Notch stress intensity factors under mixed mode loadings: an overview of recent advanced methods for rapid calculation

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ABSTRACT. Recently some methods for the rapid calculation of notch stress intensity factors (NSIFs) have been developed and three of them are compared in this work. First, the criteria proposed by Lazzarin et al. and Treifi et al. have been reviewed. The former is based on the calculation of the mean value of SED on two different control volume (characterized by two different radius values) centred at the stress singularity point, whereas the latter takes advantage of the strain energy density averaged within two control volumes (semi-circular sector) centred at the notch tip. Then, a new method based on the evaluation of the total and deviatoric SED averaged in a single control volume has been proposed. Finally, plate specimens weakened by different notch geometries have been subjected to the application of the above mentioned methods and the obtained values of the NSIFs have been compared with those derived according to Gross and Mendelson.

KEYWORDS. NSIFs; SED; Control volume; Mixed mode; FE analysis.

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INTRODUCTION

Because of the detrimental effects played by notches on mechanical behavior [1-3] , several criteria have been proposed in literature for the assessment of material strength of components [4,5]. Concerning brittle and quasi-brittle materials, Notch stress intensity factor (NSIF) has been reported to be an effective criterion for the evaluation of static strength [6], and also for high cycle fatigue strength of structural materials weakened by sharp V-notches [7] and of welded joints [8,9].
Gross and Mendelson [10] expressed mode I and mode II NSIFs, which define the intensity of the asymptotic stress field ahead of the sharp V-notch tip.

$$K_1 = \sqrt{2\pi}\lim_{r\to 0}\left[\left(\sigma_{\theta\theta}\right)_{\theta=0} r^{(1-\lambda_1)}\right] \tag{1}$$

$$K_2 = \sqrt{2\pi}\lim_{r\to 0}\left[\left(\tau_{r\theta}\right)_{\theta=0} r^{(1-\lambda_2)}\right] \tag{2}$$

where r and θ is the radial and angular coordinate of a polar coordinate system centred at the notch tip (Fig. 1), $\sigma_{\theta\theta}$ and $\tau_{r\theta}$ are the stress components according to the coordinate system and the mode I and mode II first eigenvalues in William's equations [11] are denoted respectively as λ_1 and λ_2. The main drawback exploiting NSIF criterion for the calculation of (1) and (2) is that very refined meshes are needed using finite element (FE) analysis, and this requirement holds also for cracked component [12-17].

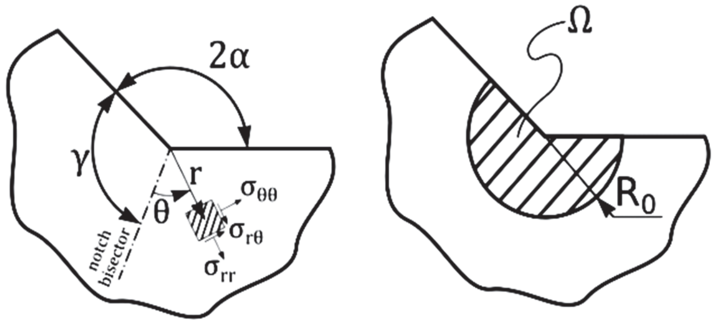

Figure 1: Polar coordinate system centred at the notch tip.

However, an approach based on the determination of the total elastic strain energy density (SED) averaged over a control volume of radius R_0 surrounding the points of stress singularity has been developed [18] and it has broadly gained interest in the last years, since it allows to exploit course meshes overcoming the main drawback of NSIF criterion both for static static [19-24] and fatigue [1, 25-27] strength evaluation.
Moreover, some approximate methods for the rapid calculation of the NSIFs, based on the averaged strain energy density, are available in literature and, referring to mixed mode loading, they required the solution of a system of two equations in two unknowns (K_1 and K_2).
In this work, two of these methods, one proposed by Lazzarin et al. [28] and the other by Treifi and Oyadiji [29], have been reviewed, and, afterwards, a new method based on the evaluation of the total and deviatoric SED averaged in a single control volume has been proposed, allowing also in this case the determination of the SIFs, K_I and K_{II}, of cracks under mixed mode loading by means of two independent equations, one linked to the total SED and the other to the deviatoric one.

