Visual Abstract

A simple and efficient eight node finite element for multilayer sandwich composite plates bending behavior analysis

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ABSTRACT. In this paper, a C^0 simple and efficient isoparametric eight-node element displacement-model based on higher order shear deformation theory is proposed for the bending behavior study of multilayer composites sandwich plates. Difficult C^1-continuity requirement is overcome efficiently by choosing seven degrees of freedom for each element node: two displacements for in-plane behavior and five bending unknowns namely: a transverse displacement, two rotations and two shear angles, which results in the approximation formulation having only first order derivative requirement. The governing equations of the element (constitutive, virtual work and equilibrium equations) are implemented for the prediction of approximate solutions of deflections and stresses of sandwich plates linear elastic problems. The formulation element is able to present a cubic in-plane displacement along both core and faces sandwich cross-sectional, as well as, the shear stresses are found to vary as quadratic field without requiring shear correction factors and independent from any transverse shear locking problems when the plate is thin. The accuracy and validity of the proposed formulation is verified through the numerical evaluation of displacements and stresses and their comparison with the available analytical 3D elasticity solutions and other published finite element results.

KEYWORDS. Third Order Shear Deformation Theory; Sandwich Composite Plates; Finite Element; Bending Behavior.

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Citation: Belkaid, K., Boutasseta, N., Aouaichia, H., Gaagaia, D. E., Boubir, B., Deliou, A., A simple and efficient eight node finite element for multilayer sandwich composite plates bending behavior analysis, Frattura ed Integrità Strutturale, 61 (2022) 372-393.

Received: 27.03.2022
Accepted: 01.06.2022
Online first: 10.06.2022
Published: 01.07.2022

INTRODUCTION

Sandwich composite plates are being increasingly used in many fields of modern technology due to their high strength, weight ratio and low maintenance cost. A good understanding of their behavior in terms of deformations and stresses distribution through the structures provide an effective vision for their applications.

Generally, sandwich composite structures are made-up of a very rigid isotropic or orthotropic face sheets and relatively soft thick core material. In mechanical analysis of the bending sandwich plate, displacement fields vary in a zigzag manner through the thickness, thus making the displacements very discontinuous at the layer interfaces due to the large variation in stress between layers. Hence, the development of a suitable computational theory is required for accurately predicting the responses of these laminated sandwich structures. In this context, a number of shear deformation theories have been developed in order to accurately model multilayer plate.

In the literature, the simplest equivalent single layer ESL laminate approach is the classical laminated plate theory (CLPT) Kirchhoff assumptions [1]. Their finite element model spatial approximations [2] contain C0-requirement for the in-plan displacements using Lagrange interpolation function, and transverse displacement C1-requirement using Hermite interpolation functions over the element. However, these elements models are characterized by a complex mathematical formulation due to the C1-requirement, and the theory is only appropriate for thin laminate plate analysis due to neglecting the effects of transverse shear deformation.

The simplest theory which takes into account the transverse shear deformation is the first order shear deformation theory [FSDT] [3, 4]. Their finite element models [5] are characterized by a simple formulation with C0-requirement for all degrees of freedom using Lagrange interpolation function. However, the theory requires shear correction factors and the transverse shear stresses show at least a quadratic distribution through the plate thickness according to Pagano three-dimensional elasticity theory [6].

Various analytical higher order theories (HSDT) have been proposed for the multilayer composite structures analysis, taking into account shear deformation effects without shear correction. Their kinematics assumption is expanded up to higher powers of the thickness coordinate and quadratic transverse shear [7-10]. Barut et al. [11] analyzed a thick sandwich plate by third and second order theories in which the in-plane and the transverse displacements show cubic and quadratic variations respectively through the thickness of the plate. Other analytical and experimental works can be found for the sandwich plate and shell analysis: Noor et al. [12], Kant et Swaminathan [13], Mantari et al. [14], Grover et al.[15], Kanematsu et al. [16], Torabizadeh and Fereidoon, [17], M. Michele et al. [18], Deliou Adel [19]. However, analytical methods are only suitable for specific simple boundary conditions and geometries. In this case, several (2D) finite element models based on higher order shear deformation theory (HSDT) have been developed [20] for the static analysis of multilayer composite sandwich plates.

B. Pandya , T. Kant [21] have presented a simple isoparametric finite element formulation based on a higher-order displacement model for flexure analysis of multilayer symmetric sandwich plates. B.S. Manjunatha, T. Kant [22] have evaluated the transverse stresses between layers of laminated composite and sandwich laminates using C0 nine and sixteen finite element formulation based on higher order theory. However, the models resort to use selective numerical integration scheme in order to overcome the shear locking problem. T. Kant and J. Kommineni, [23] presented a simple C0 quadrilateral Lagrange finite element formulation with nine-nodes and nine degrees of freedom per node based on refined higher-order shear deformation theory for the linear and geometrically non-linear analysis of fiber reinforced composite and sandwich plates. However, the selective integration scheme based on Gauss quadrature rules is introduced in order to overpass the shear locking problem. C.-P. Wu and C.-C. Lin [24] have presented the stress and displacement analysis of the thick sandwich plates using an interlaminar stress mixed nine-node finite element based on high order deformation theory. However, the formulation element possesses eleven nodal field variables in each node. R.P. Khandelwal et al. [25] have developed an efficient C0 continuous nine-node finite element model with eleven nodal field variables for each node based on combined theories refined higher order shear deformation theory (RHSDT) and least square error (LSE) method for the static analysis of soft core sandwich plates, the model satisfied the continuity of transverse shear stress condition between layer interfaces and zero transverse shear stress at the top and bottom of the sandwich plate. M.K. Pandit et al. [26] proposed a computationally efficient C0 nine-node finite element based on improved higher order zigzag theory for the static analysis of laminated sandwich plate with soft compressible core. However, the element has elven nodal field variables for each node adopting a reduced integration technique for the evaluation of stiffness matrix. T.M. Tu , T.H. Quoc [27] have developed a nine-nodded rectangular element with nine degrees of freedom at each node for the bending and vibration analysis of laminated and sandwich composite plates. The theory accounts for parabolic distribution of the transverse shear strains through the thickness of the plate and rotary inertia effects. A. Nayak et al. [28] analyzed the bending behavior of isotropic, laminated composite and sandwich plates using two C0 quadrilateral finite element formulations based on higher-

order theory where the element possess seven nodal field variables in each node. However, the formulations introduced assumed strain interpolations for the transverse shear strain in order to overcome the shear locking problem. Chalak et al. [29] presented an improved C0 2D nine-node finite element with eleven field variables per node. The model is based on higher order zigzag plate theory and has been applied to the analysis of laminated composites and sandwich plates. R. Sahoo and B. Singh [30] suggested an efficient C0 eight nodded isoparametric element with seven degrees of freedom per node based on a new inverse trigonometric zigzag theory for the static analysis of laminated and sandwich plates. However, the selective integration scheme is used in order to solve the locking shear problem. According to this literature survey, HSDT finite element models impose inconvenients such as: large number of nodal field variables, often encounter a locking problem when the plate is thin and resort to impose stiffness penalty in the formulation to remedy this problem.
On the other hand, single layer Reddy's theory is one of the higher-order theories used most often for analyzing multilayer plates, being able to evaluate stresses and transverse shear strains with a small variables number, not depending on the number of layers [9]. However, Reddy's theory encounter formulation complications when the finite element requires C1 second-order derivatives. The same problem also arises in the classical theory of thin plates [31]. Therefore, many finite-element models (2D) based on Reddy's third order theory have been proposed in the literature [20] for the bending behavior analysis of isotropic and multilayer composite plates. Furthermore, Reddy finite elements usually use conforming and non-conforming formulation where the C1 transverse displacement and its derivatives are interpolated by a modified bicubic Hermite functions, while the in-plane displacement and shear rotations are interpolated C0 Lagrange functions (JN Reddy [32], Phan and Reddy [33], Averill and Reddy [34], J. Ren, . Hinton [35], Ine-Wei Liu [36])
The objective of this work is to propose an efficient plate bending elements based on Reddy's shear deformation theory, which has a simple formulation that overcome the difficult C1 requirement with small nodal field variables and that does not need to impose any stiffness penalty in the formulation and is also able to predict accurately the response of multilayer plates. Based on the recently proposed displacement-model [37], a serendipity isoparametric eight nodes finite element is formulated for the study of multilayer sandwich plate bending behavior. In the formulation of the element, seven nodal field variables are chosen in an efficient manner so that there is no need to impose any stiffness penalty and present simple mathematical formulation. In this work, the present sandwich plate element is used to solve many multilayer sandwich plates problems for various parameters such as, different loadings, geometry, boundary conditions, and materials.

