Visual Abstract

Description of fatigue sensitivity curves and transition to critical states of polymer composites by cumulative distribution functions

O.A. Staroverov, A.I. Mugatarov, A.S. Yankin, V.E. Wildemann
Center of Experimental Mechanics, Perm National Research Polytechnic University, Russia
cem_staroverov@mail.ru, https://orcid.org/0000-0001-6095-0962
cem_mugatarov@mail.ru, https://orcid.org/0000-0002-2229-8181
yas.cem@yandex.ru, https://orcid.org/0000-0002-0895-4912
wildemann@pstu.ru, https://orcid.org/0000-0002-6240-4022

ABSTRACT. In this paper, a novel model is presented to describe the composite mechanical properties degradation during cyclic loading. The model is based on cumulative distribution functions using. Weibull probability distribution law and beta distribution are considered. The dependences of the fatigue sensitivity coefficient on the preliminary cyclic exposure are derived. The damage value function derivative using is proposed to define damage accumulation stages boundaries. Model parameters are obtained using experimental data. Determination coefficients are calculated. A high descriptive capability is noted. Rationality and expediency of using cumulative distribution functions as the approximation of experimental data on mechanical characteristics reduction after preliminary cyclic exposure is concluded.

KEYWORDS. Composite, Damage accumulation, Mechanical properties degradation, Cumulative distribution functions.

OPEN ACCESS

Citation: Staroverov, O.A., Mugatarov, A.I., Yankin, A.S., Wildemann, V.E., Description of fatigue sensitivity curves and transition to critical states of polymer composites by cumulative distribution functions, Frattura ed Integrità Strutturale, 63 (2023) 91-99.

Received: 06.09.2022
Accepted: 15.10.2022
Online first: 31.10.2022
Published: 01.01.2023

INTRODUCTION

Predicting the remaining life of composite materials used in structures that can operate under cyclic loading represents a relevant scientific mission of deformable solid mechanics. A number of papers dedicates to experimental studies of mechanical characteristics of various classes of composites exposed to cyclic effects [1-5]. Staging of damage accumulation processes is noted in many of them [6-9]. The initial stage, also called the initiation stage, suggests fast damage accumulation. Multiple fatigue damages related to matrix cracking, damage to phase interfaces, and rupture of some fibers were found in [10-13]. The second stage, also called the stabilization stage, suggests slow damage accumulation. Some authors suggest that it involves matrix cracking processes. The stabilization stage is followed by intensified accumulation of damages and transition to the third stage, or the final breakdown stage, which implies fiber destruction and macro-failure of the specimen. Damage accumulation in the composites frequently leads to mechanical properties (Young's modulus, ultimate strength, etc.) reduction [14-18].

A number of models which enable damage value calculations under fatigue loading exist. The most famous of them are the Palmgren–Miner model (linear summing of damages) [19-22] and the Marco-Starkie model (non-linear summing) [23-27]. However, neither of these models takes into account the aforementioned three stages, so formally they are not suitable to describe damage accumulation processes in composites. An approach where the damage parameter is related to the varying mechanical characteristic takes place in [5, 9, 28-29]. Moreover, some papers [8, 30-32] model the degradation of composite mechanical characteristics by setting random values of strength and deformation parameters of the reinforcing component and matrix. The modeling results are good, but their disadvantage is the high number of required constants.
This paper proposes a new model to describe the degradation of composite mechanical characteristics after preliminary cyclic loading. The model is based on experimental data approximation by probability distribution functions.

MATERIAL AND METHODS

Twenty-six fiber-glass laminate specimens with the reinforcement pattern of $[0/90]_8$ were used in the experimental studies. The test method is based on the existing standards of quasi-static and fatigue tension of polymer composite materials. Nominal values of ultimate strength σ_u and elasticity modulus E were taken from quasi-static uniaxial tension tests (ASTM D3039). The maximum number of cycles to failure N_{max} was found for uniaxial cyclic tension at the maximum stress value $\sigma_{max} = 0.5 \cdot \sigma_u$, the asymmetry coefficient $R = 0.1$, and the frequency $\nu = 20\ Hz$ (ASTM D3479). Three specimens were tested for quasi-static and cyclic tension. The other 20 specimens were exposed to preliminary cyclic loading and then statically tested. Preliminary cyclic exposure was implemented within 0.1 to 0.8 nominal fatigue life N_{max}. The test method is schematically shown in Fig. 1.

