Fuzzy Fixed Point Theorem for Some Types of Fuzzy Jungck Contractive Mappings in Hilbert Space

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Abstract

In this paper, developed Jungck contractive mappings into fuzzy Jungck contractive and proved fuzzy fixed point for some types of generalize fuzzy Jungck contractive mappings.

Keywords: fuzzy mapping, fuzzy fixed point and Jungck contractive.

1. Introduction

The concept of fuzzy set was introduced by L.Zadeh [3]in (1965).After that a lot of work has been done regarding fuzzy set and fuzzy mappings. The concept of fuzzy mapping was first introduced by Heilpern [4].In (2001) , Estruch and Vidal [1] proved a fuzzy fixed point theorem for fuzzy contractive mappings .Jungck, G.[2][(1976) introduced Jungck contractive mapping and proved fixed point theorems .

In this paper , we introduced Jungck contractive mapping and studied some results of fuzzy fixed point theorems for some types of generalized fuzzy Jungck contractive mapping in Hilbert space.

Preliminaries

In this section, we recall some basic definitions and preliminaries that will be needed in this paper.

Definition 2.1[3]: Let H be a Hilbert space and F(H) be a collection of all fuzzy sets in H . Let $A \in F(H)$ and $\alpha \in [0, 1]$ the $\alpha -$ level set of A, denoted by A_α is defined by

$$A_\alpha = \{u : A(u) \geq \alpha\} \text{ if } \alpha \in [0,1]$$

$$A_0 = \overline{\{u : A(u) > \alpha\}}$$

Where $\overline{B}$ denotes the closure of a set B.

Definition A fuzzy set A is said to be an approximate quantity if and only if A_α is compact and convex for each $\alpha \in [0,1]$, and

$\sup_{u \in X} A(u) = 1$ .When A is an approximate quantity and $A(u_0)=1$ for some $u_0 \in H$, A is identified with an approximate of u_0 .

The collection of all fuzzy sets in H is denoted by F(H) and W(H) is the sub collection of all approximate quantities.

Definition Let A , B $\in W(H)$ and $\alpha \in [0,1]$. Then

i. $\delta_\alpha(A,B) = \inf_{u \in A_\alpha,\ v \in B_\alpha} \|u - v\|$
ii. $D_\alpha(A,B) = dis(A_\alpha, B_\alpha)$, where "dis" is the Hausdorff distance
iii. $D(A , B) = \sup_\alpha D_\alpha(A,B)$
iv. $\delta (A , B) = \sup_\alpha \delta_\alpha(A,B)$.

It is to be noted that for any 'α', δ_α is a non decreasing as well as continuous function.

Definition Let A , B $\in W(H)$. An approximate quantity A is said to be more accurate than B (denoted by $A \subset B$) if and only if $A(u) \leq B(u),\ \forall\, u \in H$.

Definition A mapping M from the set H into W(H) is said to be fuzzy mapping.

Definition The point $u \in H$ is called fixed point for the fuzzy mapping M if $\{u\} \subset Tu$. If $u_\alpha \subset Tu$ is called fuzzy fixed point of M .

We shall use the following lemmas due to Helipern.

Lemma $\delta_\alpha(u,B) \leq \|u - v\| + \delta_\alpha(v, B),\ \forall\, u, v \in H$.

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Lemma If $\{u_0\} \subset A$, then $\delta_\alpha(u_0,B) \leq D_\alpha(A,B)$, $\forall B \in W(H)$.

Lemma Let $A \in W(H)$ and $u_0 \in H$, if $\{u_0\} \subset A$ then $\delta_\alpha(u_0, A) = 0$, for each $\alpha \in [0,1]$.

Lemma Let H be a Hilbert space and M fuzzy mapping from H into W(H) and $u_0 \in H$, then there exists $u_1 \in H$ such that $\{u_1\} \subset Tu_0$.

Fuzzy fixed point theorem for types of fuzzy Jungck

The scope of the function in this section, Ω the class of all functins $\Psi: [0,\infty) \to [0,\infty)$, where Ψ is non-decreasing and

$\sum_{n=1}^{\infty} \Psi^n(t) < \infty$, for each $t > 0$ and Ψ^n is $n -$ th iteration of Ψ and

$\Psi(0) = 0$.

First of all, we introduce the following definitions and examples.

