Complete Arcs in Projective Plane PG (2,11) Over Galois field

Mahmood S. Fiadh

Department of computer/ College of Education / The Iraqi University

Received in: 19 September 2012 , Accepted in:3 February 2013

Abstract

In this work, we construct complete (K, n)-arcs in the projective plane over Galois field GF (11), where $2 \le n \le 12$,by using geometrical method (using the union of some maximum(k,2)- Arcs , we found (12,2)-arc, (19,3)-arc , (29,4)-arc, (38,5)-arc , (47,6)-arc, (58,7)-arc, (68,6)-arc, (81,9)-arc, (96,10)-arc, (109,11)-arc, (133,12)-arc, all of them are complete arc in PG(2, 11) over GF(11).

Key words : Algebraic geometry, complete arcs, Galois field.

Introduction

Yasin 1986, [8] gave the construction and classification of(k,3)-arcs over the Galois field GF(8), Ahmed (1999) [1] studied the complete arcs in projective plane over Galois field GF (9), also sawan (2001) [2], studied the construction of (k ,n)-arcs form (k , m)-arcs in PG(2,p)

The aim of this paper is to find the complete arcs in projective plane PG (2, 11) over Galois field GF (11).This paper is divided into twelve sections, section one consists of the basic theorems and definition of projective plane.From In section two to section twelve, the construction of complete (k, n)-arcs for $2 \le n \le 12$ in PG (2, 11) is given

1.1Definition [3]:

A projective PG(2, p) over Galois field , where p is a prime number , consists of $p^2 + p + 1$points and $p^2 + p + 1$lines, every line contains $p+1$ points and every point is on $p+1$ lines. Any point a of the plane has the form of a triple ((X_0, X_1, X_2), where X_0, X_1, X_2 are elements in GF(p) with the exception of a triple consisting of three zero elements. Two triples (X_0, X_1, X_2) and (y_0, y_1, y_2) represent the same point if there exists λ in GF (p) \ {0}, s.t. (y_0, y_1, y_2) = λ (X_0, X_1, X_2).

There exists one point of the form (1,0,0) , there exists p points of the form $(X,1,0)$, there exists p^2 points of the form $(X,Y,1)$, similarly for the lines .

A point P (X_0, X_1, X_2) is incident with the line [y_0, y_1, y_2] iff :

$X_0 y_0 + X_1 y_1 + X_2 y_2 = 0$

The projective plane PG(2, p) satisfying the following axioms :

a) Any two distinct lines intersected in a unique point.
b) Any two distinct points are contained in a unique line.
c) There exists at least four points such that no three of them are collinear.

The projective plane PG(2, 11) contains 133 points , 133 lines , every line contains 12 point and every points is on 12 lines. Let p_i and L_i , i= 1, 2,… ,133 be the points and lines of PG (2, 11) respectively , all the points and lines of PG (2, 11) are given in table (1).

1.2Definition [5]:

A (k, n)-arc K in PG (2, P) is a set of k points such that some n, but no n+1 of them are collinear.

1.3Definition [5]:

(A k, n)-arc in PG (2, P) k is complete if it is not contained in a (K+1,n)-arc .

1.4Definition [3]:

An (n-secant) of a (k,n)-arc is a line intersects K in n points . a o-secant is called an external line of K, 1-secant is called unsecant line, a 2-secant is called a bisecant line and 3-secant is called a trisecant line.

1.5Definition [3]:

A point which is not on a (k, n)-arc K has index i denoted by N_i , if there are exactly i (n-secant) of K thought N_i . Let $C_i = |N_i|$ be the number of the points N_i of index i.

1.6Remark [4]:

The (k, n)-arc K is complete if and only if $C_0 = 0$, thus K is complete if every point of PG(2, p) lies on some n-secants of K.

1.7Definition [5]:

A k, n)-arc K in PG(2. p) is maximal arc if k =(n-1)p+n.

1.8Definition [3]:

The maximum number of points that can be a (K, 2)-arc in PG(2, p) is m(2, p)- this arc called an oval.

1.9Definition [5]:

A polynomial F in $K[X_1, X_2, ... X_n]$ is called homogenous or a form of degree d if all its terms have the same degree d. A subset V of PG (n, k) is variety over K if there exists forms $F_1, F_2, ..., F_R$ in $K[X_1, X_2, ... X_n]$ such that V = {P(A) in PG(n, k), $F_1(A) = F_2(A) = ... = F_R(A) = 0\} = V(F_1, F_2, ..., F_R)$.

1.10Definition [5]:

A variety V(F) in PG(n, k) is called a primal. The order or degree of a primal V(F) is the degree of F.

