On the Riesz Means of Expansions by Riesz Bases Formed by Eigenfunctions for the Ordinary Differential Operator of 4-th Order

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Abstract

The aim of this paper is prove a theorem on the Riesz mean of expansions with respect to Riesz bases, which extends the previous results of Loi and Tahir on the Schrödinger operator to the operator of 4-th order.

Introduction

The aim of this paper is to prove a theorem on the Riesz means of expansions with respect to Riesz bases, which extends the previous results of (1) and (2) of the Shrödinger operator to the operator of 4-th order.

Let $G \subset \mathbf{R}$ be an arbitrary finite open interval, $q(x) \in L^1(G)$ an arbitrary complex function and consider the operator $L\, u_r := u_r^{(4)} + q(x)\, u_r$.

Given a complex number λ, the function $u_{-1} : G \longrightarrow \mathbf{C}$, $u_{-1} \equiv 0$ is called an eigenfunction of order -1 of the operator L with the eigenvalue λ. A function $u_r : G \longrightarrow \mathbf{C}$, $u_{-1} \not\equiv 0$ ($r = 0, 1, \ldots$) is said to be an eigenfunction of order r of the operator L with the eigenvalue λ if u_r together with its derivative is absolutely continuous on every compact subinterval of G and if for almost all $x \in G$ the equation $L\, u_r(x) = \lambda\, u_r(x) - u_{r-1}(x)$ holds, where $u_{r-1}(x)$ is an eigenfunction of order $(r-1)$ with the same λ.

Let us now give a Riesz basis $(u_r(x)) \subset L^2(G)$ of the operator L.

Let λ_r (resp. o_r) denotes the eigenvalue (resp. the order) of u_r and assume that the following conditions are satisfied:

$$\sup_r o_r < \infty \qquad \text{...........[1]}$$

in case $o_r > 0$, $\lambda_r u_r - L u_r = u_{r-1}$ ....[2]

Suppose the biorthogonal system (υ_r) of the system (u_r) have the property

$$\sum_{|\mu-\rho_r|\le 1} {}_r \|\upsilon_r\|^2_{L^\infty(G)} < V < \infty \; (\mu \ge 1; \rho_k = \operatorname{Re}\sqrt{\lambda_k}, \rho_k \ge 0) \ldots[3]$$

Now consider the Riesz means of the biorthogonal series

$$\sigma^s_\mu(f,x) := \sum_{|\rho_r|<\mu} \langle f, \upsilon_r \rangle u_r(x)(1-\frac{\rho_r^2}{\mu^2})^s \; .....[4]$$

$(f \in L^1(G)$, $x \in G$, $\mu > 0$, $0 \le s < 1/2)$, where (υ_r) is the dual system of (u_r), i.e., $(\upsilon_r) \subset L^2(G)$ and $<\upsilon_r, u_j> = \delta_{rj}$.

Given any compact interval $K \subset G$, denote by R an arbitrary number from the interval $(0, \operatorname{dist}(K, \partial G))$.

Now, fix $x \in K$ arbitrary and define W^s_R: G $\longrightarrow$ **R** by

$$W^s_R(t) = \begin{cases} a(s)\mu^{\frac{1}{2}-s} |t|^{-s-\frac{1}{2}} J_{s+1/2}(\mu|t|) & \text{if } |t| \le R, \\ 0 & \text{otherwise,} \end{cases} \quad ...[5]$$

where $a(s) := 2^s(2\pi)^{-1/2}\,\Gamma(s+1)$.

Moreover, for any $f \in L^1(G)$, $x \pm R \in G$ define

$$S^s_\mu(f,x) = \int_{x-R}^{x+R} W^s_R(y-x) f(y)\,dy \; .....[6]$$

Denote by $\theta^s(x,y,\mu)$ the spectral function of the Riesz mean (i.e.)

$$\theta^s(x,y,\mu) := \sum_{\rho_r<\mu} u_r(x)\overline{\upsilon_r(y)}(1-\frac{\rho_r^2}{\mu^2})^s \; ......[7]$$

where $x, y \in G$.

