 https://doi.org/ 10.30526/2017.IHSCICONF.1812

On Estimating the Survival Function for the Patients Suffer from the Lung Cancer Disease

Abbas N. Salman
abbasnajim66@yahoo.com
Dept. of Mathematics/College of Education for Pure Science
(Ibn AL-Haitham), University of Baghdad
Ibtehal H. Farhan
Ministry of Education, 2nd Al- Rusafa Education Directorate
Maymona M.Ameen
University of Fallujah
Adel Abdulkadhim Hussein
Dept. of Mathematics/ College of Education for Pure Science
(Ibn AL-Haitham)/ University of Baghdad

Abstract

In this paper, the survival function has been estimated for the patients with lung cancer using different parametric estimation methods depending on sample for completing real data which explain the period of survival for patients who were ill with the lung cancer based on the diagnosis of disease or the entire of patients in a hospital for a time of two years (starting with 2012 to the end of 2013). Comparisons between the mentioned estimation methods has been performed using statistical indicator mean squares error, concluding that the estimation of the survival function for the lung cancer by using pre-test singles stage shrinkage estimator method was the best .

Keywords: Survival Function, Lung Cancer Disease, Complete Real Data, Maximum Likelihood Method, Shrinkage Method, Mean Squares Error and Mean Absolute Percentage Error.

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1. Introduction

"Survival study is one of the broadly used technique in health check statistics; its importance also arises in various fields such as medicine, engineering, epidemiology, biology, economics, physics, public health and or event history analysis in sociology. Survival analysis involves the modelling of time to event data; in this context, death or failure is considered an "event" in the survival analysis literature – traditionally only a single event occurs for each subject, after which the organism or device is lifeless or broken down. Recurring event or repeated event models relax that assumption. The study of recurring events is applicable in systems reliability, and in many areas of social science and medical research. In a clearer way, the analyses of survival function include modelling time. This is to say that the study of patient case since the case diagnosis up to the event started. The events correspond to the death in the literature of survival analyses in the medical experiments", [9].

"Cancer is a category of diseases when a cell or group of cells display uncontrolled growth, invasion and sometimes spread to other locations in the body via lymph or blood (metastasis). It causes about 13% of all human deaths in 2007 with a total of 7.6 million affecting people at all ages. Although there are many causes of cancer, 90-95% of cancer caused due to lifestyle and environmental factors and 5-10% are due to genetics", [3].

"Lung cancer is the most common cancers in the world and the cause of cigarette smoking in most types of lung cancer, the more the number of cigarettes smoked per day more and more beginning was in the habit of smoking at the age of the youngest whenever the risk of lung cancer is the biggest, as well as the high levels of air pollution and exposure radiation and asbestos may also increase the risk of lung cancer";[14].

The aim of this paper is concerned with finding and estimating the survival function $S(t)$ using some parametric methods after the survival time of the patients suffer from Lung Cancer diseases in Kadhimia Hospital (Jawadain Center of Cancerous Diseases) in Baghdad, Iraq, to show and study how long the patients remain alive for this diseases.

The lifetime data for the parametric method under the influence distributed as three parameters Weibull distribution based on complete data which needs to make estimation for the three parameters of the Weibull distribution by using three methods which are Maximum Likelihood Estimator method (MLE), Shrinkage Estimator method (SHE) and Pre-Test single stage shrinkage estimator method (PRE).

And then estimate the survival function. Finally, comparisons of the above estimation methods were made using statistical indicators (mean squared error MSE and mean absolute percentage error MAPE) in the sense of real survival function.

Many authors studied the survival function depending on three – parameters Weibull model like, Heo J. H. et al. [7] , Al–Helaly F.S.A. [4] , Denial I. and Somani K.[6] , Ahamd M.R., Ali A.S. and Assad A.M.[2] ,Surucu B. and Sazak H.S.[17] , Jasim, Sh. A. [8] and Majeed. D.F. [12].

The survival function $S(t)$ is the probability that the patient will stay alive till time t.

$S(t) = Pr(T > t)$, T refers to the time of death

Survival probability is frequently assumed to approach zero as age increases.

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i.e.; $(0) = 1$, $\lim_{t\to\infty} s(t) = 0$. and $S(t)$ is non – increasing and continuous from right side. Another characteristic of survival data is that the survival time cannot be negative [14]. See Figure (1), which includes the curve of the survival function.

