Accuracy evaluation of pendulum gravity measurements of Robert Daublebsky von Sterneck

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Abstract

The accuracy of first pendulum gravity measurements in the Czech territory was determined using both original surveying notebooks of Robert Daublebsky von Sterneck and modern technologies. Since more accurate methods are used for gravity measurements nowadays, the work [3] is mostly important from the historical point of view. In previous works [5], the accuracy of Sterneck's gravity measurements was determined using only a small dataset. Here we process all Sterneck's measurements from the Czech territory (a dataset ten times larger than in the previous works [5]), and we complexly assess the accuracy of these measurements. Locations of the measurements were found with the help of original notebooks. Gravity in the site was interpolated using gravity model EGM08, resultant gravity is in actual system S–Gr10. Finally, the accuracy of Sterneck's measurements was evaluated on the base of the differences between the measured and interpolated gravity.

Keywords: Robert Daublebsky von Sterneck, relative pendulum measurements, gravity.

1. Introduction

Robert Daublebsky von Sterneck (* 7.2.1839, † 2.11.1910) was born in Prague, he acted as geodesist, astronomer and geophysicist. He was Head of Astronomical Observatory Institute in Vienna in 1880-1884 and also he was the first to make gravimetrical measurements in the Austria-Hungary. Although he worked in army all his life, he also did various surveying and astronomical measurements. His work was recognized and his name is famous also nowadays. The pendulum instrument built by Sterneck himself was used for gravity measurements, and its improved version was also used in other countries in Europe. Daublebsky used a relative method to measure gravity. Only the time of swing of the pendulum was measured with four implemented corrections. The initial gravity point was located in the cellar of Military Geographical Institute in Vienna, with value g = 980 876 mGal [2]. We divided Sterneck's measurements to two datasets. The first rule for division was different localities of the measurements (measurements on hilltops near trigonometrical points, and measurements in buildings in towns). The second rule was the time of measurements (there is a 3 year gap between the two datasets).

2. Localization of Sterneck's gravity measurements

The original Daublebsky's surveying notebooks [6] and a summary of results in technical report [4] were used for gravity measurement localization. The technical report contains approximate astronomical coordinates of the measurements, whereas detailed information about the measurement process and locations is given in the notebooks. From the technical report, were used these informations: year of measurement, number and title of point (Czech and Germany), latitude and longitude, elevation and the measured value of gravity. Only the details about the locations were used from the notebooks. These details were not registered for all measured points, - 15 points measured in towns haven't had any information about their location (these points were locallized only by approximate coordinates and heights). The measurements were divided into two groups: both by measurement location and by the time measurement. In 1889 – 1895, - 106 points were determined in the Czech territory, as is shown in Figure 1. The first group of points is located on hilltops close to know trigonometric points – hilltop dataset (blue circles in Figure 1). In 1889 – 1891 were determined 35 points in close trigonometric points and 6 points with differently locations in the Czech territory. In 1894 – 1895 (after a 3 year gap), the second group of 65 points was measured in buildings inside towns in the Moravian territory – building dataset (green squares in Figure 1).

