Ibn Al Haitham Journal for Pure and Applied Sciences

Journal homepage: http://jih.uobaghdad.edu.iq/index.php/j/index

Simulation and Analysis the Effect of the Lorentz Force in a Free Electron Laser

Thair Abdulkareem Khalil Al-Aish
thair.ak.i@ihcoedu.uobaghdad.edu.iq
Department of Physics, College of Education for Pure Sciences Ibn Al-Haitham, University of Baghdad, Baghdad, Iraq.

Hanady Amjed Kamil
hanadyamjedkamil@gmail.com
Directorate of Education of First Karkh, Ministry of Education, Baghdad, Iraq.

Article history: Received,16, January, 2022, Accepted,20, February, 2022, Published in April 2022.

Doi: 10.30526/35.2.2775

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Due to the scientific and technical development in the free electron laser devices and the accompanying industrial and technological progress in various fields of civil and military life, it became necessary to expand the understanding of the mechanism of interaction of electrons (as an effective medium) with the magnetic field that they pass through to form coherent photons.

In this paper, the Lorentz force effect is simulated and analysed. The results showed that the Lorentz force originates from the magnetic field, making the electron move through it oscillate. This sinusoidal motion of the electron causes it to emit two photons for every electron wavelength. It has been concluded that the electron velocity directly affects the Lorentz power and the wavelength and power of the output laser.

Keywords free electron laser, Lorentz force, wavelength, power.

1. Introduction

At the beginning of the seventies the last century, John Madey [1] invented the free electron laser FEL, a new type of laser, that differs in its mechanism of action from the common lasers. A free electron beam is passed through an opposite periodic magnetic field, which causes the electron to oscillate by the force of Lorentz, to produce coherent photons that make up the laser beam.

Through a wide review of the literature dealing with the subject of the free electron laser, it was noted that the effect of the Lorentz force did not receive enough attention despite its great importance in making the electron's oscillatory motion and thus generating the coherent

photons. Therefore, the focus of this work was on analyzing the Lorentz force in terms of its origin and its effect on the path of electron movement through the undulator [2-6].
In this work, an executable program ELFFEL was constructed using Matlab 2019 software it as shown in **Figure (1).** contained several parameters to simulate the change in the movement electrons from accelerated linear motion to sinusoidal motion of the synchrotron beam formation. All factors affecting Lorentz force formation were studied. This was done by dividing the electron wave movement into regions that differed according to the direction of the magnetic field.

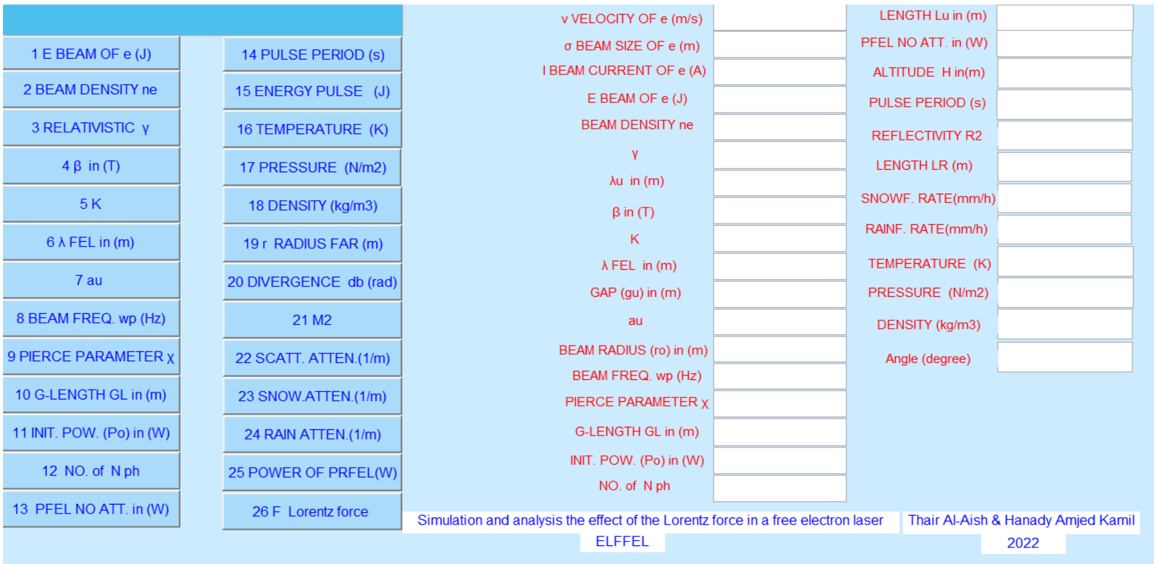

Figure 1: The implementation of executable program ELFFEL to Simulation and analysis the effect of the Lorentz force in a free electron laser.

