

ALGEBRAIC DESCRIPTION OF TECHNICAL CHEMICAL

SYSTEMS III.

TRANSFORMATIONS OF MATERIAL SYSTEMS

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Material systems and the changes occurring in them were described with algebraic methods in the previous paper [1]. However, there is a basic connection between the two subjects: the changes occur on the material systems, and produce new ones from them. This may be termed transformation and is described in the following manner:

$$v : \begin{pmatrix} a_0 \\ a_v \end{pmatrix} \quad (1)$$

This expression means that the material system a_0 is, as a consequence of the change v_1 , transformed to material system a_v . In the following the changes will be discussed and the relations between the mentioned changes and other material systems will be examined.

In technical chemical processes, the changes occur in operational units [2]. The material system of the operational unit is a composition of the starting material system with that of being produced. In the following this will be termed a quasi union of the two material systems and is designated by:

$$\hat{a} = a_0 \cup a_v$$

The material system of the operational unit and the change occurring in it is also a description of the technical chemical process. This is termed a change-material composition and is designated by:

$$v_1 \diamond \hat{a} \quad (3)$$

The content of Equations (1) and (3) being identical:

$$v_1 \cdot \left(\begin{array}{c} \hat{a} \\ a_c \\ a_v \end{array} \right) = v_1 \diamond \hat{a} \quad (4)$$

may be written, provided that:

$$\hat{a} = a_c \cup a_v$$

Expression (4) is called a Z technical chemical transformation, or - briefly - a Z transformation, and its designation is:

$$Z = [v_1 : \left(\begin{array}{c} \hat{a} \\ a_c \\ a_v \end{array} \right) \diamond v_1 \diamond \hat{a}] \quad (5)$$

An explanation of the concepts so far presented and a few new concepts are given in the following.

Starting and Resultant Material System

Let $A \cup a_i$ be a set of material systems. Four relations are interpreted in connection with this set according to the following:

Elements a_i and a_j are in relation φ_1 if - and only if - they are material streams continuously entering into an operational unit. Relation φ_1 defines a partial set A_0 of the set A:

$$a_i, a_j \in A$$

if

$$a_i \varphi_1 a_j$$

we may write

$$a_i, a_j \in A_0$$

However, instead of designation $A_0 = \{a_{0,1}; a_{0,2}; \dots a_{0,n}\}$ the following can be introduced:

$$A_0 = a_{0,1} \circ a_{0,2} \circ \dots \dots \circ a_{0,n} \quad (7)$$

Elements a_i and a_j are in relation φ_2 if - and only if - they are material streams continuously discharging from an operational unit. In a manner totally similar to the previous case we can write:

$$A_v = a_{v,1} \circ a_{v,2} \circ \dots \dots \circ a_{v,n} \quad (8)$$

Elements a_i and a_j are in relation φ_3 if - and only if - they represent the material system of a given operational unit at the beginning of the process:

$$A_0^x = a_{0,1}^x \circ a_{0,2}^x \circ \dots \dots \circ a_{0,n}^x \quad (9)$$

Elements a_i and a_j are in relation φ_4 if - and only if - they represent the material system of a given operational unit at the end of the process:

$$A_v^x = a_{v,1}^x \circ a_{v,2}^x \circ \dots \dots \circ a_{v,n}^x \quad (10)$$

In the following, A_0 and A_0^x will be termed starting material systems, A_v and A_v^x resultant material systems.

Change

The definition of ZADEK [3] can be applied to material systems. According to this, the system is the totality of objects which are connected by interactions and mutual connections. The following items of information will be considered as the objects of the material system: crystal structure, chemical structure, biological structure, state, dimensions, distribution, form, temperature, pressure, homogeneous connection and heterogenous con-

nection. The material system is the structure of these, the structure being defined by the mutual connections. It is the objects that alter during a change. The set of objects which form the material system a_1 will be denoted by $\eta(a_1)$. In accordance with this, the connection between the change and the material systems will be described by the following equation:

$$v = [\eta(A_0) \cup \eta(A_0^x)] \setminus [\eta(A_v) \cup \eta(A_v^x)] \quad (11)$$

(The symbols that are usual in the theory of sets are used: $A \cap B$ is the common part of sets A and B ; $A \cup B$ is the combination of sets A and B ; $A \setminus B$ is the difference of sets A and B .)

The change:

$$v = \eta(A_0) \setminus \eta(A_v) \quad (12)$$

will in the following be termed stationary, whereas the change:

$$v^x = \eta(A_0^x) \setminus \eta(A_v^x) \quad (13)$$

will be termed intermitted and the change:

$$v^o = [\eta(A_0) \cup \eta(A_0^x)] \setminus [\eta(A_v) \cup \eta(A_v^x)] \quad (14)$$

unstationary.

The changes pertaining to the object difference of the material system were given in the previous paper [1]. The change is termed elementary if the sum of the difference set is one. In this case, the resultant material system produced by the change differs from the starting material system in one object only.

Transformation

On the basis of the aforesaid, the transformation according to Equation (1) can in general be given in the following form: the expression

$$v_1 : \begin{pmatrix} A_O + A_O^x \\ A_V + A_O^x \end{pmatrix} \quad (15)$$

is termed transformation if the condition:

$$v_1 = [\eta(A_O) \cup \eta(A_O^x)] \setminus [\eta(A_V) \cup \eta(A_V^x)]$$

is fulfilled.