KINEMATICS

The displacement field of the plate according to Reddy's third order shear deformation theory (TSDT) [9] can be expressed as follows:

$$u_1 = u + z\psi_x - \frac{4z^3}{3h}\left(\psi_x + \frac{\partial w}{\partial x}\right)$$

$$u_2 = v + z\psi_y - \frac{4z^3}{3h}\left(\psi_y + \frac{\partial w}{\partial y}\right) \tag{1}$$

$$u_3 = w$$

where: u_1, u_2, u_3 are the displacements field in the x, y and z directions respectively.
u, v displacement of a point (x, y) on the mid-plane of the plate. ψ_x , ψ_y are rotations about the axes y and x respectively, and h is the thickness of the plate.
The strain associated with displacement field (1) are given as follows:

$$\varepsilon_1 = \frac{\partial u_1}{\partial x} = \varepsilon_1^0 + z\left(\kappa_1^0 + z^2\kappa_1^2\right)$$

$$\varepsilon_2 = \frac{\partial u_2}{\partial y} = \varepsilon_2^0 + z\left(\kappa_2^0 + z^2\kappa_2^2\right)$$

$$\varepsilon_3 = \frac{\partial u_3}{\partial z} = 0 \tag{2}$$

$$\varepsilon_4 = \left(\frac{\partial u_2}{\partial z} + \frac{\partial u_3}{\partial y} + \frac{\partial u_3}{\partial y}\frac{\partial u_3}{\partial z} \right) = \varepsilon_4^0 + z^2 \kappa_4^2$$

$$\varepsilon_5 = \left(\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x} + \frac{\partial u_3}{\partial x}\frac{\partial u_3}{\partial z} \right) = \varepsilon_5^0 + z^2 \kappa_5^2$$

$$\varepsilon_6 = \left(\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x} + \frac{\partial u_3}{\partial x}\frac{\partial u_3}{\partial y} \right) = \varepsilon_6^0 + z\left(\kappa_6^0 + z^2 \kappa_6^2 \right)$$

where:

$$\varepsilon_1^0 = \frac{\partial u}{\partial x};\ \kappa_1^0 = \frac{\partial \psi_x}{\partial x};\ \kappa_1^2 = -\frac{4}{3h^2}\left(\frac{\partial \psi_x}{\partial x} + \frac{\partial^2 w}{\partial x^2} \right)$$

$$\varepsilon_2^0 = \frac{\partial u}{\partial y};\ \kappa_2^0 = \frac{\partial \psi_y}{\partial y};\ \kappa_2^2 = -\frac{4}{3h^2}\left(\frac{\partial \psi_y}{\partial y} + \frac{\partial^2 w}{\partial y^2} \right)$$

$$\varepsilon_4^0 = \psi_y + \frac{\partial w}{\partial y};\ \kappa_4^2 = -\frac{4}{h^2}\left(\psi_y + \frac{\partial w}{\partial y} \right)$$

$$\varepsilon_5^0 = \psi_x + \frac{\partial w}{\partial x};\ \kappa_5^2 = -\frac{4}{h^2}\left(\psi_x + \frac{\partial w}{\partial x} \right)$$

$$\varepsilon_6^0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y};\ \kappa_6^0 = \frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x};$$

$$\kappa_6^2 = -\frac{4}{3h^2}\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} + 2\frac{\partial^2 w}{\partial x \partial y} \right)$$

CONSTITUTIVE EQUATIONS

The laminate is usually made of several orthotropic layers (Fig 1). Each layer must be transformed into the laminate coordinate system (x, y, z) [2]. The stress–strain relationship is given as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & \bar{C}_{16} \\ \bar{C}_{12} & \bar{C}_{22} & \bar{C}_{26} \\ \bar{C}_{16} & \bar{C}_{26} & \bar{C}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix};\ \begin{Bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix}_k = \begin{bmatrix} \bar{C}_{44} & \bar{C}_{45} \\ \bar{C}_{45} & \bar{C}_{55} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_{xz} \\ \varepsilon_{yz} \end{Bmatrix} \tag{3}$$

where $\bar{C}_{ij}$ are the transformed material constants:

$$\bar{C}_{11} = C_{11}c^4 + 2\left(C_{12} + 2C_{66}\right)c^2s^2 + C_{22}s^4$$

$$\bar{C}_{12} = \left(C_{11} + C_{22} - 4C_{66}\right)c^2s^2 + C_{12}(c^4 + s^4)$$

$$\bar{C}_{16} = \left(C_{11}c^2 + \left(C_{12} + 2C_{66}\right)\left(s^2 - c^2\right) - C_{22}s^2\right)cs$$

$$\bar{C}_{22} = C_{11}s^4 + 2\left(C_{12} + 2C_{66}\right)c^2s^2 + C_{22}c^4$$

$$\bar{C}_{26} = \left(C_{11}s^4 + \left(C_{12} + 2C_{66}\right)(c^2 - s^2\right) - C_{22}c^4)cs$$

$$\bar{C}_{66} = \left(C_{11} + C_{22} - 4C_{12}\right)c^2s^2 + C_{66}(c^2 - s^2)$$

$$\bar{C}_{44} = C_{44}c^2 + C_{55}s^2$$

$$\bar{C}_{45} = \left(C_{55} - C_{44}\right)cs$$

$$\bar{C}_{55} = C_{55}c^2 + C_{44}s^2$$

where $c = \cos\theta$, $s = \sin\theta$ and θ is angle between global axis and local axis for each laminate layer.

$$C_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}};\ C_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}};\ C_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}};\quad C_{66} = G_{12};\ C_{55} = G_{13};\ C_{44} = G_{23}$$

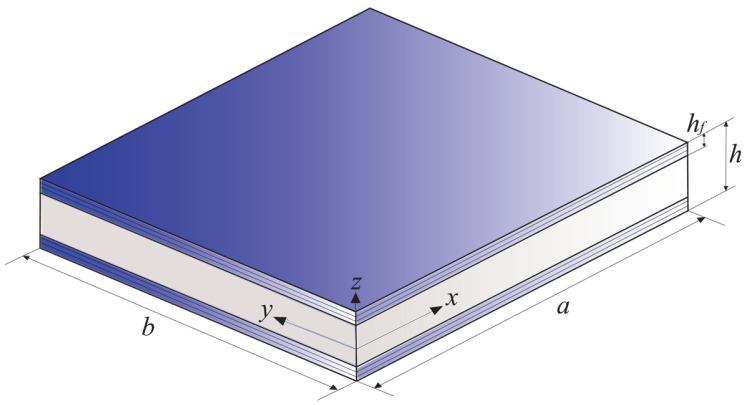

Figure 1: Geometry and coordinate system of the sandwich plate

The differential equilibrium equations for transverse stress analysis are given as follows [2]:

$$\sigma_{xz} = -\sum_{k=1}^{n}\int_{h_{k-1}}^{h_k}\left(\frac{\partial\sigma_{xx}^k}{\partial x} + \frac{\partial\sigma_{xy}^k}{\partial y}\right)dz$$

$$\sigma_{yz} = -\sum_{k=1}^{n}\int_{h_{k-1}}^{h_k}\left(\frac{\partial\sigma_{xy}^k}{\partial x} + \frac{\partial\sigma_{yy}^k}{\partial y}\right)dz \tag{4}$$

VIRTUAL WORK PRINCIPLE

The static equations of the theory can be derived from the virtual work principle [9] by expressing the strain energy variation as follows:

$$\int_V \left(\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} + \sigma_{xy}\delta\varepsilon_{xy} + \sigma_{xz}\delta\varepsilon_{xz} + \sigma_{yz}\delta\varepsilon_{yz}\right)dV + \int_V q\delta W dV = 0 \tag{5}$$

According to the substitution of equations (2) in the static equation (5) we obtain:

$$\begin{aligned}&\int_A \Bigg[N_{xx}\frac{\partial\delta u}{\partial x} + M_{xx}\frac{\partial\delta\psi_x}{\partial x} + P_{xx}\left(-\frac{4}{3h^2}\left(\frac{\partial\delta\psi_x}{\partial x} + \frac{\partial^2\delta w}{\partial x^2}\right)\right) + N_{yy}\frac{\partial\delta v}{\partial y} + M_{yy}\frac{\partial\delta\psi_y}{\partial y} + P_{yy}\left(-\frac{4}{3h^2}\left(\frac{\partial\delta\psi_y}{\partial y} + \frac{\partial^2\delta w}{\partial y^2}\right)\right) + \\ &N_{xy}\left(\frac{\partial\delta u}{\partial y} + \frac{\partial\delta v}{\partial x}\right) + M_{xy}\left(\frac{\partial\delta\psi_x}{\partial y} + \frac{\partial\delta\psi_y}{\partial x}\right) + P_{xy}\left(-\frac{4}{3h^2}\left(\frac{\partial\delta\psi_x}{\partial y} + \frac{\partial\delta\psi_y}{\partial x} + 2\frac{\partial^2\delta w}{\partial x\partial y}\right)\right) + Q_{xx}\left(\delta\psi_x + \frac{\partial\delta w}{\partial x}\right) + \\ &R_{xx}\left(-\frac{4}{h^2}\left(\delta\psi_x + \frac{\partial\delta w}{\partial x}\right)\right) + Q_{yy}\left(\delta\psi_y + \frac{\partial\delta w}{\partial y}\right) + R_{yy}\left(-\frac{4}{h^2}\left(\delta\psi_y + \frac{\partial\delta w}{\partial y}\right)\right) + q\delta W\Bigg] dA = 0\end{aligned} \tag{6}$$

where the resultants forces are defined as follows:

$$\begin{bmatrix} N_{xx} & M_{xx} & P_{xx} \\ N_{yy} & M_{yy} & P_{yy} \\ N_{xy} & M_{xy} & P_{xy} \end{bmatrix} = \sum_{k=1}^{n}\int_{h_{k-1}}^{h_k} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} \left(1, z, z^3\right) dz \; ; \; \begin{bmatrix} Q_{xx} & R_{xx} \\ Q_{yy} & R_{yy} \end{bmatrix} = \sum_{k=1}^{n}\int_{h_{k-1}}^{h_k} \begin{Bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} \left(1, z^2\right) dz \tag{7}$$