2. Theory of the Free Electron Laser FEL

A free electron laser has an effective medium represented by a beam of electrons, which distinguishes it from other conventional lasers. That led to its possession of important advantages, including the ability to tune the wavelength of the external laser to cover a wide range of the electromagnetic spectrum, as well as the high power of the laser beam. The free electron laser consists of four main components, as shown in **Figure (2).**[7-10]

* The Electrons gun.
* The Linear accelerator for electrons.
* The undulator is a set of magnets arranged oppositely.
* The resonator is two mirrors, one of which is fully reflective and the other partially reflective.

The electrons are launched from the gun to be accelerated through the linear accelerator. When entering the undulator, the magnetic field of the magnet will be affected by the force of Lorentz, which causes it to oscillate in a sine wave to emit the coherent photons and form the output laser beam, which will be explained later in detail. [10-13]

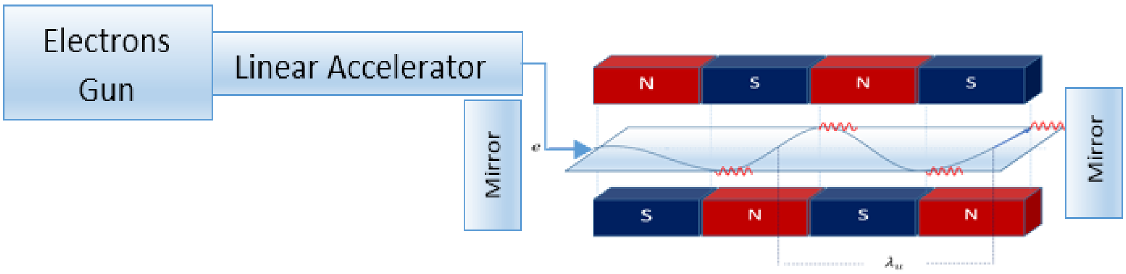

Figure 2. Components of free electron laser FEL

After performing several calculations, the equations for the wavelength λ and power P of the output laser beam are derived as shown below:[2,6,7,8]

$$\lambda = 4.095 \times 10^{-14} \times \left(\frac{\lambda_u}{{E_e}^2}\right)\left(1 + \left(4354.77 \times {\lambda_u}^2 B^2\right)\right) \quad (1)$$

$$P = 1.67 \times 10^{23} \ {E_e}^2 \ exp^{\left(2L_r + 0.021\left(\frac{L_u}{\lambda_u}\right)\right)} \quad (2)$$

Where: λ_u is the wavelength of the electron, E is the energy of the electrons beam; B is the magnetic field,

L_u is the length of the undulator and L_r is the length of the resonator.

3. Results and discussion of Simulation

The electron is launched from the electronic gun to be accelerated by a linear accelerator to reach values v close to the speed of light, then it enters the undulator towards the index finger according to the rule of the left hand as shown in **Figure (3-a)**.

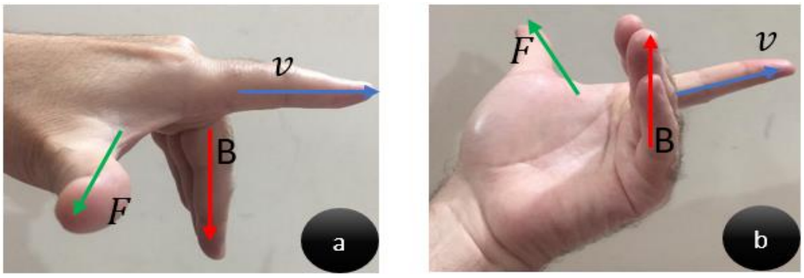

Figure 3. The rule of the left hand

Figure (4) represents an undulator part divided into four regions to analyze and simulate the movement of an electron through two rows of magnets, arranged periodically and oppositely.