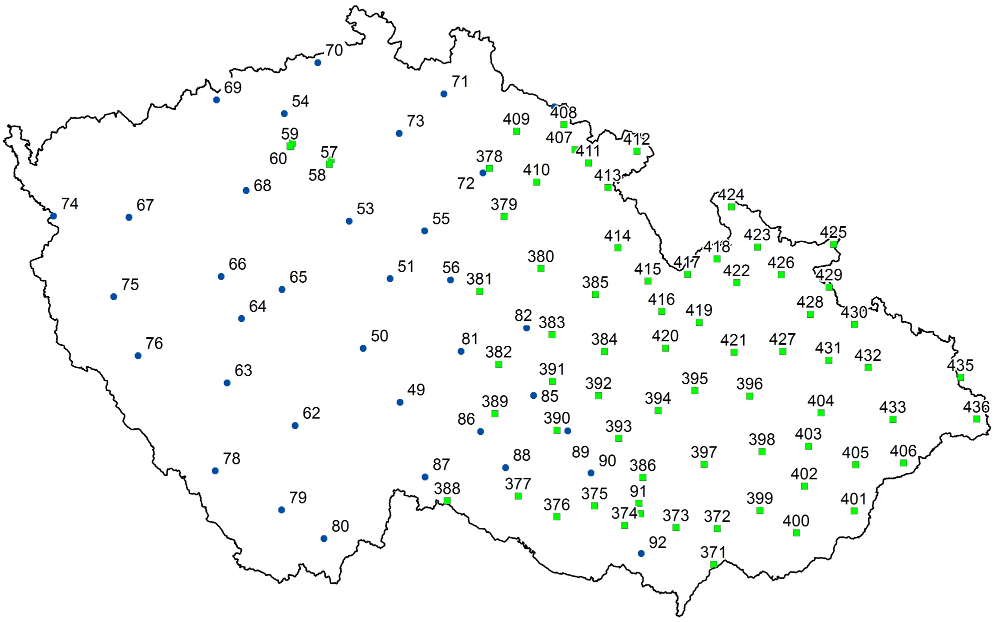

Figure 1: Locations of Sterneck's gravity measurements.

3. Determination of the gravity differences

We used the ArcMap program to determine the coordinates of the measurements with joint WMS provided by The Czech Office for Surveying, Mapping and Cadastre (ČÚZK). Coordinates of the locations with error estimates and corrections for heights (e.g. measurement in a building or on top of a lookout tower) was provided by The Department of Gravimetry, Land Survey Office (ZÚ). They intepolated the complete Bouguer anomaly using the methods of ordinary kriging. The results of interpolation are the most probable values of gravity for the referenced locations, given with their upper and lower estimate limits. The gravity value is found in this interval with 95% probability. The limits are affected by the uncertainty in elevation and position. The estimated interval isn't symmetrical and it is different for each of the measured points. Throughout this work, only the most probable gravity values were used. The gravity differences are calculated as the difference between Sterneck's measured gravity and the interpolated gravity. These differences were used to evaluate the accuracy of Daublebsky's pendulum gravity measurements.

4. Data analysis

The differences between the measured and interpolated gravity values are distinctly different for the hilltop and the building dataset. The differences gravity in the building dataset show a systematical offset +21.7 mGal, shown in Figure 2. This displacement represents a 72 meters error in elevation. The cause of this displacement isn't known, therefore both datasets were processed separately. A surprising fact about building dataset is that the gravity differences for points without precise location information (only approximate coordinates and heights) and points with these information weren't significantly different. This is illustrated in Figure 3.

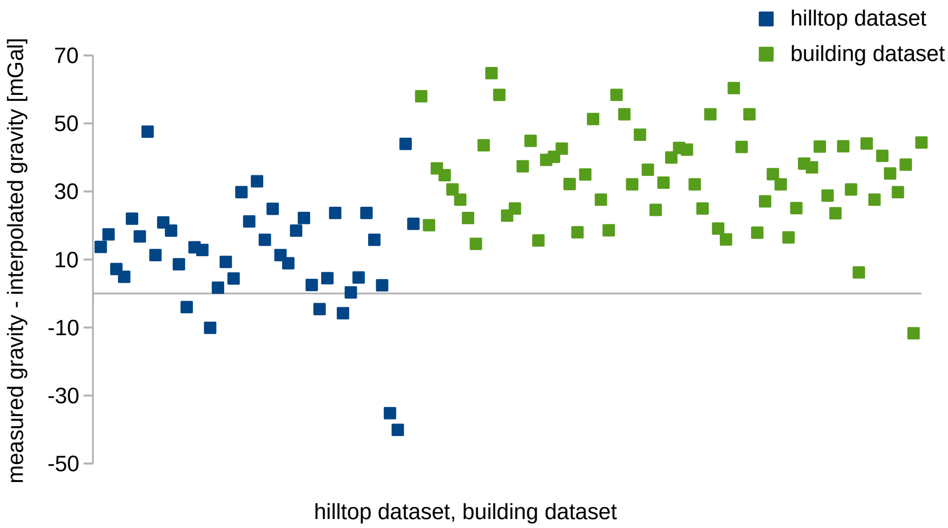

Figure 2: Differences between measured and interpolated gravity for both datasets.