Therefore, from the eq (7) and eq (3), we obtain the generalized relations resultants forces as follows [9]:

$$\begin{Bmatrix} \{N\} \\ \{M\} \\ \{P\} \end{Bmatrix} = \begin{bmatrix} \left[A_{ij}\right] & \left[B_{ij}\right] & \left[E_{ij}\right] \\ sym & \left[D_{ij}\right] & \left[F_{ij}\right] \\ sym & sym & \left[H_{ij}\right] \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa^0 \\ \kappa^2 \end{Bmatrix}$$

$$\begin{Bmatrix} \{Q\} \\ \{R\} \end{Bmatrix} = \begin{bmatrix} \left[A_{ij}^S\right] & \left[D_{ij}^S\right] \\ sym & \left[F_{ij}^S\right] \end{bmatrix} \begin{Bmatrix} \{\gamma^s\} \\ \{\kappa^s\} \end{Bmatrix} \tag{8}$$

where:

$$\left[\varepsilon_1^0 \quad \varepsilon_2^0 \quad \varepsilon_6^0 \quad \kappa_1^0 \quad \kappa_2^0 \quad \kappa_6^0 \quad \kappa_1^2 \quad \kappa_2^2 \quad \kappa_6^2\right]^T = \left[\varepsilon^0 \quad \kappa^0 \quad \kappa^2\right]^T$$

$$\left[\varepsilon_4^0 \quad \varepsilon_5^0 \quad \kappa_4^2 \quad \kappa_5^2\right]^T = \left[\gamma^s \quad \kappa^s\right]^T$$

$$\left(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}\right) = \sum_{k=1}^{n}\int_{h_{k-1}}^{h_k} \bar{C}_{ij}\left(1, z, z^2, z^3, z^4, z^6\right) dz, \; i, j = 1, 2, 6$$

$$\left(A_{ij}^S, D_{ij}^S, F_{ij}^S\right) = \sum_{k=1}^{n}\int_{h_{k-1}}^{h_k} \bar{C}_{ij}\left(1, z^2, z^4\right) dz, \; i, j = 4, 5$$

Substituting the resultant forces (8) in equation (6), the static equation becomes

$$\int_A (\delta\varepsilon^{0T}[A]\varepsilon^0 + \delta\varepsilon^{0T}[B]\kappa^0 + \delta\varepsilon^{0T}[E]\kappa^2 + \delta\kappa^{0T}[B]\varepsilon^0 + \delta\kappa^{0T}[D]\kappa^0 + \delta\kappa^{0T}[F]\kappa^2 + \delta\kappa^{2T}[E]\varepsilon^0 + \delta\kappa^{2T}[F]\kappa^0 + \delta\kappa^{2T}[H]\kappa^2 + \delta\gamma^{sT}[A^s]\gamma^s + \delta\gamma^{sT}[D^s]\kappa^s + \delta\kappa^{sT}[D^s]\gamma^s + \delta\kappa^{sT}[F^s]\kappa^s + q\delta w)dA = 0 \quad (9)$$

FINITE ELEMENT FORMULATION

In this work, a C0 isoparametric serendipity eight-node element (Fig.2) is employed for the bending analysis of sandwich plates based on Reddy's third order shear deformation. In the present formulation element the complexities associated with C1 continuous plate are overcome with efficient manner by choosing seven nodal degrees of freedom (DOF) [37, 38] as follows: two displacements (u, v) for the membrane behavior and five displacements $(w, \psi_x, \psi_y, \theta_x, \theta_y)$ for describing the bending behavior, where $\theta_x = \left(\psi_x + \frac{\partial w}{\partial x}\right), \theta_y = \left(\psi_y + \frac{\partial w}{\partial y}\right)$ are shear angles.

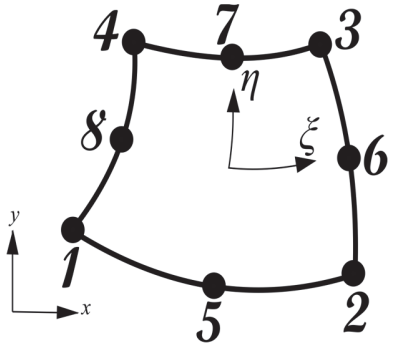

Figure 2: Eight-node isoparametric finite element

The generalized field variable and element geometry of the model at any point may be expressed in terms of nodal approximation as follows:

$$x(\xi,\eta) = \sum_{i=1}^{8} \bar{N}_i(\xi,\eta) x_i;\ y(\xi,\eta) = \sum_{i=1}^{8} \bar{N}_i(\xi,\eta) y_i$$

$$u(\xi,\eta) = \sum_{i=1}^{8} \bar{N}_i(\xi,\eta) u_i;\ v(\xi,\eta) = \sum_{i=1}^{8} \bar{N}_i(\xi,\eta) v_i;\ \psi_x(\xi,\eta) = \sum_{i=1}^{8} \bar{N}_i(\xi,\eta) \psi_{xi};\ \psi_y(\xi,\eta) = \sum_{i=1}^{8} \bar{N}_i(\xi,\eta) \psi_{yi};$$

$$\theta_x(\xi,\eta) = \sum_{i=1}^{8} \bar{N}_i(\xi,\eta) \theta_{xi};\ \theta_y(\xi,\eta) = \sum_{i=1}^{8} \bar{N}_i(\xi,\eta) \theta_{yi}$$

where corner nodes are defined as follows:

$$\bar{N}_i(\xi,\eta) = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i + \eta\eta_i - 1), i = 1,2,3,4$$

and mid side nodes as:

$$\overline{N}_i(\xi,\eta)=\frac{1}{2}\left(1-\xi^2\right)(1+\eta\eta_i),\ i=5,7$$

$$\overline{N}_i(\xi,\eta)=\frac{1}{2}\left(1-\xi\xi_i\right)(1+\eta^2),\ i=6,8$$

$\overline{N}_i(\xi,\eta)$ are the bi-nonlinear interpolation functions of Lagrange type corresponding to node i=1-8 [39].
The strain displacements vectors can be formed as derivative elementary nodal matrix *[B]* multiplied by the proposed nodal field variable $\{\delta\}$ as follows:

$$\varepsilon^0=\left[B_\varepsilon^0\right]\{\delta\};\ \kappa^0=\left[B_\kappa^0\right]\{\delta\},\ \kappa^2=\left[B_\kappa^2\right]\{\delta\},\ \gamma^s=\left[B_\varepsilon^s\right]\{\delta\},\ \kappa^s=\left[B_\kappa^s\right]\{\delta\} \tag{10}$$

$$\left[B_\varepsilon^0\right]=\begin{bmatrix}\overline{N}_{i,x} & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \overline{N}_{i,y} & 0 & 0 & 0 & 0 & 0\\ \overline{N}_{i,y} & \overline{N}_{i,x} & 0 & 0 & 0 & 0 & 0\end{bmatrix};\quad \left[B_k^0\right]=\begin{bmatrix}0 & 0 & \overline{N}_{i,x} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \overline{N}_{i,y} & 0 & 0 & 0\\ 0 & 0 & \overline{N}_{i,y} & \overline{N}_{i,x} & 0 & 0 & 0\end{bmatrix}$$

$$\left[B_\kappa^2\right]=c_1\begin{bmatrix}0 & 0 & 0 & 0 & 0 & \overline{N}_{i,x} & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \overline{N}_{i,y}\\ 0 & 0 & 0 & 0 & 0 & \overline{N}_{i,y} & \overline{N}_{i,x}\end{bmatrix};\ \left[B_\varepsilon^s\right]=\begin{bmatrix}0 & 0 & \overline{N}_{i,y} & 0 & \overline{N}_i & 0 & 0\\ 0 & 0 & \overline{N}_{i,x} & \overline{N}_i & 0 & 0 & 0\end{bmatrix}$$

$$\left[B_\kappa^s\right]=c_2\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & \overline{N}_i\\ 0 & 0 & 0 & 0 & 0 & \overline{N}_i & 0\end{bmatrix}$$

$$\{\delta\}^T=\left\{u_i,v_i,w_i,\psi_{xi},\psi_{yi},\theta_{xi},\theta_{yi}\right\},i=1-8$$

with:

$c_1=-4/3h^2$, $c_2=-4/h^2$

According to the substitution of strain matrix (10) in the static equation (9), the elementary stiffness matrix $[K]_e$ is deduced according the static system $[K]_e\{\delta\}=\{F\}$ [37] as follows:

$$[K]_e=\int_{-1}^{1}\int_{-1}^{1}([B_\varepsilon^0]^T[A][B_\varepsilon^0]+[B_\varepsilon^0]^T[B][B_\kappa^0]+[B_\varepsilon^0]^T[E][B_\kappa^2]+[B_\kappa^0]^T[B][B_\varepsilon^0]+[B_\kappa^0]^T[D][B_\kappa^0]+[B_\kappa^0]^T[F][B_\kappa^2]+$$
$$[B_\kappa^2]^T[E][B_\kappa^0]+[B_\kappa^2]^T[F][B_\kappa^0]+[B_\kappa^2]^T[H][B_\kappa^2]+[B_\varepsilon^s]^T[A^s][B_\varepsilon^s]+[B_\varepsilon^s]^T[D^s][B_\kappa^s]+[B_\kappa^s]^T[D^s][B_\varepsilon^s]+ \tag{11}$$
$$[B_\kappa^s]^T[F^s][B_\kappa^s])\det[J]d\xi d\eta$$

where $\{F\},\{\delta\}$ are elementary nodal vectors of forces and degrees of freedom, respectively.
The analytical integration can be converted to Gauss's numerical integration [40]. Full integration scheme quadrature rules, namely (3×3) is employed in the energy expression for the evaluation of the element stiffness property.

NUMERICAL EXAMPLES AND COMPARISON STUDIES

In this section, several sandwich plates examples are solved to verify the effectiveness of the proposed formulation in the prediction of displacements and stresses results including different parameters such as, thickness ratios, materials, loading distribution, laminated face sheet, boundary conditions. The results of proposed model are compared with the three-dimensional elasticity solutions and other element models available in the literature.

Bending of a simply supported sandwich square plate subject to a doubly sinusoidal transverse load

For this example, the proposed element convergence and ability of the bending behavior is studied by considering a simply supported (f/c/f) symmetrical square sandwich plate subject to a doubly sinusoidal transverse load $q(x,y)=q_0\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$ for different thickness ratios a/h = 4,10,50,100. The properties of the core and those of the sheets are shown in (Table 1). It is noted that normalized central deflection displacement results converge for different uniform mesh sizes toward Pagano 3D-elasticity solution [6] (Table 2). The increasing meshes indicate the accuracy and convergence rate of central transverse displacements results with a fast decrease of relative errors for both thin and moderately thick sandwich plates with no shear locking in the thin plates (a/h=50, 100).

The simply supported boundary conditions used for the bending example are as follows:

$x=\pm a/2\rightarrow v=w=\psi_y=\theta_y=0\ ;\ y=\pm b/2\ \rightarrow u=w=\psi_x=\theta_x=0$

The normalized deflection, stresses and in-plan are defined by:

$$\bar{w}=w\left(\frac{a}{2},\frac{b}{2},0\right)\left(\frac{h^3E_2}{q_0a^4}\right)10^2,\ \bar{\sigma}_{xx}=\sigma_{xx}\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)\left(\frac{h^2}{q_0a^2}\right),\bar{\sigma}_{yy}=\sigma_{yy}\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)\left(\frac{h^2}{q_0a^2}\right),$$

$$\bar{\sigma}_{xy}=\sigma_{xy}\left(0,0,\frac{h}{2}\right)\left(\frac{h^2}{q_0a^2}\right),\ \bar{\sigma}_{xz}=\sigma_{xz}\left(0,\frac{b}{2},0\right)\left(\frac{h}{q_0a}\right),\ \bar{\sigma}_{yz}=\sigma_{yz}\left(\frac{a}{2},0,0\right)\left(\frac{h}{q_0a}\right)$$

$$\bar{u}=u\left(\frac{100h^3E_2}{q_0a^4}\right),\ \bar{v}=v\left(\frac{100h^3E_2}{q_0a^4}\right)$$

Proprieties		E_1	E_2	G_{12}	G_{13}	G_{23}	ν_{12}	Thickness
Sandwich plate	Core	275.8 MPa	275.8 MPa	110.32 MPa	413.68 MPa	413.68 MPa	0.25	0.8h
	Sheet	172369.9 MPa	6894.76 MPa	3447.38 MPa	1378.95 MPa	3447.38 MPa	0.25	0.1h

Table 1: Mechanical proprieties of a sandwich plate.

Reference	Theory	$\bar{w}\left(\frac{a}{2},\frac{b}{2},0\right)$			
		a/h=4(err%)	a/h=10(err %)	a/h=50(err %)	a/h=100(err %)
Present (2×2)	HSDT(Q8)	6.5668 (13.55)	1.8739 (14.83)	0.60962 (34.78)	0.3439 (61.43)
Present (4×4)		7.1009 (6.52)	2.07035 (5.91)	0.91441 (2.181)	0.8604 (3.5)
Present (6×6)		7.1377 (6.03)	2.081172 (5.41)	0.92644 (0.89)	0.8845 (0.796)
Present (8×8)		7.1461 (5.927)	2.08326 (5.32)	0.92844 (0.68)	0.8888 (0.323)
Present (10×10)		7.1492 (5.88)	2.08396 (5.29)	0.92901 (0.61)	0.89005 (0.185)
Present (12×12)		7.1508 (5.86)	2.08426 (5.27)	0.92923 (0.59)	0.89053 (0.131)
Pagano[6]	Elasticity solution	7.5962	2.2004	0.9348	0.8917

Table 2: Normalized center deflection convergence of a simply supported square sandwich plate subject to a doubly sinusoidal transverse load.

In Fig 3 the central normalized $\overline{w}$ deflection is plotted for different a/h ratios. In Fig 4, the in-plane displacements u1(a/2,0,0) and u2(0,b/2,0) are described through the plate thickness as a cubic variation. It can be seen that the obtained bending responses by the present formulation are in excellent agreement with the elasticity solution of Pagano [6]and with those obtained by T. Kant and K. Swaminathan analytical solution [13].

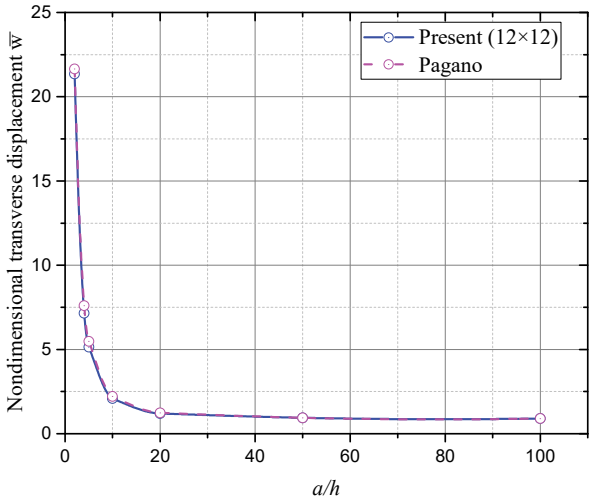

Figure 3: Evolution of normalized central transverse deflection $\overline{w}$ with aspect ratio a/h of a simply supported sandwich plate $\left(0^{\circ}/C/0^{\circ}\right)$ subject to a doubly sinusoidal transverse load.

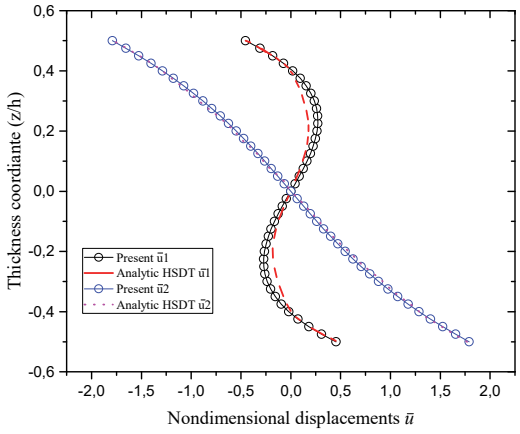

Figure 4: Evolution of normalized in-plane displacements $\overline{u}_1, \overline{u}_2$ as function of the thickness of a simply supported $\left(0^{\circ}/C/0^{\circ}\right)$ sandwich plate subject to a doubly sinusoidal transverse load a/h=4.