1.11Definition [5]:

A quadric Q in PG(n-1, p) is a primal of order two , so if Q is a quadric then Q=F(V) where F is Quadric form , that is $\sum_{\substack{i \le j \\ i,j=1}}^{n} a_{ij} X_i X_j = a_{11} X_1^2 + a_{12} X_1 X_2 + ... + a_{nn} X_n^2$

1.12Definition [5]:

Let Q(2, p) be the set of quadrics in PG(2, p) that is the varieties V(F), where $F = a_{11} X_1^2 + a_{22} X^2{}_2 + a_{33} X_3^2 + a_{12} X_1 X_2 + a_{13} X_1 X_3 + a_{23} X_2 X_3$

If $A = \begin{bmatrix} a_{11} & \frac{a_{12}}{2} & \frac{a_{13}}{2} \\ \frac{a_{21}}{2} & a_{22} & \frac{a_{23}}{2} \\ \frac{a_{31}}{2} & \frac{a_{32}}{2} & a_{33} \end{bmatrix}$ is non-singular, then the quadric is a conic.

1.13Theorem [3]:

In PG (2, p) , with p odd, every oval is a conic.

1.14Theorem [6]:

Let m be a point of a (K, 2)-arc K and let t(m) be the number of unisecants through m in PG(2p) then $t = t(m) = p + 2 - k$

1.15Corollary [6]:

If k is an oval then t(m) =1

1.16Theorem [5]:

Let k be a (k, 2)-arc in PG (2, p) and let T_i be the number of i-secants of k in the plane, that is T_2 is the number of bisecants, T_1 is the number of unisecant, and T_0 is the number of external line $t = p + 2 - k$, then:

a) $T_2 = \dfrac{k(k-1)}{2}$

b) $T_1 = kt$

c) $T_0 = \dfrac{p(p-1)}{2} + \dfrac{t(t-1)}{2}$

1.17Lemma [7]:

Let C_i be the number of points Q of index i . Then

a) $\sum_{\alpha}^{\beta} C_i = p^2 + p + 1 - k$

b) $\sum_{\alpha}^{\beta} iC_i = \dfrac{k(k-1)}{2(p-1)}$

where α is smallest I for which $C_i \neq 0$, and β is the largest I for which $C_i \neq 0$.

1.118Theorem [2]:

A (k, n)-arc K is maximal if and only if every line in PG (2, p) is a 0-secant or n-secant,

2. Complete (K, 2)-arc in PG(2, 11)

Let A={1,2,13,25} be the set of unit and reference points in PG(2,11) as in the table (1) such that :

1=(1,0,0) , 2=(0,1,0), 13=(0,0,1), 25=(1,1,1), A is (4,2)-arc, since no three points of A are collinear,

The general equation of the conic is:

$$F = a_1 X_1^2 + a_2 X_2^2 + a_3 X_3^2 + a_4 X_1 X_2 + a_5 X_1 X_3 + a_6 X_2 X_3 = 0 \qquad \ldots (1)$$

By substituting the points of the arc A in (1), so (1) becomes:

$$a_4 X_1 X_2 + a_5 X_1 X_3 + a_6 X_2 X_3 = 0 \qquad \ldots (2)$$

If $a_4 = 0$, then the conic is degenerated. Therefore $a_4 \neq 0$, similarly $a_5 \neq 0$ and $a_6 \neq 0$

Dividing equation (2) by a_4, we get:

$$X_1 X_2 + \alpha X_1 X_3 + \beta X_2 X_3 = 0 \qquad \ldots (3)$$

Where $\alpha = \dfrac{a_5}{a_4}$, $\beta = \dfrac{a_6}{a_4}$, so that $1 + \alpha + \beta = 0 \pmod{11}$

β = - (1+ α), then (3) can be writer as: $X_1 X_2 + \alpha X_1 X_3 - (1+\alpha) X_2 X_3 = 0$

Where $\alpha \neq 0$ and $\alpha \neq 10$ for $\alpha = 0$ or $\alpha = 10$, we degerenated conics, can be obtaiened thus $\alpha = 1,2,3,4,5,6,7,8,9$. The ovals in PG(2,11) throught the reference and the unit points has the following points :

$C_1 = \{1,2,13,25,40,53,63,77,87,100,104,116\}$ which are the points of $V_1(X_1X_2 + X_1X_3 + 9X_2X_3)$

$C_2 = \{1,2,13,25,42,50,59,78,84,96,110,131\}$ which are the points of $V_2(X_1X_2 + 2X_1X_3 + 8X_2X_3)$