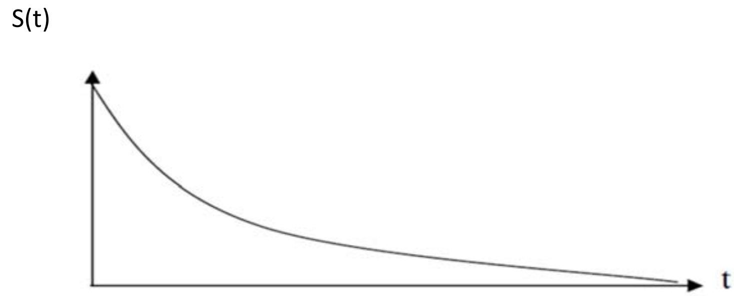

Figure (1): The curve of the survival function

"Weibull distribution broadly used in the reliability engineering and life data analysis to model failure times . It was developed in 1939 by Waloddi Weibull and it was introduced to a greater population in 1951 through the paper (a statistical distribution function of wide applicability)";[4, 10 and 16].

Let T be a r.v. denote to the failure (death) time.

The p.d.f of the three – parameters Weibull distribution [T~ Wei(α,β,γ)] is:

$$f(t;\alpha,\beta,\gamma) = \begin{cases} \frac{\alpha}{\gamma}\left(\frac{t-\beta}{\gamma}\right)^{\alpha-1} exp\left[-\left(\frac{t-\beta}{\gamma}\right)^{\alpha}\right], & t \geq 0 \ \ ,t > \beta \\ 0 & \text{other wise} \end{cases} \quad (1)$$

Here; α ,β,γ refer to the shape parameter, location parameter , scale parameter respectively , and the parameter space is:

$$\Omega = \{(\alpha\,,\beta\,,\gamma): \ \alpha > 0 \ , \ 0 < \beta < \infty \ , \ \gamma > 0 \ \}$$

And the survival function of the three – parameters Weibull distribution will be:

$$S(t) = \exp\left[-\left(\frac{t-\beta}{\gamma}\right)^{\alpha}\right] \quad (2)$$

2. Estimation Methods

2.1 Maximum Likelihood Estimation Method (ML)

The maximum likelihood estimation (MLE) is one of the most well-liked and dependable methods to obtain a point estimator of parameters in any distribution [1, 4, 5 and 11]

The likelihood function of Weibull distribution with three parameters is:

$L = \prod_{i=1}^{n} f(t_i\ ;\ \alpha, \beta, \gamma)$

$$L = \prod_{i=1}^{n} \left[\frac{\alpha}{\gamma} \left(\frac{t_i - \beta}{\gamma} \right)^{\alpha - 1} exp \left[- \left(\frac{t_i - \beta}{\gamma} \right)^{\alpha} \right] \right]$$

$$L = \left(\frac{\alpha}{\gamma} \right)^{n} \prod_{i=1}^{n} \left[\left(\frac{t_i - \beta}{\gamma} \right)^{\alpha - 1} \right] exp \left[- \sum_{i=1}^{n} \left(\frac{t_i - \beta}{\gamma} \right)^{\alpha} \right]$$

The logarithm for L will be:

$$LnL = n\ Ln\ \alpha - n\ Ln\ \gamma + \alpha \sum_{i=1}^{n} Ln\ (t_i - \beta) - \sum_{i=1}^{n} Ln\ (t_i - \beta) - \alpha \sum_{i=1}^{n} Ln\ \gamma + \sum_{i=1}^{n} Ln\ \gamma - \sum_{i=1}^{n} \left(\frac{t_i - \beta}{\gamma} \right)^{\alpha}$$

Take the partial derivatives for Ln L w.r.t the parameters , β and γ we obtained

$$\frac{\partial Ln\ L}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} Ln\ (t_i - \beta) - \sum_{i=1}^{n} Ln\ \gamma - \sum_{i=1}^{n} \left(\frac{t_i - \beta}{\gamma} \right)^{\alpha} Ln\ \left(\frac{t_i - \beta}{\gamma} \right)$$

By equality the above partial derivative to zero, we obtain:

$$\frac{n}{\hat{\alpha}} + \sum_{i=1}^{n} Ln \left(\frac{t_i - \hat{\beta}}{\hat{\gamma}} \right) - \sum_{i=1}^{n} \left(\frac{t_i - \hat{\beta}}{\hat{\gamma}} \right)^{\hat{\alpha}} Ln\ \left(\frac{t_i - \hat{\beta}}{\hat{\gamma}} \right) = 0$$