Introduce the operation $D_{R_0} : L^1(G) \longrightarrow$ **R**

$$D_{R_0}[f] = \frac{2}{R_0}\int_{\frac{R_0}{2}}^{R_0} f(R)\,dR \quad ....[8]$$

where $R_0 \in (0, \operatorname{dist}(K, \partial G))$.

Theorem

Given any compact interval K ⊂ G, for all $0 \le s < 1/2$, $\mu > 0$ and $f \in L^1(G)$ the estimate

$$\sigma_\mu^s(f,x) - S_\mu^s(f,x) = O(1)\mu^{-s} \qquad ......[9]$$

Hold the uniformly on the compact interval K ⊂ G.

Remark

For the case $s = 0$, $f \in L^2(G)$ (resp. $f \in L^1(G)$) this estimate was proved in (3) (resp.(4)) without the condition $\left|\operatorname{Im}\sqrt{\lambda_r}\right| \le c$.(c is constant)

In (1) the same estimate was proved with the condition $\left|\operatorname{Im}\sqrt{\lambda_r}\right| \le c\left|\sqrt{\lambda_r}\right|^{-s}$.

The validity of [9] without this condition was proved in (2).

For the proof of this theorem we shall choose the 4-th roots $\mu_{r,j}$ (i=1,…,4) of λ_r, such that Re $\mu_{r,1} \ge \mu_{r,2} \ge \mu_{r,3} \ge \mu_{r,4}$ and put $\mu_r := \mu_{r,2}$, $\rho_r :=$ Re μ_r , $v_r := |\operatorname{Im} \mu_r|$.

Now , we have (M.B.Tahir,(5))

$$0 = \begin{vmatrix} \hat{u}_r(x) & \hat{u}_r(x-t)+\hat{u}_r(x+t) & \hat{u}_r(x-2R)+\hat{u}_r(x+2R) \\ 1 & 2\cos\mu_r t & 2\cos 2\mu_r R \\ 1 & 2Ch\,\mu_r t & 2Ch\,2\mu_r R \end{vmatrix}$$

Expanding this determinant according to the first row and using

$$\hat{u}_r(y) := u_r(y) + \int_x^y \frac{Sh\,\mu_r(y-\xi) - \sin\mu_r(y-\xi)}{2\mu_r^3} Q(\xi)\,d\xi$$

where $Q(\xi) := q(\xi)\,u_r(\xi) - u_r^*(\xi)$

and $u_r^* := L\,u_r - \lambda_r\,u_r$.

We have

$2(\text{Ch } 2\mu_r R - \cos 2\mu_r R)\,(u_r(x-t) + u_r(x+t)) =$

$4\,u_r(x)(\cos \mu_r t\, Ch\, 2\mu_r R - \cos 2\mu_r R\, Ch\, \mu_r t) + 2\,(Ch\, \mu_r t - \cos \mu_r t)\cdot$
$[u_r(x+2R) + u_r(x-2R)] + 2(Ch\, \mu_r t - \cos \mu_r t)\cdot$

$$\int_{x-2R}^{x+2R} \frac{Sh\,\mu_r(2R-|x-\xi|) - \sin\mu_r(2R-|x-\xi|)}{2\mu_r^3} Q(\xi)\,d\xi -$$

$$2\,(Ch\, 2\mu_r R - \cos 2\mu_r R) \int_{x-t}^{x+t} \frac{Sh\,\mu_r(t-|x-\xi|) - \sin\mu_r(t-|x-\xi|)}{2\mu_r^3} Q(\xi)\,d\xi,$$

dividing by $e^{2\mathrm{Re}\mu_{r,1}R}$ we obtain:

$$[u_r(x-t) + u_r(x+t)]\, d(\mu_r,R) = 2\,u_r(x)\, d_0(\mu_r,R,t) + d_2(\mu_r,R,t)\,[u_r(x-2R) + u_r(x+2R)] + \int_{x-2R}^{x+2R} \frac{D(\mu_r,R,t,|x-\xi|)}{e^{2\mathrm{Re}\mu_{r,1}R}} Q(\xi)\,d\xi, \quad \text{.......[10]}$$

where $d(\mu_r,R) := \dfrac{2(Ch\,2\mu_r R - \cos t\, 2\mu_r R)}{e^{2\mathrm{Re}\mu_{r,1}R}}$,