Definition

Let H be a Hilbert space and M, N:H→ W(H). A fuzzy mappings M and N are called fuzzy Jungck contraction mapping if there exists $\xi \in [0,1]$, such that

$$D^2(T(u), T(v)) \leq \xi D^2\big(S(u), S(v)\big), \text{for all } u, v \in H.$$

Example Let H =[0 ,1] , let us define M,N :H→ W(H) by

$$S(u)(s) = T(v)(s) = \begin{cases} 0 & , 0 \leq s < \frac{1}{2} \\ \frac{5}{4} & , \frac{1}{2} \leq s < \frac{u+1}{2} \\ \frac{3}{4} & , \frac{u+1}{2} \leq s \leq 1 \end{cases}$$

And let $\xi = 1$. Then, M and N are fuzzy Jungck contraction mapping .

Definition

Let H be a Hilbert space and M, N:H→ W(H). A fuzzy mappings {M , N} are said to be fuzzy $R^* -$ weakly commuting if for each $x, y \in H$ and $R \geq 0$, such that $D^2(S(u), T(v)) = R\|u - v\|^2$.

Example

Let H=[0,1] and define M,N:H→ W(H) be a fuzzy mapping such that

M=N for all x ∈ [0,1] , T(u) is a fuzzy set on H given by ,

$$T(0)(s) = \begin{cases} 1, & s = 0 \\ \alpha, & s \in \left(0, \frac{1}{3}\right] \\ \frac{\alpha}{3}, & s \in (\frac{1}{3}, 1] \end{cases}$$

$$T(1)(s) = \begin{cases} 1, & s = 0 \\ 3\alpha, & s \in \left(0, \frac{1}{3}\right] \\ \frac{\alpha}{3}, & s \in (\frac{1}{3}, 1] \end{cases}$$

For all $z \in (0, 1)$

$$T(z)(s) = \begin{cases} 1, & s = 0 \\ \alpha, & s \in \left(0, \frac{1}{3}\right] \\ 0, & s \in (\frac{1}{3}, 1] \end{cases}$$

When $0 \leq \alpha \leq \frac{1}{3}$, then $[M(0)]_1 = [M(z)]_1 = [M(1)]_1 = \{0\}$

$[M(0)]_\alpha = [M(z)]_\alpha = [M(1)]_\alpha = \left[0, \frac{1}{3}\right]$, $[M(0)]_{\frac{\alpha}{3}} = [M(1)]_{\frac{\alpha}{3}} = [0, 1]$, $[M(z)]_{\frac{\alpha}{3}} = [0, \frac{1}{3}]$

and $[N(0)]_1 = [N(z)]_1 = [N(1)]_1 = \{0\}$

$[N(0)]_\alpha = [N(z)]_\alpha = [N(1)]_\alpha = \left[0, \frac{1}{3}\right]$, $[N(0)]_{\frac{\alpha}{3}} = [N(1)]_{\frac{\alpha}{3}} = [0, 1]$,

$[N(z)]_{\frac{\alpha}{3}} = \left[0, \frac{1}{3}\right]$. Consequently

$D_1^2(N(u), M(v)) = H^2([N(u)]_1, [M(v)]_1) = 0, \forall\ u, v \in H,$

$D_\alpha^2(N(u), M(v)) = H^2([N(u)]_\alpha, [M(v)]_\alpha) = 0, \forall\ u, v \in H,$

$D_{\frac{\alpha}{3}}^2(N(u), M(v)) = H^2\left([N(u)]_{\frac{\alpha}{3}}, [M(v)]_{\frac{\alpha}{3}}\right) = 0, \forall\ u, v \in (0,1)$ and $u, v \in \{0,1\}$,

$D_{\frac{\alpha}{3}}^2(N(u), M(v)) = H^2\left([N(u)]_{\frac{\alpha}{3}}, [M(v)]_{\frac{\alpha}{3}}\right) = \frac{1}{3}, \forall\ v \in (0,1)$ and $u \in \{0,1\}$.

Now,

$D^2(N(u), M(v)) = \sup_\alpha\{D_\alpha^2(N(u), M(v))\} = \frac{1}{3}$

$D^2(N(u), M(v)) = R\|u - v\|^2$.

Then N and M is fuzzy $R^* -$ weakly commuting .

We prove the following theorem:

Theorem

Let H be a Hilbert space and M,N be a fuzzy Jungck contractive mappings satisfy the following conditions:

1. N is continuous fuzzy mapping.
2. $M(H) \subset N(H)$.
3. $\{N, M\}$ are fuzzy $R^* -$ weakly commuting .

Then , there exists $u \in H$ such that u_α is a common fuzzy fixed point of M and N .