And

$$\hat{\beta} = \frac{\sum_{i=1}^{n} (t_i) - \left[\frac{\hat{\gamma}^{\hat{\alpha}}\ (\hat{\alpha} - 1)}{\hat{\alpha}} \right]^{\frac{1}{\hat{\alpha}}}}{n}$$

$$\frac{\partial LnL}{\partial \gamma} = -\frac{n}{\gamma} - \alpha \sum_{i=1}^{n} \frac{1}{\gamma} + \sum_{i=1}^{n} \frac{1}{\gamma} + \frac{\alpha}{\gamma} \sum_{i=1}^{n} \left(\frac{t_i - \beta}{\gamma} \right)^{\alpha}$$

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$$\frac{\partial LnL}{\partial \gamma} = -\frac{n}{\gamma} - \frac{\alpha\, n}{\gamma} + \frac{n}{\gamma} + \frac{\alpha}{\gamma} \sum_{i=1}^{n} \left(\frac{t_i-\beta}{\gamma}\right)^{\alpha} = 0$$

$$\frac{\hat{\alpha}}{\hat{\gamma}} \left[-n + \sum_{i=1}^{n} \left(\frac{t_i-\hat{\beta}}{\hat{\gamma}}\right)^{\hat{\alpha}} \right] = 0 \text{ , we get } \hat{\gamma} = \left[\frac{\sum_{i=1}^{n} (t_i-\hat{\beta})^{\hat{\alpha}}}{n}\right]^{\frac{1}{\hat{\alpha}}}$$

We use the numerical analysis method as Newton – Raphson to solve the nonlinear equations simultaneously

The steps of this method are as follows:

$$\begin{bmatrix} \alpha_{K+1} \\ \beta_{k+1} \\ \gamma_{k+1} \end{bmatrix} = \begin{bmatrix} \alpha_K \\ \beta_k \\ \gamma_k \end{bmatrix} - J_{k_i}^{-1} \begin{bmatrix} f_1(\alpha) \\ f_2(\beta) \\ f_3(\gamma) \end{bmatrix} \qquad i = 1,2,3$$

Where,

$$f_1(\alpha) = \frac{n}{\alpha} + \sum_{i=1}^{n} Ln \left(\frac{t_i-\beta}{\gamma}\right) - \sum_{i=1}^{n} \left(\frac{t_i-\beta}{\gamma}\right)^{\alpha} Ln \left(\frac{t_i-\beta}{\gamma}\right)$$

$$f_2(\beta) = -(\alpha-1) \sum_{i=1}^{n} (t_i-\beta)^{-1} + \frac{\alpha}{\gamma} \sum_{i=1}^{n} \left(\frac{t_i-\beta}{\gamma}\right)^{\alpha-1}$$

$$f_3(\gamma) = \frac{\alpha}{\gamma} \left[-n + \sum_{i=1}^{n} \left(\frac{t_i-\beta}{\gamma}\right)^{\alpha} \right]$$

And ,the Jacobean matrix J_{k_1} is :

$$J_{k_1} = \begin{bmatrix} \frac{\partial f_1(\alpha)}{\partial \alpha} & \frac{\partial f_1(\alpha)}{\partial \beta} & \frac{\partial f_1(\alpha)}{\partial \gamma} \\ \frac{\partial f_2(\beta)}{\partial \alpha} & \frac{\partial f_2(\beta)}{\partial \beta} & \frac{\partial f_2(\beta)}{\partial \gamma} \\ \frac{\partial f_3(\gamma)}{\partial \alpha} & \frac{\partial f_3(\gamma)}{\partial \beta} & \frac{\partial f_3(\gamma)}{\partial \gamma} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 Ln\, L}{\partial \alpha^2} & \frac{\partial^2 Ln\, L}{\partial \alpha \partial \beta} & \frac{\partial^2 Ln\, L}{\partial \alpha \partial \gamma} \\ \frac{\partial^2 Ln\, L}{\partial \beta \partial \alpha} & \frac{\partial^2 Ln\, L}{\partial \beta^2} & \frac{\partial^2 Ln\, L}{\partial \beta \partial \gamma} \\ \frac{\partial^2 Ln\, L}{\partial \gamma \partial \alpha} & \frac{\partial^2 Ln\, L}{\partial \gamma \partial \beta} & \frac{\partial^2 Ln\, L}{\partial \gamma^2} \end{bmatrix}$$