The datasets were tested for data quality. Dependencies between various quantities were tested for this purpose using hypothesis verification. The computed correlation coefficient was compared with its critical value. The tested hypotheses are: gravity falls with growing elevation – (H1), gravity grows with growing latitude – (H2), and gravity and longitude are independent – (H3). All three hypotheses were verified for the hilltop dataset. In the building dataset, H2 and H3 were also verified, but H1 not. Because all of the tested quantities in the building dataset are all right, we think that the elevation values are also affected by an error different from Gaussian noise. Still, the building dataset was used in other processing.

The accuracy of Sterneck's measurements was evaluated by several methods. First, we deter-

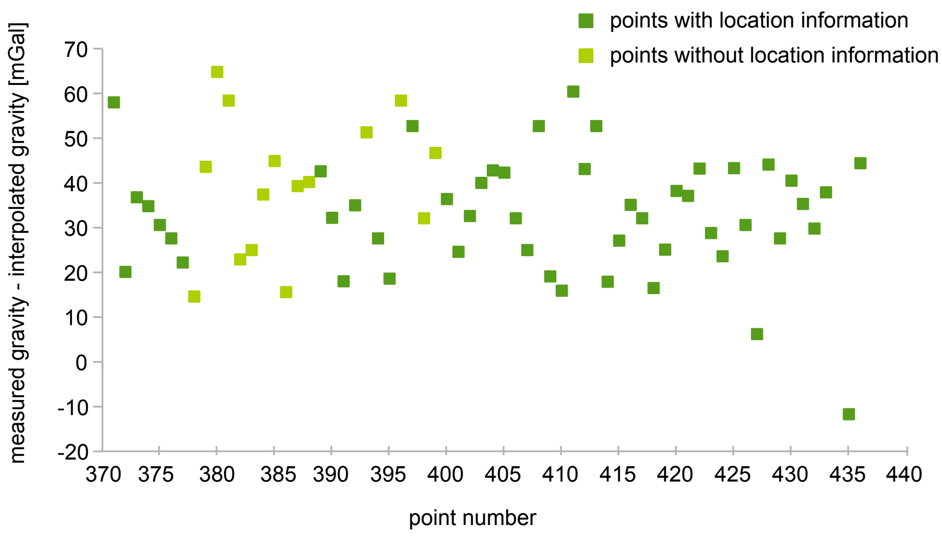

Figure 3: Differences between measured and interpolated gravity for the building dataset.

mined the value mean gravity difference. This value shows the magnitude of the difference between Sterneck's measured gravity and the interpolated gravity. The hilltop dataset has mean gravity difference +11.6 mGal, and +33.3 mGal is for the building dataset. These means are apparently affected by an unknown displacement of the used gravity systems. However, the computed differences are only valid with the assumption of null displacement between the gravity systems. If we want to compare the accuracy of the past and recent measurements, we must calculate the mean difference from absolute value of gravity difference. The datasets are characterized by the mean absolute value of gravity difference of 12.9 mGal for the hilltop dataset and 33.3 mGal for the building dataset. The second method is to evaluate the precision of the measurements using standard deviation of the mean gravity difference. This value shows precision of the measurement method and removes the systematic displacement between the two datasets. Both datasets have identical value of standard deviation equal to 10.3 mGal. The conclusion is that both datasets have identical measurement accuracy, although they were determined with different conditions and in a different environment.