In Table 3, the bending sandwich plate $\left(0^{\circ}/C/0^{\circ}\right)$ is studied for different aspect ratios a/h in order to present the normalized transverse deflection and maximum stresses using the constitutive equation. Furthermore, the transverse shear stresses are evaluated using the equilibrium equations. A close agreement has been found between the obtained results by the proposed element and those obtained using Pagano elasticity-3D solution [6], as well as with those obtained by finite element models based on different theories.

Furthermore, it's noting that in thin sandwich plate cases (a/h=50,100), the results of the proposed model do not have locking shear problems with additional better accuracy in comparison with other model elements that have used stiffness penalty with a high number of nodal variables than the proposed model (e.g. B. Pandya , T. Kant [21], T. Kant and J. Kommineni, [23], M.K. Pandit et al. [26], Chalak et al. [29], R. Sahoo and B. Singh [30]).

a/h	Reference	Theory	$\bar{w}$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$\bar{\sigma}_{xz}$	$\bar{\sigma}_{yz}$	$\bar{\sigma}_{xy}$
2	Present (12×12)	HSDT(Q8)	21.356	2.770	0.386	0.2174* 0.1880+	0.1535* 0.1317+	0.2288
	Pagano [6]	Elasticity solution	21.653	2.6530	0.3920	0.1850	0.1400	0.2340
	Ramtekkar et al. [41]	MFEM-3D-LW	-	2.6840	0.3960	0.1860	0.1420	0.2360
	Kant et Kommineni [23]	FEM-Q9-TSDT	21.3707	2.7985	-	-	-	0.2371
4	Present (12×12)	HSDT(Q8)	7.1508	1.4929	0.2375	0.279* 0. 2418+	0.1154* 0.0997+	0.1377
	Pagano [6]	Elasticity solution	7.5962	1.5160	0.2595	0.2390	0.1072	0.1440
	Ramtekkar et al. [41]	MFEM-3D-LW	--	1.5700	0.2600	0.2400	0.108	0.149
	Wu et Lin [24]	MFEM-3D-LW	--	1.5480	0.2413	0.2497	- -	0.1339
	Pandya et Kant [21]	FEM-Q9-HSDT	0.6947	1.247	0.2338	0.2382	0.1132	0.1343
	Manjunatha et Kant [22]	FEM-Q9-HSDT	7.1596	-	- -	0.2750	0.1137	-
	Kant et Kommineni [23]	FEM-Q9-TSDT	7.1502	1.4989	--	- -	--	0.1428
	Kant et Swaminathan [13]	HSDT	7.0551	1.5137	0.2648	--	--	0.1379
	IGA [42]	TSDT	7.0872	1.4244	0.2361	0.2708	0.1169	0.1383
	IGA [42]	HSDT	7.0686	1.4791	0.2391	0.3074	0.1274	0.1406
5	Present (12×12)	HSDT(Q8)	5.1389	1.334	0.1955	0.299* 0.2592^{+}	0.0986* 0.0851^{+}	0.1149
	Pagano [6]	Elasticity solution	5.4746	1.3704	0.2094	0.2569	0.0918	--
	Khandelwal et al. [25]	FEM-Q9-HZZT	5.4464	1.3617	0.2216	0.2530	0.1025	--
10	Present (12×12)	HSDT(Q8)	2.08426	1.1419	0.1035	0.3454* 0.2986+	0.05797* 0.0493+	0.0672
	Pagano [6]	Elasticity solution	2.2004	1.1531	0.1104	0.3000	0.0530	0.0707
	Pandit et al.[26]	FEM-Q9-HZZT	2.2002	1.1483	0.1086	0.3158	0.0570	0.0709
	Tu et al.[27]	FEM-Q9-TSDT	2.2027	1.1466	0.1105	0.3181	0.0532	0.0715
	Khandelwal et al. [25]	FEM-Q9-HZZT	2.1786	1.1539	0.1184	0.3185	0.0598	-
	Chalak et al.[26]	FEM-Q9-HZZT	2.1775	1.1528	0.1143	0.3058	0.0575	0.0705
	Ramtekkar et al. [41]	MFEM-3D-LW	- -	1.1590	0.1110	0.3030	0.0550	0.0720
	Wu et Lin [24]	MFEM-3D-LW	- -	1.2100	0.1115	0.3177	- - - -	0.0713
	Pandya et Kant [21]	FEM-Q9-HSDT	2.023	1.110	0.1017	0.2841	0.05593	0.0666
	Kant et Kommineni[23]	FEM-Q9-TSDT	2.0864	1.1657	-	- -	--	0.0692

	Kant et Swaminathan[13]	HSDT-Anal	2.0798	1.1523	0.1100	0.3465	0.0538	0.0685
	Nayak et al. [28]	FEM-Q4-HSDT	- -	1.1410	0.1034	0.3506	0.0534	0.0685
	Nayak et al. [28]	FEM-Q9-HSDT	- -	1.1510	0.1043	0.2815	0.0532	0.0689
	IGA [42]	TSDT	2.0629	1.1299	0.1028	0.3302	0.0578	0.0679
	IGA [42]	HSDT	2.0515	1.1424	0.1031	0.3781	0.0628	0.0682
20	Present (12×12)	HSDT(Q8)	1.1937	1.1019	0.0678	0.3648* 0.31374+	0.04227* 0.03401+	0.0493
	Pagano [6]	Elasticity solution	1.2264	1.1100	0.0700	0.3174	0.0361	0.0511
	Pandit et al. [26]	FEM-Q9-HZZT	1.2254	1.1055	0.0694	0.3342	0.0392	0.0509
	Khandelwal et al. [25]	FEM-Q9-HZZT	1.2128	1.1113	0.0769	0.3374	0.0415	-
	Chalak et al.[29]	FEM-Q9-HZZT	1.2121	1.1103	0.0742	0.3272	0.0399	0.0508
	Ramtekkar et al. [41]	MFEM-3D-LW	- -	1.1150	0.0700	0.3170	0.0360	0.0510
	Wu et Lin [24]	MFEM-3D-LW	--	1.1730	0.0724	0.3530	- - -	0.0525
	Kant et Kommineni [23]	FEM-Q9-HSDT	1.1947	1.1246	-	- -	--	0.0506
	Kant et Swaminathan [13]	HSDT	1.1933	1.1110	0.0705	--	--	0.0504
	IGA [42]	TSDT	1.1876	1.1027	0.0678	0.3467	0.0408	0.0501
	IGA [42]	HSDT	1.1850	1.1061	0.0678	0.3974	0.0443	0.0502
50	Present (12×12)	HSDT(Q8)	0.92923	1.0918	0.0563	0.3799* 0.3147+	0.4283* 0.02646+	0.0435
	Pagano [6]	Elasticity solution	0.9348	1.0990	0.0569	0.3230	0.0306	0.0446
	Pandit et al. [26]	FEM-Q9-HZZT	0.9341	1.0948	0.0566	0.3403	0.0333	0.0445
	Chalak et al. [29]	FEM-Q9-HZZT	0.9248	1.0997	0.0611	0.3300	0.0321	0.0443
	IGA [42]	TSDT	0.9284	1.0965	0.0565	0.3520	0.0352	0.0444
	IGA [42]	HSDT	0.9280	1.0971	0.0565	0.4036	0.0383	0.0445
	Kant et Kommineni [23]	FEM-Q9-HSDT	0.9299	1.1118	-	-	-	0.0448
100	Present (12×12)	HSDT(Q8)	0.8905	1.0904	0.0546	0.4118* 0.30167+	0.05132* 0.02157+	0.0427
	Pagano [6]	Elasticity solution	0.8917	1.098	0.0550	0.324	0.0297	0.0433
	Rosalin Sahoo, B.N. Singh [30]	ITZZT-Q8	0.8919	1.1088	0.0555	0.3433	0.0276	0.044
	Chalak et al.[29]	HOZT-Q9	0.8814	1.0982	0.0592	0.3426	0.0332	0.0433
	Pandit et al. [26]	HOZT-Q9	0.8917	1.1093	0.0547	0.3412	0.0324	0.0434
	IGA [42]	TSDT	0.8908	1.0957	0.0548	0.3528	0.0344	0.0436

IGA [42]	HSDT	0.8907	1.0958	0.0548	0.4046	0.0374	0.0436
Pandya et Kant [21]	FEM-Q9-HSDT	0.891	1.108	0.0554	0.3001	0.03362	0.044
Kant et Kommineni [23]	FEM-Q9-HSDT	0.8915	1.1058	-	-	-	0.044

*Constitutive
+Equilibre

Table 3: Normalized transverse displacement, plane stresses, transverse shear stresses of a simply supported sandwich plate $\left(0^{\circ}/C/0^{\circ}\right)$ under doubly sinusoidal loading.