Proof Let $u_0 \in H$, there exists $u_1 \in [Nu_1]_\alpha \subset [Mu_0]_\alpha$. In general, choose

u_n such that for $n = 1, 3, 5, \ldots \ldots u_{2n+1} \in [N\, u_{2n+1}]_\alpha \subset [Mu_{2n}]_\alpha$ and

$\| u_n - u_{n+1}\|^2 = \delta_\alpha^2(u_n, M\, u_n) \leq D^2(M\, u_{n-1}, M\, u_n)$

So $\| u_n - u_{n+1}\|^2 \leq D^2(M\, u_{n-1}, M\, u_n)$

Since M,N are a fuzzy Jungck contractive mappings , then

$\| u_n - u_{n+1}\|^2 \leq D^2(M\, u_{n-1}, M\, u_n) \leq \xi\, D^2(N\, u_{n-1}, N\, u_n)$

Sine $u_{n-1} \in [N\, u_{n-1}]_{\alpha}$ and $u_n \in [N\, u_n]_{\alpha}$.
Then , $\| u_n - u_{n+1}\|^2 \le \xi\, D^2(N\, u_{n-1}, N\, u_n) = \xi \| u_{n-1} - u_n\|^2$
Whenever $\xi \in (0, 1)$

$$\| u_n - u_{n+m}\|^2 \le \sum_{j=n}^{n+m-1} \|u_{j+1} - u_j\|^2$$
$$\le \sum_{j=n}^{n+m-1} (\xi^j \|u_1 - u_0\|^2)$$
$$\le \frac{\xi^n}{1-\xi} \|u_1 - u_0\|^2$$

If $n \to \infty$, then ξ^n converge to 0. Therefore the sequence $\{ u_n\}$ is a Cauchy sequence in H .
So by completeness of H, $\{ u_n\}$ converge to $u \in H$.
Now,
Since N is continuous fuzzy mapping , then $[N\, u_{2n+1}]_{\alpha}$ also converges on H.
Finally, we show that $\delta_\alpha^2(u, Mu) = 0$
$\delta_\alpha^2(u, M(u)) \le \|u_n - u\|^2 + D^2(N\, u_n, Mu)$
Since , $\{N, M\}$ are fuzzy $R^* -$ weakly commuting .Then
$\delta_\alpha^2(u, N(u)) \le \|u_n - u\|^2 + R\| u_n - u\|^2$.
Hence , $\delta_\alpha^2(u, N(u)) = 0$ and $u_\alpha \subset Nu$.
Clearly u_α is a common fuzzy fixed point of the
fuzzy mappings N and M. In particular if $\alpha = 1$, then u is
a common fixed point of N and M. ■

Theorem

Let H be a Hilbert space and T, S be a fuzzy mappings satisfy the following conditions:

1. $D^2(N(u), N(v)) \le \Psi(\beta \max\{m(u, v)\})$ .Such that, $\beta \in (0,1)$
   and
   $$m(u, v) = \{D^2(M(u), M(v)), \frac{D^2(M(u),N(v)) + D^2(M(v),N(u))}{2}, \frac{D^2(M(v), N(v)) + D^2(M(v), N(u))}{2}\}$$
2. M is continuous fuzzy mapping.
3. $N(H) \subset M(H)$.
4. $\{N, M\}$ are fuzzy $R^* -$ weakly commuting .

Then , there exists $u \in H$ such that u_α is a common fuzzy fixed point of M and N .

Proof Let $u_0 \in H$, there exists $u_1 \in [Mu_1]_{\alpha} \subset [Nu_0]_{\alpha}$. In general, choose

u_n such that for $n = 1, 3, 5, \ldots\ldots u_{2n+1} \in [M\, u_{2n+1}]_{\alpha} \subset [Nu_{2n}]_{\alpha}$ and
$\| u_n - u_{n+1}\|^2 = \delta_\alpha^2(u_n, N\, u_n) \le D^2(N\, u_{n-1}, N\, u_n)$
So $\| u_n - u_{n+1}\|^2 \le D^2(N\, u_{n-1}, N\, u_n)$
By condition 1 , then
$\| u_n - u_{n+1}\|^2 \le D^2(N\, u_{n-1}, N\, u_n) \le \Psi(\beta \max\{m(u_{n-1}, u_n)\})$

$$m(u_{n-1}, u_n) = \{D^2(M(u_{n-1}), M(u_n)), \frac{D^2(M(u_{n-1}),N(u_{n-1})) + D^2(M(u_n),N(u_{n-1}))}{2}, \frac{D^2(M(u_n), N(u_n)) + D^2(M(u_n), N(u_{n-1}))}{2}\}$$