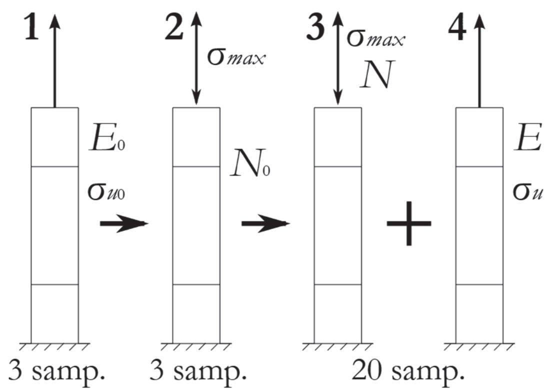

Figure 1: The order of experimental tests procedure: 1 – quasi-static tension tests; 2 – fatigue tests; 3 – preliminary cyclic loading; 4 – quasi-static tension after preliminary cyclic loading.

For each specimen, the fatigue sensitivity coefficients are found using the formula

$$K_E = \frac{E}{E_0};\ K_B = \frac{\sigma_u}{\sigma_{u0}} \tag{1}$$

where E is Young's modulus; E_0 is the mean Young's modulus for a non-damaged material; σ_u is the ultimate strength of the material; σ_{u0} is the ultimate strength of a non-damaged material. The fatigue sensitivity coefficient takes values from 0 (completely failed material) to 1 (non-damaged material). Fatigue sensitivity coefficients correspond to the following damages values

$$\omega_E = 1 - K_E;\ \omega_B = 1 - K_B \tag{2}$$

The preliminary cyclic exposure is found using the formula

$$n = \frac{N}{N_0} \tag{3}$$

where N is the number of preloading cycles; N_0 is the fatigue life for this loading cycle.
The test data set represents the dependency of the fatigue sensitivity coefficient K_B (K_E) on the preliminary cyclic exposure n. The results are processed using the model below.

MODEL DESCRIPTION

Typical points of the fatigue sensitivity curve in the coordinates of preliminary cyclic exposure vs. fatigue sensitivity coefficient (n – K_B) are shown in Fig. 2a. The following conversion can be made: the same points are built in the coordinates of damage value vs. preliminary cyclic exposure (ω_B – n) as shown in Fig. 2b. Some features of this dependency can be noted. First, it is limited by zero and one. Second, if the "healing" of the material is absent, this dependency is monotone-increasing. Third, the characteristic segment of slow damage accumulation in the diagram middle can be noted. These features also have some probability distribution integral functions. In the case of non-damaged material before preliminary cyclic exposure, the $n(\omega_B)$ function passes through the coordinate system center. Therefore, consideration of the two-parameter Weibull law of probability distribution [33] and beta distribution is convenient. As an example, the integral curve of the Weibull distribution law is given in Fig. 2c.

