Mechanical behavior of unidirectional composites according different failure criteria

Adel Deliou

University of M^{ed} Seddik Benyahia (UMSB of Jijiel), Department of Mechanical Engineering
Laboratory of Materials and Reactive Systems LMSR, University Djillali, Liabes, Sidi Bel-Abbes, Algeria.
del032003@yahoo.fr, adel.deliou@univ-jijel.dz, deliouadel15@gmail.com

Benattou Bouchouicha

Laboratory of Materials and Reactive Systems LMSR, University Djillali, Liabes, Sidi Bel-Abbes, Algeria.
benattou_b@yahoo.fr

ABSTRACT. This work is about study of the mechanical behaviour of unidirectional Kevlar / Epoxy composite laminates according to different failure criteria. Varying strength parameters values, makes it possible to compare the ultimate mechanical characteristics obtained by the criteria of Tsai-Hill, Norris, Fisher, Ashkenazi and Tsai-Wu.

The epoxy matrix of the material in question is reinforced with up to 60% of its volume by aramid fibers. The stack of four layers composing the arbitrarily oriented and alternating $[+\theta/-\theta]_S$ materials results in balanced symmetrical laminates. The laminate is subjected to uniaxial tensile membrane forces. Estimate of their ultimate strengths and the plotting of the failure envelope constitute the principal axis of this study. Using the theory of maximum stress, we can determine the various modes of damage of the composite. The different components of the deformation are presented for different orientations of fibers.

KEYWORDS. Unidirectional Kevlar/Epoxy composite; Failure criterion; Membrane stress; Deformations; Failure envelope.

INTRODUCTION

Composite materials, in the most common sense of the term, are a set of synthetic materials designed and used mainly for structural applications; the mechanical function is dominant.

The mechanical behaviors of the composite, as well as the degradation mechanisms leading to its rupture depend on the nature of the constituents and on the architecture of the fiber preform [1].

The profile is required because it guides the engineer in designing structures with precise properties in relation to the needs.

The study of the stability of these structures requires, among other things, knowledge of the limiting behavior of the material. This behavior is expressed by a failure criterion. Which expresses the relationships between the components of the tensor of the stresses; indeed, when checked locally, they translate the beginning of the failure.

Despite the complexity of the failure mechanism for composite materials, in particular due to the heterogeneity and the anisotropy of their structure, some work has attempted to simplify this, by giving a single failure criterion, applicable for any type of stress [2].

The Tsai-Hill criterion [3-4], initially based on the idea of Von-Mises for isotropic metallic materials and extended to the case of anisotropic materials, does not take account the difference in behavior, in tension and in compression.

The other criterion most commonly used is Tsai-Wu criterion [5]; it is based on the invariant tensor theory. It appears in quadratic form and it takes into account the interactions between the various components of the stress tensor.

The tensor coefficients of the rupture matrix are evaluated by means of tensile, compressive and shear tests, at rupture [6-7]. The difference between the many criteria, comes mainly from the types of tests and hypotheses used to evaluate these coefficients. For example, certain criteria like those of Tsai-Hill, Fisher, Ashkenazi and Norris admit the equivalence between the behaviors in tension and in compression, in order to limit even more the number of coefficients.

Damage to composite materials investigated by the use of failure criteria, is the subject of numerous studies [8-12].

Christensen [13] developed a mathematical model to predict the strength and the macromechanical fracture characteristics of unidirectional reinforced composite materials; and thus crack propagation can be optimized by the finite element method. Sauder et al. [14] found that this approach is limited, given the restrictive assumptions regarding the composition of the material; however, it allows to obtain reliable results for particular types of composites.

Reference [15] shows that the number of parameters required for the Tsai-Wu criterion, can be reduced from seven to five for composite materials that do not rupture at specific hydrostatic or transverse pressure levels.

Arola [16] presents a finite element model from failure envelopes during drilling tests of the carbon / epoxy composite, representing the values of the Tsai-Hill failure criterion. Then Mahdi [17] studied the influence of the mesh on the prediction of cutting forces as a function of the orientation angle of fibers using the same model as Arola.

M.A. Mbacke in his thesis [18], describes the sizing approach for coil reservoirs and multiform reservoirs designed by braiding fibers on the liner side. To assess the mechanical strength of the tanks, several failure criteria, such as Tsai-Wu, Tsai-Hill criteria, maximum stresses and strains were used.