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$$m(u_{n-1}, u_n) = \{ \| u_{n-1} - u_n\|^2, \frac{\| u_{n-1} - u_n\|^2 + \| u_n - u_n\|^2}{2}, \frac{\| u_n - u_{n-1}\|^2 + \| u_n - u_n\|^2}{2} \}$$

$$m(u_{n-1}, u_n) = \{ \| u_{n-1} - u_n\|^2, \frac{\| u_{n-1} - u_n\|^2}{2}, \frac{\| u_{n-1} - u_n\|^2}{2} \}$$

Hence,

$$\| u_n - u_{n+1}\|^2 \le \Psi\left(\beta \max\{\| u_{n-1} - u_n\|^2, \frac{\| u_{n-1} - u_n\|^2}{2}, \frac{\| u_{n-1} - u_n\|^2}{2}\}\right)$$

$$\| u_n - u_{n+1}\|^2 \le \Psi(\beta\| u_{n-1} - u_n\|^2) .$$

Therefore, $\| u_n - u_{n+1}\|^2 \le \Psi(\| u_{n-1} - u_n\|^2) =$

$$\Psi(D^2(N(u_{n-2}), N(u_{n-1})))$$

$$\| u_n - u_{n+1}\|^2 \le \Psi^2(\| u_{n-2} - u_{n-1}\|^2)$$

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$$\| u_n - u_{n+1}\|^2 \le \Psi^n(\| u_0 - u_1\|^2)$$

$$\| u_n - u_{n+m}\|^2 \le \Psi^n(\| u_0 - u_1\|^2 + \cdots \dots \dots \dots + \Psi^{n+m-1}\| u_0 - u_1\|^2)$$

$$\| u_n - u_{n+m}\|^2 \le \sum_{j=n}^{n+m-1} \Psi^j(\| u_0 - u_1\|^2) .$$

Since, $\sum_{n=1}^{\infty} \Psi^n(\| u_0 - u_1\|^2) < \infty$

Therefore, the sequence $\{ u_n\}$ is a Cauchy sequence in H. So by completeness of H, $\{ u_n\}$ converges to x in H.

Now,

Since M is continuous fuzzy mapping , then $[M u_{2n+1}]_{\alpha}$ also converges on H.

Finally, we show that $\delta_\alpha^2(u, Nu) = 0$

$$\delta_\alpha^2(u, N(u)) \le \|u_n - u\|^2 + D^2(N u_{n-1}, Nu)$$

$$\le \|u_n - u\|^2 + \Psi(\beta \max\{m(u_{n-1}, u)\})$$

$$m(u_{n-1}, u) = \{D^2(M(u_{n-1}), M(u)), \frac{D^2(M(u_{n-1}), N(u_{n-1})) + D^2(M(u), N(u_{n-1}))}{2}, \frac{D^2(M(u), N(u)) + D^2(M(u), N(u_{n-1}))}{2}\}$$

Since , $\{M , N\}$ are fuzzy R^* − weakly commuting .Then

$$\delta_\alpha^2(u, N(u)) \le \|u_n - u\|^2 + \Psi(\| u_{n-1} - u\|^2).$$

Hence , $\delta_\alpha^2(u, N(u)) = 0$ and $u_\alpha \subset Nu$.

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Clearly u_α is a common fuzzy fixed point of the fuzzy mappings N and M .In particular if $\alpha = 1$, then u is a common fixed point of T and S. ■

Theorem Let H be a Hilbert space and N_1, N_2, N_3 be a fuzzy mappings satisfies the following conditions:

1. $D^2(N_1(u), N_1(v)) \leq \beta\, D^2\big(N_1(u), N_2(u)\big) + \xi\, D^2\big(N_1(v), N_3(v)\big) +$

   $\lambda\, D^2(N_1(u), N_2(v)) + \gamma\, D^2(N_1(u), N_3(v)) + \eta\, D^2(N_1(v), N_2(v))$

   For all u, v ∈ H where $\gamma, \eta, \beta, \lambda, \xi \geq 0$ with

   $\gamma + \eta + \beta + \lambda + \xi < 1$.