A transformation transforming a set A_O to A_V is termed stationary (v); that transforming A_O^x to A_V^x is termed intermittent (v^x); that transforming $(A_O + A_O^x)$ to $(A_V + A_V^x)$ is termed unstationary (v^o). The transformation can be regarded as the internal transformation of set A , since:

$$A_O, A_O^x, A_V, A_V^x, A_V^x \leq A \quad (16)$$

If the objects of the starting and resultant material systems are identical, the transformation is, in the algebraic sense of the word, a permutation. For example, the transformation:

$$v_9 \wedge \delta_4 : \begin{pmatrix} K_1 \Rightarrow K_2 \rightarrow K_3 \\ K_1 \Rightarrow K_3 \rightarrow K_2 \end{pmatrix}$$

is a permutation.

The transformation is termed a multiple one, if more than one of the objects of the starting system are changed. A multiple transformation may be homogeneous, when the same change occurs more than once, for example:

$$(v_9 \wedge \delta_2)^2 : \begin{pmatrix} K_1 \rightarrow K_2 \rightarrow K_3 \\ K_1 \Rightarrow K_2 \Rightarrow K_3 \end{pmatrix}$$

As can be seen, the homogeneous transformation is designated by v^m . Here the exponent shows how many times the change occurs. If the starting material system is such that one given change may occur m_t times, but $m < m_t$, the transformation is selective with respect to the material system. For example:

$$v_0 \wedge \delta_2 : \left(\begin{array}{c} K_1 \rightarrow K_2 \rightarrow K_3 \\ K_1 \implies K_2 \rightarrow K_3 \end{array} \right)$$

It is apparent from the above that the starting and resultant material systems define the change in an unequivocal way, whereas the reverse is not true.

The multiple transformation may be heterogenous, when more than one change occurs. For example:

$$(v_1 \wedge \delta_1) \wedge (v_0 \wedge \delta_2) : \left(\begin{array}{c} K_1 \rightarrow K_2 \rightarrow K_3, T_1 \\ K_1 \implies K_2 \rightarrow K_3, T_2 \end{array} \right)$$

(T_1 and T_2 represent temperatures.)

The transformation is of identical order if the number of the materials of the starting and the resultant material system is the same; if the number of the starting materials is higher, the transformation is of the combining, if lower, it is of the decomposing type:

$$v : \left(\begin{array}{cc} a_{0,1} & a_{0,2} \\ a_{v,1} & a_{v,2} \end{array} \right); v : \left(\begin{array}{cc} a_{0,1} & a_{0,2} \\ & a_{v,1} \end{array} \right); v : \left(\begin{array}{cc} a_{0,1} & \\ & a_{v,2} \end{array} \right) \quad (16)$$

If it is continuous and not discrete objects that are altered during the change, the degree of change can be given by designating it by s and writing it in the exponent. Two such continuous transformations can be:

$$v_1^{s_1} : \left(\begin{array}{c} a_0 \\ a_{v,1} \end{array} \right) \quad \text{and} \quad v_2^{s_2} : \left(\begin{array}{c} a_0 \\ a_{v,2} \end{array} \right) \quad (17)$$

and then we set the following postulations:

$$0 < s_1 < 1 : \quad 0 < s_2 < 1$$

if $s_1 = 0$, $a_0 = a_{v,1}$

if $s_1 > s_2$, the deviation of $a_{v,1}$ from a_0 is greater than that of $a_{v,2}$; i.e. if the changing object is x , we may write

$$|x_{v,1} - x_0| > |x_{v,2} - x_0|$$

The transformation $v^s: (a_1^2)$ is termed the inverse transformation of $v^{-s}: (a_2^1)$. In the case of discrete changes, s is an integral number.

The transformation $v_1: (A_{v,1}^0, 1)$ and $v_2: (A_{v,2}^0, 2)$ are termed similar if:

$$v_1 = v_2 \quad (18)$$

The two transformations mentioned in the above are equal if:

$$A_{0,1} = A_{0,2} \quad (19)$$

$$A_{v,1} = A_{v,2}$$

Quasi Union

The quasi union of two material systems is not defined unequivocally; it depends on the properties of the material systems.

The steps of the definition of quasi union are the following:

The Concept of Mean Material Systems

A mean temperature and pressure are supposed for the material systems present in the operational unit; only one state pertains to a given chemical structure and the information as to distribution is disregarded. Accordingly a material system \bar{a} is obtained, whose information content is the following: crystal structure, chemical structure, biological structure, dimensions, form, homogeneous and heterogeneous connection.

The Combination (Union) of Material Systems

The following two rules are valid:

$$a_1 \cup a_1 = a_1 \quad (20)$$

$$a_1 \cup a_2 = \begin{cases} a_1 \Rightarrow a_2 \\ a_2 \Rightarrow a_1 \\ a_1 \rightarrow a_2 \\ a_2 \rightarrow a_1 \end{cases} \quad (21)$$

knowing the material systems it is possible to decide which equality is the appropriate one.