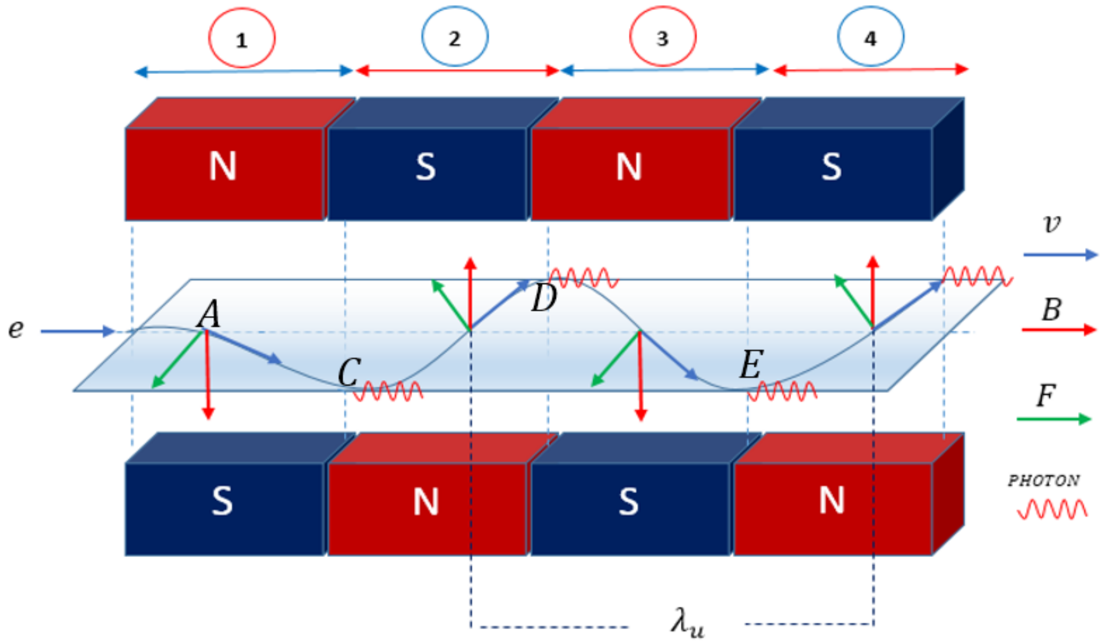

Figure 4. An electron oscillates through the undulator.

In region (1) of the undulator, the direction of the magnetic field B is towards the thumb according to the rule of the left hand as shown in **Figure (3-a)**, so the electron will be deflected as a result of being affected by the Lorentz force F.

Lorentz force F acts on an electric charge q moving in a magnetic field, which is discovered by Dutch scientist Hendrik Lorentz. In a magnetic field, the Lorentz force is the most significant when the direction of electron motion is perpendicular to the magnetic field lines. If the electron moves in a direction parallel to the direction of the magnetic field lines, the Lorentz force does not arise. The Lorentz force is always perpendicular to the direction of the electron's motion v and the magnetic field lines$B$. The Lorentz force value is given by the following equation:[14-20]

$$F = q(v \times B) = evB \sin \emptyset \qquad (3)$$

Where $\emptyset$ is the angle between the v and B. The Lorentz force causes the electron to move in a sine wave due to the presence of the term ($\sin \emptyset$). when the angle is 90^0 at point A, the Lorentz magnetic force will be at its greatest value, causing a large deviation in the electron's motion at the beginning of its entry into the undulator, but the velocity direction remains perpendicular to the direction of the Lorentz force. As a result of changing the angle from 90^0 to 45^0, the Lorentz magnitude will decrease to a minimum value at point C, causing a change in the electron's motion to make 45^0 with the magnetic field lines. Consequently, the electron's velocity slows down, and the electron loses part of its energy in the form of a photon.