$C_3 = \{1,2,13,25,41,48,64,76,89,95,115,132\}$ which are the points of
$V_3(X_1X_2 + 3X_1X_3 + 7X_2X_3)$
$C_4 = \{1,2,13,25,44,56,65,72,82,108,118,125\}$ which are the points of
$V_4(X_1X_2 + 4X_1X_3 + 6X_2X_3)$
$C_5 = \{1,2,13,25,43,51,67,71,99,103,119,127\}$ which are the points of
$V_5(X_1X_2 + 5X_1X_3 + 5X_2X_3)$
$C_6 = \{1,2,13,25,45,52,62,88,98,105,114,126\}$ which are the points of
$V_6(X_1X_2 + 6X_1X_3 + 4X_2X_3)$
$C_7 = \{1,2,13,25,38,55,75,81,94,106,122,129\}$ which are the points of
$V_7(X_1X_2 + 7X_1X_3 + 3X_2X_3)$
$C_8 = \{1,2,13,25,39,60,74,86,92,111,120,128\}$ which are the points of
$V_8(X_1X_2 + 8X_1X_3 + 2X_2X_3)$
$C_9 = \{1,2,13,25,54,66,70,83,93,107,117,130\}$ which are the points of
$V_9(X_1X_2 + 9X_1X_3 + 1X_2X_3)$
Thus there are nine compelete (12,2)-arcs (conics) in PG(2,11). Hence each arc is a maximum arc, since each line is 0-secant or 2-secant.

3.The construction of compelete (k,3)-arcs in PG(2,11)

In this section, we try to get a compelete (k,3)-arc throught following steps.

a) We take the union of two maximal (k,2)-arcs, say C_1 and C_2 denoted by E, $E = C_1 \cup C_2 - \{59,78,96\}$, we notice that $E = \{1,2,13,25,40,53,63,77,87,100,104,116,42,50,84,110,131\}$ is incompelete (k,3)-arc since there exists the points {3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36,37,38,39,41,43,44,45,46,47,48,49,51,52,54,55,56,57,58,59,60,61,62,64,65,66, 67,68,69,70,71,72,73,74,75,76,78,79,80,81,82,83,85,86,88,89,90,91,92,93,94,95,96, 97,98,99,101,102,103,105,106,107,108,109,111,112,113,114,115,117,118,119,120, 121,122,123,124,125,126,127,128,129,130,132,133} which are the points of index zero for E .

b) We add two points of index zero which are {3,46}. Then $E' = \{1,2,3,13,25,40,42,46,50,53,63,77,84,87,100,104,110,116,131\}$ is a compelete (19,3)-arc since $C_0 = 0$.

4.The construction of compelete (k,4)-arcs in PG(2,11)

In this section, we try to get a compelete (k,4)-arc through following steps.

a) We take the union of three maximal (k,2)-arcs, say C_1 , C_2 and C_3 denoted by E_1, $E_1 = C_1 \cup C_2 \cup C_3 - \{48,64,89,132\}$, we notice that $E_1 = \{1,2,13,25,40,53,63,77,87,100,104,116,42,50,59,78,84,96,110,131,41,76,95,115\}$ is incompelete (k,4)-arc since there exists the points {3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36,37,38,39,43,44,45,46,47,48,49,51,52,54,55,56,57,58,60,61,62,64,65,66,67, 68,69,70,71,72,73,74,75,79,80,81,82,83,85,86,88,89,90,91,92,93,94,97,98,99,101,102, 103,105,106,107,108,109,111,112,113,114,117,118,119,120,121,122,123,124, 125,126,127,128,129,130,132,133} which are the points of index zero for E_1.

b) We add five points of index zero which are {3,33,34,38,108}. Then $E_1' = \{1,2,3,13,25,33,34,38,40,41,42,50,53,59,63,76,77,78,84,87,95,96,100,104,108,110, 115,116,131\}$. is a compelete (29,4)-arc since $C_0 = 0$.

5.The construction of compelete (k,5)-arcs in PG(2,11)

In this section, we try to get a compelete (k,5)-arc throught following steps.

a) We take the union of four maximal (k,2)-arcs, say C_1 ,C_2 ,C_3 and C_4 denoted by E_2,

$E_2 = C_1 \cup C_2 \cup C_3 \cup C_4 - \{44,56,82,108,118\}$,

$E_2 = \{1,2,13,25,40,53,63,77,87,100,104,116,42,50,59,78,84,96,110,131,41,48,64,76,89,95, 115,132,65,72,125\}$ E_2 is incompelete (k,5)-arc since there exists the points {3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36,37,38,39,43,44,45,46,47,49,51,52,54,55,56,57,58,60,61,62,66,67,68,69,70 ,71,73,74,75,79,80,81,82,83,85,86,88,89,90,91,92,93,94,97,98,99,101,102,103,105 ,106,107,108,109,111,112,113,114,117,118,119,120,121,122,123,124,126,127,128 ,129,130,133} which are the points of index zero for E_2.

b) We add seven points of index zero which are {9,10,12,19,31,39,68}. Then $E_2' = \{1,2,9,10,12,13,19,25,31,39,40,41,42,48,50,53,59,63,64,65,68,72,76,77,78,84,87,89, 95,96,100,104,110,115,116,125,131,132\}$. E_2' is a compelete (38,5)-arc since $C_0 = 0$.