Accordingly, the quasi union of the operational unit can be defined, taking the aforesaid into consideration, in the following manner:

$$\hat{a} = A_o^x \cup A_o^x \cup A_v^x \cup A_v^x \quad (22)$$

The particular components of the material system \hat{a} present in the operational unit are the auxiliary materials. A filling auxiliary material (\hat{a}'') is termed that is added to the system at the beginning of the process and can be removed in an unchanged state at the end of the process:

$$\hat{a}'' = A_o^x \cap A_v^x \quad (23)$$

A material system introduced continuously into the operational unit and leaving it in an unchanged state is termed recirculating auxiliary material (\hat{a}'):

$$\hat{a}' = A_o \cap A_v \quad (24)$$

The material system changing during the process can be described by:

$$\hat{a}' = A_c^x \setminus A_v^x \cup A_v^x \setminus A_o^x \cup A_c^x \setminus A_v^x \cup A_v^x \setminus A_o^x \quad (25)$$

On the basis of Equations (22) to (25) we can write:

$$\hat{a} = \hat{a}' \cup \hat{a}'' \cup \hat{a}'' \quad (26)$$

Permutations

According to the algebraic interpretation, the following transformations can be regarded as permutations:

$$\begin{array}{ll} (v_5 \wedge \delta_1) \wedge (v_5 \wedge \delta_1)^{-1} & \text{heat exchange} \\ (v_7 \wedge \delta_1) \wedge (v_7 \wedge \delta_1)^{-1} & \text{e.g. rectification} \\ v_9 \wedge \delta_4 & \text{e.g. hypersorption} \end{array}$$

None of the other transformations is a permutation.

Connection Between the Starting Material Systems and the Change

Certain starting material systems postulate the occurrence of a given change.

Such are the following:

A_0 postulates v_1 . If there are two entering material streams ($A_{0,1} \circ A_{0,2}$) they postulate $v_8 \wedge \delta_2$ in order to reach the state $A(K_1 \leftrightarrow K_2)$. The reverse of the above is that two leaving material streams postulate $v_8 \wedge \delta_3$.

If the temperature of the entering material streams is different, the v_5 temperature change will occur, whereas in the case of a difference in pressure the result will be the v_6 change in pressure.

A homogeneous system may be formed, i.e. the change $v_9 \wedge \delta_2$ may occur if the entering material system or that present in the operational unit is heterogeneous. If both material systems are gases, the change $v_9 \wedge \delta_2$ always occurs; if they are liquids, the change occurs in most of the cases; if they are solids, the change does not occur.

Connection Between the Material System and the Change

The starting and the resultant material systems and the change are in such a connection as to determine the third one, if the other two are given. However, this system cannot in all cases be totally free, there are some restrictions. These are summarized in the following.

A considerable part of the change may act on any type of material system, there being no restrictions. Such changes are: transportation ($v_1 \wedge \delta_1$), change in scattering ($v_2 \wedge \delta_1$), increasing or decreasing the dimensions ($v_3 \wedge \delta_1$), any type of change in temperature or pressure [$v_5 \wedge (\delta_1 \vee \delta_2 \vee \delta_3)$, $v_6 \wedge (\delta_1 \vee \delta_2 \vee \delta_3)$].

The combination of material streams is possible only in the case of two input material streams, the separation only in the case of two output material streams. The same holds for transformations $v_8 \wedge \delta_2$ and $v_8 \wedge \delta_3$; however, there are further restrictions. The heterogeneous system resulting from the change $v_8 \wedge \delta_2$ may be $\beta_a \rightarrow \beta_b$, $\beta_a \leftrightarrow \beta_b$ or $\beta_b \rightarrow \beta_a$, depending on material properties and the quantitative relations. In the case of solid-liquid systems, the heterogeneous system may be changed on addition or on removal of one of the materials; this holds both for $v_8 \wedge \delta_2$ and $v_8 \wedge \delta_3$.

A change in the form is possible only in the case of a solid system, production of form in the case of a "fictive solid material", the demolition of form in the case of a "fictive liquid or gas".

It is self-evident that the change which transforms heterogeneous ones: $v_3 \wedge \delta_2$ postulates a heterogeneous input system which - except for the case of a solid-solid heterogeneous system - may be changed to a homogeneous one. The case is just the opposite with the change of the $v_3 \wedge \delta_3$ type, with the difference that this change can separate neither a solid-solid nor a gas-gas homogeneous system.

The change $v_3 \wedge \delta_3$ transposes a heterogeneous connection, and accordingly the starting material consists of at least two mate-

rial streams, at least one of them being a heterogeneous system. The same holds true of the resultant material system. Consequently, a minimum of three components must be present, from which two or less may be solid, because such a type of change is not possible between exclusively solid materials. Furthermore, not more than one of the components may be a gas, since a heterogeneous gas-gas system cannot exist. The change $v_9 \Delta \delta_4$ transposes a homogeneous connection in the following general system:

$$\begin{aligned} A_0\{[K_1(\beta_a) \Rightarrow K_2(\beta_b)] + K_3(\beta_c)\} + v_9 \Delta \delta_4 = \\ = A_v\{K_1(\beta_a) + [K_2(\beta_b) \Rightarrow K_3(\beta_c)]\} \end{aligned} \quad (27)$$

where a, b and c may be 1, 2 and 3.