After point C, due to the momentum and continuity of the electron, the electron will enter region 2 as shown in **Figure (4).** The poles of the magnet will be reversed, and thus the direction of the magnetic field will be reversed. According to the left-hand rule (**Figure(3-b)**), the velocity is towards the index finger and is perpendicular to the direction of the magnetic field. Therefore, the Lorentz magnetic force will form, which is of great value at the point where the angle is 90 between the velocity of the electron and the Lorentz force. it causes the electron to deflect, after which the value of Lorentz force decreases due to a change The angle is from 90 to 45 down to point D. the Lorentz magnitude will decrease to a minimum value at point D, causing a change in the electron's motion to make 45^0 with the magnetic field lines. Consequently, the electron's velocity slows down, and the electron loses part of its energy in the form of a photon.

After point D, due to the momentum and continuity of the electron, the poles of the magnet will be reversed and thus the direction of the magnetic field will be reversed electron will enter region 3 as shown in the **Figure (4)**, the same simulations are repeated in Region 1. As a result, the electron will oscillate in a sine wave and have a wavelength λ_u. Moreover, a new photon is released due to the slowing down of the electron. Thus, a photon will be emitted in every undulator region (1, 2, 3, and 4).. For a certain wavelength λ_u of the electron, two photons will emit within that distance.

The number of the emitted photons is determined according to what the electron traverses from regions along the undulator (each region emits one photon) or what the electron possesses of one wavelength (every single wavelength through which two photons are emitted), as shown in **Figure (4)**.

Since each region is similar to the other region inside the undulator, as each has similar values for both the magnetic field and the electron velocity, the same magnetic force will be created in each region according to equation (3), which notes that the reversal of the electron's direction as a result of the polarity reflection of the magnets which closely resembles the electron's behavior (its wavelength and the loss of its energy in each region), will produce coherent photons in each region. These coherent photons are faster than an electron because they have the speed of light, so they will precede the electrons that produced them, gather in the resonator, and then leave the partial reflection mirror to produce the laser beam at a certain threshold.

The electron energy E_e value is a function of the electron path in the undulator. High-energy electrons have a shorter path than low-energy electrons, so coherent photons will be generated faster as the electron energy increases. The total energy of an electron E_e represents the sum of the electron's kinetic energy E_K and its rest energyE_0, as shown in the equation below:[16-19]

$$E_e = E_K + E_0 = \gamma \ m_e \ c^2 \qquad (4)$$

Where the ($\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$) is the relativistic factor.

$$E_e = \frac{m_e \ c^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (5)$$

$$v = c \ \sqrt{1-\frac{{m_e}^2 \ c^4}{{E_e}^2}} \qquad (6)$$

Table (1), shows the effect of changing the electrons' velocity v about the Lorentz force F upon entering the magnetic field. As the energy $E_e$of the electrons increases as a result of an increase in electrons velocity v according to the equation (5), the values of the Lorentz force F will increase according to the equation (3). **Figure (5)** shows the sinewave behavior of the Lorentz force for different velocities of electrons.

Table 1: The Values of Lorentz force F in the undulator.

When $v = 2.5 \times 10^8\ m/s$				When $v = 2.9 \times 10^8\ m/s$		
$\emptyset$	F	PARAMETERS		F	PARAMETERS	
90	0 (B=0 out undulator)	E_e J	1.48163e-13	0 (B=0 out undulator)	E_e J	3.19874e-13
105	0 (B=0 out undulator)	B T	0.0407825	0 (B=0 out undulator)	B T	0.0407825
120	0 (B=0 out undulator)	λ_u m	0.02	0 (B=0 out undulator)	λ_u m	0.02
135	0 (B=0 out undulator)	λ m	0.000658896	0 (B=0 out undulator)	λ m	6.70054e-07
150	0 (B=0 out undulator)	P w	23844.5	0 (B=0 out undulator)	P w	1.06253e+19
165	0 (B=0 out undulator)			0 (B=0 out undulator)		
180	0 (B=0 out undulator)			0 (B=0 out undulator)		
195	4.22212-e-13			-4.89765e-13		
210	8.1565-e-13			-9.46154e-13		
225	1.1535-e-12			-1.33806e-12		
240	1.41275-e-12			-1.63879e-12		
255	1.57571-e-12			-1.82783e-12		
270	1.6313-e-12			-1.89231e-12		
285	1.57571-e-12			-1.82783e-12		
300	1.41275-e-12			-1.63879e-12		
315	1.1535-e-12			-1.33806e-12		
330	8.1565-e-13			-9.46154e-13		
345	4.22212-e-13			-4.89765e-13		
360	0 (sin360=0)			0 (sin360=0)		
375	4.22212e-13			4.89765e-13		
390	8.1565e-13			9.46154e-13		
405	1.1535e-12			1.33806e-12		
420	1.41275e-12			1.63879e-12		
435	1.57571e-12			1.82783e-12		
450	1.6313e-12			1.89231e-12		
465	1.57571e-12			1.82783e-12		
480	1.41275e-12			1.63879e-12		
495	1.1535e-12			1.33806e-12		
510	8.1565e-13			9.46154e-13		
525	4.22212e-13			4.89765e-13		
540	0 (sin540=0)			0 (sin540=0)		

