dependence of $k_0$ on the phase shift.

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