The following abbreviations were applied in the description of the combinations, for example:

$$A\{[K_1(\beta_2) \Rightarrow K_2(\beta_1)] + K_3(\beta_3)\} = (2,1,3) \quad (28)$$

Accordingly, the possible triple combinations are the following:

$$(1,1,1) \quad (2,2,2) \quad (3,3,3) \quad (28.a)$$

$$(1,1,2) \quad (1,1,3) \quad (2,2,1) \quad (2,2,3) \quad (3,3,1) \quad (3,3,2) \quad (28.b)$$

$$(1,2,3) \quad (28.c)$$

The laws decreasing the number of the possible combinations are the following:

- a) No change in state occurs, and consequently the starting and the resultant materials are of the same combination;
- b) A gas-gas heterogeneous system cannot exist;
- c) A solid-solid homogeneous system cannot be decomposed by a solid;
- d) In the case of the homogeneous connection of materials of the same state, the direction \Rightarrow is optional, $\beta_a \Rightarrow \beta_b = \beta_b \Rightarrow \beta_a$, whereas in the case of materials of different states, the direction is determined by the form of appearance;

- e) A solid material may enter from a solid homogeneous system only into another homogeneous system;
- f) A solid material present in a gas may not be exchanged for another solid;
- g) A solid-solid system may be formed only from a homogeneous system;
- h) Only one change may take place;
- i) A change must take place;
- j) A solid present in a gas may not be exchanged by a liquid, neither can a liquid be exchanged by a gas.

The combination remains unaltered by the change, only the system goes over from one permutation into another; taking the prohibitive laws into consideration, the remaining changes are the following:

From the line (28.a) there remains only the combination (2,2,2):

$$[\beta_2 \Rightarrow \beta_2'] + \beta_2'' = [\beta_2 \Rightarrow \beta_2''] + \beta_2'$$

The permutations of the second combination of line (28.b) are the following:*

	(1,1,2)	(1,2,1)	(2,1,1)
(1,1,2)	d	e	+
(1,2,1)	e	+	d
(2,1,1)	+	d	+

The permutations of the second combination of line (28.b) are the following:

	(1,1,3)	(1,3,1)	(3,1,1)
(1,1,3)	d	e	+
(1,3,1)	e	+	d
(3,1,1)	+	e	f

*The cases which do occur in reality are designated by + in the Table, whereas those prohibited are indicated by the code letter of the prohibiting law.

The permutations of the third combination of line (28.b) are the following:

	(2,2,1)	(2,1,2)	(1,2,2)
(2,2,1)	d	+	+
(2,1,2)	+	+	d
(1,2,2)	+	d	+

The permutations of the fourth combination of line (28.b) are the following:

	(2,2,3)	(2,3,2)	(3,2,2)
(2,2,3)	d	+	+
(2,3,2)	+	+	d
(3,2,2)	+	d	+

The permutations of the fifth combination of (28.b) are:

	(3,3,1)	(3,1,3)	(1,3,3)
(3,3,1)	d	b	+
(3,1,3)	b	b	b
(1,3,3)	+	b	+

The permutations of the sixth combination of (28.b) are:

	(3,3,2)	(3,2,3)	(2,3,3)
(3,3,2)	d	b	+
(3,2,3)	b	b	b
(2,3,3)	+	b	+

The permutations of the combinations of line (28.c) are:

	(1,2,3)	(1,3,2)	(2,1,3)	(2,3,1)	(3,1,2)	(3,2,1)
(1,2,3)	i	+	d	h	h	+
(1,3,2)	+	i	h	+	d	h
(2,1,3)	d	h	i	+	+	h
(2,3,1)	h	+	+	i	h	d
(3,1,2)	h	d	+	h	i	j
(3,2,1)	+	h	h	d	j	i