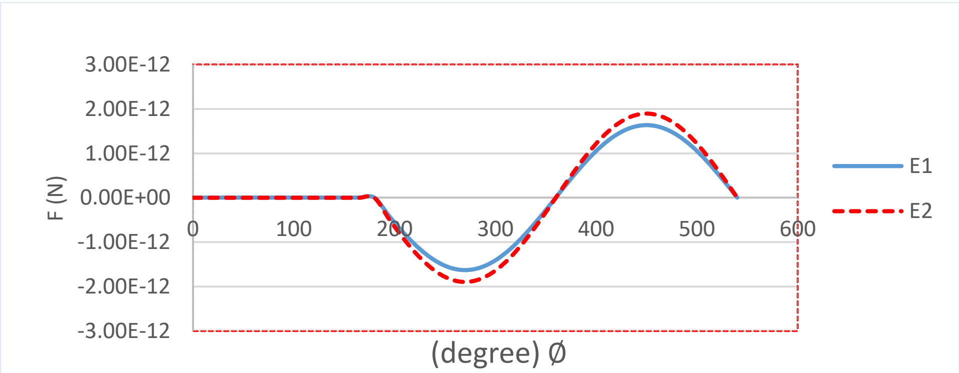

Figure 5: shows the sinewave behavior of the Lorentz force for different $\emptyset$

In **Figure (6),** it can be seen that increasing the electron's velocity and approaching the speed of light will lead to an exponential increase in the electron's energy E_e and the power P of the output laser according to equation (5) and equation (2), respectively. While the increase is linear and gradual in Lorentz's force according to Equation (3). Finally, the wavelength of the output laser will decrease linearly and gradually according to equation (1).