Thus:

$$\frac{\partial f_1(\alpha)}{\partial \alpha} = -\frac{n}{\alpha^2} - \sum_{i=1}^{n} \left(\frac{t_i-\beta}{\gamma}\right)^{\alpha} \left[Ln \left(\frac{t_i-\beta}{\gamma}\right)\right]^2$$

$$\frac{\partial f_1(\alpha)}{\partial \beta} = -\sum_{i=1}^{n} (t_i-\beta)^{-1} + \sum_{i=1}^{n} \left(\frac{t_i-\beta}{\gamma}\right)^{\alpha} (t_i-\beta)^{-1} + \frac{\alpha}{\gamma} \sum_{i=1}^{n} \left(\frac{t_i-\beta}{\gamma}\right)^{\alpha-1} Ln \left(\frac{t_i-\beta}{\gamma}\right)$$

$$\frac{\partial f_1(\alpha)}{\partial \gamma} = -\frac{n}{\gamma} + \frac{1}{\gamma}\sum_{i=1}^{n}\left(\frac{t_i-\beta}{\gamma}\right)^{\alpha} + \frac{\alpha}{\gamma}\sum_{i=1}^{n}\left(\frac{t_i-\beta}{\gamma}\right)^{\alpha} Ln\left(\frac{t_i-\beta}{\gamma}\right)$$

$$\frac{\partial\, f_2(\beta)}{\partial \alpha} = -\sum_{i=1}^{n}(t_i-\beta)^{-1} + \frac{\alpha}{\gamma}\sum_{i=1}^{n}\left(\frac{t_i-\beta}{\gamma}\right)^{\alpha-1} Ln\left(\frac{t_i-\beta}{\gamma}\right) + \frac{1}{\gamma}\sum_{i=1}^{n}\left(\frac{t_i-\beta}{\gamma}\right)^{\alpha-1}$$

$$\frac{\partial\, f_2(\beta)}{\partial \beta} = -(\alpha-1)\sum_{i=1}^{n}(t_i-\beta)^{-2} - \frac{\alpha(\alpha-1)}{\gamma^2}\sum_{i=1}^{n}\left(\frac{t_i-\beta}{\gamma}\right)^{\alpha-2}$$

$$\frac{\partial\, f_2(\beta)}{\partial \gamma} = -\frac{\alpha^2}{\gamma^2}\sum_{i=1}^{n}\left(\frac{t_i-\beta}{\gamma}\right)^{\alpha-1}$$

$$\frac{\partial f_3(\gamma)}{\partial \alpha} = -\frac{n}{\gamma} + \frac{\alpha}{\gamma}\sum_{i=1}^{n}\left(\frac{t_i-\beta}{\gamma}\right)^{\alpha} Ln\left(\frac{t_i-\beta}{\gamma}\right) + \frac{1}{\gamma}\sum_{i=1}^{n}\left(\frac{t_i-\beta}{\gamma}\right)^{\alpha}$$

$$\frac{\partial f_3(\gamma)}{\partial \beta} = -\frac{\alpha^2}{\gamma^2}\sum_{i=1}^{n}\left(\frac{t_i-\beta}{\gamma}\right)^{\alpha-1}$$

$$\frac{\partial f_3(\gamma)}{\partial \gamma} = \frac{\alpha\, n}{\gamma^2} - \frac{\alpha(\alpha+1)}{\gamma^2}\sum_{i=1}^{n}\left(\frac{t_i-\beta}{\gamma}\right)^{\alpha}$$

Then, the error term is symbolized by ϵ , formulated as:

$$\begin{bmatrix}\epsilon_{k+1}(\alpha)\\ \epsilon_{k+1}(\beta)\\ \epsilon_{k+1}(\gamma)\end{bmatrix} = \left|\begin{bmatrix}\alpha_{k+1}\\ \beta_{k+1}\\ \gamma_{k+1}\end{bmatrix} - \begin{bmatrix}\alpha_k\\ \beta_k\\ \gamma_k\end{bmatrix}\right|$$

So, the maximum likelihood estimators for the parameters α, β and γ are respectably $\hat{\alpha}_{ML}, \hat{\beta}_{ML}$ *and* $\hat{\gamma}_{\mathrm{ML}}$

Thus, the maximum likelihood estimator for the survival function is defined as:

$$\hat{S}_{ML}(t) = \exp\left[-\left(\frac{t-\hat{\beta}_{ML}}{\hat{\gamma}_{ML}}\right)^{\hat{\alpha}_{ML}}\right] \quad (3)$$