Starting material	Change	
A_o	$v_1 \wedge \delta_1 \vee v_2 \wedge \delta_1 \vee v_3 \wedge (\delta_2 \vee \delta_3) \vee (v_6 \vee v_5) \wedge (\delta_1 \vee \delta_2 \vee \delta_3)$	1
$A_{0,1} \circ A_{y0,2}$	$v_1 \wedge \delta_2$	2
A_{y0}	$v_1 \wedge \delta_3$	3
$A\{\beta_a\} \circ A\{\beta_b\}$	$v_8 \wedge \delta_2$	4
$A\{\beta_a \leftrightarrow \beta_b\} \circ A\{\beta_b\}$	$v_8 \wedge \delta_2$	5
$A\{\beta_a + \beta_b\} \circ A\{\beta_b\}$	$v_8 \wedge \delta_2$	6
$A\{\beta_a \leftrightarrow \beta_b\}$	$v_8 \wedge \delta_3$	7
$A\{\beta_a \leftrightarrow \beta_b\}$	$v_8 \wedge \delta_3$	8
$A\{\beta_2 \rightarrow \beta_1\}$	$v_8 \wedge \delta_3$	9
$A\{K[\beta_1, (\alpha_5)_1]\}$	$v_4 \wedge \delta_1$	10
$A\{K^+[\beta_1, \alpha_4, 0]\}$	$v_4 \wedge \delta_2$	11
$A\{K^+[\beta_a, 0, \alpha_5]\}$	$v_4 \wedge \delta_3$	12
$A\{\beta_a\}$	$v_7 \wedge \delta_1$	13
$A\{\beta_a\}$	$v_7^{-1} \wedge \delta_1$	14
$A\{\beta_1\}$	$(v_7 \wedge \delta_1)^2$	15
$A\{\beta_3\}$	$(v_7 \wedge \delta_1)^{-2}$	16
$A\{\beta_a + \beta_b\}$	$v_9 \wedge \delta_2$	17
$A\{\beta_a \Rightarrow \beta_b\}$	$v_9 \wedge \delta_3$	18
$A\{\beta_a + \beta_b\} \circ A\{\beta_c\}$	$v_8 \wedge \delta_4$	19
$A\{\beta_a \Rightarrow \beta'_a + \beta_b\}$	$v_9 \wedge \delta_4$	20
$A\{\beta_a \Rightarrow \beta'_a + \beta_b\}$	$v_9 \wedge \delta_4$	21
$A\{\beta_a \Rightarrow \beta_b + \beta'_b\}$	$v_9 \wedge \delta_4$	22
$A\{\beta_a \Rightarrow \beta_b + \beta_c\}$	$v_9 \wedge \delta_4$	23
$A\{\beta_a \Rightarrow \beta_b + \beta_c\}$	$v_9 \wedge \delta_4$	24
$A\{K[(x_{14})_1, s_1]\}$	$v_{13} \wedge \delta_1$	25
$A\{K[(x_{15})_1]\}$	$v_{11} \wedge \delta_1$	26
$A\{K[(x_{15})_1] \rightarrow K[(x_{15})_2]\}$	$v_{11} \wedge \delta_2$	27
$A\{K[(x_{15})_1]\}$	$v_{11} \wedge \delta_3$	28
$A\{K[(x_{15})_1] \rightarrow K[(x_{15})_1]\}$	$v_{11} \wedge \delta_4$	29
$A\{K[(x_{16})_1, s_1]\}$	$v_{11} \wedge \delta_1$	30
$A\{K[(x_{16})_1, s_{16}, s_1]\}$	$v_{11} \wedge \delta_1$	31
$A\{K[(x_{16})_1, s_{16}, s_1]\}$	$v_{11} \wedge \delta_1$	32
$A\{K[(x_{16})_1, s_1]\}$	$v_{11} \wedge \delta_1$	33

	Product	Remark
1	A_v	any material
2	A_v	any material
3	$A_{v,1} \circ A_{v,2}$	any material
4	$A\{\beta_a \rightleftharpoons \beta_b\}$	$a = 1,2,3$ $b = 1,2,3$
5	$A\{\beta_b \rightarrow \beta_a\}$	$a = 1,2$ $b = 1,2$
6	$A\{\beta_a \leftrightarrow \beta_b\}$	$a = 1,2$ $b = 1,2$
7	$A\{\beta_a\} \cdot A\{\beta_b\}$	$a = 1,2,3$ $b = 1,2,3$
8	$A\{\beta_a \rightarrow \beta_b\} \cdot A\{\beta_b \rightarrow \beta_a\}$	$a = 1,2$ $b = 1,2$
9	$A\{\beta_1 \leftrightarrow \beta_2\} \circ A\{\beta_2\}$	
10	$A\{K[\beta_1, (\alpha_5)_2]\}$	
11	$A\{K[\beta_1, \alpha_4, \alpha_5]\}$	
12	$A\{K[\beta_a, \circ, \circ]\}$	$a = 2,3$
13	$A\{\beta_{a+1}\}$	$a = 1,2$
14	$A\{\beta_{a-1}\}$	$a = 2,3$
15	$A\{\beta_2\}$	
16	$A\{\beta_1\}$	
17	$A\{\beta_a \Rightarrow \beta_r\}$	$a = 1,2,3$ $r = 1,2,3$ $a \neq r \neq 1$
18	$A\{\beta_a \rightarrow \beta_r\}$	$a = 1,2,3$ $r = 1,2,3$ $a \neq r \neq 3$
19	$A\{\beta_a\} \cdot A\{\beta_b \rightarrow \beta_c\}$	$a, b, c = 1,2,3$ (one only twice, 3 only once)
20	$A\{\beta_a \Rightarrow \beta'_a = \beta_r \rightarrow (\beta'_a \cdot \beta_r)\}$	$a = 2,3$ $r = 1,2,3$
21	$A\{\beta_r \Rightarrow \beta'_a + \beta_a\}$	$a, b = 1,2,3$ $a \neq r$
22	$A\{\beta_a \Rightarrow \beta'_a + \beta_r\}$	$a = 1,2; b = 1,2,3; a \neq r; a = 3; r = 2$
23	$A\{\beta_a \Rightarrow \beta_c \rightarrow \beta_r\}$	$a = 1,2; b, c = 1,2,3; a \neq r \neq c$
24	$A\{\beta_c \Rightarrow \beta'_c \cdot \beta_a\}$	$a, r, c = 1,2,3$ $a \neq r \neq c$
25	$A\{K[(\alpha_2, 4)_2, \beta_1]\}$	
26	$A\{K[(\alpha_1, 5)_2]\}$	
27	$A\{K[(\alpha_1, 5)_3]\}$	
28	$A\{K[(\alpha_1, 5)_2] \Rightarrow K[(\alpha_1, 5)_1, 1]\}$	
29	$A\{K[(\alpha_1, 5)_2] \Rightarrow K[(\alpha_1, 5)_1, 1]\}$	
30	$A\{K[(\alpha_1, 5)_1, \beta_1]\}$	
31	$A\{K[(\alpha_1, 5)_1, \beta_1, \beta_1]\}$	$\beta_1 = 1,2,3$
32	$A\{K[(\alpha_1, 5)_1, \beta_1, \beta_1, \beta_1]\}$	$\beta_1 = 1,2,3$
33	$A\{K[(\alpha_1, 5)_1, \beta_1, \beta_1, \beta_1, \beta_1]\}$	$\beta_1 = 1,2,3$
34	$A\{K[(\alpha_1, 5)_1, \beta_1, \beta_1, \beta_1, \beta_1, \beta_1]\}$	$\beta_1 = 1,2,3$
35	$A\{K[(\alpha_1, 5)_1, \beta_1, \beta_1, \beta_1, \beta_1, \beta_1, \beta_1]\}$	$\beta_1 = 1,2,3$