6.The construction of compelete (k,6)-arcs in PG(2,11)

In this section, we try to get a compelete (k,6)-arc through following steps.

a) We take the union of five maximal (k,2)-arcs, say C_1 ,C_2 ,C_3, C_4 and C_5 denoted by E_3, ,

$E_3 = C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 - \{82,118\}$,

$E_3 = \{1,2,13,25,40,53,63,77,87,100,104,116,42,50,59,78,84,96,110,131,41,48,64,76,89, 95,115,132,44,56,65,72,108,125,43,51,67,71,99,103,119,127\}$. E_3 is incompelete (k,6)-arc since there exists the points {3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36,37,38,39,45,46,47,49,52,54,55,57,58,60,61,62,66,68,69,70,73,74,75,79,80,81, 82,83,85,86,88,90,91,92,93,94,97,98,101,102,105,106,107,109,111,112,113,114,117,118,120, 121,122,123,124,126,128,129,130,133} which are the points of index zero for E_3.

b) We add six points of index zero which are {9,10,12,15,16,85}. Then $E_3' = \{1,2,9,10,12,13,15,16,25,40,41,42,43,44,48,50,51,53,56,59,63,64,65,67,71,72,76,77, 78,84,85,87,89,95,96,99,100,103,104,108,110,115,116,119,125,127,131,132\}$. E_3' is a compelete (48,6)-arc since $C_0 = 0$.

7.The construction of compelete (k,7)-arcs in PG(2,11)

In this section, we try to get a compelete (k,6)-arc through following steps.

a)We take the union of six maximal (k,2)-arcs, say C_1 ,C_2 ,C_3, C_4, C_5 and C_6 denoted by E_4 , $E_4 = C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6$,

E_4 = {1,2,13,25,40,53,63,77,87,100,104,116,42,50,59,78,84,96,110,131,41,48,64,76,89, 95,115,132,44,56,65,72,82,108,118,125,43,51,67,71,99,103,119,127,45,52,62,88,98,105, 114,126}. E_4 is incompelete (k,6)-arc since there exists the points {3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34 ,35,36,37,38,39,46,47,49,54,55,57,58,60,61,66,68,69,70,73,74,75,79,80,81,83,85,86 ,90,91,92,93,94,97,101,102,106,107,109,111,112,113,117,120,121,122,123,124,,128 ,129,130,133} which are the points of index zero for E_4.

b) We add six points of index zero which are {4,19,27,31,34,81}. Then E_4' = {1,2,4,13,19,25,27,31,34,40,41,42,43,44,45,48,50,51,52,53,56,59,62,63,64,65,67,71, 72,76,77,78,81,82,84,87,88,89,95,96,98,99,100,103,104,105,108,110,114,115,116,118,119, 125,126,127,131,132}. E_4' is a compelete (58,7)-arc since $C_0 = 0$

8.The construction of compelete (k,8)-arcs in PG(2,11)

In this section, we try to get a compelete (k,8)-arc through following steps.

a) We take the union of seven maximal (k,2)-arcs, say C_1 ,C_2 ,C_3,C_4,C_5 C_6,and C_7 denoted by E_5 , $E_5 = C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7$,

E_5 = {1,2,13,25,40,53,63,77,87,100,104,116,42,50,59,78,84,96,110,131,41,48,64,76,89, 95,115,132,44,56,65,72,82,108,118,125,43,51,67,71,99,103,119,127,45,52,62,88,98,105, 114,126,38,55,75,81,94,106,122,129} E_5 is incompelete (k,8)-arc since there exists the points {3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36,3 7,39,46,47,49,54,57,58,60,61,66,68,69,70,73,74,79,80,83,85,86,90,91,92,93,97,101,102,107,1 09,111,112,113,117,120,121,123,124,,128,130,133} which are the points of index zero for E_5 .

b) We add eight points of index zero which are {3,4,5,6,7,11,19,31}. Then E_5' = {1,2,3,4,5,6,7,11,13,19,25,31,38,40,41,42,43,44,45,48,50,51,52,53,55,56,59,62,63, 64,65,67,71,72,75,76,77,78,81,82,84,87,88,89,94,95,96,98,99,100,103,104,105,106,108, 110,114,115,116,118,119,122,125,126,127,129,131,132} E_5' is a compelete (68,8)-arc since $C_0 = 0$.