2.2 Thompson-Type Shrinkage Estimation Method (SH)

"The shrinkage estimation method is one of the Bayesian approach depending on prior information concerning the value of the specific parameter θ from past experiences or previous studies. However, in certain situations, prior information is available only from of an initial guess value (natural origin) $\theta_\circ$ of θ. In such a situation, it is natural to start with an estimator $\hat{\theta}$ (e.g., MLE) of θ and modify it by moving it closer to $\theta_\circ$.Thompson in 1968 has suggested the problem of shrinking the usual estimator $\hat{\theta}$ of the parameter θ toward prior information (a natural origin) $\theta_\circ$by single stage shrinkage estimator $\psi(\hat{\theta})\hat{\theta} + (1 - \psi(\hat{\theta}))\theta_\circ$, $0 \le \psi(\hat{\theta}) \le 1$, which is more efficient than $\hat{\theta}$ if $\theta_\circ$ is close to θ and less efficient than $\hat{\theta}$ otherwise";[18] .

According to Thompson, $\theta_\circ$ is a natural origin and, as such, may arise for any one of a number of reasons—e.g., we are estimating θ and (a) we believe $\theta_\circ$ is closed to the true value of θ, or (b) we fear that $\theta_\circ$ may be near the true value of θ, that is mean, something bad happens if $\theta_\circ = \theta$, and we do not know about it (that is, something bad happens if $\theta_\circ \approx \theta$ and we do not use $\theta_\circ$).
Where, $\psi(\hat{\theta})$ is so called shrinkage weight factor; $0 \leq \psi(\hat{\theta}) \leq 1$ which represents the belief of $\hat{\theta}$,and (1- $\psi(\hat{\theta})$) represent the belief of $\theta_\circ$.Thompson noting that the shrinkage weight factor may be a function of $\hat{\theta}$ or may be constant and the chosen of the shrinkage weight factor is(ad hoc basis).
Also, the shrinkage weight function $\psi(\hat{\theta})$ can be founded by minimizing the mean square error of $\tilde{\theta}$.

In this paper, we take a constant shrinkage weight factor as below:

i.e.; $\psi(\hat{\theta}) = K_1 = e^{-\frac{10}{n}}$

So, the Thompson-Type shrinkage estimator of the survival function is

$$\hat{S}_{SH}(t) = \exp\left[-\left(\frac{t-\hat{\beta}_{SH}}{\hat{\gamma}_{SH}}\right)^{\hat{\alpha}_{SH}}\right] \tag{4}$$

2. 3 Pre-Test Singles Stage Shrinkage Estimator Method (PR)

As Thompson recommended shrinking the natural estimator $\hat{\theta}$ of θ towards the prior guess point θ_0 ,the pre-test shrinkage estimator defined as:

$$\tilde{\theta} = \begin{cases} k_2\hat{\theta} + (1-k_2)\theta_0 & if\, \hat{\theta} \in R \\ \hat{\theta} & if\ \hat{\theta} \notin R \end{cases} \tag{5}$$

Where , R refers to the pre- test region for acceptance the null hypothesis $H_0:\theta = \theta_0$ beside $H_1: \theta \neq \theta_0$, $\hat{\theta}$ is the usual estimator of θ, K_2 is a shrinkage weight factor such that $0 \leq K_2 \leq 1$ which may be a function of $\hat{\theta}$ or may be a constant;[13],[15] and[18] .
In this paper, we may assume the regions R as follow:

$$R = [\alpha_0 - \varepsilon\ ,\alpha_0 + \varepsilon\ ] \quad ,\varepsilon = 0.01$$

And,

$$k_2 = e^{-\frac{1}{n}} \qquad ,0 \leq k_2 \leq 1$$

Where, θ may be referred to α, β or γ, and n=127(sample size).

Thus, the pre-test shrinkage estimator of the survival function is defined

$$\hat{S}_{PR}(t) = \exp\left[-\left(\frac{t-\hat{\beta}_{PR}}{\hat{\gamma}_{PR}}\right)^{\hat{\alpha}_{PR}}\right] \quad (6)$$

3. Discussions and Result Analysis

1. As an expected the values of survival function of all estimation methods which are proposed in this paper has been decreasing gradually at increasing failure times, that means there is an reverse correlation between failure time and survival function.

