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Modified Ranking Function to Compute Fuzzy Matrix Games

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Abstract

Game theory problems (GTP) frequently occur in Economy, Business Studies, Sociology, Political Science, Military Activities, and so on are some of the subjects covered. To tackle the uncertainty in Games, the analysis of games in which the payoffs are represented by fuzzy numbers (FN) will benefit from fuzzy set theory (FST).

The purpose of this paper is to develop an efficient technique for solving constraint matrix games (MG) with payoff trapezoidal fuzzy numbers (TFN). The description of the new ranking method is introduced for a constrained matrix with TFN and values. Stock market forecasting has been one of the most important research areas for decades. Stock market values are volatile, non-linear, complicated and chaotic. Based on a ranking function (RF), we used a new algorithm to solve the fuzzy game problem (FGP) employing TFN and also to try to get a desirable gain. Centered on the latest proposed ranking algorithm, the Fuzzy decision method is designed to analyze possible stock opportunists.

The paper considers a zero-sum game for two persons in which TFN are fuzzy payoffs. A ranking method (RM) is proposed to convert TFN into crisp numbers (CN) and it is used to solve FGP. The fuzzy game (FG) issue with concept strategies pure minimax maximin is presented. This problem is converted into the crisp problem by a new RF and then solved using the arithmetic (oddment) method. With the help of numerical examples, the suggested technique is explained. This paper finalizes the conclusion and includes an outlook for future study in this direction.

Keywords: Ranking of Trapezoidal Fuzzy Numbers (RTFN), Game Theory (GTH), Saddle point (SP), Pure Strategy (PS).

1. Introduction

Fuzzy Game (FG) has been used in many areas, including operations analysis, control theory and management sciences, etc. When J.Newmann and O.Morgenstern [1] published the influential essay 'Theory of Games and Economic Performance' in 1944, the mathematical treatment of G TH became available.
The important paper ' Economic results and G TH ' by J.Newmann and O.Morgenstern [1] published in 1944 presented the first statistical treatment of Game Theory (G TH).

FG is a body of information that discusses decision-making (DM) while two and most experienced and rational competitors are in dispute and competition conditions. G TH has played a significant role in decision-making analysis fields like economics and management. We need to determine the right values of payoffs when we can use G TH to design any specific problems we face in real life. Knowing the actual value with payoffs, on the other hand, is difficult, and we could only know the calculated solution. In such situations, modeling the issues as games with uncertain payoffs is useful. All parameters are FN in a FGP. Standard or irregular triangular, trapezoidal, or octagonal numbers can be fuzzy.

The vague FGP is transformed to a crisp estimation problem using the RF and then solved using the traditional approach.

Some researchers extend FGP applications to crisp games and suggest appropriate solution methods and principles to solve FGP with ranking features [2-12].

Recently, the theory of fuzzy sets (FS) has been established and is the most suitable to date theory to deal with uncertainties. Many important G TH applications have been extended by embedding the FS ideas [13-18].

Several authors [19 -23] investigated two-person zero sum games with fuzzy payoffs and fuzzy aims G TH.

We have taken two people's zero-sum games in this article, in which Trapezoidal fuzzy numbers (TFN) are imprecise values. We have explained it using the ranking technique to convert it to a crisply valued game problem. Using TFN, we have exposed a fuzzy game problem.

The paper is structured as follows. Section 2 presents the concept of certain basic definitions and preliminaries regarding the crisp matrix games (CMG). In section 3, we present the mathematical formulation of the FGP. Section 4 introduces the proposed ranking technique. Section 5 presents the procedure for solving FGP using the matrix oddment method. In section 6, a numerical illustration is given to demonstrate the proposed method's efficacy. Finally, Section 7 presents the conclusions of this work.

2. Basic concepts:

Definition (2.1): Fuzzy set (FS) [24]

If the membership function $Ŋ_{ǫ} : Ʈ \rightarrow [0,1]$ of a FS $\widetilde{Đ}$ defined on the set of points $Ʈ$ has the following properties, it is said to be a FN.

(i) $\widetilde{Đ}$ is an upper semi- continuous membership function;

(ii) $\widetilde{Đ}$ is convex, i. e.,$Ŋ_{Q}Ę(ℏ\,\vartheta + (1-ℏ))nim\,\{Ŋ_{Q}(\vartheta),\ Ŋ_{Q}(v)\},ℏ \in [0,1]$, For all $\vartheta, v \in Ţ$.