Cazeneuve et al. [19] studied the behavior of Carbon/Epoxy and Kevlar/Epoxy tubes. They used their experimental results to modify the Tsai criterion and to predict better the failure of these high-performance composites. In the same vein, Vicario and Rizzo [20] and Herring et al. [21] studied the distribution of stresses in Boron/Epoxy tubes and determined the mechanical characteristics allowing comparison to conventional models.

R.M. Jones [22] verifies that the Tsai-Hill criterion used, is in good agreement with the experimental results for unidirectional E-glass / Epoxy composites, than that obtained by the maximum stress theory

A study known as the "World-Wide Failure Exercise (WWFE)" [23] was conducted with the aim of comparing the different failure models in the case of continuous fiber composite materials. This study is the most complete to date. 18 models were compared using 14 test cases, to assess different types of loads and according to the stacking sequence.

Among the criteria that give good results in tension, we have those of Tsai-Hill and Tsai-Wu.

Recently, S. Li [24] systematically re-examines from a mathematical point of view, the quadratic function of Tsai -Wu, guided by the principles of analytical geometry in the context of unidirectional composites.

The major objective of our work is to contribute to the analysis of the strength studies, by different failure criteria of unidirectional laminate in Kevlar/Epoxy, according to the stacking sequence under the effect of uniaxial tension. Moreover, we must bear in mind that our laminate composite is composed of four balanced and symmetrical layers.

PREDICTION OF MATERIAL FAILURE

A failure criterion is characterized by the knowledge of a scalar function $\emptyset(\sigma)$. There is no rupture of the material if prevailing stresses do not exceed the ultimate stress value; that is to say, as long as the following inequality is satisfied:

$$\emptyset(\sigma) \leq 1 \tag{1}$$

When the equality is satisfied, we obtain the failure envelope or the limiting surface [25].

Tsai-Hill Criterion

The evaluation of the resistance of the composite material working in tension, is ensured by the rupture criterion of Tsai-Hill. It allows us to predict the ultimate resistance of the least resistant ply, in the case of the plane stress [4-6,25]:

$$\left(\frac{\sigma_{11}}{X}\right)^2 + \left(\frac{\sigma_{22}}{Y}\right)^2 - \frac{K}{XY}\sigma_{11}.\sigma_{22} + \left(\frac{\tau_{12}}{S}\right)^2 = 1 \tag{2}$$

With:

$$K = \frac{Y}{X}$$

X and Y are, respectively, the ultimate tensile strength stresses of the ply [0 °] and [90 °]. S is the ultimate shear stress in the plane (1,2) of the [0°] layer.
There is therefore no rupture of the material if the prevailing stresses do not exceed the ultimate constraints.

Norris Criterion

Norris [25 - 29] assumes that in the constraint field, the point:

$$\sigma_{11} = X \ , \sigma_{22} = Y \ , \tau_{12} = 0 \tag{3}$$

lies on the fracture surface.
So after substitution of (3) in relation (2) we find: $K = 1$.

Fisher's criterion

Fisher's criterion is applied to orthotropic materials and is based on Norris analysis.
Fisher assumes that the point [25,27,28,29,30]: $\sigma_{11} = P$, $\sigma_{22} = -P$ and $\tau_{12} = 0$ lies on the failure surface.
In this case we have:

$$K = A_1 + A_2 \tag{4}$$

with:

$$A_1 = E_1\left(1 + \nu_{21}\right)$$

$$A_2 = E_2\left(1 + \nu_{12}\right)$$

E_1, E_2 : Young's modulus in directions 1 and 2.
ν_{21} , ν_{21} : Poisson's ratio.

Ashkenazi Criterion

This criterion is used for unidirectional composite materials. Ashkenazi [25,29,31] assumes that the points:

$$\sigma_{11} = \frac{T}{2} \ , \sigma_{22} = \frac{T}{2} \ , \tau_{12} = \frac{T}{2} \tag{5}$$

lie on the failure surface.
T: Ultimate tensile strength at 45 ° from the direction of the fibers. It must satisfy the stability condition:

$$\left| \frac{4}{T^2} - \left(\frac{1}{X} + \frac{1}{Y} \right)^2 \right| < \frac{1}{S^2}$$

The coefficient K is then:

$$K = \frac{Y}{X} + \frac{X}{Y} + XY \left(\frac{1}{S^2} - \frac{1}{T^2} \right) \tag{6}$$