9.The construction of compelete (k,9)-arcs in PG(2,11)

In this section, we try to get a compelete (k,9)-arc through following steps.

a) We take the union of eight maximal (k,2)-arcs, say C_1 ,C_2 ,C_3,C_4,C_5 C_6,C_7 and C_8 denoted by E_6 , $E_6 = C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8$,

E_6={1,2,13,25,40,53,63,77,87,100,,104,116,42,50,59,78,84,96,110,131,41,48,64,76, 89,95,115,132,44,56,65,72,82,108,118,125,43,51,67,71,99,103,119,127,45,52,62,88, 98,105,114,126,38,55,75,81,94,106,122,129,39,60,74,86,92,111,120,128}

E_6 is incompelete (k,9)-arc since there exist the points {3,4,5,6,7,8,9,10,11,12,14,15, 16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36,37,46,47,49,54,57,58, 61,66,68,69,70,73,79,80,83,85,90,91,93,97,101,102,107,109,112,113,117,121,123, 124,130,133} which are the points of index zero for E_6.

b) We add thirteen points of index zero whichare{4,5,6,7,10,11,12,18,22,37,49,61, 109}Then E_6' ={1,2,4,5,6,7,10,11,12,13,18,22,25,37,38,39,40,41,42,43,44,45,48,49,50,51,52,53,55,56,59,60,61,62,63,64,65,67,71,72,74,75,76,77,78,81,82,84,86,87,88,89, 92,94,95,96,98,99,100,103,104,105,106,108,109,110,111,114,115,116,118,119,120, 122,125,126,127,128,129,131,132}is complete (81,9) –arc,since $C_0 = 0$.

10.The construction of compelete (k,10)-arcs in PG(2,11)

In this section, we try to get a compelete (k,10)-arc through following steps.

a)We take the union of nine maximal (k,2)-arcs, say C_1 ,C_2 ,C_3,C_4,C_5,C_7 C_8 and C_9 denotedby E_7, $E_7 = C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8 \cup C_9$, E_7 ={1,2,13,25,40,53,63,77,87,100,,104,116,42,50,59,78,84,96,110,131, 41,48,64,76,89,95,115,132,44,56,65,72,82,108,118,125,43,51,67,71,99,103,119,127, 45,52,62,88,98,105,114,126,38,55,75,81,94,106,122,129,39,60,74,86,92,111,120,128,54,66,70,83,93,107,117,130} E_7 is incompelete (k,10)-arc since there exists the points {3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36,37,46,47,49,57,58,61,68,69,73,79,80,85,90,91,97,101,102,109,112,113,121,123,124,133} which are the points of index zero for E_7.

b) We add twenty points of index zero which are{3,4,5,6,7,8,9,10,14,15,16,22,23,61 69,73,85,97,109,124}Then E_7' ={1,2,3,4,5,6,7,8,9,10,13,14,15,16,22,23,25,38,39,40, 41,42,43,44,45,48,50,51,52,53,54,55,56,59,60,61,62,63,64,65,66,67,69,70,71,72,73, 74,75,76,77,78,81,82,83,84,85,86,87,88,89,92,93,94,95,96,97,98,99,100,103,104,105, 106,107,108,109,110,111,114,115,116,117,118,119,120,122,124,125,126,127,128, 129,130,131,132}is complete (96,10) –arc,since $C_0 = 0$.

11.The construction of compelete (k,11)-arcs in PG(2,11)

In this section, we take (96,10)-arc E_7' which is incomplete (k,11)-arc, since there exist points of index zero for E_7' which are{11,12,17,18,19,20,21,24,26,27,28 ,29,30,31,32,33,34,35,36,37,46,47,49,57,58,68,79,80,90,91,101,102,112,113,121,123, 133} ,i.e. $C_0 \neq 0$.

Now ,we add to E_7' thirteen points of index zero which are {11,17,18,19,20,26,27,35, 36,47,102,121,133}.

Then E_8' ={1,2,3,4,5,6,7,8,9,10,11,13,14,15,16,17,18,19,20,22,23,25,26,27,35,36,38,39,40,41,42,43,44 ,45,47,48,50,51,52,53,54,55,56,59,60,61,62,63,64,65,66,67,69,70,71, 72,73,74,75,76,77,78,81,82,83,84,85,86,87,88,89,92,93,94,95,96,97,98,99,100,102, 103,104,105,106,107,108,109,110,111,114,115,116,117,118,119,120,121,122,124, 125,126,127,128,129,130,131,132,133}is a complete (109,11) –arc,since $C_0 = 0$.