The chemical change v_{11} postulates a homogeneous material system, $v_{11}\wedge\delta_2$ acts in the case of at least two input, and $v_{11}\wedge\delta_3$ at least two output components. $v_{11}\wedge\delta_4$ acts in the case of two input and two output components.

Micro-biological changes (v_{12}) postulate a solid material.

The connections between the changes and the material systems are summarized in the Table.

Total Changes

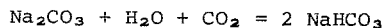
Let the sum of elementary changes, consisting of such a minimum number of terms as not to permit the occurrence of a fictive material system among the products, be termed a total change. The following principles may be defined for the production of total changes on the basis of the elementary changes:

- a) The starting material and the product of the elementary chemical changes may be homogeneous only, and the chemical change is to be complemented to a total change in accordance with this.
- b) If the product of the chemical reaction is a solid, β_1^+ and it is complemented by supplying a crystal structure.
- c) If a solid, is produced in the change, β_1^{+++} , i.e. it is complemented by supplying a crystal structure, shape and dimensions; the reverse is true in the case of a disappearance.
- d) If the product of the change is a liquid, β_2^+ , it is complemented by supplying dimensions.
- e) In the line $0 \rightarrow$ only one step is possible in the course of the elementary change.

For the sake of brevity, only those signs were given which are of some importance in the examination.

The chemical changes are always written as quasi-homogeneous ones and they are complemented with elementary changes, depending on the phases, so as to obtain total changes.

For example:



The description of a chemical reaction as a quasi-homogeneous change:

$$\begin{aligned} A\{K[\text{H}_2\text{O}, \beta_2] \Rightarrow K[\text{Na}_2\text{CO}_3, \beta_1] \Rightarrow K[\text{CO}_2, \beta_3]\} + v_{11}\Delta\delta_2 &= \\ &= A\{K[\text{H}_2\text{O}, \beta_2] \Rightarrow K[\text{NaHCO}_3, \beta_1]\} \end{aligned}$$

The supplementary elementary changes are the following:

$$\begin{aligned} A\{K[\text{H}_2\text{O}, \beta_2]\} \circ A\{K[\alpha_{14}, \text{Na}_2\text{CO}_3, \beta_1, \alpha_4, \alpha_5]\} + v_2\Delta\delta_2 &= \\ &= A\{K[\text{H}_2\text{O}, \beta_2] \rightarrow K[\alpha_{14}, \text{Na}_2\text{CO}_3, \beta_1, \alpha_4, \alpha_5]\} \end{aligned}$$

$$\begin{aligned} A\{K[\text{H}_2\text{O}, \beta_2] \rightarrow K[\alpha_{14}, \text{Na}_2\text{CO}_3, \beta_1, \alpha_4, \alpha_5]\} + v_8\Delta\delta_2 &= \\ &= A\{K[\text{H}_2\text{O}, \beta_2] \Rightarrow K^{+++}[\alpha_{14}, \text{Na}_2\text{CO}_3, \beta_1, \alpha_4, \alpha_5]\} \end{aligned}$$

$$\begin{aligned} A\{K[\text{H}_2\text{O}, \beta_2] \Rightarrow K^{+++}[\alpha_{14}, \text{Na}_2\text{CO}_3, \beta_1, \alpha_4, \alpha_5]\} + v_{11}\Delta\delta_1 &= \\ &= A\{K[\text{H}_2\text{O}, \beta_2] \Rightarrow K^{++}[\text{C}, \text{Na}_2\text{CO}_3, \beta_1, \alpha_4, \alpha_5]\} \end{aligned}$$

$$\begin{aligned} A\{K[\text{H}_2\text{O}, \beta_2] \Rightarrow K^{++}[\text{C}, \text{Na}_2\text{CO}_3, \beta_1, \alpha_4, \alpha_5]\} + v_4\Delta\delta_3 &= \\ &= A\{K[\text{H}_2\text{O}, \beta_2] \Rightarrow K^+[\text{C}, \text{Na}_2\text{CO}_3, \beta_1, \alpha_4, 0]\} \end{aligned}$$