Through Figs (5, 6, 7, 8, 9, 10,11), the normal $\bar{\sigma}_{xx},\bar{\sigma}_{yy},\bar{\sigma}_{xy}$ and transverse shear $\bar{\sigma}_{xz},\bar{\sigma}_{yz}$ stresses states are described through thickness of the sandwich plate in bending behavior for aspect ratios a/h=10,4 according to the constitutive and equilibrium equations. The obtained results stresses are in close agreement with those obtained by Pagano elasticity solution [13] and by TSDT numerical solution [42].

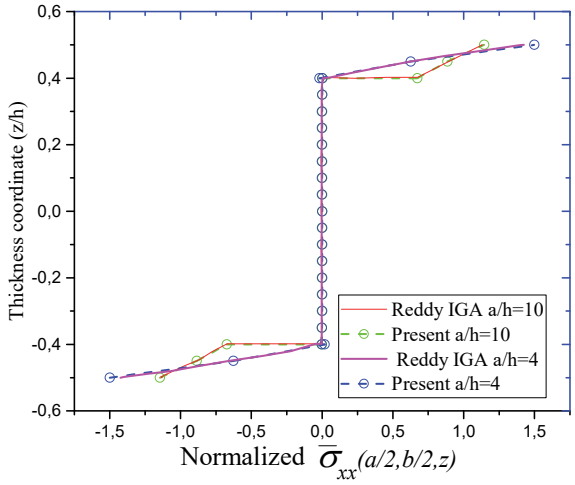

Figure 5: Normal stress distribution $\bar{\sigma}_{xx}$ through the thickness of a simply supported sandwich plate $\left(0^{\circ}/C/0^{\circ}\right)$ subject to a sinusoidal load (a/h =10, 4).

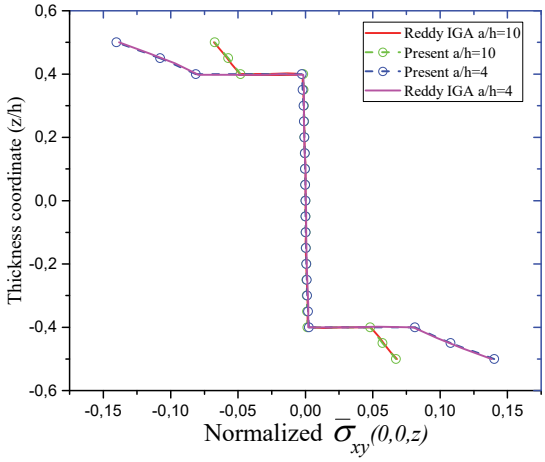

Figure 6: Normal stress distribution $\bar{\sigma}_{yy}$ through the thickness of a simply supported sandwich plate $\left(0^{\circ}/C/0^{\circ}\right)$ subject to a sinusoidal load (a/h =10, 4).

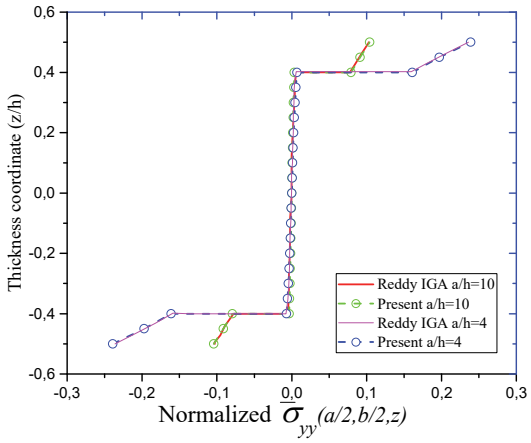

Figure 7: Normal stress distribution $\bar{\sigma}_{xy}$ through the thickness of a simply supported sandwich plate $\left(0^{\circ}/C/0^{\circ}\right)$ subject to a sinusoidal load (a/h =10, 4).

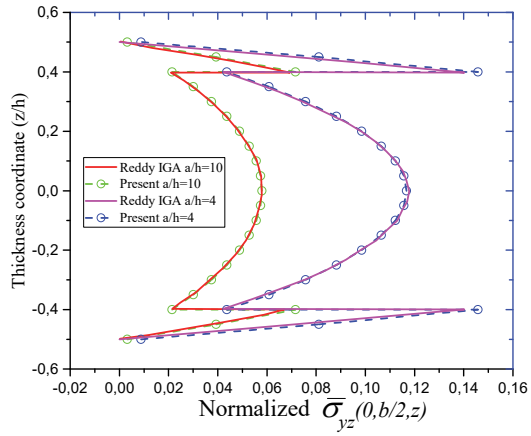

Figure 8: Transverse shear stress distribution $\bar{\sigma}_{yz}$ through the thickness of a simply supported sandwich plate $\left(0^{\circ}/C/0^{\circ}\right)$ subject to a sinusoidal load (a/h =10, 4) (Constitutive equations).

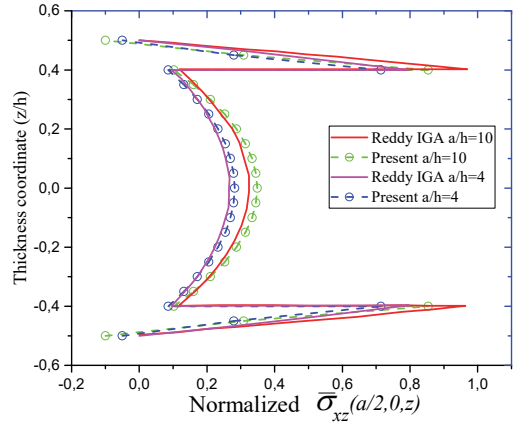

Figure 9: Transverse shear stress distribution $\bar{\sigma}_{xz}$ through the thickness of a simply supported sandwich plate $\left(0^{\circ}/C/0^{\circ}\right)$ subject to a sinusoidal load (a/h =10, 4)(Constitutive equations).

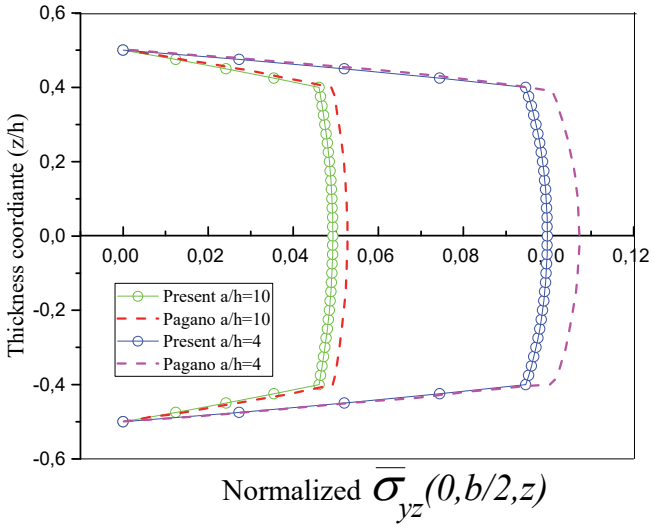

Figure 10: Transverse shear stress distribution $\bar{\sigma}_{yz}$ through the thickness of a simply supported sandwich plate $\left(0^{\circ}/C/0^{\circ}\right)$ subject to a sinusoidal load (a/h =10, 4) (Equilibrium equations).

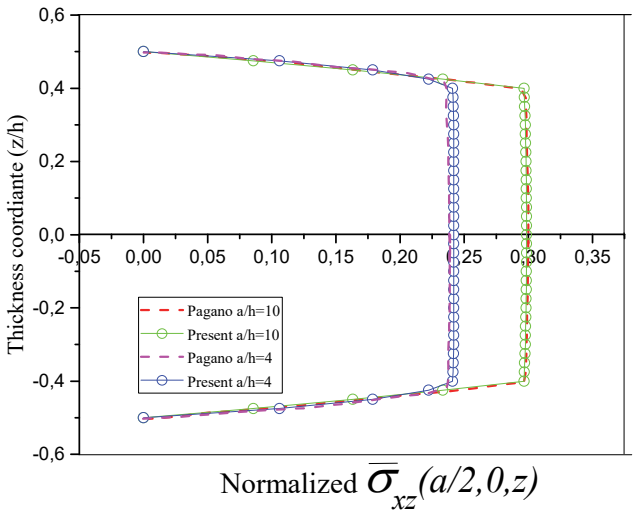

Figure 11: Transverse shear stress distribution $\bar{\sigma}_{xz}$ through the thickness of a simply supported sandwich plate $\left(0^{\circ}/C/0^{\circ}\right)$ subject to a sinusoidal load (a/h =10, 4)(Equilibrium equations).