APPROXIMATE METHODS

Lazzarin et al. approach

The first method has been proposed by Lazzarin et al. [28] and it is based on the evaluation of the averaged SED on two different control volumes (circular sectors) centred at the notch tip and characterized by the radii R_a and R_b (Fig. 2a). Known the SED values ($\overline{W}_a$ and $\overline{W}_b$), by means of a FE analysis, and defined the control radii (R_a and R_b), it is possible to obtain a system of two equations in two unknowns (K_1 and K_2):

$$\begin{cases} \overline{W}_{a,FE} = \dfrac{1}{2E}\left[\dfrac{I_1}{2\lambda_1\gamma}\dfrac{K_1^2}{R_a^{2(1-\lambda_1)}} + \dfrac{I_2}{2\lambda_2\gamma}\dfrac{K_2^2}{R_a^{2(1-\lambda_2)}}\right] = C_a K_1^2 + D_a K_2^2 \\ \overline{W}_{b,FE} = \dfrac{1}{2E}\left[\dfrac{I_1}{2\lambda_1\gamma}\dfrac{K_1^2}{R_b^{2(1-\lambda_1)}} + \dfrac{I_2}{2\lambda_2\gamma}\dfrac{K_2^2}{R_b^{2(1-\lambda_2)}}\right] = C_b K_1^2 + D_b K_2^2 \end{cases} \tag{3}$$

where E is the Young's modulus of the material while I_1 and I_2 are the integrals of the angular stress functions [18], which depend on the notch opening angle, $2a = 2\pi - 2\gamma$, and the Poisson's ratio ν. This method cannot be applied to a crack subjected to mixed mode loading, since an indeterminate system of equations would be obtained. Solving the system of equations, the values of the NSIFs can be determined:

$$K_1 = \sqrt{\frac{D_a \overline{W}_{b,FE} - D_b \overline{W}_{a,FE}}{D_a C_b - D_b C_a}} \tag{4}$$

$$K_2 = \sqrt{\frac{\overline{W}_{a,FE} - C_a K_1^2}{D_a}} \tag{5}$$

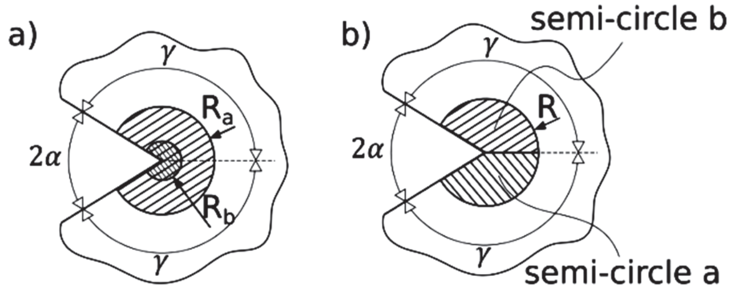

Figure 2: Control volumes in the Lazzarin et al. approach (a) and in the Treifi et al. approach (b).

Treifi et al. approach

The second method has been proposed by Treifi et al. [29] and it is based on the evaluation of the averaged SED on two different control volumes (semi-circular sectors with a central angle equal to γ) centred at the notch tip and characterized by a radius R (Fig. 2b). Known the SED values ($\overline{W}_a$ and $\overline{W}_b$) by means of a FE analysis, and defined the control radius (R), it is possible to obtain a system of two equations in two unknowns (K_1 and K_2):

$$\begin{cases} \overline{W}_{a,FE} = \dfrac{1}{2E}\left[\dfrac{I_{1,s}}{\lambda_1\gamma}\dfrac{K_1^2}{R^{2(1-\lambda_1)}} + \dfrac{I_{2,s}}{\lambda_2\gamma}\dfrac{K_2^2}{R^{2(1-\lambda_2)}} + \dfrac{2I_{12,s}}{(\lambda_1+\lambda_2)\gamma}\dfrac{K_1K_2}{R^{2-\lambda_1-\lambda_2}}\right] = MK_1^2 + NK_2^2 + QK_1K_2 \\ \overline{W}_{b,FE} = \dfrac{1}{2E}\left[\dfrac{I_{1,s}}{\lambda_1\gamma}\dfrac{K_1^2}{R^{2(1-\lambda_1)}} + \dfrac{I_{2,s}}{\lambda_2\gamma}\dfrac{K_2^2}{R^{2(1-\lambda_2)}} - \dfrac{2I_{12,s}}{(\lambda_1+\lambda_2)\gamma}\dfrac{K_1K_2}{R^{2-\lambda_1-\lambda_2}}\right] = MK_1^2 + NK_2^2 - QK_1K_2 \end{cases} \tag{6}$$