$$\begin{aligned} A\{K[\text{H}_2\text{O}, \beta_2] \Rightarrow K^+[\text{C}, \text{Na}_2\text{CO}_3, \beta_1, \alpha_4, 0]\} + v_3\Delta\delta_3 &= \\ &= A\{K[\text{H}_2\text{O}, \beta_2] \Rightarrow K[\text{C}, \text{Na}_2\text{CO}_3, \beta_1, 0, 0]\} \end{aligned}$$

$$\begin{aligned} A\{K[\text{H}_2\text{O}, \beta_2] \Rightarrow K[\text{C}, \text{Na}_2\text{CO}_3, \beta_1, 0, 0]\} \circ A\{K[\text{CO}_2, \beta_3]\} + v_3\Delta\delta_2 &= \\ &= A\{K[\text{H}_2\text{O}, \beta_2] \Rightarrow K[\text{C}, \text{Na}_2\text{CO}_3, \beta_1, 0, 0] + K[\text{CO}_2, \beta_3]\} \end{aligned}$$

$$\begin{aligned} A\{K[H_2O, \beta_2] \Rightarrow K[O, Na_2CO_3, \beta_1, O, O] + K[CO_2, \beta_3]\} + v_9 \Delta \delta_2 = \\ = A\{K[H_2O, \beta_2] \Rightarrow K[O, Na_2CO_3, \beta_1, O, O] \Rightarrow K[CO_2, \beta_3]\} \end{aligned}$$

$$\begin{aligned} A\{K[H_2O, \beta_2] \Rightarrow K[O, NaHCO_3, \beta_1, O, O]\} + v_3 \Delta \delta_3 = \\ = A\{K[H_2O, \beta_2] + K^{+++}[O, NaHCO_3, \beta_1, O, O]\} \end{aligned}$$

$$\begin{aligned} A\{K[H_2O, \beta_2] + K^{+++}[O, NaHCO_3, \beta_1, O, O]\} + v_{10} \Delta \delta_2 = \\ = A\{K[H_2O, \beta_2] + K^{++}[\alpha_{14}, NaHCO_3, \beta_1, O, O]\} \end{aligned}$$

$$\begin{aligned} A\{K[H_2O, \beta_2] + K^{++}[\alpha_{14}, NaHCO_3, \beta_1, O, O]\} + v_4 \Delta \delta_2 = \\ = A\{K[H_2O, \beta_2] + K^+[\alpha_{14}, NaHCO_3, \beta_1, O, \alpha_5]\} \end{aligned}$$

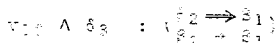
$$\begin{aligned} A\{K[H_2O, \beta_2] + K^+[\alpha_{14}, NaHCO_3, \beta_1, O, \alpha_5]\} + v_3 \Delta \delta_2 = \\ = A\{K[H_2O, \beta_2] + K[\alpha_{14}, NaHCO_3, \beta_1, \alpha_4, \alpha_5]\} \end{aligned}$$

$$\begin{aligned} A\{K[H_2O, \beta_2] + K[\alpha_{14}, NaHCO_3, \beta_1, \alpha_4, \alpha_5]\} + v_2 \Delta \delta_3 = \\ = A\{K[H_2O, \beta_2]\} \circ A\{K[\alpha_{14}, NaHCO_3, \beta_1, \alpha_4, \alpha_5]\} \end{aligned}$$

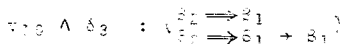
$$\begin{aligned} A\{K[H_2O, \beta_2]\} \circ A\{K[\alpha_{14}, NaHCO_3, \beta_1, \alpha_4, \alpha_5]\} + v_1 \Delta = \\ = A\{K[\alpha_{14}, NaHCO_3, \beta_1, \alpha_4, \alpha_5]\} \circ A\{K[H_2O, \beta_2]\} \end{aligned}$$

Partial Transformations

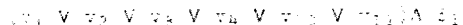
In the overwhelming majority of cases, the total transformations do not go on to completion, but only partially. For example, the symbol of the transformation corresponding to crystallization:



expresses that according to the "picture" there is no solid (β_1) dissolved (\Longrightarrow) in the liquid (δ_2). If the transformation occurs only partially, crystallization can be expressed by the following symbol:



If the crystallization is continued on the produced material system $\beta_2 \Longrightarrow \beta_1 + \beta_1$, a material system $\beta_2 \Longrightarrow \beta_1 + \beta_1$ is obtained again, despite the fact that the two systems differ from each other. This difference is a quantitative one; however, up to now, quantitative discriminations were not made. Neither will the definition of the concrete quantity of the materials be needed in the following algebraic description; it is only necessary that a system of definitions be applied which enables the materials of different quantities to be discriminated. This was already carried out in transformations where the change is continuous by the introduction of the degree of change. Such transformations are the following:



There is no need for a separate discrimination of quantity in the case of these transformations. Similarly, it is not necessary to introduce such a designation in the transformations which do not occur individually.



The other transformations are dealt with in more detail.

In the formation and dissolution of a heterogeneous connection, the output material system is equal to the unified material

system in such a way that the quantity of one of the phases is changed. The quantitative parameter is designated by p and q and the only reservations are that $0 < p, q < 1$ and that β_a^{P1} ; β_a^{P2} if $p_1 > p_2$ and β_a^{P1} means in this case a larger quantity of material than β_a^{P2} .