Tsai-Wu criterion
We will retain the Tsai-Wu criterion

$$F_{ij}\sigma_i\sigma_j + F_i\sigma_i = 1 \qquad \left(i,j = 1,\dots 6 \right) \tag{7}$$

where the constants F_i and F_{ij} are the components of two tensors, respectively of rank 2 and 4, with the parameters F_{ij} [32,33]:

$$F_1 = \frac{1}{X_t} - \frac{1}{X_c}$$

$$F_2 = \frac{1}{Y_t} - \frac{1}{Y_c}$$

$$F_{11} = \frac{1}{X_t X_c}$$

$$F_{22} = \frac{1}{Y_t Y_c} \tag{8}$$

$$F_6 = 0 \quad \text{and} \quad F_{66} = \frac{1}{S^2}$$

$$F_{12} = -\frac{1}{2}\sqrt{F_{11}F_{22}}$$

X_t, X_c: Ultimate tensile strengths along the longitudinal axis, in tension and compression respectively.
Y_t, Y_c: Ultimate tensile strengths along the transverse axis, respectively in tension and in compression.
S: Stress at failure in shear, in the plane of the layer.

Maximum stress criterion
This criterion states that the structure withstands the conditions of use if the calculated stresses meet the conditions below.

$$-X_c \leq \sigma_{11} \leq X_t$$

$$-Y_c \leq \sigma_{22} \leq Y_t \tag{9}$$
$$|\tau_{12}| \leq S$$

If one of the inequalities is no longer true, the limit state is reached; the failure is then attributed to the stress present into this inequality.

A failure envelope [34] is a three-dimensional plot of the combinations of the normal and shear stresses that can be applied to an angle laminate just before failure. One may develop failure envelopes for constant shear stress τ_{12} and then use the two normal stresses σ_{11} and σ_{22} as the two axes. Then, if the applied stress is within the failure envelope, the laminate is safe; otherwise, it has failed.

METHODS

In the case of stresses planes (thin plate), the tensor $\{\sigma\}$ as a function of that of the strains for a unidirectional composite layer, is given by the following relation:

$$
\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}
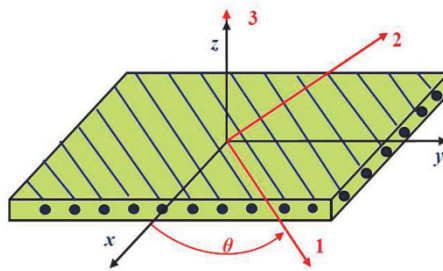$$

(10)

Figure 1: Definition of axis systems for a single stacking order layer.

Values of the reduced stiffness matrix in membrane [Q_{ij}] as a function of the elastic constants [35] are as follows:

$$
Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}
$$

$$
Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = Q_{21} = \nu_{21}Q_{22}
$$

(11)

$$
Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}
$$

$$
Q_{66} = G_{12}
$$

E_1 : Young's Modulus in the longitudinal to fiber direction.

E_2 : Young's Modulus in the transverse direction to the fiber.

ν_{12} , ν_{21} : Poisson's ratios of the composite material.

G_{12} : Shear modulus of the composite material.

The indices 1 and 2 refer to the direction longitudinal and perpendicular to the fibers.

This constitutive law for a layer of stacking order k in the laminate (Fig. 1) ,is not ,in general, that of the structure. When the orientation of the fibers changes, the matrices base change, makes it possible to express the tensor of the stresses in the reference mark of the plate (x, y) according to the transformed reduced stiffness matrix:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{11} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{12}$$

In the case of a symmetrical composite plate working only as a membrane, its forces will be expressed in the form [6]:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{yx}^0 \end{Bmatrix} \tag{13}$$

$\begin{bmatrix} A_{ij} \end{bmatrix}$: Membrane stiffness matrix.

with:

$$A_{ij} = \sum_{k=1}^{n} (h_k - h_{k-1}) \bar{Q}_{ij} \tag{14}$$

h_k , h_{k-1} : Coordinates of the layer k along the z axis.

n : Total number of layers.