12.The construction of compelete (k,12)-arcs in PG(2,11)

In this section, we take from E_8' the complete (109,11)-arc which is incomplete (k,12)-arc E_8' since there exist points of index zero for E_8' which are{12, ,21,24,28,29, 30,31,32,33,34,37,46,49,57,58,68,79,80,90,91,101,112,113, 123,} ,i.e. $C_0 \neq 0$.

We add the points of index zero to E'_8 denoted by E'_9 then E'_9 contains all the points of the plane i.e., E'_9={1,2,3,…,131,132,133} is a complete (133,12)-arc since $C_0 = 0$.
This arc is the whole plane PG(2,11), since each line in it contains 12 points . Hence this arc is maximal.

References

1. Ahmed, A.m (1999). "Complete Arcs in Projective Plane Over GF(9)", Accepted to puplish , Ibn Al-Hitham, Iraq.
2. Swsan,J.K (2001). M.SC.Thesis,University of Baghdad, Ibn-Al-Haitham College, Iraq.
3. Hirschfeld, J.W.P (1979). "Projective Geometries Over Finite Fields", Oxford Press.
4. Saleh,R.A (1999). "Complete Arcs in Projective Plane Over Galois filed",M.S.C.Thesis,College of Education Ibn-Al-Haitham, University of Baghdad,Iraq.
5. Hughes,D.R. and Piper, F.C (1973)."Projective Plane", Springer Village,New York, Inc.
6. Al-Mukhtar, A.S (2001). Mathematics and Physics.J.Vol.17.
7. بان عبد الكريم ، (2001) القيد الأعلى للاقواس (k,n)-arc، رسالة ماجستير ، جامعة الموصل.
8. Yasin, A. L (1986). "Cubic Arcs in the Projective Plane of Order Eight", Ph.D. Thesis, Uinversity of Sussex, England.

Table (1):Points and lines of PG(2,11)

i	Pi	Li											
1	(1,0,0)	2	13	24	35	46	57	68	79	90	101	112	123
2	(0,1,0)	1	13	14	15	16	17	18	19	20	21	22	23
3	(1,1,0)	12	13	34	44	54	64	74	84	94	104	114	124
4	(2,1,0)	7	13	29	45	50	66	71	87	92	108	113	129
5	(3,1,0)	9	13	31	38	56	63	70	88	95	102	120	127
6	(4,1,0)	10	13	32	40	48	67	75	83	91	110	118	126
7	(5,1,0)	4	13	26	39	52	65	78	80	93	106	119	132
8	(6,1,0)	11	13	33	42	51	60	69	89	98	107	116	125
9	(7,1,0)	5	13	27	41	55	58	72	86	100	103	117	131
10	(8,1,0)	6	13	28	43	47	62	77	81	96	111	115	130
11	(9,1,0)	8	13	30	36	53	59	76	82	99	105	122	128
12	(10,1,0)	3	13	25	37	49	61	73	85	97	109	121	133
13	(0,0,1)	1	2	3	4	5	6	7	8	9	10	11	12
14	(1,0,1)	2	23	34	45	56	67	78	89	100	111	122	133
15	(2,0,1)	2	18	29	40	51	62	73	84	95	106	117	128
16	(3,0,1)	2	20	31	42	53	64	75	86	97	108	119	130
17	(4,0,1)	2	21	32	43	54	65	76	87	98	109	120	131
18	(5,0,1)	2	15	26	37	48	59	70	81	92	103	114	125
19	(6,0,1)	2	22	33	44	55	66	77	88	99	110	121	132
20	(7,0,1)	2	16	27	38	49	60	71	82	93	104	115	126
21	(8,0,1)	2	17	28	39	50	61	72	83	94	105	116	127
22	(9,0,1)	2	19	30	41	52	63	74	85	96	107	118	129
23	(10,0,1)	2	14	25	36	47	58	69	80	91	102	113	124
24	(0,1,1)	1	123	124	125	126	127	128	129	130	131	132	133
25	(1,1,1)	12	23	33	43	53	63	73	83	93	103	113	123
26	(2,1,1)	7	18	34	39	55	60	76	81	97	102	118	123
27	(3,1,1)	9	20	27	45	52	59	77	84	91	109	116	123