Table (1): Estimated Values for the Survival Function

No.	Time/d	$\hat{S}_{ML}(t)$	$\hat{S}_{SH}(t)$	$\hat{S}_{PR}(t)$
1	221	**0.997689774009338**	**0.997652052126603**	**0.997308477502722**
2	221	**0.997689774009338**	**0.997652052126603**	**0.997308477502722**
3	233	**0.980097162067915**	**0.979972118696028**	**0.978863674962477**
4	240	**0.966665156535477**	**0.966495555128340**	**0.964998855406156**
5	241	**0.964614626294358**	**0.964439002891875**	**0.962889927101286**
6	243	**0.960427393254456**	**0.960239980440228**	**0.958588431665180**
7	249	**0.947246796518457**	**0.947026032800463**	**0.945085064668406**
8	254	**0.935651565172396**	**0.935405286051186**	**0.933243367968516**
9	266	**0.906049355307675**	**0.905750014433746**	**0.903130048872226**
10	273	**0.887883318525875**	**0.887558137413452**	**0.884715942201407**
11	276	**0.879938933017485**	**0.879603780815122**	**0.876676014002889**
12	277	**0.877272106137868**	**0.876933773804755**	**0.873978739474084**
13	278	**0.874596457399901**	**0.874255015894708**	**0.871273334605233**
14	281	**0.866519604152638**	**0.866169255871672**	**0.863111310958356**
15	290	**0.841905573501910**	**0.841532172105296**	**0.838277439421037**
16	301	**0.811250697391143**	**0.810856150237242**	**0.807422102414005**
17	301	**0.811250697391143**	**0.810856150237242**	**0.807422102414005**
18	301	**0.811250697391143**	**0.810856150237242**	**0.807422102414005**
19	302	**0.808442063262001**	**0.808045955027022**	**0.804598744286670**
20	304	**0.802816674234362**	**0.802417617019756**	**0.798945583016250**
21	304	**0.802816674234362**	**0.802417617019756**	**0.798945583016250**
22	306	**0.797181696216187**	**0.796779917384459**	**0.793285031901063**
23	307	**0.794361064044440**	**0.793958008335780**	**0.790452424686892**
24	307	**0.794361064044440**	**0.793958008335780**	**0.790452424686892**
25	308	**0.791538571778995**	**0.791134294439630**	**0.787618491371743**
26	313	**0.777404158311710**	**0.776994582966160**	**0.773434699190564**
27	313	**0.777404158311710**	**0.776994582966160**	**0.773434699190564**
28	314	**0.774574036117023**	**0.774163559500373**	**0.770596230029599**
29	318	**0.763247987159542**	**0.762834415689601**	**0.759241716227755**
30	330	**0.729294717478616**	**0.728876476933493**	**0.725247532995825**
31	331	**0.726471893851405**	**0.726053557813177**	**0.722424126082410**
32	332	**0.723650669281255**	**0.723232280556182**	**0.719602729885345**
33	332	**0.723650669281255**	**0.723232280556182**	**0.719602729885345**
34	334	**0.718013484662405**	**0.717595116985774**	**0.713966417466740**
35	334	**0.718013484662405**	**0.717595116985774**	**0.713966417466740**
36	335	**0.715197753729259**	**0.714779458794354**	**0.711151720649880**
37	335	**0.715197753729259**	**0.714779458794354**	**0.711151720649880**
38	335	**0.715197753729259**	**0.714779458794354**	**0.711151720649880**
39	335	**0.715197753729259**	**0.714779458794354**	**0.711151720649880**
40	338	**0.706763343794751**	**0.706345510034843**	**0.702722747302462**