According to Eqn. (13) the plane strain $\{\varepsilon^0\}$ is equal to:

$$\begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{yx}^0 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix}^{-1} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \tag{15}$$

In the case of the balanced symmetrical laminate, working in uni-axial tension we have:

$$\begin{aligned} \varepsilon_{xx}^0 &= A_p(1,1) N_x^t \\ \varepsilon_{yy}^0 &= A_p(2,1) N_x^t \\ \gamma_{xy}^0 &= A_p(3,1) N_x^t \end{aligned} \tag{16}$$

and:

$$\begin{bmatrix} A_p \end{bmatrix} = \begin{bmatrix} A_{ij} \end{bmatrix}^{-1} \tag{17}$$

By substituting Eqns. (16) in the matrix form (12), and for a layer of order *K*, we find that:

$$\begin{Bmatrix} \sigma^t_{xx} \\ \sigma^t_{yy} \\ \tau^t_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{60} \end{bmatrix} \begin{Bmatrix} A_p(1,1) \\ A_p(2,1) \\ A_p(3,1) \end{Bmatrix}_k (N^t_x)_k$$

In the (natural) orthotropic plane we have:

$$\begin{Bmatrix} \sigma^t_{11} \\ \sigma^t_{22} \\ \tau^t_{12} \end{Bmatrix}_k = [T][\overline{Q}_{ij}]_k \begin{Bmatrix} A_p(1,1) \\ A_p(2,1) \\ A_p(3,1) \end{Bmatrix}_k (N^t_x)_k \tag{18}$$

with:

$$[T] = \begin{bmatrix} C^2 & S^2 & 2SC \\ S^2 & C^2 & -2SC \\ -SC & SC & C^2 - S^2 \end{bmatrix} \tag{19}$$

where $C = \cos\theta$ and $S = \sin\theta$
We put:

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_{12} \end{Bmatrix}_k = [T][\overline{Q}]_k \begin{Bmatrix} A_p(1,1) \\ A_p(2,1) \\ A_p(3,1) \end{Bmatrix}_k \tag{20}$$

So, formula (18) can be rewritten in the following form:

$$\begin{Bmatrix} \sigma^t_{11} \\ \sigma^t_{22} \\ \tau^t_{12} \end{Bmatrix}_k = \begin{Bmatrix} R_1 \\ R_2 \\ R_{12} \end{Bmatrix}_k (N^t_x)_k \tag{21}$$

and by the computation of the tensor of stresses in the orthotropic coordinate system, one can use the energy criteria to predict the limits of membrane forces that the laminate can withstand.
These criteria must be applied successively to each ply constituting the laminate, for orientations from 0° to 90° with a step of 1.
The membrane force applied to each ply constituting the laminate will be obtained as follows:

$$(N^t_x)_k = \frac{1}{\sqrt{\dfrac{R_1^2}{X^2} + \dfrac{R_2^2}{Y^2} - \dfrac{K}{XY} R_1 R_2 + \dfrac{R_3^2}{S^2}}} \tag{22}$$

The use of the Ashkenazi criterion is possible if the breaking stress of the bend oriented at 45° is introduced.
To determine the ultimate membrane force either in tension or in compression, we can use the tensor criterion of Tsai-Wu and find the solution to the following equation:

$$(F_{11}R_1^2 + F_{22}R_2^2 + F_{12} R_1 R_2 + F_{66}R_3^2)^2 (N^t_x)_k^2 + (F_1 R_1^2 + F_2 R_2^2)(N^t_x)_k - 1 = 0 \tag{23}$$

Using the theory of the maximum stress, we determine the various modes of failure by the substitution of the components of the tensor of the stresses in the equation defining the criterion of the maximum stress theory:
We have the rupture forces as:

$$\left(N_x^t\right)_k = \frac{X}{R_1}$$

$$\left(N_x^t\right)_k = \frac{Y}{R_2}$$

$$\left(N_x^t\right)_k = \frac{S}{R_3}$$

(24)

Therefore, the membrane force applied to each layer, $\left(N_x^t\right)_k$, is the minimum of the forces obtained by the last three modes. Then, the limit force capable of avoiding the breaking of the least resistant layer, is determined for each failure criterion.

RESULTS AND DISCUSSION

Material used

The material examined is a unidirectional composite of Kevlar fiber and Epoxy resin. It is currently one of the most industrially developed composite families, especially in the production of high-performance parts. Its advantages are: low density, high tensile strength, low cost, and high impact resistance. Its drawbacks include low compressive properties and degradation in sunlight [34]

The elastic constants and the mechanical characteristics obtained experimentally for unidirectional Kevlar / Epoxy composites (h: The total thickness of the composite plate , $V_f = 0.6$ is the volume fraction of the fiber) are [33]:

ρ(g/m³)	E_1(GPa)	E_2(GPa)	G_{12}(GPa)	v_{21}	X_T(GPa)	Y_T(GPa)	X_C(GPa)	Y_C(GPa)	S(GPa)	T(GPa)	h(mm)
1380	80	5.5	2.2	0.34	1.4	0.335	0.03	0.1358	0.049	48.9	8

Table 1: Properties of Kevlar/Epoxy.