$$d_0(\mu_r,R,t) := \frac{2(\cos\mu_r t\; Ch\,2\mu_r R - \cos 2\mu_r R\; Ch\,\mu_r t)}{e^{2\mathrm{Re}\mu_{r,1}R}},$$

$$d_2(\mu_r,R,t) := \frac{2(\,Ch\,\mu_r t - \cos\mu_r t)}{e^{2\mathrm{Re}\mu_{r,1}R}}$$

and

$$D(\mu_r,S,t,|x-\xi|) := \begin{cases} d_2(\mu_r,R,t)\displaystyle\sum_{p=1}^{2} \frac{\omega_p\, Sh\,\omega_p \mu_r(2R-|x-\xi|)}{2\mu_r^3}, \\ -d(\mu_r,R)\displaystyle\sum_{p=1}^{2} \frac{\omega_p\, Sh\,\omega_p \mu_r(t-|x-\xi|)}{2\mu_r^3}, & \text{if } 0 \le |x-\xi| \le t, \\ d_2(\mu_r,R,t)\displaystyle\sum_{p=1}^{2} \frac{\omega_p\, Sh\,\omega_p \mu_r(2R-|x-\xi|)}{2\mu_r^{2m-1}}, & \text{if } t \le |x-\xi| \le 2R \end{cases}$$

we want to prove the following estimate.

$$\theta^s(x,y,\mu) - W_R^s(|y-x|) = O(1)\mu^{-S}, \quad \text{.........[11]}$$

We count the Fourier coefficients of the function $W_R^s(y-x)$ with respect to the system (u_r):

$$\left\langle u_r, D_{R_0} W_R^s \right\rangle = D_{R_0} \int_0^R W_R^s(t)[u_r(x-t)+u_r(x+t)]\,dt$$

$$= D_{R_0} \int_0^R W_R^s(t) \left[2u_r(x) \frac{d_0(\mu_r,R,t)}{d(\mu_r,R)} + \{u_r(x-2R)+u_r(x+2R)\} \right.$$

$$\left. \frac{d_2(\mu_r,R,t)}{d(\mu_r,R)} + \frac{e^{-2\operatorname{Re}\mu_{r,1}R}}{d(\mu_r,R)} \int_{x-2R}^{x+2R} W_R^s D(\mu_r,R,t,|x-\xi|) Q(\xi)\,d\xi \right] dt$$

since

$$D_{R_0} W_R^s(y-x) = \sum \overline{\upsilon_r(y)} \left\langle u_r, D_{R_0} W_R^s \right\rangle$$

and from ((1),p.67)

$$D_{R_0} W_R^s(t) = W_R^s(t) + O(1)\mu^{-S}$$

We obtain

$$\theta^s(x,y,\mu) - W_R^s(y-x) = O(1)\mu^{-S} + \theta^s(x,y,\mu) - D_{R_0} W_R^s(y-x),$$

By using the definition of $\theta^s(x,y,\mu), D_{R_0} W_R^s(y-x)$ and the relation $\int_0^\infty W_R^s(t)\cos\rho_r t\,dt = \left(1-\frac{\rho_r^2}{\mu^2}\right)$ (see 1), p.61)

We have the following:

$$\theta^s(x,y,\mu) - W_R^s(|y-x|) = O(1)\mu^{-S} + c\sum_r u_r(x)\overline{\upsilon_r(y)} \ldots [12]$$

$$\left[D_{R_0} \int_R^\infty W_R^s(t)\cos\rho_r t\,dt - D_{R_0} \int_0^R W_R^s(t)\cos\rho_r t \left(\frac{Ch\,2\,\mu_r R\,Ch\,\nu_r t)}{e^{2\operatorname{Re}\mu_{r,1}R} d(\mu_r,R)} - 1 \right) dt - \right.$$

$$\left. i\,D_{R_0} \int_R^\infty \frac{W_R^s(t)\sin\rho_r t\,Sh\,\nu_r t}{d(\mu_r,R)e^{2\operatorname{Re}\mu_{r,1}R}} dt - D_{R_0} \int_0^R W_R^s(t) \frac{\cos 2\,\mu_r R\,Ch\,\mu_r t)}{d(\mu_r,R)e^{2\operatorname{Re}\mu_{r,1}R}} dt \right] -$$