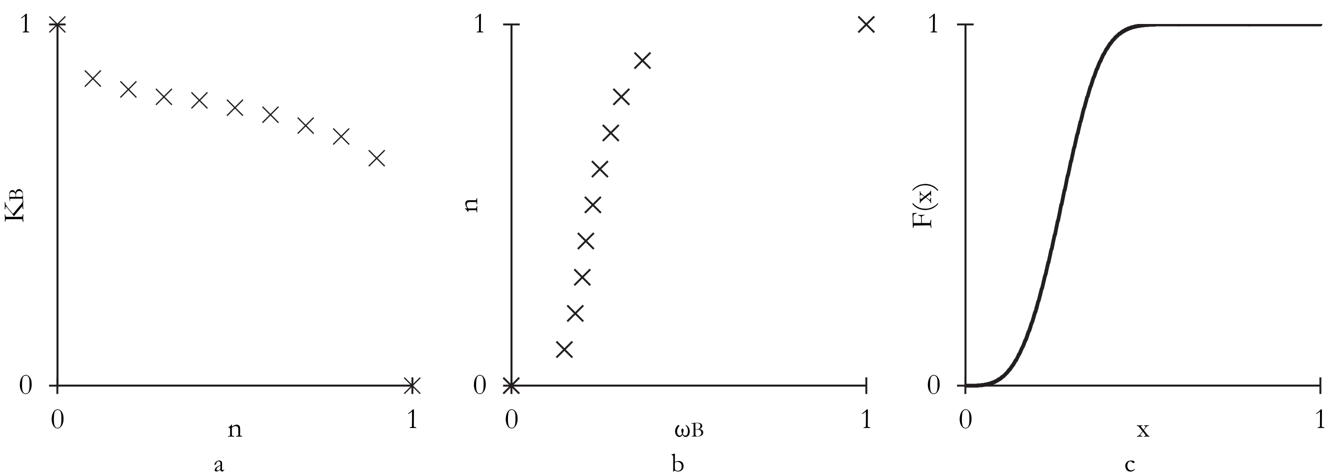

Figure 2: Residual properties dependence on preliminary cyclic exposure in the coordinates of "K_B–n" (a) and "n–ω_B" (b); the integral curve of the two-parameter Weibull distribution law (c)

Two-parameter Weibull distribution

The dependency of the preliminary cyclic exposure on the damage can be described by the following equation:

$$n\left(\omega_B\right)=1-e^{-\left(\frac{\omega_B}{\lambda}\right)^{\kappa}} \tag{4}$$

where $\lambda>0$ is the scale parameter; $\varkappa>0$ is the form parameter. Both these values are material properties characterizing its ability to keep strength and rigidity after some operating time. In a general case, these parameters can depend on the temperature, exposure amplitude, frequency, etc. The dependency of the residual strength coefficient on the preliminary cyclic exposure can be described as:

$$K_B\left(n\right)=1-\lambda\left(-\ln\left(1-n\right)\right)^{\frac{1}{\kappa}} \tag{5}$$

The parameters λ and $\varkappa$ can be defined both numerically and using the method of least squares from the equation of a straight line approximating data in logarithmic coordinates (these approaches give close but slightly different results):

$$\ln\left(1-K_B\right)=\ln\lambda+\frac{1}{\kappa}\ln\left(-\ln\left(1-n\right)\right) \tag{6}$$

An example of this dependence is shown in Fig. 3.

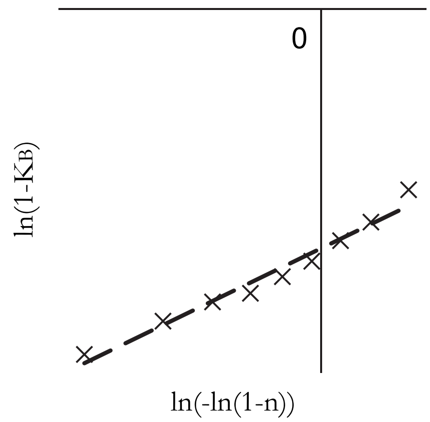

Figure 3: Model parameters receiving example

To divide the fatigue sensitivity curve into damage accumulation stages, the derivative of the damage value function can be considered:

$$\omega_B' = \frac{\lambda}{\kappa}\left(-\ln\left(1-n\right)\right)^{\frac{1}{\kappa}-1}\frac{1}{1-n} \quad (7)$$

The graph of this function is given in Fig. 4a. The physical sense of this function is the damage accumulation rate. For a low number of cyclic exposures, the intensity of damage accumulation is high (stage I), this stage is followed by an area of slow damage accumulation (stage II); when the number of cyclic exposures approaches the limit, the damage accumulation rate rapidly grows (stage III).
Earlier, the authors in [9] proposed a definition of boundaries for these stages using the points n_{s1} and n_{s2}, where $\omega_B'=0.3$. Such division is conditional and may vary depending on the material (and its class). The values of n_{s1} and n_{s2} are defined by solving the transcendent Eqn. (7). An example of a $K_B(n)$ curve with the highlighted stages is given in Fig. 4b. The values of the fatigue sensitivity coefficient in exposures n_{s1} and n_{s2} are designated as K_{Bs1} and K_{Bs2}, respectively. The characteristic of the material defining the average rate of fatigue sensitivity coefficient reduction in the area of slow damage accumulation ψ can be introduced as:

$$\psi = \frac{K_{Bs1} - K_{Bs2}}{n_{s2} - n_{s1}}. \quad (8)$$

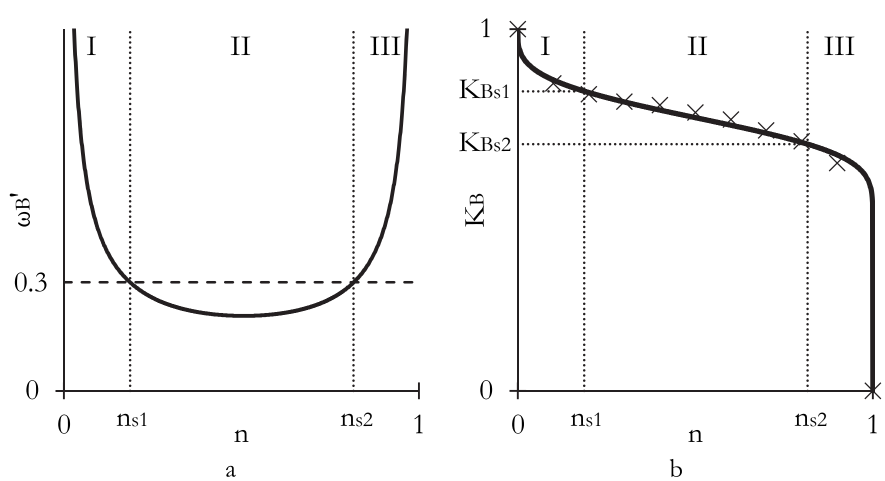

Figure 4: Damage accumulation rate curve (a) and fatigue sensitivity coefficient vs. preliminary cyclic exposure graph (b) with highlighted stages

The model has some disadvantages. First, when $n \to 1$, K_B becomes less than 0. However, this will take place only for the number of cycles close to the limit, so this specific feature can be neglected. Second, taking the logarithm is not suitable to describe data where the values of K_B and n exceed 1. This problem can be solved by a numerical search of the parameters from Eqn. (5), where taking the logarithm is not required.

Beta distribution

The dependency of $K_B(n)$ can be described as follows

$$K_B(n) = 1 - \frac{B_n(\alpha,\beta)}{B(\alpha,\beta)};\ B_n(\alpha,\beta) = \int_0^n t^{\alpha-1}(1-t)^{\beta-1}\,dt;\ B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,dt \qquad (9)$$

where $B_n(\alpha,\beta)$ is an incomplete beta-function; $B(\alpha,\beta)$ is a beta-function. The parameters α and β must exceed zero and are numerically searched. A derivative of the damage function:

$$\omega_B'(n) = \frac{n^{\alpha-1}(1-n)^{\beta-1}}{B(\alpha,\beta)} \qquad (10)$$

In comparison with the previous one, this function has some advantages. First, for n=1, the value K_B=1. Second, all experimental points can be used to find the parameters α and β without the logarithm taking.

No	*N*, cycles	*n*	*E, GPa*	K_E	σ_u, *MPa*	K_B
1	0	-	2.19	-	369	-
2	0	-	2.32	-	392	-
3	0	-	2.36	-	382	-
4	25000	0.1171	2.11	0.9214	351	0.9213
5	25000	0.1171	2.10	0.9170	332	0.8714
6	25000	0.1171	2.00	0.8734	343	0.9003
7	25000	0.1171	2.08	0.9083	327	0.8583
8	25000	0.1171	1.96	0.8559	327	0.8583
9	75000	0.3514	2.03	0.8865	287	0.7533
10	75000	0.3514	2.00	0.8734	317	0.8320
11	75000	0.3514	1.96	0.8559	307	0.8058
12	75000	0.3514	1.94	0.8472	318	0.8346
13	75000	0.3514	2.02	0.8821	310	0.8136
14	125000	0.5856	1.81	0.7904	296	0.7769
15	125000	0.5856	1.84	0.8035	300	0.7874
16	125000	0.5856	1.95	0.8515	310	0.8136
17	125000	0.5856	1.94	0.8472	306	0.8031
18	125000	0.5856	1.97	0.8603	299	0.7848
19	175000	0.8199	1.87	0.8166	274	0.7192
20	175000	0.8199	1.81	0.7904	269	0.7060
21	175000	0.8199	1.80	0.7860	264	0.6929
22	175000	0.8199	1.72	0.7511	269	0.7060
23	175000	0.8199	1.86	0.8122	250	0.6562
24	182321	-	0.00	-	0	-
25	228810	-	0.00	-	0	-
26	229212	-	0.00	-	0	-

Table 1: Experimental data.