28	(4,1,1)	10	21	29	37	56	64	72	80	99	107	115	123
29	(5,1,1)	4	15	28	41	54	67	69	82	95	108	121	123
30	(6,1,1)	11	22	31	40	49	58	78	87	96	105	114	123
31	(7,1,1)	5	16	30	44	47	61	75	89	92	106	120	123
32	(8,1,1)	6	17	32	36	51	66	70	85	100	104	119	123
33	(9,1,1)	8	19	25	42	48	65	71	88	94	111	117	123
34	(10,1,1)	3	14	26	38	50	62	74	86	98	110	122	123
35	(0,2,1)	1	68	69	70	71	72	73	74	75	76	77	78
36	(1,2,1)	11	23	32	41	50	59	68	88	97	106	115	124
37	(2,2,1)	12	18	28	38	48	58	68	89	99	109	119	129
38	(3,2,1)	5	20	34	37	51	65	68	82	96	110	113	127
39	(4,2,1)	7	21	26	42	47	63	68	84	100	105	121	126
40	(5,2,1)	6	15	30	45	49	64	68	83	98	102	117	132
41	(6,2,1)	9	22	29	36	54	61	68	86	93	111	118	125
42	(7,2,1)	8	16	33	39	56	62	68	85	91	108	114	131
43	(8,2,1)	10	17	25	44	52	60	68	87	95	103	122	103
44	(9,2,1)	3	19	31	43	55	67	68	80	92	104	116	128
45	(10,2,1)	4	14	27	40	53	66	68	81	94	107	120	133
46	(0,3,1)	1	90	91	92	93	94	95	96	97	98	99	100
47	(1,3,1)	10	23	31	39	47	66	74	82	90	109	117	125
48	(2,3,1)	6	18	33	37	52	67	71	86	90	105	120	124
49	(3,3,1)	12	20	30	40	50	60	70	80	90	111	121	131
50	(4,3,1)	4	21	34	36	49	62	75	88	90	103	116	129
51	(5,3,1)	8	15	32	38	55	61	78	84	90	107	113	130
52	(6,3,1)	7	22	27	43	48	64	69	85	90	106	122	127
53	(7,3,1)	11	16	25	45	54	63	72	81	90	110	119	128
54	(8,3,1)	3	17	29	41	53	65	77	89	90	102	114	126
55	(9,3,1)	9	19	26	44	51	58	76	83	90	108	115	133
56	(10,3,1)	5	14	28	42	56	59	73	87	90	104	114	132
57	(0,4,1)	1	101	102	103	104	105	106	107	108	109	110	111
58	(1,4,1)	9	23	0	37	55	62	69	87	94	101	119	126
59	(2,4,1)	11	18	27	36	56	65	74	83	92	101	121	130
60	(3,4,1)	8	20	26	43	49	66	72	89	95	101	118	124
61	(4,4,1)	12	21	31	41	51	61	71	81	91	101	121	132
62	(5,4,1)	10	15	34	42	50	58	77	85	93	101	120	128
63	(6,4,1)	5	22	25	39	53	67	70	84	98	101	115	129
64	(7,4,1)	3	16	28	40	52	64	76	88	100	101	113	125
65	(8,4,1)	7	17	33	38	54	59	75	80	96	101	117	133
66	(9,4,1)	4	19	32	45	47	60	73	86	99	101	114	127
67	(10,4,1)	6	14	29	44	48	63	78	82	97	101	116	131
68	(0,5,1)	1	35	36	37	38	39	40	41	42	43	44	45
69	(1,5,1)	8	23	29	35	52	58	75	81	98	104	121	127
70	(2,5,1)	5	18	32	35	49	63	77	80	94	108	122	125
71	(3,5,1)	4	20	33	35	48	61	74	87	100	102	115	128
72	(4,5,1)	9	21	28	35	53	60	78	85	92	110	117	124
73	(5,5,1)	12	15	25	35	56	66	76	86	96	106	116	126
74	(6,5,1)	3	22	34	35	47	59	71	83	95	107	119	131
75	(7,5,1)	6	16	31	35	50	65	69	84	99	103	118	133
76	(8,5,1)	11	17	26	35	55	64	73	82	91	111	120	129