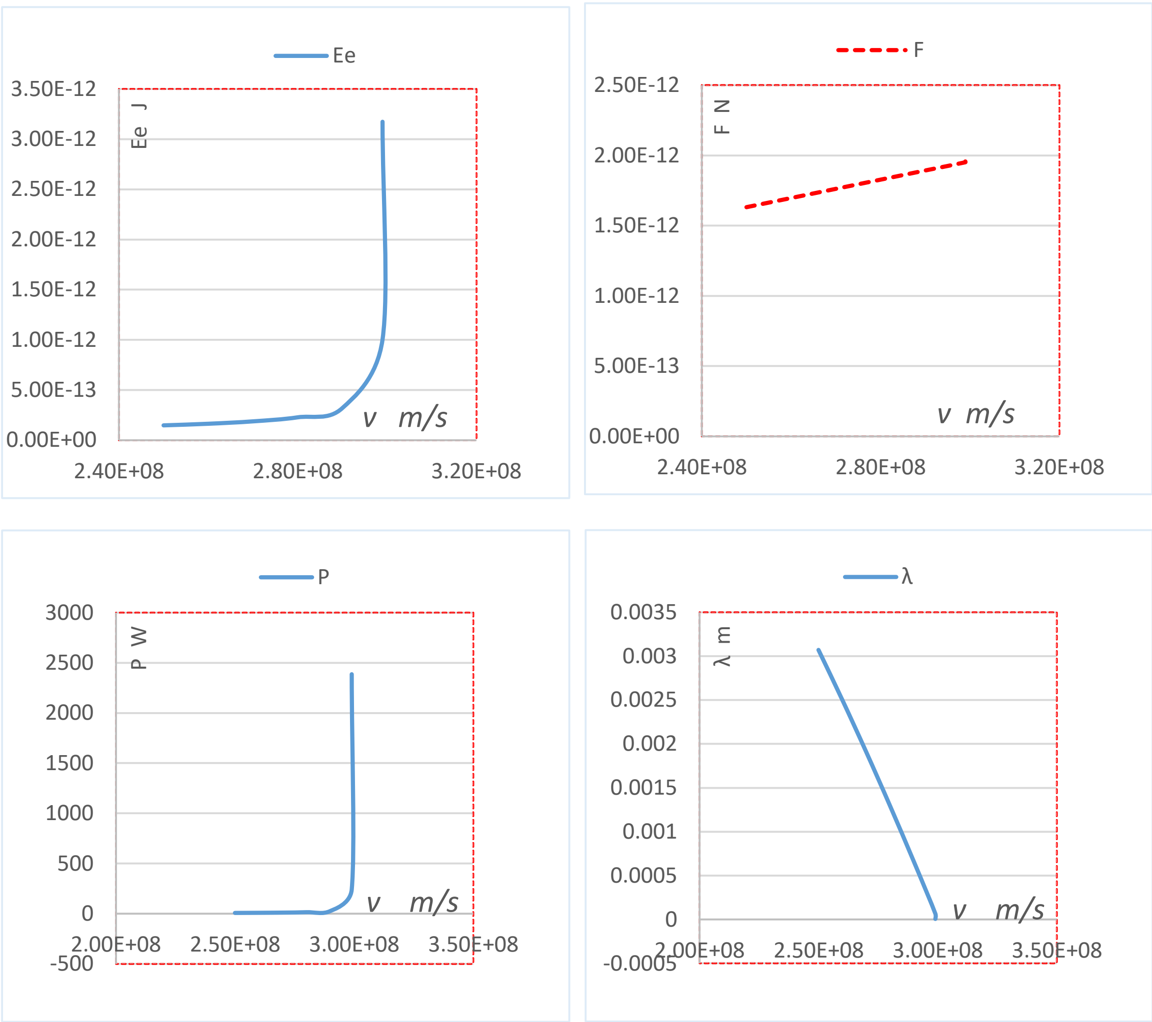

Figure 6: The effects of increasing the electron's velocity.

Now, the analytical methods will be illustrated below to show the effect of the Lorentz force in FEL. The beam-radiation interaction in the undulator is described by classical physics while in quantum physics effects are expected to be small. Consider an ultra-relativistic electron traversing in the undulator magnetic field described by the equation below.

$$B = B_0 \sin(k_u x) \qquad (7)$$

Where $\left(k_u = \frac{2\pi}{\lambda_u}\right)$ and B_0 is the magnetic pole field. When the velocity v is assumed to be equal to the speed of light c, and based on equations (3,4 and 7), we obtain

$$\gamma\ m_e \frac{dv}{dt} \approx -e(v \times B)$$

Yields two coupled equations

$$\frac{dv_x}{dt} \approx -\frac{ev_z B_y}{\gamma\, m_e} \quad and \quad \frac{dv_z}{dt} \approx -\frac{ev_x B_y}{\gamma\, m_e} \qquad (8)$$

Integrating equation (8) and $(v_z \approx v = \beta c = constant)$, $(v_x \ll v_z)$, $\left(\frac{dv_z}{dt} \approx 0\right)$, leads to obtaining the transverse velocity and the solution for $x(t), z(t)$.

$$v_x \approx \frac{eB_0}{\gamma\, m_e k_u} \cos(k_u z) \tag{9}$$

$$x(t) \approx \frac{eB_0}{\gamma\, m_e \beta c k_u^2} \sin(k_u \beta c t) \tag{10}$$

$$z(t) \approx \beta c t \tag{11}$$

From the equations above, an important dimensionless undulator parameter K has been obtained, which is equal to

$$K = \frac{eB_0}{m_e c\, k_u} = \frac{eB_0\, \lambda_u}{2\pi\, m_e c\, k_u} = 0.934\, B_0(Tesla\ un.)\, \lambda_u(cm\ un.) \tag{12}$$

in Figure (7), it can be seen that increasing the magnetic field will lead to an exponential increase in the wavelength of the output laser and a linear increase in the undulator parameter K according to equation (1) and equation (12), respectively. While an exponential decrease in the power P of the output laser according to Equation is observed (2).