$$c\sum_r \overline{\upsilon_r(y)}D_{R_0}\int_0^R W_R^s(t)(u_r(x+2R)+u_r(x-2R))\frac{d_2(\mu_r,R,t)}{d_1(\mu_r,R,t)}dt -$$

$$c\sum_r \overline{\upsilon_r(y)}D_{R_0}\int_0^R W_R^s(t)\frac{e^{-2\text{Re}\mu_{r,1}R}}{d(\mu_r,R)}\int_{x-2R}^{x+2R} D(\mu_r,R,t,|x-\xi|)Q(\xi)d\xi dt$$

Let

$$\theta^s(x,y,\mu)-W_R^s(|y-x|)=O(1)\mu^{-S}+\sum_{i=1}^{6}H_i \text{ ....[13]}$$

where H_1, …, H_6 denote the first, …, sixth integral of the right hand side of [12]

we want to find the estimates of H_i ($i = 1, \ldots, 6$).

Firstly, we know that (M.B.Tahir (5)), for $m = 2$.

$$\left|\frac{D(\mu_r,R,t,|x-\xi|)}{e^{2\text{Re}\mu_{r,1}R}}\right| \le c|\mu_r|^{-3}e^{\rho_r R}\min\{1,|\mu_r|t\} \text{ ....[14]}$$

and

$$\int_R^{2R}\left|\frac{d(\mu_r,\alpha)}{e^{2\text{Re}\mu_{r,1}R}}\right|d\alpha \ge \delta>0 \quad (\delta=\delta(R_0)) \text{........[15]}$$

if $R_0 > 0$, $\frac{R_0}{2} \le R \le R_0$ and $|\mu_r| \ge A = A(R_0) \ge 1$.

From ((1), P.63), we have

$$|H_1| \le \mu^{-s}\frac{c(R_0,s)}{1+|\mu-\rho_r|^2}$$

$$|H_2| \le \left|D_{R_0}\int_0^R W_R^s(t)\cos\rho_r t\left(\frac{Ch\,2\,\mu_r R\; Ch\,\nu_r t\, e^{-2\nu_r R}}{e^{2\text{Re}\mu_{r,1}R}d(\mu_r,R)}\right)dt\right|$$

$$\le c\,e^{2(\rho_r-\nu_r)R_0}\left|D_{R_0}\int_0^R W_R^s(t)\cos\rho_r t\, Ch\,\nu_r t\,dt\right|$$

(see [15])

But from ([1], p.63) this integral less than

$$\frac{\mu^{-s}c(R_0,s)Ch\,\nu_r R_0}{1+|\mu-\rho_r|^2}$$

Hence

$$|H_2|\le \mu^{-s}e^{2(\rho_r-\nu_r)R_0}\frac{c'(R_0,s)Ch\,\nu_r R_0}{1+|\mu-\rho_r|^2}\le \mu^{-s}\frac{c''(R_0,s)e^{|\nu_r|R_0}}{1+|\mu-\rho_r|^2}$$

H_2 is the only place in the proof where N.H.Lio (1) used his additional assumption on the distributional of $\lambda_r:\left(\left|\operatorname{Im}\sqrt{\lambda_r}\right|\le c\left|\sqrt{\lambda_r}\right|^{-s}\right)$. M.B. Tahir (2) proved the theorem without using this assumption and also we shall not need this assumption.

$$|H_3|\le c\,e^{-2\nu_r R_0}\left|D_{R_0}\int_0^R W_R^s(t)\sin\rho_r t\,Sh\,\nu_r t\,dt\right|$$

$$=\mu^{-s}c\,e^{-2\nu_r R_0}\left|D_{R_0}\int_0^R \mu^{\frac{1}{2}}t^{-s-\frac{1}{2}}J_{s+\frac{1}{2}}(\mu t)\sin\rho_r t\,Sh\,\nu_r t\,dt\right|$$

From the definition of $W_R^s(t)$.