5. Discussion and conclusion

Sterneck's measurements were divided into two dataset differing by both the type of the measurement locations and the time of their acquisition. The statistical processing and evaluation was done separately because of these differences. The building dataset is displaced systematically by about +21.7 mGal from the hilltop dataset (mean gravity difference 11.6 mGal for the hilltop dataset and 33.3 mGal for the building dataset). The cause of this systematic displacement is unknown. The building dataset was determined after 3 year gap. During this time some parameters of the pendulum instrument or some changes in way of calculating corrections could be changed. These changes probably can cause the systematic displacement between both of datasets. Therefore the accuracy of Sterneck's measurements is better assessed by standard deviation of the mean difference. That is 10.3 mGal and is identical for both datasets. This value can be compared with Sterneck's precision estimate of

10 mGal [4]. The mean of gravity difference 11.6 mGal for the hilltop dataset and 33.3 mGal for the building dataset can be compared to measurements in Hungary where the errors of Sterneck's measurements are up to ±20 mGal [5], (but the difference for some points is up to 25 mGal [1]).

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References

[1] Alexandr Drbal and Milan Kocáb. "Významný rakouský generálmajor Dr.h.c. Robert Daublebsky von Sterneck". In: *Geodetický a kartografický obzor* 56(98).2 (2010), pp. 40–46. URL: http://archivnimapy.cuzk.cz/zemvest/cisla/Rok201002.pdf.

[2] Martin Lederer. "Historie kyvadlových měření na území České republiky". In: *Geodetický a kartografický obzor* 58(100).6 (2012), pp. 129–133. URL: http://archivnimapy.cuzk.cz/zemvest/cisla/Rok201206.pdf.

[3] Alena Pešková. "Hodnocení přesnosti kyvadlových tíhových měření R. Sternecka". Master thesis. Czech Technival University in Prague, 2015. URL: http://geo.fsv.cvut.cz/proj/dp/2015/alena-peskova-dp-2015.pdf.

[4] Zdeněk Šimon. *Kyvadlová měření v letech 1956 - 1962*. Tech. rep. Geodetický a topografický ústav v Praze, 1962.

[5] V. B. Staněk and J. Potoček. "Vývoj a způsob měření intensity tíže v Čechách a na Moravě". In: *Zeměměřičský obzor* 1(28).6 (1940), pp. 81–87.

[6] Robert Sterneck. "Měřické sešity 1889 - 1895". Vojenský zeměpisný ústav ve Vídni. Unpublished.

Table 1: Input – Part 1

Year of measurement	Number of point	Latitude [° ']	Longitude from Ferro [° ']	Altitude [m]	Measured gravity [mGal]
1889	49	49 24	32 38	738	980 856
	50	49 36	32 20	712	980 887
	51	49 55	32 27	545	980 938
	52	50 44	33 24	1602	980 762
	53	50 08	32 08	356	981 016
	54	50 33	31 36	835	980 924
	55	50 08	32 39	213	981 070
	56	49 57	32 51	470	980 952
	57	50 22	31 57	205	981 076
	58	50 23	31 57	459	981 019
	59	50 25	31 40	202	981 060
	60	50 26	31 41	417	980 998
	61	50 25	31 41	250	981 055
1890	62	49 14	31 58	624	980 846
	63	49 22	31 29	585	980 851
	64	49 39	31 31	842	980 855
	65	49 48	31 45	659	980 911
	66	49 49	31 20	716	980 893
	67	50 01	30 40	822	980 922
	68	50 12	31 25	534	980 983
	69	50 34	31 08	921	980 920
	70	50 48	31 47	748	980 963
	71	50 44	32 39	1010	980 915
	72	50 25	32 59	430	981 016
	73	50 32	32 23	565	980 989
	74	49 58	30 10	939	980 862
	75	49 40	30 39	537	980 937
	76	49 26	30 52	724	980 877
	78	49 00	31 29	1362	980 663
	79	48 52	31 57	1084	980 716
	80	48 46	32 15	869	980 760
1890	81	49 39	32 59	709	980 849
	82	49 47	33 24	662	980 895
1891	85	49 30	33 30	693	980 881
	86	49 19	33 11	732	980 873
	87	49 05	32 51	731	980 819
	88	49 10	33 22	710	980 861
	89	49 22	33 45	639	980 841
	90	49 11	33 56	513	980 846
	91	49 05	34 16	201	981 004
	92	48 52.0	34 19.0	550	980 853