Ultimate tensile strengths of laminates

Figs. 2 and 3 represent the variation of the ultimate tensile force as a function of the orientation of the fibers of the layers composing the laminate [θ/-θ]$_S$ ranging from 0° to 90°.

It is noted that with the orientation of the layers θ = 0 °, the ultimate tensile forces obtained by all the criteria are similar and are maximum.

In the interval of 0 ° < θ <28 °, we notice that there are different spectra of curves which represent the variation of the ultimate tensile force for each criterion.

With this interval of angle of orientation, we notice on Fig. 3 a very fast reduction of the forces of membrane; and with breaks of the fibers and shears of the matrix, the criteria of rupture considered, do not produce the same ultimate values and become very different as the degree of anisotropy increases.

Once θ reaches the value 28 °, and with the exception of the theory of maximum stress, the curves tend to have the same values.

As one moves away from θ = 28 ° and approaches 90 °, the membrane forces become more and more equal, monotonous and take more or less the form of a straight line. They then converge parallel to the axis of the abscissa. At this stage, the tensile breaking stresses of the matrix, are responsible.

Moreover, according to Fig. 2, the energy failure criteria provide continuous curves, unlike the maximum stress theory (Fig. 3) based on three distinct failure equations, where we have a discontinuity. On the other hand, the tensor criterion and the energy criteria have only one equation, therefore only one aspect of the curves.

Nevertheless, it is the Tsai-Hill criterion which gives good results comparing to experimental results for unidirectional composites in tension [22, 23, 35, 36, 37].

These three equations (Formula 9) represent the three failure modes of the material. It is the outer layers oriented at $+\theta$ that rupture. The first mode is concerned with the orientation angles between 0 ° and 10 °, where we have the tensile breakage of the fibers. When the arrangement of the layer changes to an orientation equal to or less than 38 °. We have a shear failure of the matrix. As we approach the 90 ° angle, the low strength of the Epoxy resin, causes it to rupture by tension; and this is the third mode of rupture of the material [35].
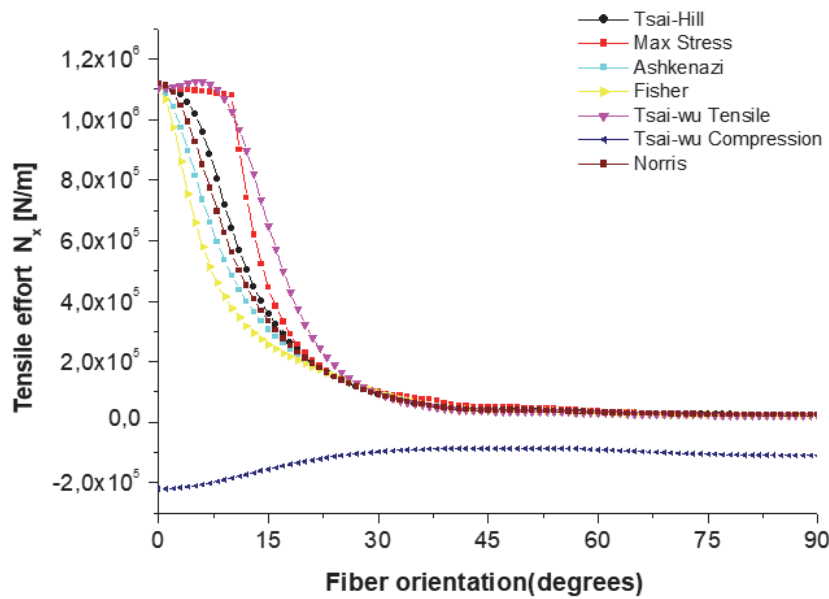