77	(9,5,1)	10	19	27	35	54	62	70	89	97	105	113	132
78	(10,5,1)	7	14	30	35	51	67	72	88	93	109	114	130
79	(0,6,1)	1	112	113	114	115	116	117	118	119	120	121	122
80	(1,6,1)	7	23	28	44	49	65	70	86	91	107	112	128
81	(2,6,1)	10	18	26	45	53	61	69	88	96	104	112	131
82	(3,6,1)	11	20	29	38	47	67	76	85	94	103	112	132
83	(4,6,1)	6	21	25	40	55	59	74	89	93	108	112	127
84	(5,6,1)	3	15	27	39	51	63	75	87	99	111	112	124
85	(6,6,1)	12	22	32	42	52	62	72	82	92	102	112	133
86	(7,6,1)	9	16	34	41	48	66	73	80	98	109	112	130
87	(8,6,1)	4	17	30	43	56	58	71	84	97	110	112	125
88	(9,6,1)	5	19	33	36	50	64	78	81	95	109	112	126
89	(10,6,1)	8	14	31	37	54	60	77	83	100	106	112	129
90	(0,7,1)	1	46	47	48	49	50	51	52	53	54	55	56
91	(1,7,1)	6	23	27	42	46	61	76	80	95	110	114	129
92	(2,7,1)	4	18	31	44	46	59	72	85	98	111	113	126
93	(3,7,1)	7	20	25	41	46	62	78	83	99	104	120	125
94	(4,7,1)	3	21	33	45	46	58	70	82	94	106	118	130
95	(5,7,1)	5	15	29	43	46	60	74	88	91	105	119	133
96	(6,7,1)	10	22	30	38	46	65	73	81	100	108	116	124
97	(7,7,1)	12	16	26	36	46	67	77	87	97	107	117	127
98	(8,7,1)	8	17	34	40	46	63	69	86	92	109	115	132
99	(9,7,1)	11	19	28	37	46	66	75	84	93	102	122	131
100	(10,7,1)	9	14	32	39	46	64	71	89	96	103	121	128
101	(0,8,1)	1	57	58	59	60	61	62	63	64	65	66	67
102	(1,8,1)	5	23	26	40	54	57	71	85	99	102	116	130
103	(2,8,1)	9	18	25	43	50	57	75	82	100	107	114	132
104	(3,8,1)	3	20	32	44	56	57	69	81	93	105	117	129
105	(4,8,1)	11	21	30	39	48	57	77	86	95	104	113	133
106	(5,8,1)	7	15	31	36	52	57	73	89	94	110	115	131
107	(6,8,1)	8	22	28	45	51	57	74	80	97	103	120	126
108	(7,8,1)	4	16	29	42	55	57	70	83	96	109	122	124
109	(8,8,1)	12	17	27	37	47	57	78	88	98	108	118	128
110	(9,8,1)	6	19	34	38	53	57	72	87	91	106	121	125
111	(10,8,1)	10	14	33	41	49	57	76	84	92	111	119	127
112	(0,9,1)	1	79	80	81	82	83	84	85	86	87	88	89
113	(1,9,1)	4	23	25	38	51	64	77	79	92	105	118	131
114	(2,9,1)	3	18	30	42	54	66	78	79	91	103	115	127
115	(3,9,1)	10	20	28	36	55	63	71	79	98	106	114	133
116	(4,9,1)	8	21	27	44	50	67	73	79	96	102	119	125
117	(5,9,1)	9	115	33	40	47	65	72	79	97	105	122	129
118	(6,9,1)	66	22	26	41	56	60	75	79	94	109	113	128
119	(7,9,1)	7	16	32	37	53	58	74	79	95	111	116	132
120	(8,9,1)	5	17	31	45	48	62	76	79	93	107	121	124
121	(9,9,1)	12	19	29	39	49	59	69	79	100	110	120	130
122	(10,9,1)	11	14	34	43	52	61	70	79	99	108	117	126
123	(0,10,1)	1	24	25	26	27	28	29	30	31	32	33	34
124	(1,10,1)	3	23	24	36	48	60	72	84	96	108	120	132
125	(2,10,1)	8	18	24	41	47	64	70	87	93	110	116	133

126	(3,10,1)	6	20	24	39	54	58	73	88	92	102	122	126
127	(4,10,1)	5	21	24	38	52	66	69	83	97	111	114	128
128	(5,10,1)	11	15	24	44	53	62	71	80	100	109	118	127
129	(6,10,1)	4	22	24	37	50	63	76	89	91	104	117	130
130	(7,10,1)	10	16	24	43	51	59	78	86	94	102	121	129
131	(8,10,1)	9	17	24	42	49	67	74	81	99	106	113	131
132	(9,10,1)	7	19	24	40	56	61	77	82	89	103	119	124
133	(10,10,1)	12	14	24	45	55	65	75	85	95	105	115	125

الأقواس الكاملة في المستوى الاسقاطي PG(2,11) حول حقل كالو

محمود سالم فياض
قسم الحاسوب / كلية التربية / الجامعة العراقية

استلم البحث في:19 ايلول 2012 ، قبل البحث في:3 شباط 2013

الخلاصة

في هذا البحث قمنا بإنشاء الأقواس الكاملة (k,n) في المستوى الأسقاطي على حقل كالوا GF(11) عندما $2\le n\le 12$ باستخدام الطريقة الهندسية عن طريق اتحاد بعض الأقواس العظمى (12,2). وجدنا الأقواس(12,2) و (19,3) و (29,4) و (38,5) و(47,6) و (58,7) و(68,8) و(81,9) و (96,10) و(109,11) و(133,12) أقواس كاملة في المستوى الاسقاطي PG(2,11)

الكلمات المفتاحية : الهندسة الجبرية ، الاقواس الكاملة ، حقل كالوا .