Three-layer sandwich square plate subject to a uniform load

In this example, the effect of the scale factor R $\left(\bar{C}_{face}=R\bar{C}_{core}\right)$ variation on deflection and stresses state of a simply supported sandwich square plate $\left(0^{\circ}/C/0^{\circ}\right)$ is studied under a uniform transverse load with aspect ratio **a**/**h** =10 and face and core layers thickness **h**c/**h**f = 8. This sandwich example has been suggested by Srinivas [43].
The material properties of core layer are defined as:

$$\bar{C}_{core}=\begin{bmatrix} 0.999781 & 0.231192 & 0 & 0 & 0 \\ 0.231192 & 0.524886 & 0 & 0 & 0 \\ 0 & 0 & 0.262931 & 0 & 0 \\ 0 & 0 & 0 & 0.26681 & 0 \\ 0 & 0 & 0 & 0 & 0.159914 \end{bmatrix}$$

The normalized deflection and stresses are defined by:

$$\bar{w}=\left(\frac{0.999781w\left(a/2,b/2,0\right)}{q_0 h}\right),\ \bar{\sigma}^{1}_{xx}=\left(\frac{\sigma^{1}_{x}\left(a/2,b/2,h/2\right)}{q_0}\right),\bar{\sigma}^{2}_{xx}=\left(\frac{\sigma^{1}_{x}\left(a/2,b/2,2h/5\right)}{q_0}\right),$$

$$\bar{\sigma}^{3}_{xx}=\left(\frac{\sigma^{2}_{x}\left(a/2,b/2,2h/5\right)}{q_0}\right),\ \bar{\sigma}^{1}_{yy}=\left(\frac{\sigma^{1}_{y}\left(a/2,b/2,h/2\right)}{q_0}\right),\bar{\sigma}^{2}_{yy}=\left(\frac{\sigma^{1}_{y}\left(a/2,b/2,2h/5\right)}{q_0}\right),$$

$$\bar{\sigma}^{3}_{yy}=\left(\frac{\sigma^{2}_{y}\left(a/2,b/2,2h/5\right)}{q_0}\right)$$

Table 4 shows the deflection and normal stresses solution by the proposed model for different factor scales R = 5, 10, 15 compared with those obtained using the exact solution reported by Srinivas [43], HSDT finite element by Pandya and Kant [21], HSDT meshfree solution by Ferreira et al. [44], TrSDT trigonometric shear deformation theory solution by Mantari

et al. [14] and IHSDT inverse hyperbolic shear deformation theory by Grover et al.[15]. It is noted that the obtained results are in very good correlation with all scale factor R cases when compared with the exact solution.

R	Theory	$\bar{w}$	$\bar{\sigma}^1_{xx}$	$\bar{\sigma}^2_{xx}$	$\bar{\sigma}^3_{xx}$	$\bar{\sigma}^1_{yy}$	$\bar{\sigma}^2_{yy}$	$\bar{\sigma}^3_{yy}$
5	Present HSDT(Q8) (12×12)	257.4323	60.1869	46.6913	9.3382	38.3327	30.07465	6.0149
	Exact [43]	258.97	60.353	46.623	9.34	38.491	30.097	6.161
	FEM-HSDT [21]	256.13	62.380	46.910	9.382	38.930	30.330	6.065
	MRBF-HSDT [44]	257.110	60.366	47.003	9.401	38.456	30.242	6.048
	CFS-IHSDT [15]	255.644	60.675	47.055	9.411	38.522	30.206	6.041
	CFS-TrSDT [14]	256.706	60.525	47.061	9.412	38.452	30.177	6.035
10	Present HSDT(Q8) (12×12)	155.9531	65.3098	49.3654	4.936	43.2633	33.42849	3.3428
	Exact [43]	159.38	65.332	48.857	4.903	43.566	33.413	3.5
	FEM-HSDT [21]	152.33	64.650	51.310	5.131	42.830	33.970	3.397
	MRBF-HSDT [44]	154.658	65.381	49.973	4.997	43.240	33.637	3.364
	CFS-IHSDT [15]	154.550	65.741	49.798	4.979	43.4	33.556	3.356
	CFS-TrSDT [14]	155.498	65.542	49.708	4.971	43.385	33.591	3.359
15	Present HSDT(Q8) (12×12)	116.9587	66.9283	49.3353	3.289	45.8775	34.9577	2.3305
	Exact [43]	121.72	66.787	48.299	3.238	46.424	34.955	2.494
	FEM-HSDT [21]	110.430	66.620	51.970	3.465	44.920	35.410	2.361
	MRBF-HSDT [44]	114.644	66.919	50.323	3.355	45.623	35.167	2.345
	CFS-IHSDT [15]	115.820	67.272	49.813	3.321	45.967	35.088	2.339
	CFS-TrSDT [14]	115.919	67.185	49.769	3.318	45.910	35.081	2.339

Table 4: The normalized deflection and stresses of square sandwich plates under uniform load.

An additional analysis is considered for the effect of different scale factors R = 5, 10, 15 on the evolution of in-plane displacements $u_1(a/2,b/2,z)$, $u_2(a/2,b/2,z)$, normal stresses $\bar{\sigma}_{xx}(a/2,b/2,z)$, $\bar{\sigma}_{yy}(a/2,b/2,z)$ and transverse shear stresses $\bar{\sigma}_{xz}(0,b/2,z)$, $\bar{\sigma}_{yz}(a/2,0,z)$ with respect to the sandwich plate thickness.

In Fig 12, it is noted for the bending behavior that the in-plane displacements are reduced along the thickness when the scale factor R is increased. In addition, in Figs 13, 14 it can be seen that the normal and transverse shear stresses have also reduced only through the core of plate, whereas they have increased through the faces.

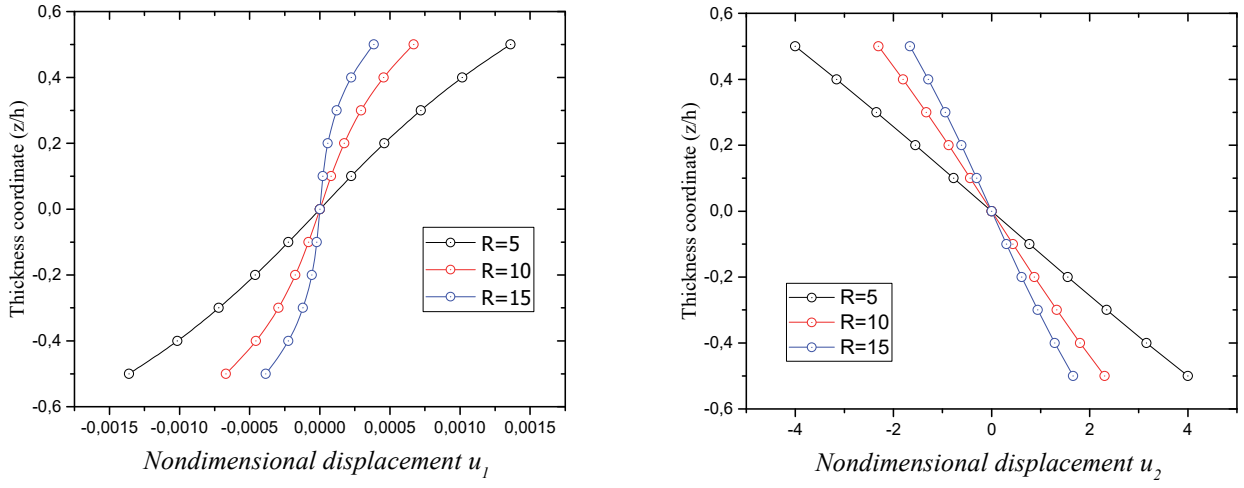

Figure 13: Effect of scale factor R variation on the distribution of normal stresses $\bar{\sigma}_{xx}$, $\bar{\sigma}_{yy}$ through the thickness of a simply supported $\left(0^\circ / C / 0^\circ\right)$ sandwich plate subject to a uniform transverse load.

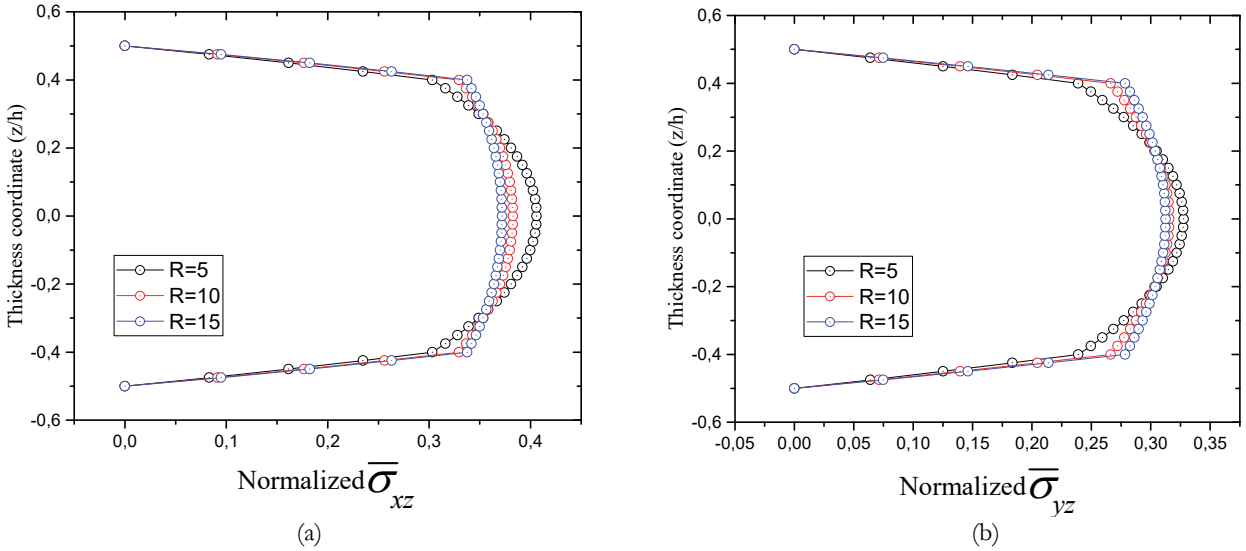

Figure 14: Effect of scale factor R variation on the distribution of transverse shear stresses $\bar{\sigma}_{xz}$, $\bar{\sigma}_{yz}$ through the thickness of a simply supported $\left(0^\circ / C / 0^\circ\right)$ sandwich plate subject to a uniform transverse load.