2. N_2 and N_3 are continuous fuzzy mapping.
3. $N_1(H) \subset N_2(H) \cap N_3(H)$.
4. $\{N_2, N_1\}$ and $\{N_2, N_1\}$ are fuzzy R^*-weakly commuting .

Then , there exists u ∈ H such that u_α is a common fuzzy

fixed point of N_2 , N_1 and N_3 .

Proof: Let $u_0 \in H$, there exists u_1 and u_2 such that

$u_1 \in [N_2u_1]_\alpha \subset [N_1u_0]_\alpha$ and $u_2 \in [N_3u_2]_\alpha \subset [N_1u_1]_\alpha$. By induction one can

construct a sequence $\{u_n\}$ in H, such that for n= 1, 3 , 5, …..
$u_{2n+1} \in [N_2u_{2n+1}]_\alpha \subset [N_1u_{2n}]_\alpha$ and
$u_{2n+2} \in [N_3u_{2n+2}]_\alpha \subset [N_1u_{2n+1}]_\alpha$.
And
$\| u_n - u_{n+1}\|^2 = \delta_\alpha^2(u_n, N_1u_n) \leq D^2(N_1 u_{n-1}, N_1 u_n)$
So $\| u_n - u_{n+1}\|^2 \leq D^2(N_1u_{n-1}, N_1u_n)$
By condition 1 , then
$\| u_n - u_{n+1}\|^2 \leq D^2(N_1(u_{n-1}), N_1(u_n)) \leq \beta\, D^2\big(N_1(u_{n-1}), N_2(u_{n-1})\big) +$
$\xi\, D^2\big(N_1(u_n), N_3(u_n)\big) + \lambda\, D^2\big(N_1(u_{n-1}), N_2(u_n)\big) +$
$\gamma\, D^2(N_1(u_{n-1}), N_3(u_n)) + \eta\, D^2(N_1(u_n), N_2(u_n))$
$\| u_n - u_{n+1}\|^2 \leq \beta \| u_{n-1} - u_n\|^2 + \xi \| u_{n+1} - u_n\|^2 +$
$\lambda \| u_n - u_n\|^2 + \gamma \| u_n - u_n\|^2 + \eta \| u_{n+1} - u_n\|^2$
Hence
$\| u_n - u_{n+1}\|^2 - \xi \| u_{n+1} - u_n\|^2 - \eta \| u_{n+1} - u_n\|^2 \leq \beta \| u_{n-1} - u_n\|^2$

$$\| u_n - u_{n+1}\|^2 \leq \frac{\beta}{1-\xi-\eta} \| u_{n-1} - u_n\|^2 .$$

Putting $q = \frac{\beta}{1-\xi-\eta} < 1$
Then, we have

$$\|u_{2n+1} - u_{2n}\|^2 \leq q\|u_{2n-1} - u_{2n}\|^2$$

Now , for any positive integer m

$$\|u_n - u_{n+m}\|^2 \leq q\|u_n - u_{n+1}\|^2 + \|u_{n+1} - u_{n+2}\|^2 + . + \|u_{n+m-1} - u_{n+m}\|^2$$
$$\leq (q^n + q^{n+1} + q^{n+2} + \cdots + q^{n+m-1}\|u_1 - u_0\|^2$$
$$\leq \frac{q^n}{1-q}\|u_1 - u_0\|^2$$

Which implies that $\|u_n - u_{n+m}\|^2 \to 0$ as $n \to \infty$
Hence $\{u_n\}$ is cauchy sequance in H, but H is a Hilbert space,
so $\{u_n\}$ is converge to u.
Now,
Since N_2 and N_3 are continuous fuzzy mapping , then $[N_2(u_{2n+1})]_\alpha$ and $[N_3(u_{2n+2})]_\alpha$ also converges on H.
Finally, we show that $\delta^2_\alpha(u, N_1(u)) = 0$

$$\delta^2_\alpha(u, N_1(u)) \leq \|u_n - u\|^2 + D^2(N_1(u_{n-1}), N_1(u))$$

$$\leq \|u_n - u\|^2 + \beta D^2\big(N_1(u_{n-1}), N_2(u_{n-1})\big) +$$

$$\xi D^2\big(N_1(u), N_3(u)\big) + \lambda D^2\big(N_1(u_{n-1}), N_2(u)\big) +$$

$$\gamma D^2(N_1(u_{n-1}), N_3(u)) + \eta D^2(N_1(u), N_2(u)).$$

Since , $\{N_1, N_2\}$ and $\{N_1, N_3\}$ are fuzzy R^* − weakly commuting .Then

$$\delta^2_\alpha(u, N(u)) \leq (\lambda+\gamma)\| u_{n-1} - u\|^2.$$

Consequently , $\delta^2_\alpha(u, N(u)) = 0$ and $u_\alpha \subset Nu$.