where $I_{1,s}$, $I_{2,s}$ and $I_{12,s}$ are the integrals of the angular stress functions [29], which depend on the notch opening angle, $2a$, the angle defined by the semi-circular sector, γ, and the Poisson's ratio ν. In this case also the contribution of the mixed term ($K_1.K_2$) is present because the integration for the strain energy evaluation is not performed on a control volume symmetrical with respect to the notch bisector line (Fig. 2b). Due to the presence of the mixed term it is possible to decouple the contributions of the loading modes, obtaining a solution of the system even in the crack case. Solving the system of equations, the values of the NSIFs can be determined:

$$K_1 = \sqrt{\frac{\left(\overline{W}_{a,FE} + \overline{W}_{b,FE}\right) \pm \sqrt{\left(\overline{W}_{a,FE} + \overline{W}_{b,FE}\right)^2 - \dfrac{4MN\left(\overline{W}_{a,FE} - \overline{W}_{b,FE}\right)^2}{Q^2}}}{4M}} \tag{7}$$

$$K_2 = \frac{\overline{W}_{a,FE} - \overline{W}_{b,FE}}{2QK_1} \tag{8}$$

New approach based on the deviatoric SED

In the present contribution a new method is proposed. It is based on the evaluation of the total and deviatoric SED averaged in a single control volume (circular sector) centred at the notch tip and characterized by a radius R (Fig. 3a). Two independent equations can be obtained: one linked to the total SED and the other to the deviatoric one. In this way it is possible to obtain a solution of the system even in the case of a crack.

As previously, knowing the SED values ($\bar{W}$ and $\bar{W}_{dev}$), by means of a FE analysis, and defining the control radius (R), it is possible to obtain a system of two equations in two unknowns (K_1 and K_2):

$$\begin{cases} \bar{W}_{FE} = \dfrac{1}{2E}\left[\dfrac{I_1}{2\lambda_1\gamma}\dfrac{K_1^2}{R^{2(1-\lambda_1)}} + \dfrac{I_2}{2\lambda_2\gamma}\dfrac{K_2^2}{R^{2(1-\lambda_2)}}\right] = SK_1^2 + TK_2^2 \\ \bar{W}_{dev,FE} = \dfrac{1+\nu}{3E}\left[\dfrac{I_{1,dev}}{2\lambda_1\gamma}\dfrac{K_1^2}{R^{2(1-\lambda_1)}} + \dfrac{I_{2,dev}}{2\lambda_2\gamma}\dfrac{K_2^2}{R^{2(1-\lambda_2)}}\right] = S_{dev}K_1^2 + T_{dev}K_2^2 \end{cases} \tag{9}$$

where $I_{1,dev}$ and $I_{2,dev}$ are the integrals of the angular stress functions related to the deviatoric strain energy density [18], which depend on the notch opening angle, 2α, and the Poisson's ratio ν. Solving the system of equations, the values of the NSIFs can be determined:

$$K_1 = \sqrt{\frac{T\bar{W}_{dev,FE} - T_{dev}\bar{W}_{FE}}{TS_{dev} - T_{dev}S}} \tag{10}$$

$$K_2 = \sqrt{\frac{\bar{W}_{dev,FE} - S_{dev}K_1^2}{T_{dev}}} \tag{11}$$

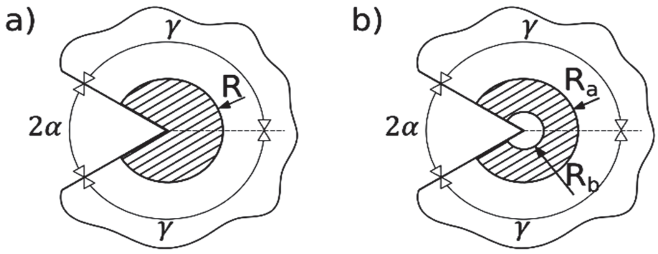

Figure 3: Control volumes in the new approach (a) and in the modified version of the new approach (b).