Sandwich Plates with Laminated Face Sheets

In this example, the proposed element is evaluated for different laminated face sheets of rectangular sandwich plates. Kanematsu et al. [16] carried out experimental solution of clamped rectangular sandwich plates (450×300 mm) with four types laminated composite orientation faces SP1, SP2, SP3 and SP4 (Fig. 15). The faces of the sandwiches are symmetrical laminated composite made of carbon/epoxy (Carbon Fiber–Reinforced Plastic-CFRP) E1=105 GPa, E2=8.74 GPa, G12= G13= G23=4.56 GPa, v=0.327, while the core is an aluminum honeycomb material (Aluminum Honeycomb Core) E1=68.6 MPa, E2=68.6 MPa, G12=26.4 MPa, G13=103 MPa, G23=62.1 MPa, v=0.3. The thickness of each layer is 0.125mm, while the core thickness is 10mm for the SP1 and SP2 types, and 7mm core thickness for the SP3 and SP4 types. The plate is subject to a uniform distributed load of intensity q=1.01 KPa. In addition, the authors provided analytical solutions based on the Rayleigh-Ritz method for the same plate problem, using two types boundary conditions, simply supported (SSSS) and clamped (CCCC). The obtained results of the transverse displacement using the proposed element are given in Table 5, compared with those obtained using analytical solutions and from experimental work given by

Kanematsu et al. [16]. The results have also been compared with those obtained by finite element models of Lee and Fan [45] and Nayak et al. [28], M. O. Belarbi et al. [46]. The comparison results show the reliability and practicality of the proposed element for the studies of laminated faces sandwich structures.

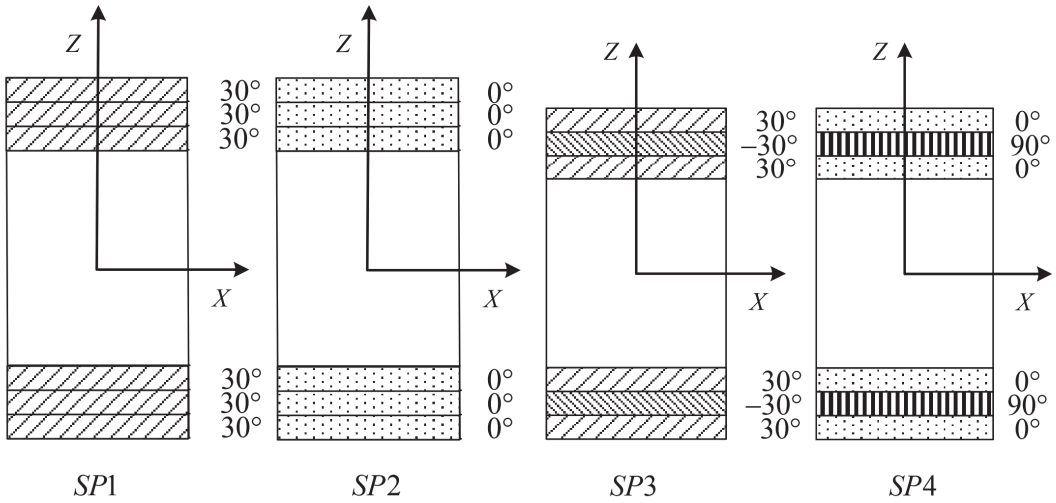

Figure 15: Different types of laminated faces sandwich structures [46]

References	Theory	Central deflection (mm)			
		SP1	SP2	SP3	SP4
		Clamped (CCCC)			
Present (12×12)	FEM-Q8-TSDT	0.04891	0.056578	0.074968	0.05469
Kanematsu et al. [16]	Analytical solution	0.05040	0.05400	0.07720	0.06130
Kanematsu et al. [16]	Experimental solution	0.06900	0.08500	0.09400	0.09000
Lee et Fan [45]	FEM-Q9-LW	0.05190	0.05524	0.07834	0.06216
Nayak et al. [28]	FEM-Q9-HSDT	-	0.05248	-	0.05797
M. O. Belarbi et al. [46]	FEM-Q4-RSFT52	0.04906	0.05647	0.07506	0.05525
		Simply supported (SSSS)			
Present (12×12)	FEM-Q8-TSDT	0.12124	0.17823	0.17213	0.2067
Kanematsu et al. [16]	Analytical solution	0.1173	0.1829	0.1794	0.2206
Lee et Fan [45]	FEM-Q9-LW	0.1213	0.1774	0.1729	0.2138
Nayak et al. [28]	FEM-Q9-HSDT	-	0.1754	-	0.2111
M. O. Belarbi et al. [46]	FEM-Q4-RSFT52	0.1160	0.1733	0.1695	0.2010

Table 5: Deflection laminated faces sandwiches plates under uniformly transverse load.

Conclusion

In this paper, an improved C^0 two-dimensional plate finite element (FE) model has been developed for the static analysis of laminated thin and thick sandwich plates. The Reddy's third order shear deformation theory (TSDT) is employed by adopting single layer approach when the warping cross-sectional of in-plane displacements is considered to be cubic for both the face sheets and the core of sandwich. The problem of C1 continuity requirement of the second order derivatives of transverse displacements is circumvented by selecting the degrees of freedom nodal field in an efficient manner. For the present analysis, an eight-node C0 finite element is successfully implemented having seven degrees of freedom for each element node: two displacements $(u,\ v)$ for in-plane behavior and five bending unknowns: a transverse displacement, two rotations and two shear angles $(w,\ \psi_x,\ \psi_y,\theta_x,\theta_y)$. Whereas element stiffness matrices, which have first order derivative requirement, are solved through a computationally (3×3) Gauss integration scheme. In the proposed formulation, there is no stiffness penalty requirement such as: shear correction factors, and numerical techniques to overcome transverse shear locking phenomenon that as used in previous element models. In order to demonstrate the effectiveness and validity of the proposed formulation, many sandwich plates numerical examples are solved and displacements as well as stresses are calculated for different problems and which give results better than other existing 2D finite element models. The results obtained by using the present FE model are successfully compared with those of analytical and numerical solutions available in the literature. The numerical results show that the performance of the present finite element model is excellent in predicting the bending response of thin and thick laminated composites sandwich structures as the error percentage with respect to the 3D elasticity solution is considerably low. The present FE model may, therefore, be recommended for use as accurate tool in other behavior analysis of laminated sandwich plates.

Statements and Declarations - Competing Interests and Funding

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
There is no funding to declare.

Nomenclature

u_1,u_2,u_3	Displacement field in the x, y and z directions respectively
u,v,w	Displacement of a point on the mid-plane
ψ_x,ψ_y	Rotations of normal to the mid-plane about the y and x axes respectively
θ_x,θ_y	Shear angles to the mid-plane about the y and x axes respectively
ε_i	Strain
a,b,h	Dimensions of the plate along the x, y and z directions respectively
σ_{ij}	Stress tensor
$\left[\overline{C}_{ij}\right]$	Constitutive matrix at the lamina level
E_i	Young modulus
$G_{12},\ G_{13},G_{23}$	Shears modulus
ν_{12},ν_{21}	Poisson's ratios
$\{N\},\ \{M\},\ \{P\},\{Q\},\ \{R\}$	Resultants forces
$A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},H_{ij}$	Extensional, coupling and flexural stiffnesses
$A_{ij}^S,D_{ij}^S,F_{ij}^S$	Transverse shear stiffnesses

$\bar{N}_i$	Interpolation functions
$\left[B_\varepsilon^0\right],\left[B_\kappa^0\right],\left[B_\kappa^2\right],\left[B_\varepsilon^s\right],\left[B_\kappa^s\right]$	Strain displacement matrices
$\{\delta\}$	Element nodal field variables
$\{F\}$	Element load vector
$[K]_e$	Element stiffness matrix
h_c, h_f	Core and face thicknesses of sandwich, respectively

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