Clearly u_α is a common fuzzy fixed point of the

fuzzy mappings N_1 , N_2, N_3 ,.In particular if $\alpha = 1$, then x is a

common fixed point of N_1 , N_2 and N_3. ■

Theorem

Let H be a Hilbert space and N_1, N_2, N_3 be a fuzzy mappings satisfy the following conditions:

1. $D^4\big(N_1(u), N_1(v)\big) \leq \beta\,[D^2\big(N_1(u), N_2(v)\big). D^2\big(N_1(v), N_2(v)\big)] +$

   $\xi\,[D^2\big(N_1(v), N_3(v)\big). D^2\big(N_1(u), N_3(u)\big) +$

   $\lambda\,[D^2\big(N_1(u), N_2(v)\big). D^2\big(N_1(v), N_2(u)\big)] + \gamma\,\|v - u\|^4$

   For all u, v ∈ H where $\gamma, \beta, \lambda, \xi \geq 0$ with

   $2\gamma + 2\beta + \lambda + 2\xi < 1$.

2. N_2 and N_3 are continuous fuzzy mapping.
3. $N_1(H) \subset N_2(H) \cap N_3(H)$.

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4. $\{N_2, N_1\}$ and $\{N_2, N_1\}$ are fuzzy R^* − weakly commuting .

Then , there exists $u \in H$ such that u_α is a common fuzzy

fixed point of N_2 , N_1 and N_3 .

Proof Similar to prove theorem 3.7 ■

Definition

Let H be a Hilbert space and N, M:H→ W(H). A fuzzy mappings T and S are called **fuzzy Jungck like contractive mapping**, if there exists $\xi \in (0,1)$, such that

$$D^2(N(u), N(v)) \le \xi\|u-v\|^2 + \Psi(D^2(M(u), M(v))), \text{for all } u, v \in H.$$

Theorem Let H be a Hilbert space and N, M be a fuzzy Jungck like contractive mappings satisfies the following conditions:

1. M is continuous fuzzy mapping.
2. N(H)⊂ M(H).
3. $\{M, N\}$ are fuzzy R^* − weakly commuting .

Then , there exists $u \in H$ such that u_α is a common fuzzy fixed point of N and M .

Proof Trivial ■

Definition Let H be a Hilbert space and N, M:H→ W(H). A fuzzy mappings T and S are called fuzzy Jungck generalized like contractive mapping, if there exists $\xi \in (0,1)$, such that

$$D^2(N(u), N(v)) \le \xi\|u-v\|^2 + \Psi(D^2(M(u), N(u)), D^2(N(u), M(v))), \text{for all } u, v \in H.$$

Theorem Let H be a Hilbert space and N, M be a fuzzy Jungck generalized like contractive mappings satisfies the following conditions:

1. M is continuous fuzzy mapping.
2. N(H)⊂ M(H).
3. $\{M, N\}$ are fuzzy R^* − weakly commuting .

Then , there exists $u \in H$ such that u_α is a common fuzzy fixed point of N and M .

Proof: Trivial ■

Definition

Let H be a Hilbert space and N, M:H→ W(H). A fuzzy mappings T and S are called fuzzy Jungck S- like contractive mapping, if there exists $\xi \in (0,1)$, such that

$$D^2(N(u), N(v)) \le \xi(m(u,v)) + \Psi(m(u,v)), \text{for all } u, v \in H$$

And $m(u,v) = \max\{\|u-v\|^2, D^2(N(u), M(v)), D^2(M(v), N(v)),$

$$\frac{D^2(M(u), N(v)) + D^2(M(v), N(u))}{2}\}$$

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Theorem Let H be a Hilbert space and N, M be a fuzzy Jungck S-like contractive mappings satisfies the following conditions:

1. M is continuous fuzzy mapping.
2. $N(H) \subset M(H)$.
3. $\{M , N\}$ are fuzzy $R^* -$ weakly commuting .

Then , there exists $u \in H$ such that u_α is a common fuzzy fixed point of N and M .

Proof Trivial ■

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