Figure 2: Evolution of the tensile membrane force N_x of the laminated composite as a function of the orientation of the fibers obtained by various criteria
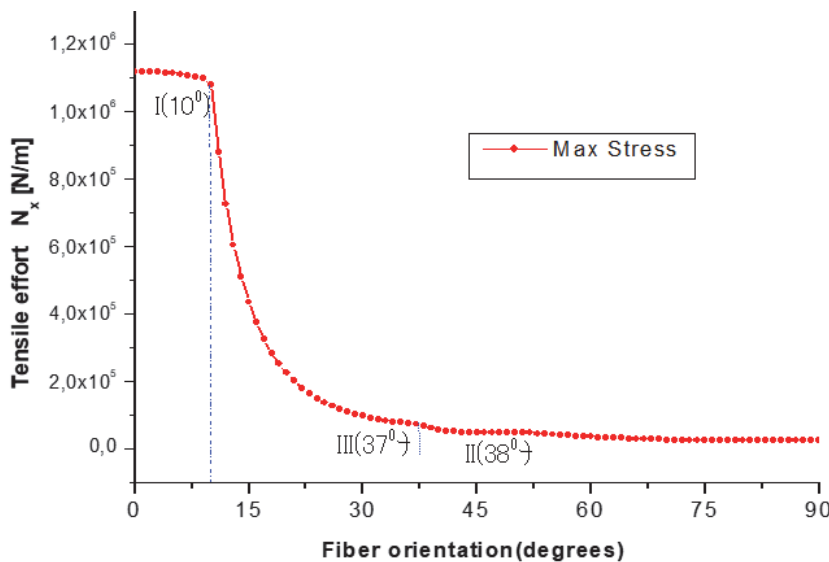

Figure 3: Evolution of the membrane force in traction N_x and the modes of damage of the laminated composite according to the orientation of the fibers obtained by the maximum stress theory.

Strain tensor

In Figs. 4, we have respectively the tensor of elastic deformations of composite plates [15/-15] s, [45/-45] s, [70/-70]s, [45/0]s and [90/0]s.

We see the absence of angular distortion for the balanced laminate [θ/-θ] s and [90/0] s. Their mechanical behavior is similar to that of isotropic material. On the other hand, the plate [45/0] s presents, in addition to the linear deformations, a significant angular deformation.

In addition, we have the absence of bending of the laminates due to the cancellation of the membrane-bending coupling matrix [B]. The shape of the curves of the components of the tensors of the deformations ε_x, ε_y and γ_{xy} is linear in form, up to the rupture as a function of the variation of the tensile force.

These curves show that the mechanical properties of composites are purely elastic (absence of plastic phase unlike metallic materials) and depend on the orientations of the fibers.

For each stacking sequence, we find more of the expansion strain in the (X) direction; contraction in the perpendicular (Y) direction more or less important.

It can be seen that the layered plate of stacking sequence [45/0]s exhibits an important angular distortion γ_{xy} with elastic linear deformations of expansion in the (x, y) plane. Besides, we have a deformation in the (X) direction, that is greater than in (Y) axis.