(iii) $\widetilde{Đ}$ is normal, i. e., $\exists\, \vartheta_0 \in Ţ$ for which $Ŋ_{Q}(\vartheta_0) = 1$.

(iv) $Supp\,(\widetilde{Đ}) = \{\vartheta \in Ţ : Ŋ_{Q}(\vartheta) > 0\}$ is the support of the $\widetilde{Đ}$, and its closure $cl\,(Supp\,(\widetilde{Đ}))$ is a compact set.

Definition (2.2): Pure strategy (PS) [25]

If a player knows precisely what the other player is going to do is called pure strategy.

Definition (2.3): Saddle point (SP) [26]

A saddle point has been said to occur when the maximum value equals the minimum value, and optimal strategies are known as the complementary strategies that give the saddle point. The crisp game value (CGV) of the game matrix (GM) is the number of payoffs at an equilibrium point.

Definition (2.4): Trapezoidal Fuzzy Numbers (TFN) [27]:

A fuzzy number $\widetilde{Đ} = (t^{\wp}, t^{ς}$, ę, ḑ) is said to be a TFN if its membership function is defined as follows:

$$Ŋ_{Q}Ę = \begin{cases} \frac{\vartheta-(t^{\wp}-ę)}{ę}, & t^{\wp} - ę \le \vartheta \le t^{\wp} \\ 1\,, & t^{\wp} \le \vartheta \le t^{ς} \\ \frac{(t^{ς}+\beta)-\vartheta}{ḑ}, & t^{ς} \le \vartheta \le t^{ς} + ḑ \\ 0\,, & other\ wise \end{cases}$$

Definition (2.5): Ranking function (RF) [28]:

The function: $Ţ: Ƒ(Ţ) \rightarrow Ţ$ that maps each FN into the real line is a RF where a natural order exists. If $\check{t}, \check{ȿ} \in Ƒ(Ţ)$, then

a) $\check{t} \ge \check{ȿ}$ if and only if $Ţ(\check{t}) \ge Ţ(\check{ȿ})$;

b) $\check{t} > \check{ȿ}$ if and only if $Ţ(\check{t}) > Ţ(\check{ȿ})$;

c) $\check{t} = \check{ȿ}$ if and only if $Ţ(\check{t}) = Ţ(\check{ȿ})$;

d) $\check{t} \le \check{ȿ}$ if and only if $Ţ(\check{t}) \le Ţ(\check{ȿ})$.

3. Mathematical Formulation of FGP:

Consider a two-player zero-sum FG in which the payoffs matrix contains only TFN. Let us suggest each player has ɱ strategies and ɲ strategies. It is presumed that each player must choose a pure strategy from the available options. The gainer is always assumed to be the player δ , and the loser is always assumed to be player ϒ. ɱ × ɲ is the payoff matrix.

$$Ƕ = \begin{pmatrix} \widetilde{g_{11}} & \widetilde{g_{12}} & \widetilde{g_{13}} & \cdots & \widetilde{g_{1n}} \\ \widetilde{g_{21}} & \widetilde{g_{22}} & \widetilde{g_{23}} & \cdots & \widetilde{g_{2n}} \\ \widetilde{g_{31}} & \widetilde{g_{32}} & \widetilde{g_{33}} & \cdots & \widetilde{g_{3n}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \widetilde{g_{m1}} & \widetilde{g_{m2}} & \widetilde{g_{m3}} & \cdots & \widetilde{g_{mn}} \end{pmatrix}$$

4. Proposed Ranking Technique:

In this section, we propose a new ranking function that depends on the idea of the Maleki ranking function with a new weight $\left(\frac{7}{8}\right)$. Assume $\tilde{g} = (t^{\wp}, t^{ς}, ę, ḑ)$ is a trapezoidal fuzzy number, using the following formula to find the ranking function of $\tilde{a}$.