41	341	**0.698350367132440**	**0.697933348707775**	**0.694318608403004**
42	342	**0.695551290931886**	**0.695134620726449**	**0.691523211883165**
43	345	**0.687171213056376**	**0.686755810122216**	**0.683156308947991**
44	349	**0.676041375564801**	**0.675628164096677**	**0.672048849840847**
45	354	**0.662207350212230**	**0.661797642596573**	**0.658250123143567**
46	357	**0.653952798780289**	**0.653545579304157**	**0.650020444966782**
47	363	**0.637556944392764**	**0.637155513179055**	**0.633682103958799**
48	364	**0.634839857843534**	**0.634439491510939**	**0.630975560470705**
49	364	**0.634839857843534**	**0.634439491510939**	**0.630975560470705**
50	366	**0.629419660942131**	**0.629021506378309**	**0.625577233062175**
51	367	**0.626716687525801**	**0.626319679006257**	**0.622885577911025**
52	368	**0.624018553521592**	**0.623622717261733**	**0.620199012450985**
53	371	**0.615953838834227**	**0.615561672508484**	**0.612170465034984**
54	373	**0.610602733534515**	**0.610213137370127**	**0.606844649715169**
55	374	**0.607934976121655**	**0.607546700886452**	**0.604189879102948**
56	380	**0.592041230420290**	**0.591661357676206**	**0.588378598346891**
57	387	**0.573754436135632**	**0.573385317797610**	**0.570197068883097**
58	392	**0.560870123632979**	**0.560509247363219**	**0.557393269972151**
59	393	**0.558311620461998**	**0.557952443971122**	**0.554851356962820**
60	397	**0.548140200138508**	**0.547787982397774**	**0.544747810567180**
61	399	**0.543092596870234**	**0.542743949536347**	**0.539735006528905**
62	400	**0.540578454288757**	**0.540231613748262**	**0.537238467892724**
63	400	**0.540578454288757**	**0.540231613748262**	**0.537238467892724**
64	401	**0.538070799419566**	**0.537725779628163**	**0.534748548980560**
65	402	**0.535569667571298**	**0.535226482159909**	**0.532265281978362**
66	407	**0.523163037484442**	**0.522829217261285**	**0.519949802042027**
67	409	**0.518247199959989**	**0.517917210167554**	**0.515071220508573**
68	419	**0.494079160943614**	**0.493768944024051**	**0.491095301272884**
69	421	**0.489329233627683**	**0.489023079287875**	**0.486384811049481**
70	421	**0.489329233627683**	**0.489023079287875**	**0.486384811049481**
71	422	**0.486964882945179**	**0.486660771587578**	**0.484040288217134**
72	422	**0.486964882945179**	**0.486660771587578**	**0.484040288217134**
73	423	**0.484607636089232**	**0.484305575150108**	**0.481702938644089**
74	427	**0.475250091064691**	**0.474956300887494**	**0.472425626547661**
75	428	**0.472928661402794**	**0.472636954892139**	**0.470124403655200**
76	430	**0.468307476484749**	**0.468019954700287**	**0.465543793495850**
77	446	**0.432392045612118**	**0.432138618147163**	**0.429958610285263**
78	450	**0.423709530273999**	**0.423464731600489**	**0.421359594918979**
79	454	**0.415146708372620**	**0.414910553777580**	**0.412880392296157**
80	461	**0.400450868849749**	**0.400229841389053**	**0.398330822664271**
81	463	**0.396319792156831**	**0.396103079658480**	**0.394241454989728**
82	470	**0.382098433238835**	**0.381896769729469**	**0.380165516713902**
83	477	**0.368246366548390**	**0.368059627720715**	**0.366457605235988**
84	481	**0.360496477687706**	**0.360318191967515**	**0.358789337420163**
85	481	**0.360496477687706**	**0.360318191967515**	**0.358789337420163**
86	483	**0.356666604945219**	**0.356492521675972**	**0.355000035791562**
87	483	**0.356666604945219**	**0.356492521675972**	**0.355000035791562**
88	497	**0.330694901231667**	**0.330549692421876**	**0.329306979946940**
89	511	**0.306173978079450**	**0.306056481020563**	**0.305053322616424**
90	512	**0.304477321487087**	**0.304361752448235**	**0.303375255569742**
91	512	**0.304477321487087**	**0.304361752448235**	**0.303375255569742**
92	516	**0.297763108549736**	**0.297655177905816**	**0.296734684169189**
93	517	**0.296102598228925**	**0.295996558393815**	**0.295092401590444**
94	519	**0.292803141902415**	**0.292700860642044**	**0.291829176906074**
95	533	**0.270503541400587**	**0.270426669853482**	**0.269774463744016**
96	534	**0.268963458167283**	**0.268888338896652**	**0.268251264862742**
97	535	**0.267430329012918**	**0.267356953387738**	**0.266734936503992**