RESULTS AND DISCUSSION

The experimental data are given in Tab. 1. The average Young's modulus of a non-damaged material E_0=2.29 *GPa*, the average ultimate strength of a non-damaged material σ_u=381 *MPa*, and the average durability for this amplitude N_0=213448 cycles. Approximation function parameters were found for the following cases: Weibull distribution, using data approximation in logarithmic coordinates (WL); Weibull distribution, the numerical search of parameters (WN); beta distribution, the numerical search of parameters (BN). For all cases, the determination coefficient was found. Tab. 2 and 3 contain the model parameters found for the full data set. Because R^2>0.7 for all cases, the model has a high descriptive capability. Close determination coefficient values for all the cases can be noted. Fig. 5 represents curves of fatigue sensitivity coefficient and damage accumulation rate for Young's modulus (a, c) and ultimate strength (b, d) decrease. The similarity of these curves can be noted.

Rationality and expediency of using probability distribution functions as the approximation of experimental data on mechanical characteristics reduction after preliminary cyclic exposure is concluded. A significant advantage of this approach is the small amount of required experimental data for parameter definition (4 tests required) and simplicity in modeling.

Method	WL	WN	BN
Parameter 1	ϰ=3.67383	ϰ=3.54769	α=0.23774
Parameter 2	λ=0.17364	λ=0.17651	β=0.04566
n_{s1}	0.0854	0.0891	0.0756
n_{s2}	0.9196	0.9125	0.8685
$n_{s2}-n_{s1}$	0.8343	0.8234	0.7929
K_{s1}	0.9101	0.9096	0.9102
K_{s2}	0.7767	0.7731	0.7797
ψ	0.1599	0.1657	0.1646
R^2	0.7164	0.7196	0.7265

Table 2: Calculated model parameters (Young's modulus reduction)

Method	WL	WN	BN
Parameter 1	ϰ=2.86946	ϰ=2.81928	α=0.31281
Parameter 2	λ=0.23938	λ=0.24205	β=0.08719
n_{s1}	0.1704	0.1767	0.1516
n_{s2}	0.7933	0.7818	0.7439
$n_{s2}-n_{s1}$	0.6229	0.6051	0.5923
K_{s1}	0.8666	0.8646	0.8704
K_{s2}	0.7195	0.7190	0.7307
ψ	0.2362	0.2405	0.2359
R^2	0.8456	0.8469	0.8536

Table 3: Calculated model parameters (ultimate strength reduction)