As discussed earlier, the total SED can be derived directly from nodal displacements, so that also coarse meshes are able to give sufficiently accurate values for it. On the other hand, the calculation of the deviatoric SED by means of a FE code is based on the von Mises equivalent stress averaged within the element [18]. This quantity is more sensitive to the refinement level of the adopted mesh, so the new proposed method could not be mesh-insensitive.

With the aim to improve the results obtained from the application of the new method (based on the deviatoric SED) in the case of coarse meshes, a modified version is proposed. The approach is similar to the previous but it is applied to a control volume consisting of a circular ring (Fig. 3b). Being the calculation of the deviatoric SED by means of a FE code based on the von Mises equivalent stress averaged within the element, that is a parameter sensitive to the refinement level of the adopted mesh, it could be useful to exclude from the calculation the area characterized by the highest stress gradient (i.e. the region close to the notch tip) in the case of coarse meshes. The control volume results to be constituted by a circular ring characterized by an outer radius R_a and by an inner radius R_b (Fig. 3b).

As before, knowing the SED values ($\bar{W}$ and $\bar{W}_{dev}$), by means of a FE analysis, and defining the control radii (R_a and R_b), it is possible to obtain a system of two equations in two unknowns (K_1 and K_2):

$$\begin{cases} \bar{W}_{FE} = \dfrac{1}{2E\gamma\left(R_a^2 - R_b^2\right)}\left[\dfrac{I_1K_1^2}{2\lambda_1}\left(\dfrac{1}{R_a^{-2\lambda_1}} - \dfrac{1}{R_b^{-2\lambda_1}}\right) + \dfrac{I_2K_2^2}{2\lambda_2}\left(\dfrac{1}{R_a^{-2\lambda_2}} - \dfrac{1}{R_b^{-2\lambda_2}}\right)\right] \\ \bar{W}_{dev,FE} = \dfrac{1+\nu}{3E\gamma\left(R_a^2 - R_b^2\right)}\left[\dfrac{I_{1,dev}K_1^2}{2\lambda_1}\left(\dfrac{1}{R_a^{-2\lambda_1}} - \dfrac{1}{R_b^{-2\lambda_1}}\right) + \dfrac{I_{2,dev}K_2^2}{2\lambda_2}\left(\dfrac{1}{R_a^{-2\lambda_2}} - \dfrac{1}{R_b^{-2\lambda_2}}\right)\right] \end{cases} \tag{12}$$

Solving this system of equations, as already shown in the previous cases, the values of the NSIFs can be determined.

RESULTS

The methods described above have been applied to five different geometries of notched plates subjected to mixed mode I+II loading. For each geometry K_1 and K_2 have been first calculated according to Gross and Mendelson (Eqs. 1 and 2), by means of FE analyses adopting very refined meshes in the close neighbourhood of the notch tip (size of the smallest element of the order of 10^{-5} mm). Afterwards the approximate methods have been applied, taking into consideration three different values of the control radius R_0 (0.1, 0.01 and 0.001 mm) and by using coarse and refined FE meshes. The mesh has been performed by means of 8-nodes elements (PLANE 183), using the FE code ANSYS® 14.5. In the FE analyses a Poisson's ratio ν equal to 0.3 and a Young's modulus E equal to 206 GPa have been adopted. In the summary tables the normalized NSIFs are reported, according to the definition:

$$K_{i,normalized} = \frac{K_i}{\sigma\sqrt{\pi}a^{1-\lambda_i}} \tag{13}$$

Case studies: geometries and results

The geometries taken into consideration (Fig. 4) consist in notched finite or infinite plates subjected to mixed mode I+II loading. The geometry of the finite plates is characterized by equal width and height, $2W = H = 10$ mm, while plates of infinite extension are characterized by $2W = H = 10$ mm. Diamond shape notches (Fig. 4a) are characterized by a projected notch depth $2a = 2$ mm and a notch inclination angle $\varphi = 45°$. Two different notch opening angle 2α have been analysed: 45° and 30°. The obtained results and the comparison between the different approaches are reported in Table 1 and Table 2. The square hole (Fig. 4b) is characterized by a projected notch depth $2a = 2$ mm and a notch opening angle $2\alpha = 90°$. The obtained results and the comparison between the different approaches are reported in Table 3.