But M.B.Tahir (2) gave the following estimate

$$\left|D_{R_0}\int_0^R \mu^{\frac{1}{2}}t^{-s-\frac{1}{2}}J_{s+\frac{1}{2}}(\mu t)\sin\rho_r t\,Sh\,\nu_r t\,dt\right|\le\frac{c(R_0,s)e^{|\nu_r|R_0}}{1+|\mu-\rho_r|^2}$$

Then

$$|H_3|\le c(R_0,s)\mu^{-s}\frac{e^{|\nu_r|R_0}}{1+|\mu-\rho_r|^2}$$

For the estimate of H_4, using the definition of $W_R^s(t)$ and the relation

$$|Ch\,\mu_r t|\le\begin{cases} c|\mu_r|t & \text{,if } 0\le t\le\frac{1}{|\mu_r|}\\ c\,e^{\rho_r t} & \text{,if } t>\frac{1}{|\mu_r|},\end{cases}$$

then we have

$$\left|H_4\right| \le c\left| D_{R_0}\int_0^{1/|\mu_r|} W_R^s(t)\left|\mu_r\right| t\,dt + D_{R_0}\int_{1/|\mu_r|}^{R} W_R^s(t) e^{\rho_r t}\,dt \right|$$

$$\le c\,\mu^{-s}\left[D_{R_0}\int_0^{1/|\mu_r|} \mu^{\frac{1}{2}}\left|J_{s+\frac{1}{2}}(\mu t)\right| t^{-s-\frac{1}{2}}dt + \int_{1/|\mu_r|}^{R} \mu^{\frac{1}{2}}\left|J_{s+\frac{1}{2}}(\mu t)\right| t^{-s-\frac{1}{2}} e^{\rho_r t} dt \right]$$

$$D_{R_0}\int_0^{1/|\mu_r|} \mu^{\frac{1}{2}}\left|J_{s+\frac{1}{2}}(\mu t)\right| t^{-s-\frac{1}{2}}dt \le c_1 \max_{0\le t\le 1/|\mu_r|} t^{-s} \le c\,R_0^{-s}$$

$$D_{R_0}\int_{1/|\mu_r|}^{R} \mu^{\frac{1}{2}}\left|J_{s+\frac{1}{2}}(\mu t)\right| t^{-s-\frac{1}{2}} e^{\rho_r t} dt \le c_2 e^{\rho_r R_0} \max_{1/|\mu_r|\le t\le R} t^{-s} \le c\,e^{\rho_r R_0} R_0^{-s}$$

$$\therefore \left|H_4\right| \le c\,(R_0,s)\,e^{\rho_r R_0}\mu^{-s}$$

To find the estimate of H_5, using the relation

$$\left|Ch\,\mu_r t - \cos\mu_r t\right| \le \begin{cases} c\left|\mu_r\right| t & ,\text{if } 0\le t\le \dfrac{1}{|\mu_r|} \\ c\,e^{\mathrm{Re}\mu_{r,1} t} & ,\text{if } t > \dfrac{1}{|\mu_r|}, \end{cases}$$

(see (5)), then

$$\left| D_{R_0}\int_0^{R} W_R^s(t)(Ch\,\mu_r t - \cos\mu_r t)dt \right|$$

$$\le c(s)\mu^{-s} D_{R_0}\left[\int_0^{1/|\mu_r|} \mu^{1/2}\left|J_{s+\frac{1}{2}}(\mu t)\right| \left|\mu_r\right| t^{-s+\frac{1}{2}} dt + c(s)\mu^{-s}\int_{1/|\mu_r|}^{R} \mu^{\frac{1}{2}}\left|J_{s+\frac{1}{2}}(\mu t)\right| e^{\nu_r t} dt \right]$$

$$\le c'(s)\mu^{-s}\max_{0\le t\le 1/|\mu_r|} t^{-s} + c''(s)\mu^{-s}\sum_{m=0}^{\infty}\frac{\nu_r^m}{m!}\max_{1/|\mu_r|\le t\le R} t^{m-s}$$