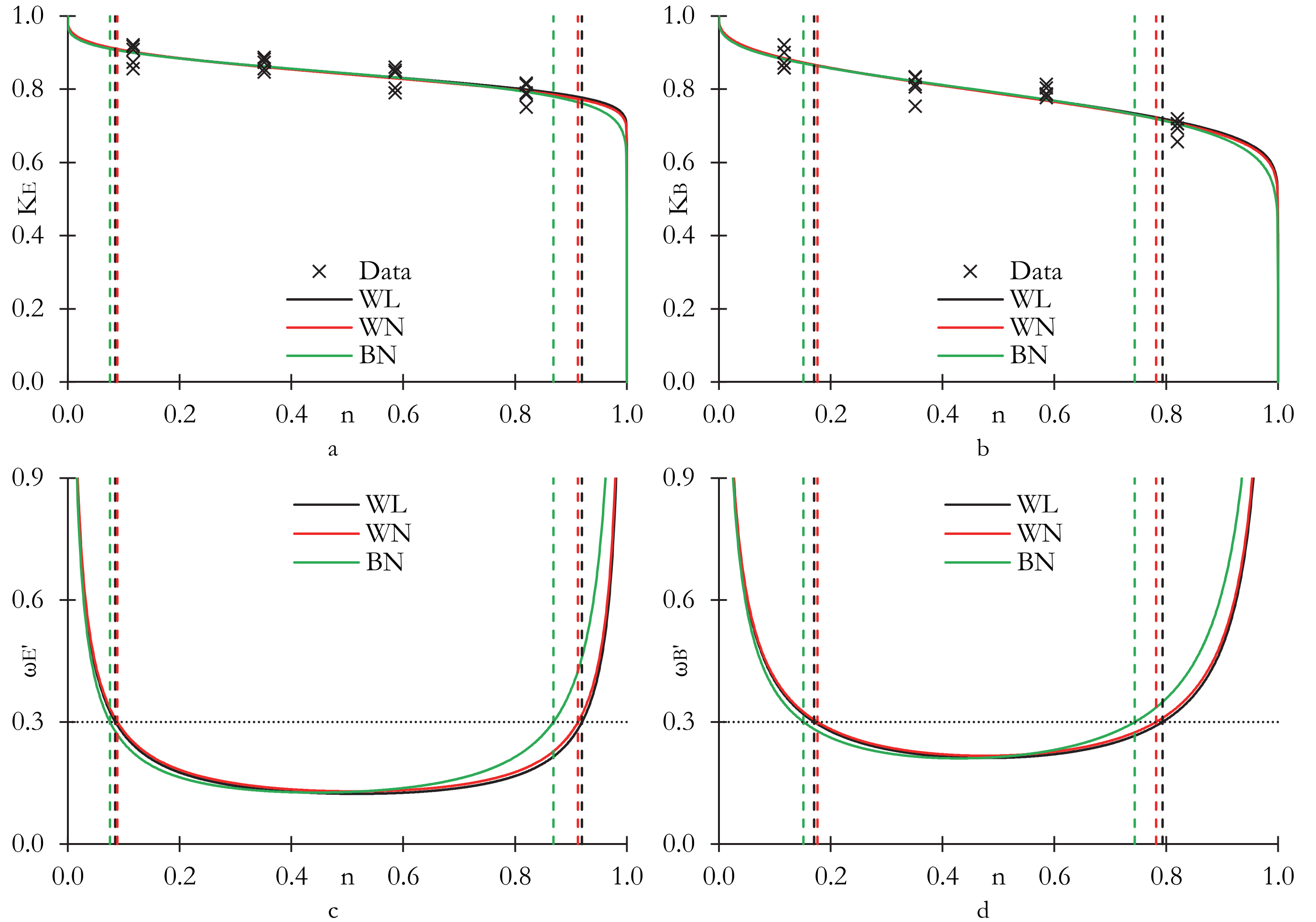

Figure 5: Fatigue sensitivity coeffitient curves for Young's modulus (a) and utimate strength (b) reduction with corresponding curves of damage accumulation rate (c, d) and highlighted stages

CONCLUSIONS

- An ability to describe experimental data on deformation and strength properties reduction of composites after preliminary cyclic exposure using probability distribution functions is considered.
- Two probability distributions are considered: two-parameter Weibull law and beta distribution. For the Weibull distribution, a method to find parameters by approximation of experimental data in logarithmic coordinates is described.
- Using of damage value function derivative is proposed to find damage accumulation stages boundaries.
- Experimental data are processed for reduced mechanical characteristics of structural fiber-glass laminate. Model parameters are obtained. Fatigue sensitivity coefficient and damage accumulation rate curves are built. Boundaries of damage accumulation stages are defined. Determination coefficients are calculated. Because R^2 exceeds 0.7, the high descriptive capability of the model can be noted.
- All parameter calculation methods showed close results. Beta distribution usage is more perspective due to the function having a value from 0 to 1. Each model requires only 4 experiment tests to define parameters (1 static test, 1 fatigue test, and 2 intermediate points on the curve). These models can be used in the modeling of the deformation and failure processes of various composite structures.
- The future planned research will contain this method's usage for the description of other experimental data and loading conditions. Moreover, experimental study of the fast reduction of properties stages seems expedient for the purpose of more competent description.

Acknowledgements

The work was carried out in Perm National Research Polytechnic University with financial support of grant of President of Russian Federation for government support of young Russian scientists (No MK-1545.2022.4) and within the State Assignment of the Ministry of Science and Higher Education of the Russian Federation (No. FSNM-2020-0027).

References

[1] Almeida, R.S.M., Besser, B., Tushtev, K., Li, Y. and Rezwan, K. (2022). Fatigue behavior and damage analysis of PIP C/SiC composite, Journal of the European Ceramic Society, 42(13), pp. 5391–5398. DOI: 10.1016/j.jeurceramsoc.2022.06.053.

[2] Mortazavian, S. and Fatemi, A. (2015). Fatigue behavior and modeling of short fiber reinforced polymer composites: A literature review, Int. J. Fatigue, 70, pp. 297–321. DOI: 10.1016/j.ijfatigue.2014.10.005

[3] Vassilopoulos, A.P. ed., (2010). Fatigue Life Prediction of Composites and Composite Structures, Woodhead Publishing.