$$Ţ(\tilde{Đ}) = \frac{7}{8}\int_0^1 [\, inf\ \tilde{t}_{ş} + sup\ \tilde{t}_{ş}\,]\, dş$$

$$Ţ(Đ) = \frac{7}{8}\int_0^1 ((t^{\wp} - ęş)\, d\gamma + (t^{ς} + ḑş))\, dş$$

$$Ţ(\tilde{Đ}) = \frac{7}{8}\left[t^{\wp}ş - \frac{1}{2}ę\, ş^2 + t^{ς}\, ş + \frac{1}{2}ḑ\, ş^2 \right|_0^1$$

$$Ţ(\tilde{Đ}) = \frac{7}{8}(t^{\wp} - \frac{1}{2}ę + t^{ς} + \frac{1}{2}ḑ)$$

$$Ţ(\tilde{Đ}) = \frac{7}{8}(t^{\wp} + t^{ς} + \frac{1}{2}(ḑ - ę))$$

$$Ţ(\tilde{Đ}) = \frac{7}{8}(t^{\wp} + t^{ς}) + \frac{7}{16}(ḑ - ę).$$

5. Procedures for solving FGP using the matrix oddment method:

The following steps are used to implement a solution procedure for solving the two-person zero-sum matrix game problems (MGP) in this section:

Step1. In the FGP, describe the TFN in its parametric form.

Step2. Translate the fuzzy game theory (FGTH) into a CV problem by applying the new RF.

Step3. Check and see if the problem has a SP. The solution can be obtained directly if it exists. Proceed to the next level if the SP does not exist.

Testing this fuzzy matrix game has SP exists or does not exist, we will continue in the following cases:

(i) Take a SP to test.

(ii) If there is no a SP, find equalizing strategies to solve the problem.

For players, the best-mixed strategies are $\delta = (\sigma_1, \sigma_2)$ in addition to the player $\Upsilon = (\mathfrak{r}_1, \mathfrak{r}_2)$

Where $\sigma_1 = \frac{g_{22} - g_{12}}{(g_{11}+g_{22})-(g_{12}+g_{21})}$, $\sigma_2 = 1- \sigma_1$

$\mathfrak{r}_1 = \frac{g_{22} - g_{21}}{(g_{11}+g_{22})-(g_{12}+g_{21})}$, $\mathfrak{r}_2 = 1- \mathfrak{r}_1$ and

Value of the game $\mathbb{C} = \frac{g_{11*}g_{22} - g_{12}*g_{21}}{(g_{11}+g_{22})-(g_{12}+g_{21})}$

Step4. To solve problems in G TH, use the arithmetic (oddment) approach to find the best strategies and the corresponding matrix game value for players.

6. Numerical examples:

Example 1:

Consider the payoff problem of the following fuzzy game as TFN.

$$\begin{array}{cc} & \Upsilon \\ \delta & \begin{pmatrix} 30 & 12 & 7 \\ 9 & 19 & 10 \\ 28 & 21 & 32 \end{pmatrix} \end{array}$$

The minimum in the first row = 7

The minimum in the second row= 9

The minimum in the third row = 21

The maximum in the first column = 30

The maximum in the second column = 21

The maximum in the third column =32

Maximum (minimum) = 21 also Minimum (maximum) =21

It has saddle point. Value of Game = 21

Consider the following fuzzy game problem (FGP):

$$\begin{array}{cc} & \Upsilon \\ \delta & \begin{pmatrix} (28,32,0.3,1.7) & (10,14,0.3,1.7) & (5,9,0.3,1.7) \\ (7,11,0.4,1.6) & (17,21,0.4,1.6) & (8,12,0.4,1.6) \\ (26,30,0.2,1.8) & (19,23,0.2,1.8) & (30,34,0.2,1.8) \end{pmatrix} \end{array}$$

Step1. We obtain the values of Ţ (Đ) for the given FGP and transform it to a CV problem, as shown in the table below.

Table 1. The TFN problem is transformed CV in example1.

g_{11} = (28,32,0.3,1.7)	Ʈ (g_{11}) = 53.1125
g_{12} = (10,14,0.3,1.7)	Ʈ (g_{12}) = 21.6125
g_{13} = (5,9,0.3,1.7)	Ʈ (g_{13}) = 12.8625
g_{21} = (7,11,0.4,1.6)	Ʈ (g_{21}) = 16.2750
g_{22} = ((17,21,0.4,1.6)	Ʈ (g_{22}) = 33.7750
g_{23} = (8,12,0.4,1.6)	Ʈ (g_{23}) = 18.0250
g_{31} = (26,30,0.2,1.8)	Ʈ (g_{31}) = 49.7000
g_{32} = ((19,23,0.2,1.8)	Ʈ (g_{32}) = 37.4500
g_{33} = (30,34,0.2,1.8)	Ʈ (g_{33}) = 56.7000