98	540	**0.259868452849521**	**0.259803664447823**	**0.259255796547725**
99	550	**0.245257147252595**	**0.245208862767228**	**0.244803479376178**
100	553	**0.241005054911949**	**0.240961543376973**	**0.240597361468829**
101	574	**0.212887148306260**	**0.212874679393736**	**0.212778421955444**
102	583	**0.201691473254446**	**0.201691021568124**	**0.201698468585515**
103	583	**0.201691473254446**	**0.201691021568124**	**0.201698468585515**
104	605	**0.176369964172040**	**0.176395633488435**	**0.176628489639365**
105	610	**0.171003966083080**	**0.171034935536213**	**0.171313530979565**
106	619	**0.161692948751512**	**0.161732876780660**	**0.162088787964821**
107	642	**0.139837843913458**	**0.139897375403770**	**0.140422509911806**
108	646	**0.136308511920629**	**0.136370987749374**	**0.136921546055529**
109	652	**0.131158759004245**	**0.131225403303716**	**0.131811960398479**
110	665	**0.120575994323559**	**0.120650681572787**	**0.121306718164700**
111	673	**0.114439806050555**	**0.114518797320601**	**0.115212029763092**
112	697	**0.097640792490440**	**0.097729961671822**	**0.098511225093927**
113	713	**0.087687486735994**	**0.087781362180940**	**0.088603396452443**
114	735	**0.075475413767259**	**0.075573385035708**	**0.076430998661455**
115	764	**0.061709430054248**	**0.061809236485303**	**0.062682994473721**
116	764	**0.061709430054248**	**0.061809236485303**	**0.062682994473721**
117	770	**0.059161036276197**	**0.059260790617839**	**0.060134158859289**
118	783	**0.053962920267320**	**0.054062123685610**	**0.054930859085850**
119	784	**0.053580738979765**	**0.053679876765286**	**0.054548054584753**
120	788	**0.052076437557697**	**0.052175281567921**	**0.053040958579770**
121	810	**0.044467341382994**	**0.044563757081821**	**0.045408639394539**
122	811	**0.044146869427402**	**0.044243145226254**	**0.045086826283784**
123	908	**0.021431127918539**	**0.021506506264741**	**0.022169744263385**
124	932	**0.017811542744258**	**0.017880702827278**	**0.018490024116964**
125	939	**0.016868439194423**	**0.016935779396530**	**0.017529304758063**
126	939	**0.016868439194423**	**0.016935779396530**	**0.017529304758063**
127	996	**0.010752780989456**	**0.010805724232380**	**0.011274021885149**

2. The mean squares error and mean absolute percentage error for estimation survival function are given in the following table (2).

Table (2): Comparing the three parametric methods

Methods	**MSE$[\hat{S}(t_i)]$**	**MAPE$[\hat{S}(t_i)]$**
ML	0.00072827218	0.08379489847
SH	0.00072536901	0.08353041768
PR	0.00070771681	0.08139638971

Where;

$$\text{MSE}[\hat{S}(t_i)] = \frac{\sum_{i=0}^{n}[\hat{S}(t_i)-S(t_i)]^2}{n} \quad , \quad \text{MAPE}[\hat{S}(t_i] = \frac{\sum_{i=0}^{n}\frac{|\hat{S}(t_i)-S(t_i)|}{S(t_i)}}{n} \qquad \dots(7)$$

 https://doi.org/ 10.30526/2017.IHSCICONF.1812

Where $S(t_i)$ $\hat{S}(t_i)$, n refers to the Real survival function, estimated survival function sample size of the patient respectively .

3. As a consequence, the computations of mentioned statistical indicators which are shown in the Table (2), above leads to the result that the mean squares error (MSE) and mean absolute percentage (MAPE) for pretest shrinkage estimator (PR) method are less than those of the ML and SH methods, so the pretest single stage shrinkage method is the best estimation method and then SH and MLE.
4. By observing figure (2) below, one can note the matching of the used estimation methods in this paper and the extent of convergence resulting accuracy of these methods, especially to real survival function methods S (t).

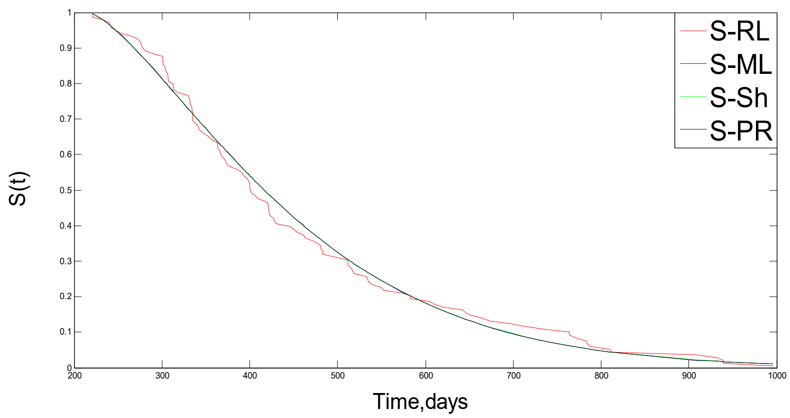

Figure (2): Plot of three estimated Survival Function.

4. Conclusions

It can be distinguished that when the prior estimator is very close to the true value of the parameter, the shrinkage estimator is accomplished better than MLE. If one has no assurance of prior estimate, then the pretest single stage shrinkage estimators (PR) will be recommended. We can carefully use the shrinkage estimator for small n at standard pretest region R and reasonable shrinkage weight factor.

 https://doi.org/ 10.30526/2017.IHSCICONF.1812

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