[4] Degrieck, J. and Van Paepegem, W. (2001). Fatigue damage modeling of fibre-reinforced composite materials: Review, Appl. Mech. Rev., 54(4), pp. 279–300. DOI: 10.1115/1.1381395.

[5] Philippidis, T.P. and Passipoularidis, V.A. (2007). Residual strength after fatigue in composites: Theory vs. experiment, Int. J. Fatigue, 29(12), pp. 2104–2116. DOI: 10.1016/j.ijfatigue.2007.01.019.

[6] De Vasconcellos, D.S., Touchard. F. and Laurence, C.-A. (2014). Tension–tension fatigue behaviour of woven hemp fibre reinforced epoxy composite: A multi-instrumented damage analysis, Int. J. Fatigue, 59, pp. 159–169. DOI: 10.1016/j.ijfatigue.2013.08.029.

[7] Van Paepegem W. and Degrieck J. (2002). A new coupled approach of residual stiffness and strength for fatigue of fibre-reinforced composites, Int. J. Fatigue, 24(7), pp. 747–762. DOI: 10.1016/S0142-1123(01)00194-3

[8] Schaff, J.R. and Davidson, B.D. (1997). Life prediction methodology for composite structures. Part I - Constant amplitude and two-stress level fatigue, Journal of Composite Materials, 31(2), pp. 128–157. DOI: 10.1177/002199839703100202.

[9] Wil'deman, V.E., Staroverov, O.A. and Lobanov, D.S. (2018). Diagram and Parameters of Fatigue Sensitivity for Evaluating the Residual Strength of Layered GFRP Composites After Preliminary Cyclic Loadings, Mechanics of Composite Materials, 54(3), pp. 313–320. DOI: 10.1007/S11029-018-9741-9.

[10] Zhao, X., Wang, X., Wu, Z. and Zhu, Z. (2016). Fatigue behavior and failure mechanism of basalt FRP composites under long-term cyclic loads, Int. J. Fatigue, 88, pp. 58–67. DOI: 10.1016/j.ijfatigue.2016.03.004.

[11] Liang, S., Gning, P.-B. and Guillaumat, L. (2014). Properties evolution of flax/epoxy composites under fatigue loading, Int. J. Fatigue, 63, pp. 36–45. DOI: 10.1016/j.ijfatigue.2014.01.003.

[12] Brod, M., Just, G., Dean, A., Jansen, E., Koch, I., Rolfes, R. and Gude, M. (2019). Numerical modelling and simulation of fatigue damage in carbon fibre reinforced plastics at different stress ratios, Thin-Walled Structures, 139, pp. 219–231 DOI: 10.1016/j.tws.2019.03.005.

[13] Brighenti, R., Carpinteri, A. and Scorza, D. (2015). Effect of fibre arrangement on the multiaxial fatigue of fibrous composites: a micromechanical computational model, Frattura ed Integrità Strutturale, 9(34), pp. 59–68. DOI: 10.3221/IGF-ESIS.34.05.

[14] Strizhius, V. (2016). Fatigue failure criterion of laminated composites under a complex stress-strain state, Mechanics of Composite Materials, 52(3), pp. 369-378. DOI: 10.1007/s11029-016-9589-9.

[15] Shokrieh, M.M. and Lessard, L.B. (2000). Progressive fatigue damage modeling of composite materials, Part II: Material characterization and model verification, Journal of Composite Materials, 34(13), pp. 1081–1116. DOI: 10.1177/002199830003401302.

[16] Epaarachchi, J.A. and Clausen, P.D. (2003). An empirical model for fatigue behavior prediction of glass fibre-reinforced plastic composites for various stress ratios and test frequencies, Composites Part A: Applied Science and Manufacturing, 34(4), pp. 313–326. DOI: 10.1016/S1359-835X(03)00052-6.

[17] Post, N.L., Case, S.W. and Lesko, J.J. (2008). Modeling the variable amplitude fatigue of composite materials: A review and evaluation of the state of the art for spectrum loading, Int. J. Fatigue, 30(12), pp. 2064–2086. DOI: 10.1016/j.ijfatigue.2008.07.002.