Step2. The given FGP is reduced in the following payoff matrix:

$$\begin{array}{cc} & \Upsilon \\ \delta & \begin{pmatrix} 53.1125 & 21.6125 & 12.8625 \\ 16.2750 & 33.7750 & 18.0250 \\ 49.7000 & 37.4500 & 56.7000 \end{pmatrix} \end{array}$$

The minimum in the first row =12.8625

The minimum in the second row = 16.2750

The minimum in the third row = 37.4500

The maximum in the first column = 53.1125

The maximum in the second column = 37.4500

The maximum in the third column = 56.7000

Maximum (minimum) = 37.4500 also Minimum (maximum) = 37.4500

It has saddle point. Value of Game = 37.4500

Example 2:

Consider the payoff problem of the following FG as TFN.

$$\begin{array}{cc} & \Upsilon \\ \delta & \begin{pmatrix} 3 & 13 & 5 \\ 7 & 14 & 8 \\ 12 & 16 & 6 \end{pmatrix} \end{array}$$

The minimum in the first row = 3

The minimum in the second row = 7

The minimum in the third row = 6

The maximum in the first column = 12

The maximum in the second column = 16

The maximum in the third column = 8

Maximum (minimum) = 7 but Minimum (maximum) = 8

Here Maximum (minimum) ≠ Minimum (maximum).

It has no SP.

The dominance approach is used. The second and third rows clearly dominate the first row, as all of the elements in the first row are lower than those in the second and third rows. As a result of removing the first row, we get

$$\begin{array}{cc} & \Upsilon \\ \delta & \begin{pmatrix} 7 & 14 & 8 \\ 12 & 16 & 6 \end{pmatrix} \end{array}$$

Since all of the elements in the second column are greater than those in the first and third columns, the second column is once again dominated by the first and third columns. As a consequence of removing the second column, we get

$$\begin{array}{cc} & \Upsilon \\ \delta & \begin{pmatrix} 7 & 8 \\ 12 & 6 \end{pmatrix} \end{array}$$

To determine the best mixed strategy and game value:

Now, $g_{11} = 7 , g_{12} = 8 , g_{21} = 12 , g_{22} = 6$.

$$\sigma_1 = \frac{g_{22} - g_{12}}{(g_{11} + g_{22}) - (g_{12} + g_{21})} = \frac{6 - 8}{(7 + 6) - (8 + 12)} = \frac{-2}{-7} = 0.285.$$

$$\sigma_2 = 1 - \sigma_1 = 1 - 0.285 = 0.715.$$

$$q_1 = \frac{g_{22} - g_{21}}{(g_{11} + g_{22}) - (g_{12} + g_{21})} = \frac{6 - 12}{(7 + 6) - (8 + 12)} = \frac{-6}{-7} = 0.857.$$

$$\mathfrak{r}_2 = 1 - \mathfrak{r}_1 = 1 - 0.857 = 0.143.$$

Strategy for player $\delta = (\sigma_1, \sigma_2) = (0.285 , 0.715)$

Strategy for player $\Upsilon = (\mathfrak{r}_1, \mathfrak{r}_2) = (0.857 , 0.143)$

Value of the game $\mathbb{C} = \frac{g_{11} * g_{22} - g_{12} * g_{21}}{(g_{11}+g_{22})-(g_{12}+a g_{21})} = \frac{(7*6)-(8*12)}{(7+6)-(8+12)} = \frac{-54}{-7} = 7.714.$

Consider the following FGP:

$$
\begin{array}{cc}
 & \Upsilon \\
\delta & \begin{pmatrix} (1,5,0.7,1.3) & (11,15,0.7,1.3) & (3,7,0.7,1.3) \\ (5,9,0.6,1.4) & (12,16,0.6,1.4) & (6,10,0.6,1.4) \\ (10,14,0.8,1.2) & (14,18,0.8,1.2) & (4,8,0.8,1.2) \end{pmatrix}
\end{array}
$$

Step1. We obtain the values of Ʈ (Đ) for the given FGP and transform it to a CV problem, as shown in the table below.