where $e^{\nu_r t} = \sum_{m=0}^{\infty}\dfrac{(\nu_r t)^m}{m!}$

$$\le c'(R_0,s)\mu^{-s}R_0^{-s} + c''(R_0,s)\mu^{-s}R_0^{-s}e^{\nu_r R_0}$$

$$\le C(R_0,s)\mu^{-s}e^{\nu_r R_0}$$

Then

$$|H_5| \le c(R_0,s)\mu^{-s}e^{|\nu_r|R_0}\|u_r\|_{L^\infty(K_{R_0})}$$

At last, for estimate of H_6, we have from [14]

$$\left|\int_{x-2R}^{x+2R}\frac{D(\mu_r,R,t,|x-\xi|)}{e^{2\mathrm{Re}\mu_{r,1}R}}Q(\xi)d\xi\right|$$

$$\le\left|\int_{x-2R}^{x+2R}\frac{D(\mu_r,R,t,|x-\xi|)}{e^{2\mathrm{Re}\mu_{r,1}R}}\right||Q(\xi)|d\xi$$

$$\le c|\mu_r|^{-3}e^{\rho_r R}\min\{1,|\mu_r|t\}\cdot\|u_r\|_{L^\infty(x-2R,x+2R)}$$

hence

$$|H_6| \le c|\mu_r|^{-3}e^{\rho_r R_0}\|u_r\|_{L^\infty(K_{R_0})}\max_{0\le t\le R}t^{-s+1}$$

$$\le c'e^{\rho_r R_0}R_0^{-s+1}\|u_r\|_{L^\infty(K_{R_0})} \le c(R_0,s)e^{\rho_r R_0}\|u_r\|_{L^\infty(K_{R_0})}$$

using [3], ((6), lemma [3]) and the Cauchy-Schwartz inequality we obtain the following estimates

$$\sum_r\left|\overline{\upsilon_r(y)}\right||u_r(x)||H_i| = O(1)\mu^{-S}, \ (i = 1, 2, 3, 4)$$

$$\sum_r\left|\overline{\upsilon_r(y)}\right||H_5| = O(1)\mu^{-S},$$

$$\sum_r\left|\overline{\upsilon_r(y)}\right||H_6| = O(1)\mu^{-S}.$$

It's clear that

16. $\theta^s(x,y,\mu) - W_R^s(|y-x|) = O(1)\mu^{-S}$

Proof of the Theorem:

Consider the operator

$$L_\mu(f,x) := \mu^s\left[\sigma_\mu^s(f,x) - S_\mu^s(f,x)\right]$$

From $L^1(G)$ into $C(K)$ for any compact $K \subset G$.

We know that

$$\sigma_\mu^s(f,x) := \sum_{|\rho_r|<\mu}\langle f,\upsilon_r\rangle u_r(x)\left(1-\frac{\rho_r^2}{\mu^2}\right)^s$$

$$S_\mu^s(f,x) := \sum_r\langle f,\upsilon_r\rangle\langle u_r,W_R^s\rangle$$

then $\sigma_{\mu}^{s}(f,x)-S_{\mu}^{s}(f,x)=\langle f,\theta^{s}-W_{R}^{s}\rangle$

$$=\sum_{r}\langle f,\upsilon_{r}\rangle\langle u_{r},\theta^{s}-W_{R}^{s}\rangle$$

By using Cauchy-Schwartz inequality and the previous result in [16], we obtain

$$\|L_{\mu}(f,x)\|_{C(K)}\leq M$$

For any fixed $K\subset G$. Further there exists $H\subset L^{1}(G)$ such that $\overline{H}$ = $L^{1}(G)$ and the relation $L_{\mu}(h,x)\longrightarrow 0$ holds uniformly in x on the K for any $h\in H$.

Hence the result follows by the Banach-Steinhaus theorem (7).

The proof is complete.

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حول متوسطات ريس للتوسيعات بوساطة قواعد ريس المصاغة بوساطة الدوال الذاتية للمؤثر التفاضلي الاعتيادي من الرتبة الرابعة

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الخلاصة

هدف هذا البحث إلى برهنة نظرية حول متوسطات ريس للتوسيعات بالنسبة لقواعد ريس والتي توسع النتائج السابقة لـ (لوي وطاهر) على مؤثر شرودنكر الى مؤثر تفاضلي من الرتبة الرابعة.