[18] Carrella-Payan, D., Magneville, B., Hack, M., Lequesne, C., Naito, T., Urushiyama, Y., Yamazaki, W., Yokozeki, T. and Van Paepegem, W. (2016). Implementation of fatigue model for unidirectional laminate based on finite element analysis: theory and practice, Frattura ed Integrità Strutturale, 38, pp. 184-190. DOI: 10.3221/IGF-ESIS.38.25.
[19] Miner, M.A. (1945). Cumulative damage in fatigue, J. Appl. Mech., 12, pp. 159–164.
[20] Fatemi, A. and Yang, L. (1998). Cumulative fatigue damage and life prediction theories: A survey of the state of the art for homogeneous materials, Int. J. Fatigue, 20(1), pp. 9-34. DOI: 10.1016/S0142-1123(97)00081-9.
[21] Liu, Y. and Mahadevan, S. (2007). Stochastic fatigue damage modeling under variable amplitude loading, Int. J. Fatigue, 29(6), pp. 1149–1161. DOI: 10.1016/j.ijfatigue.2006.09.009.
[22] Santecchia, E., Hamouda, A.M.S., Musharavati, F., Zalnezhad, E., Cabibbo, M., El Mehtedi, M. and Spigarelli, S. (2016). A Review on Fatigue Life Prediction Methods for Metals, Advances in Materials Science and Engineering, 2016, No 9573524. DOI: 10.1155/2016/9573524.
[23] Marco, S.M. and Starkey, W.L. (1954). A concept of Fatigue damage, Trans. ASME, 76(4), pp. 627–640. DOI: 10.1115/1.4014922.
[24] Okutan, B. (2002). The effects of geometric parameters on the failure strength for pin-loaded multi-directional fiber-glass reinforced epoxy laminate, Composites Part B: Engineering, 33(8), pp. 567–578. DOI: 10.1016/S1359-8368(02)00054-9.
[25] Aeran, A., Siriwardane, S.C., Mikkelsen, O. and Langen, I. (2017). A new nonlinear fatigue damage model based only on S-N curve parameters. Int. J. Fatigue, 103, pp. 327–341. DOI: 10.1016/j.ijfatigue.2017.06.017.
[26] Nouri, H., Meraghni, F. and Lory, P. (2009). Fatigue damage model for injection-molded short glass fibre reinforced thermoplastics, Int. J. Fatigue, 31(5), pp. 934–942. DOI: 10.1016/j.ijfatigue.2008.10.002.
[27] Zuo, F.-J., Huang, H.-Z., Zhu, S.-P., Lv, Z. and Gao, H. (2015). Fatigue life prediction under variable amplitude loading using a non-linear damage accumulation model, International Journal of Damage Mechanics, 24(5), pp. 767–784. DOI: 10.1177/1056789514553042.
[28] Van Paepegem, W., Degrieck, J. and De Baets, P. (2001). Finite element approach for modelling fatigue damage in fibre-reinforced composite materials, Composites Part B: Engineering, 32(7), pp. 575–588. DOI: 10.1016/S1359-8368(01)00038-5.
[29] Carraro, P.A and Quaresimin, M. (2018). Fatigue damage and stiffness evolution in composite laminates: a damage-based framework, Procedia engineering, 213, pp. 17–24. DOI: 10.1016/j.proeng.2018.02.003.
[30] Hwang, W. and Han, K.S. (1986). Cumulative Damage Models and Multi-Stress Fatigue Life Prediction, Journal of Composite Materials, 20(2), pp. 125–153. DOI: 10.1177/002199838602000202.
[31] Kawai, M. and Ishizuka, Y. (2018). Fatigue life of woven fabric carbon/epoxy laminates under alternating R-ratio loading along non-proportional path in the σm-σa plane, Int. J. Fatigue, 112, pp. 36–51. DOI: 10.1016/j.ijfatigue.2018.02.036.
[32] Subramanian, S., Reifsnider, K.L. and Stinchcomb, W.W. (1995). A cumulative damage model to predict the fatigue life of composite laminates including the effect of a fibre-matrix interphase, Int. J. Fatigue, 17(5), pp. 343–351. DOI: 10.1016/0142-1123(95)99735-S.
[33] Weibull, W. (1951). A statistical distribution function of wide applicability, J. Appl. Mech., 18(3), pp. 293–297. DOI: 10.1115/1.4010337.