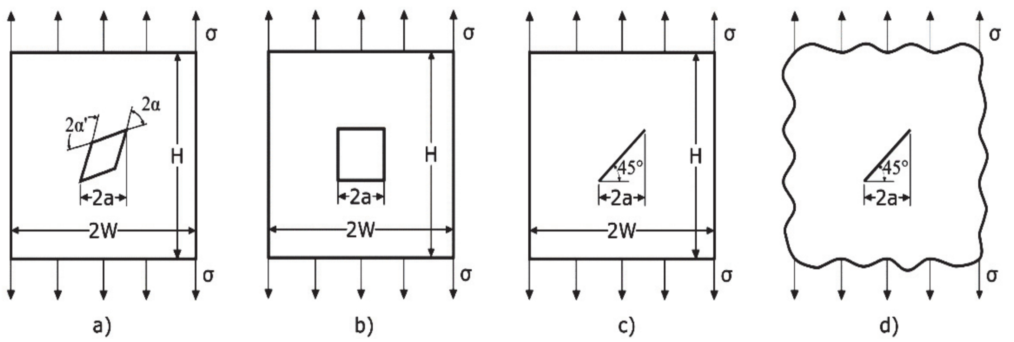

Figure 4: Case studied for the application of the method: diamond shape notch (a), square hole (b), central tilted crack in a finite plate (c) and central tilted crack in a plate of infinite extension (d).

DISCUSSION

Tabs. 1-5 show the results obtained from different geometries of notched plates subjected to mixed mode I+II loading, by adopting coarse and refined meshes for the application of the approximate methods. All the considered approaches allow to obtain very good approximations when refined meshes are adopted in the FE analyses, being the deviations, with respect to the values calculated according to Gross and Mendelson [10], lower than 1% in most of the cases.

The obtained results become more interesting when coarse meshes, which require shorter calculation time, are adopted in the FE analyses. It can be observed that the approach of Lazzarin et al. [28] enables to obtain very good approximations, being the deviations lower than 1% in most of the cases analyzed in the present contribution. It should be noted that this method has not been applied to cracks subjected to mixed mode loading (Tables 4, 5), since an indeterminate system of equations would be obtained. The percentage error increases to about 3-6% in the case of Treifi et al. approach [29], which reaches a maximum percentage deviation of 12-18% in the case of tilted cracks (Table 5).

The new proposed method, based on the evaluation of the total and deviatoric SED, allows to obtain a percentage error close to that observed in the case of Treifi et al. [29]. The deviation still remains greater than that observed in the case of Lazzarin et al. [28], because of the dependence of the deviatoric SED on the mesh size. This problem is overcome by the modified version of the new method that, through a control volume consisting of a circular ring, enables to exclude the region characterized by the highest stress gradient making the method less sensitive to the refinement level of the adopted mesh.
The new method, particularly the modified version, provides very good approximations and a greater applicability than the approach of Lazzarin et al. [28] and [30], so it could be useful for rapid calculation of the NSIFs.

		Coarse mesh (64 finite elements)				*Refined mesh (3128 finite elements)*			
R_0 [mm]	*Method*	K_1	K_2	ΔK_1 (%)	ΔK_2 (%)	K_1	K_2	ΔK_1 (%)	ΔK_2 (%)
	Gross and Mendelson	0.656	0.911						
0.1 and 0.075	Lazzarin et al.	0.650	0.919	-0.91	0.88	0.650	0.919	-0.91	0.88
0.1	Treifi et al.	0.694	0.878	5.79	-3.62	0.693	0.878	5.64	-3.62
0.1	New method	0.681	0.891	3.81	-2.20	0.666	0.905	1.52	-0.66
0.1	New modified method	0.664	0.908	1.22	-0.33				
0.01 and 0.0075	Lazzarin et al.	0.657	0.909	0.15	-0.22	0.655	0.912	-0.15	0.11
0.01	Treifi et al.	0.665	0.895	1.37	-1.76	0.663	0.897	1.07	-1.54
0.01	New method	0.671	0.883	2.29	-3.07	0.657	0.909	0.15	-0.22
0.01	New modified method	0.656	0.911	0.00	0.00				
0.001 and 0.00075	Lazzarin et al.	0.659	0.901	0.46	-1.10	0.656	0.909	0.00	-0.22
0.001	Treifi et al.	0.658	0.907	0.31	-0.44	0.657	0.908	0.15	-0.33
0.001	New method	0.671	0.856	2.29	-6.04	0.658	0.901	0.30	-1.10
0.001	New modified method	0.658	0.905	0.30	-0.66				