These different curves allow us to choose the cross laminate [90/0]s as the best stacking sequence, which must be taken into consideration.
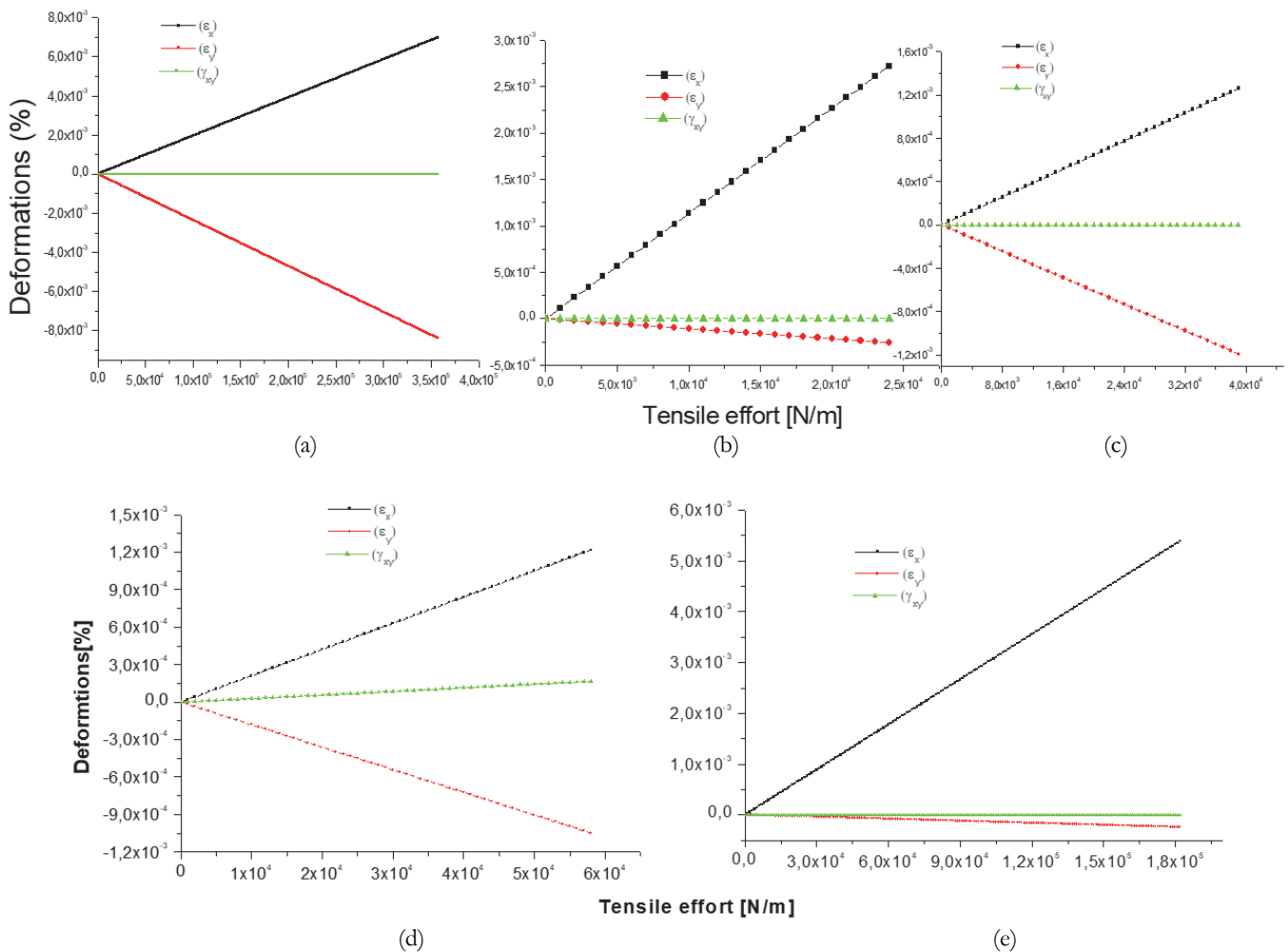

Figure 4: The components of the strain vector as a function of the tensile forces for the composite plate (a): [15/ -15]s,(b): [45/-45] s (c): [70/-70]s (d): [45/0]s and (e):[90/0]s

Curves of boundary surfaces

The boundary surface curve (failure envelope) of our composite, allows us to determine the surface where one of the stresses can be applied without breaking the material.

In Fig. 5, we have the failure envelopes obtained by the Tsai-Hill criterion in a plane form for shear stresses $\tau_{12} = 0$, 20 GPa and 48.50 GPa ; they have elliptical shapes as can be seen (Interaction between the normal stresses).

The transverse stress is obtained as a function of the longitudinal stress and the different values of τ_{12}. The boundary surface curve depends on the orientation of the fibers of the broken layer.

It can be noticed that the increase in the shear stress causes the reduction of the surface of the rupture envelope disappears and when it reaches the ultimate shear stress.

In addition, and unlike its configuration that obtained by the theory of maximum stress, indicates that the behavior of the material is not asymmetrical.

The failure envelope (Fig. 6) determined by the theory of maximum stress is characterized by the absence of the interaction between the two stresses σ_{11} and σ_{22}, which clearly means that it is not a function of the shear stress. The curve consists only of horizontal lines and verticals exhibiting the shape of a rectangle.
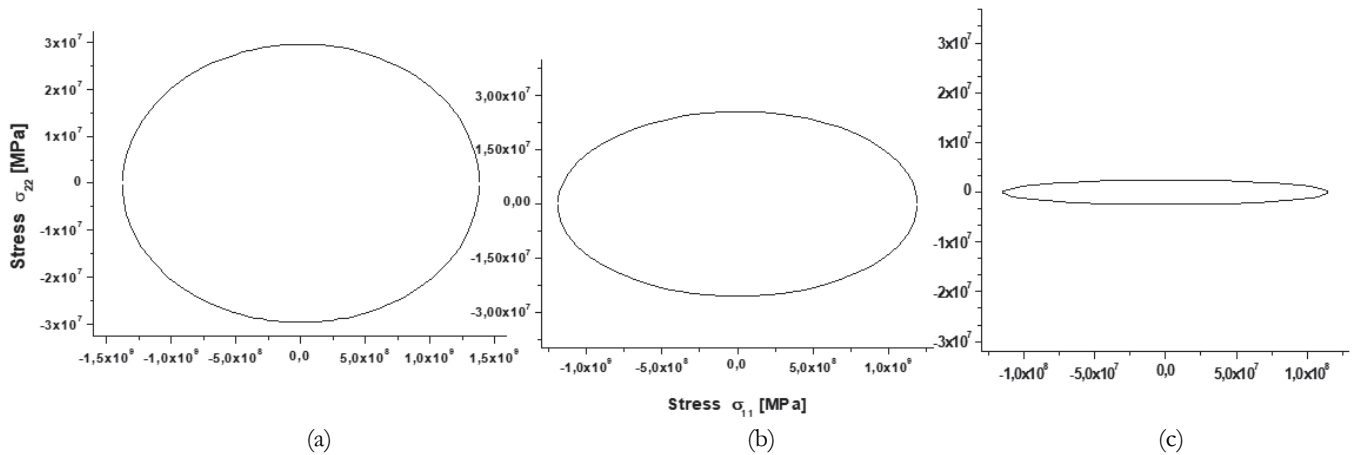