Table 2: Input – Part 2

Year of measurement	Number of point	Latitude [° ']	Longitude from Ferro [° ']	Altitude [m]	Measured gravity [mGal]
1894	371	48 51.3	34 47.7	160	980 943
	372	49 00.6	34 47.8	193	980 917
	373	48 59.7	34 31.5	226	980 943
	374	48 58.9	34 11.3	181	980 957
	375	49 03.0	33 58.8	246	980 961
	376	48 59.1	33 44.5	355	980 937
	377	49 03.3	33 28.5	465	980 925
1895	378	50 26.3	33 01.3	273	981 057
	379	50 14.5	33 09.5	228	981 068
	380	50 02.3	33 26.8	214	981 076
	381	49 54.6	33 03.5	263	981 054
	382	49 36.5	33 14.7	428	980 946
	383	49 45.7	33 34.3	569	980 935
	378	50 26.3	33 01.3	273	981 057
	379	50 14.5	33 09.5	228	981 068
	380	50 02.3	33 26.8	214	981 076
	381	49 54.6	33 03.5	263	981 054
	382	49 36.5	33 14.7	428	980 946
	383	49 45.7	33 34.3	569	980 935
	384	49 42.9	33 55.9	555	980 955
	385	49 57.3	33 49.7	287	981 030
	386	49 11.7	34 16.5	235	980 962
	387	49 02.3	34 17.1	191	980 979
	388	48 59.9	33 01.0	506	980 911
1895	389	49 23.7	33 15.5	514	980 940
	390	49 21.3	33 40.7	425	980 955
	391	49 33.7	33 36.6	574	980 922
	392	49 31.4	33 55.5	554	980 942
	393	49 21.0	34 05.3	270	980 999
	394	49 29.3	34 19.7	396	980 969
	395	49 35.4	34 33.3	410	980 953
	396	49 35.4	34 55.3	225	981 026
	397	49 16.7	34 40.0	254	981 001
	398	49 21.5	35 02.3	200	980 983
	399	49 06.3	35 03.7	209	980 958
	400	49 01.4	35 18.8	248	980 932
	401	49 08.4	35 40.7	390	980 892
	402	49 13.7	35 20.2	231	980 959
	403	49 24.0	35 20.5	316	980 972
	404	49 32.9	35 24.2	256	981 010
	405	49 20.4	35 39.7	340	980 954

Table 3: Input – Part 3

Year of measurement	Number of point	Latitude [° ']	Longitude from Ferro [° ']	Altitude [m]	Measured gravity [mGal]
1895	406	49 21.9	35 58.5	510	980 906
	407	50 33.8	33 34.9	415	981 052
	408	50 39.8	33 29.1	610	981 045
	409	50 36.7	33 10.4	462	981 052
	410	50 24.3	33 21.0	335	981 039
	411	50 30.8	33 41.0	359	981 097
	412	50 35.2	33 59.9	405	981 085
	413	50 25.1	33 49.8	337	981 069
	414	50 09.9	33 56.6	321	981 014
	415	50 02.2	34 10.0	368	981 007
	416	49 54.8	34 16.8	387	981 002
	417	50 05.1	34 25.6	567	980 972
	418	50 09.8	34 36.8	536	980 969
	419	49 53.0	34 32.3	301	981 000
	420	49 45.5	34 19.9	350	981 002
	421	49 46.3	34 47.3	235	981 025
	422	50 04.2	34 45.6	489	981 005
	423	50 13.9	34 52.5	441	981 023
	424	50 23.5	34 40.4	339	981 043
	425	50 16.5	35 22.9	238	981 081
	426	50 07.4	35 03.1	519	981 003
	427	49 47.7	35 06.6	550	980 944
	428	49 58.0	35 16.2	550	980 999
	429	50 05.4	35 22.7	313	981 041
	433	49 32.9	35 52.9	406	980 973
	435	49 45.1	36 18.3	308	980 972
	436	49 34.7	36 26.0	386	980 973