Table 1: Comparison between approximate methods for NSIFs evaluation of diamond-shaped notch ($2a = 45°$).

		Coarse mesh (64 finite elements)				*Refined mesh (3063 finite elements)*			
R_0 [mm]	*Method*	K_1	K_2	ΔK_1 (%)	ΔK_2 (%)	K_1	K_2	ΔK_1 (%)	ΔK_2 (%)
	Gross and Mendelson	0.654	0.813						
0.1 and 0.075	Lazzarin et al.	0.648	0.818	-0.92	0.62	0.650	0.817	-0.61	0.49
0.1	Treifi et al.	0.670	0.803	2.45	-1.23	0.681	0.796	4.13	-2.09
0.1	New method	0.686	0.792	4.89	-2.58	0.663	0.808	1.38	-0.62
0.1	New modified method	0.660	0.810	0.92	-0.37				
0.01 and 0.0075	Lazzarin et al.	0.657	0.811	0.46	-0.25	0.655	0.813	0.15	0.00
0.01	Treifi et al.	0.621	0.847	-5.05	4.18	0.638	0.829	-2.45	1.97
0.01	New method	0.677	0.789	3.52	-2.95	0.657	0.811	0.46	-0.25
0.01	New modified method	0.656	0.812	0.31	-0.12				
0.001 and 0.00075	Lazzarin et al.	0.660	0.807	0.92	-0.74	0.656	0.811	0.31	-0.25
0.001	Treifi et al.	0.673	0.784	2.91	-3.57	0.660	0.805	0.92	-0.98
0.001	New method	0.675	0.779	3.21	-4.18	0.659	0.807	0.76	-0.74
0.001	New modified method	0.658	0.809	0.31	-0.49				

Table 2: Comparison between approximate methods for NSIFs evaluation of diamond-shaped notch ($2a = 30°$).

		Coarse mesh (48 finite elements)				Refined mesh (2206 finite elements)			
R_0 [mm]	*Method*	K_1	K_2	ΔK_1 (%)	ΔK_2 (%)	K_1	K_2	ΔK_1 (%)	ΔK_2 (%)
	Gross and Mendelson	0.618	1.209						
0.1 and 0.075	Lazzarin et al.	0.613	1.229	-0.81	1.65	0.613	1.229	-0.81	1.65
0.1	Treifi et al.	0.604	1.249	-2.27	3.31	0.632	1.184	2.27	-2.07
0.1	New method	0.635	1.175	2.75	-2.81	0.625	1.200	1.13	-0.74
0.1	New modified method	0.629	1.196	1.78	-1.08				
0.01 and 0.0075	Lazzarin et al.	0.617	1.205	-0.16	-0.33	0.617	1.210	-0.16	0.08
0.01	Treifi et al.	0.617	1.211	-0.16	0.17	0.618	1.200	0.00	-0.74
0.01	New method	0.601	1.394	-2.75	15.30	0.617	1.212	-0.16	0.25
0.01	New modified method	0.618	1.192	0.00	-1.41				
0.001 and 0.00075	Lazzarin et al.	0.618	1.166	0.00	-3.56	0.618	1.182	0.00	-2.23
0.001	Treifi et al.	0.618	1.213	0.00	0.33	0.617	1.202	-0.16	-0.58
0.001	New method	0.627	1.167	1.46	-3.54	0.618	1.178	0.00	-2.56
0.001	New modified method	0.619	1.130	0.16	-6.53				

Table 3: Comparison between approximate methods for NSIFs evaluation of diamond-shaped notch ($2a$ = 90°).