(a)   (b)   (c)

Figure 5: Failure envelope by a plane of shears (a): $\tau_{12} = 0$, (b): $\tau_{12} = 20\,\text{GPa}$, (c): $\tau_{12} = 48.7\,\text{GPa}$ obtained with the criterion of Tsai-Hill.
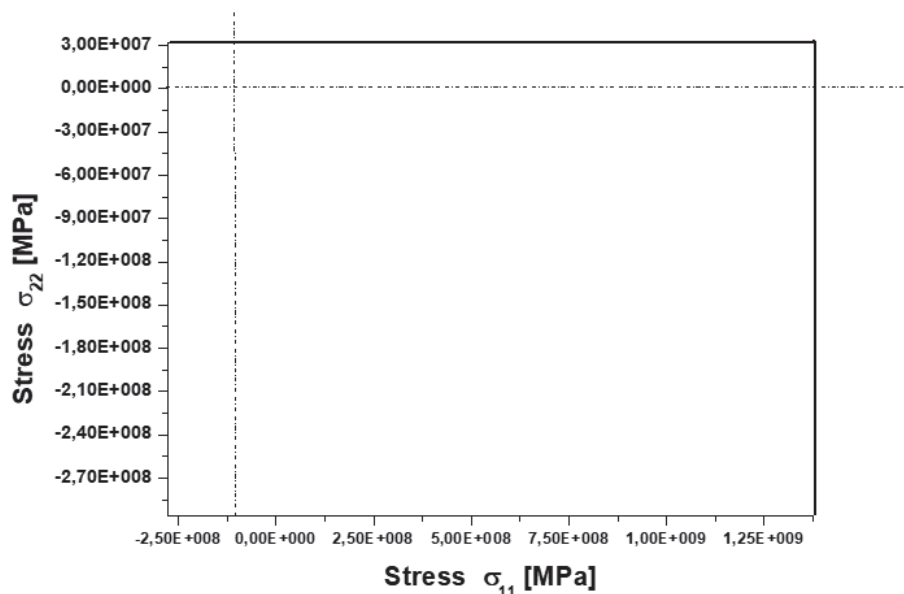

Figure 6: Failure envelope obtained by the maximum stress theory

CONCLUSION

T he examined composite material is unidirectional Kevlar / Epoxy reinforcement. It is composed, in our case, of four layers whose arrangement constitutes symmetrical laminates [θ/-θ]$_S$.

Our study allowed us to observe an elastic phase of the material until rupture, with the absence of the plastic phase. In addition, we noticed the absence of coupling between the tensile membrane force and the angular distortion of the material due to the balanced arrangement of the laminates.

The study of the deformations for different stacking sequences made us choose the cross laminate as the best configuration.

Among variety of failure criteria, we have chosen that of Tsai-Hill for unidirectional composites, especially when the orientation of the layers is in the vicinity of 0° and smaller than 28°. In addition, results obtained with then different criteria, are similar when moving forward 90°. The maximum stress theory makes it possible to determine three modes of rupture of the outer plies which are resistant.

The curves of the envelopes of failure obtained by the criterion of Tsai-Hill, present elliptical shapes; on the other hand the theory of the maximum stress theory allowed us to obtain rectangles which are independent of the influence of the tangential component of the tensor of the stresses applied to the external layer.

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