Accordingly:

$$v_8 \Delta \delta_i: \left(\begin{array}{l} \beta_a \rightarrow \beta_b^{P1} \beta^{Q1} \\ \beta_a \rightarrow \beta_b^{P2} \beta^{Q2} \end{array} \right) \begin{array}{l} p_1 > p_2 \quad i=3 \text{ and } p_2=0 \\ p_1 < p_2 \quad i=2 \text{ and } p_2=1 \end{array} \begin{array}{l} \text{corresponds to a perfect} \\ \text{decomposition} \\ \text{corresponds to a perfect} \\ \text{combination} \end{array}$$

The following equations are valid:

$$p_1 + q_1 = 1$$

$$p_2 + q_2 = 1$$

$$p_1 + p_2 = 1$$

In cases where the above Equations hold, q_1 and q_2 will not be written in the following.

In the case of the formation and dissolution of the homogeneous connection, in a way similar to the above, we may write:

$$v_9 \Delta \delta_i: \left(\begin{array}{l} \beta_a \rightarrow \beta_b^{P1} \rightarrow \beta_b \\ \beta_a \rightarrow \beta_b^{P2} \rightarrow \beta_b \end{array} \right) \begin{array}{l} p_1 > p_2 \quad i=3 \quad p_2=0 \\ p_1 < p_2 \quad i=2 \quad p_2=1 \end{array} \begin{array}{l} \text{perfect decomposition} \\ \text{perfect combination} \end{array}$$

In the case of chemical combination and decomposition:

$$v_{11} \Delta \delta_i: \left(\begin{array}{l} [\beta_a \rightarrow \beta_b]^{P1} \rightarrow \beta_c \\ [\beta_a \rightarrow \beta_b]^{P2} \rightarrow \beta \end{array} \right) \begin{array}{l} p_1 > p_2 \quad i=2 \quad p_1=1 \\ p_1 < p_2 \quad i=3 \quad p_1=0 \end{array} \begin{array}{l} \text{total combination} \\ \text{total decomposition} \end{array}$$

In changes of state, materials of both the states are present in the starting and resultant material systems:

$$(v_7 \wedge \delta_1)_i: \begin{cases} \beta_1^{p_1} \Rightarrow \beta_2 & p_1 > p_2 \quad i=1 \quad p_2=0 \quad \text{total change} \\ \beta_1^{p_2} \Rightarrow \beta_2 & p_1 < p_2 \quad i=-1 \quad p_2=1 \quad \text{total change} \end{cases}$$

In the translocation of a heterogeneous connection:

$$v_8 \wedge \delta_4: \begin{cases} \beta_a \rightarrow \beta_b^{p_1} \circ \beta_c \rightarrow \beta_b \\ \beta_a \rightarrow \beta_b^{p_2} \circ \beta_c \rightarrow \beta_b \end{cases} \quad \text{if} \quad p_2 = \begin{cases} 0 \\ 1 \end{cases} \quad \text{total change}$$

In the translocation of a homogeneous connection:

$$v_9 \wedge \delta_4: \begin{cases} \beta_a \Rightarrow \beta_b^{p_1} \rightarrow \beta_b \Rightarrow \beta_c^{q_1} \rightarrow \beta_c \\ \beta_a \Rightarrow \beta_b^{p_2} \rightarrow \beta_b \Rightarrow \beta_c^{q_2} \rightarrow \beta_c \end{cases} \quad \begin{matrix} p_2 = 1 & q_2 = 0 \\ p_2 = 0 & q_2 = 1 \end{matrix} \quad \text{total change}$$

In chemical exchange:

$$v_{11} \wedge \delta_4: \begin{cases} [\beta_a \Rightarrow \beta_b]^{1-p_1} \Rightarrow [\beta_c \Rightarrow \beta_d]^{p_1} \\ [\beta_a \Rightarrow \beta_b]^{1-p_2} \Rightarrow [\beta_c \Rightarrow \beta_d]^{p_2} \end{cases} \quad p_2 = \begin{cases} 0 \\ 1 \end{cases} \quad \text{total change}$$

Finally, in changes of the type $v_1 \wedge (\delta_2 \vee \delta_3)$ the quantitative parameter relates to the ratio of the split material stream, i.e.:

$$v_1 \wedge \delta_i: \begin{cases} a_1^{p_1} \circ a_1^{1-p_1} & p_2 = 1 \quad i = 2 \\ a_1^{p_2} \circ a_1^{1-p_2} & p_1 = 1 \quad i = 3 \end{cases}$$

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РЕЗЮМЕ

В предыдущей сообщении этой темы (1) алгебраическими методами описывались материальные системы и происходящие в них изменения. Основная зависимость образуется между происходящими в материальной системе изменениями и получающимися вследствие их новыми системами. Это называют преобразованием и обозначают следующим образом:

$$v_1: \begin{matrix} a_0 \\ a_v \end{matrix} \quad 1.$$

Вышеуказанное выражение означает, что под действием изменения v_1 материальная система a_0 преобразуется в систему a_v . Далее авторы рассматривают различные изменения и исследуют зависимости, которые имеют место между указанными изменениями и остальными материальными системами.