		Coarse mesh (64 finite elements)				Refined mesh (3395 finite elements)			
R_0 [mm]	*Method*	K_1	K_2	ΔK_1 (%)	ΔK_2 (%)	K_1	K_2	ΔK_1 (%)	ΔK_2 (%)
	Gross and Mendelson	0.655	0.638						
0.1	Treifi et al.	0.636	0.642	-2.90	0.63	0.660	0.637	0.76	-0.16
0.1	New method	0.697	0.620	6.41	-2.82	0.639	0.645	-2.44	1.10
0.1	New modified method	0.639	0.645	-2.44	1.10				
0.01	Treifi et al.	0.613	0.654	-6.41	2.51	0.635	0.647	-3.05	1.41
0.01	New method	0.708	0.616	8.09	-3.45	0.653	0.640	-0.31	0.31
0.01	New modified method	0.653	0.640	-0.31	0.31				
0.001	Treifi et al.	0.624	0.651	-4.73	2.04	0.644	0.644	-1.68	0.94
0.001	New method	0.712	0.615	8.70	-3.61	0.662	0.636	1.07	-0.31
0.001	New modified method	0.657	0.639	0.31	0.16				

Table 4: Comparison between approximate methods for NSIFs evaluation of central tilted crack ($2a$ = 0°) in a plate of finite extension.

CONCLUSIONS

In the present contribution three methods for the rapid calculation of the NSIFs, based on the averaged strain energy density, are compared. The first method, proposed by Lazzarin et al., is based on the calculation of the SED averaged in two different control volumes centred at the notch tip. This approach cannot be applied to notch with zero opening angle (cracks) subjected to mixed mode loading. The second method, presented by Treifi et al., overcomes this problem taking advantage of the strain energy density averaged within two control volumes (semi-circular sector) centred at the notch tip. Then a new method based on the evaluation of the total and deviatoric strain energy density has been proposed.

		Coarse mesh (64 finite elements)				Refined mesh (3395 finite elements)			
R_0 [mm]	*Method*	K_1	K_2	ΔK_1 (%)	ΔK_2 (%)	K_1	K_2	ΔK_1 (%)	ΔK_2 (%)
	Gross and Mendelson	0.595	0.595						
0.1	Treifi et al.	0.667	0.564	12.10	-5.21	0.703	0.547	18.15	-8.07
0.1	New method	0.649	0.572	9.08	-3.87	0.596	0.595	0.17	0.00
0.1	New modified method	0.598	0.594	0.50	-0.17				
0.01	Treifi et al.	0.582	0.599	-2.18	0.67	0.603	0.592	1.34	-0.50
0.01	New method	0.649	0.571	9.08	-4.03	0.597	0.594	0.34	-0.17
0.01	New modified method	0.598	0.594	0.50	-0.17				
0.001	Treifi et al.	0.575	0.602	-3.36	1.18	0.593	0.595	-0.34	0.00
0.001	New method	0.649	0.571	9.08	-4.03	0.612	0.588	2.86	-1.18
0.001	New modified method	0.598	0.594	0.50	-0.17				

Table 5: Comparison between approximate methods for NSIFs evaluation of central tilted crack ($2a$ = 0°) in a plate of infinite extension.

The described methods have been applied to plates subjected to mixed mode I+II loading and weakened by different V-notch geometries. The values of the NSIFs derived according to Gross and Mendelson have been compared with those obtained by means of the approximate methods taking into consideration three different values of the control radius R_0 (0.1, 0.01 and 0.001 mm) and by using coarse and refined FE meshes. The comparison of the results shown that the new proposed method provides the best combination between the degree of approximation and the level of applicability, so it could be useful for rapid calculation of the NSIFs. Central tilted cracks in a plate of finite (Fig. 4c) or infinite (Fig. 4d) extension are characterized by a projected crack length $2a$ = 2 mm and a crack inclination angle φ = 45°. The obtained results and the comparison between the different approaches are reported in Tables 4, 5. It is worth noting that the case of the infinite plate has been modelled as a finite plate characterized by width and height two orders of magnitude greater than the crack length.

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