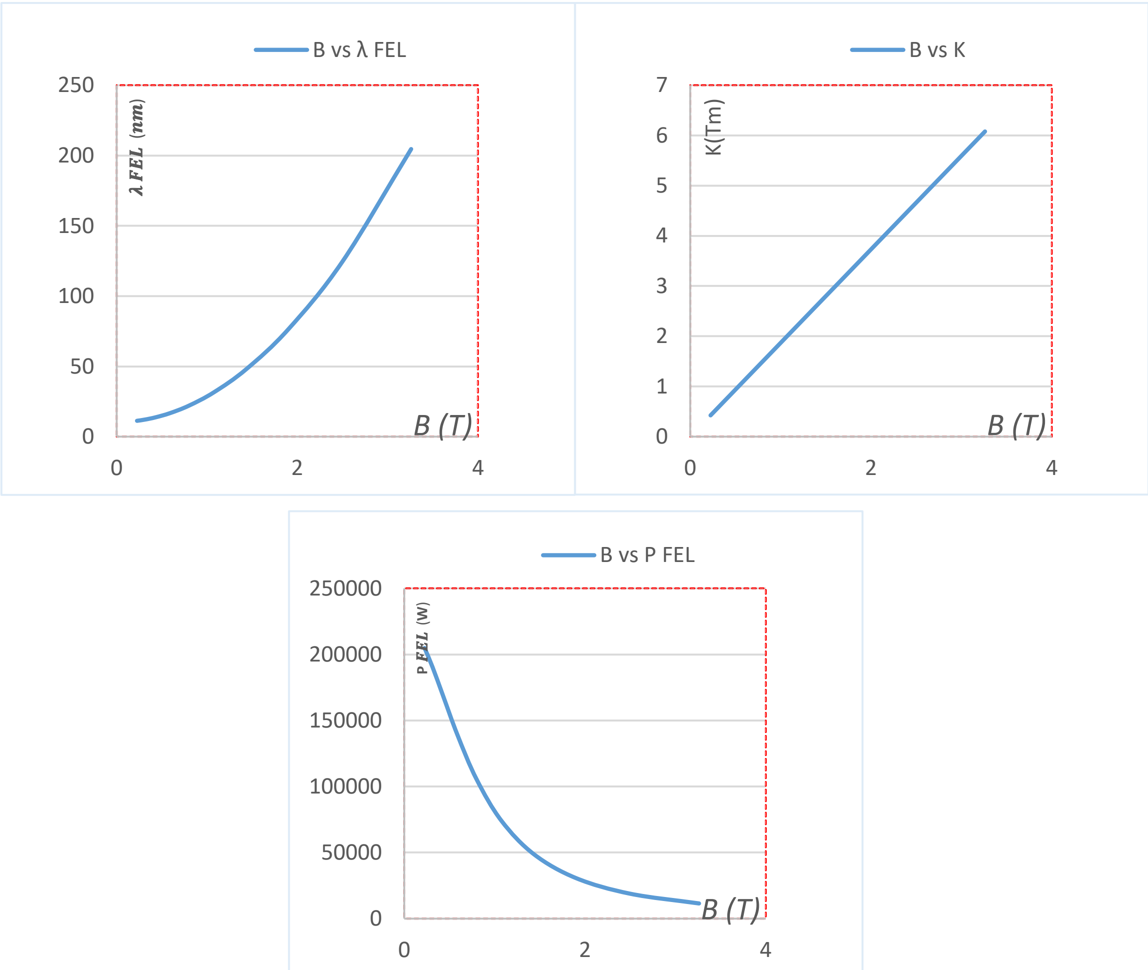

Figure 7: The effects of increasing the magnetic field.

4. Conclusions

From the simulation results obtained, it can be concluded that the Lorentz force is responsible for generating the laser beam in the free electron laser. Coherent photons are emitted as the electron gains an oscillatory movement that accelerates and decelerates it according to the direction of the magnetic field. Lorentz's power can be controlled by the velocity of the electrons released from the electron launcher and the resulting change in the wavelength power of the output laser.

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