Table 2. The TFN problem is transformed CV in the example**.**

$g_{11} = (1,5,0.7,1.3)$	Ʈ $(g_{11}) = 5.5125$
$g_{12} = (11,15,0.7,1.3)$	Ʈ $(g_{12}) = 23.0125$
$g_{13} = (3,7,0.7,1.3)$	Ʈ $(g_{13}) = 9.0125$
$g_{21} = (5,9,0.6,1.4)$	Ʈ $(g_{21}) = 12.6000$
$g_{22} = (12,16,0.6,1.4)$	Ʈ $(g_{22}) = 24.8500$
$g_{23} = (6,10,0.6,1.4)$	Ʈ $(g_{23}) = 14.3500$
$g_{31} = (10,14,0.8,1.2)$	Ʈ $(g_{31}) = 21.1750$
$g_{32} = (14,18,0.8,1.2)$	Ʈ $(g_{32}) = 28.1750$
$g_{33} = (4,8,0.8,1.2)$	Ʈ $(g_{33}) = 10.6750$

Step 2. The given FGP is reduced in the following payoff matrix:

$$
\begin{array}{cc}
 & \Upsilon \\
\delta & \begin{pmatrix} 5.5125 & 23.0125 & 9.0125 \\ 12.6000 & 24.8500 & 14.3500 \\ 21.1750 & 28.1750 & 10.6750 \end{pmatrix}
\end{array}
$$

The minimum in the first row = 5.5125

The minimum in the second row = 12.6000

The minimum in the third row = 10.6750

The maximum in the first column = 21.1750

The maximum in the second column = 28.1750

The maximum in the third column = 14.3500

Maximum (minimum) = 12.6000 but Minimum (maximum) = 14.3500

Here Maximum (minimum) ≠ Minimum (maximum).

It has no SP.

The dominance approach is used. The second and third rows clearly dominate the first row, as all of the elements in the first row are lower than those in the second and third rows. As a result of removing the first row, we get

$$\begin{array}{cc} & \Upsilon \\ \delta & \begin{pmatrix} 12.6000 & 24.8500 & 14.3500 \\ 21.1750 & 28.1750 & 10.6750 \end{pmatrix} \end{array}$$

Since all of the elements in the second column are greater than those in the first and third columns, the second column is once again dominated by the first and third columns. As a consequence of removing the second column, we get

$$\begin{array}{cc} & \Upsilon \\ \delta & \begin{pmatrix} 12.6000 & 14.3500 \\ 21.1750 & 10.6750 \end{pmatrix} \end{array}$$

To determine the best-mixed strategy and game value:

Now, $g_{11} = 12.6000\,, g_{12} = 14.3500\,, g_{21} = 21.1750\,, g_{22} = 10.6750\,.$

$$\sigma_1 = \frac{g_{22} - g_{12}}{(g_{11} + g_{22}) - (g_{12} + g_{21})} = \frac{10.6750 - 14.3500}{(12.6000 + 10.6750) - (14.3500 + 21.1750)}$$
$$= \frac{-3.6750}{-12.2500} = 0.3000.$$

$\sigma_2 = 1 - \sigma_1 = 1 - 0.3000 = 0.7000.$

$$\mathfrak{r}_1 = \frac{g_{22} - g_{21}}{(g_{11} + g_{22}) - (g_{12} + g_{21})} = \frac{10.6750 - 21.1750}{(12.6000 + 10.6750) - (14.3500 + 21.1750)}$$
$$= \frac{-10.5000}{-12.2500} = 0.8571.$$

$\mathfrak{r}_2 = 1 - \mathfrak{r}_1 = 1 - 0.8571 = 0.1429.$

Strategy for player $\delta = (\sigma_1, \sigma_2) = (0.3000\,, 0.7000)$

Strategy for player $\Upsilon = (\mathfrak{r}_1, \mathfrak{r}_2) = (0.8571\,, 0.1429)$

Value of the game $\mathbb{C} = \frac{g_{11*}g_{22}-g_{12}*g_{21}}{(g_{11}+g_{22})-(g_{12}+g_{21})} = \frac{(12.6000*10.6750)-(14.3500*21.1750)}{(12.6000+10.6750)-(14.3500+21.1750)} = \frac{-169.3562}{-12.2500} =$ 13.8250.

7. Conclusion

This research evaluates the effectiveness of trapezoidal fuzzy numbers in stock market prediction using a proposed ranking technique based on ranking functions. Our model can achieve a more suitable partition of the universe by using trapezoidal fuzzy numbers, significantly boosting forecasting results. Furthermore, we struggle with games of fuzzy payoffs issues when stock market data is uncertain. We use the trapezoidal membership function to get the best gains, making the data fuzzier. The currently proposed ranking function algorithm uses trapezoidal fuzzy numbers for the decision-maker. We suggest an arithmetic oddment strategy to solve players (fuzzy game problem strategies, all trapezoidal fuzzy numbers) are implemented using the ranking of fuzzy numbers (FN).

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