Annsemimaximal and Coannsemimaximal Modules

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Received in : 20 September 2010
Accepted in : 8 February 2011

Abstract

Some authors studied modules with annihilator of every nonzero submodule is prime, primary or maximal. In this paper, we introduce and study annsemimaximal and coannsemimaximal modules, where an R-module M is called annsemimaximal (resp. coannsemimaximal) if $ann_R N$ (resp. $ann_R \frac{M}{N}$) is semimaximal ideal of R for each nonzero submodule N of M.

Keywords: Annsemimaximal module, semisimple module, semisimple ring, semiprime module, max-module, uniform module, Z-regular module, F-regular module, Artinian module, flat module, coprime module, coannsemimaximal module.

Introduction

Let R be a commutative ring with unity and let M be an R-module. Muntaha A.R.H. in [1] introduced and studied quasi-prime modules where an R-module M is quasi-prime if $ann_R N$ is a prime ideal of R for every nonzero submodule N of M. Adwia J.A.A. in [2] introduced and studied quasi-primary modules, where an R-module M is called quasi-primary if $ann_R N$ is a primary ideal of R, for each nonzero submodule N of M. Adwia J.A.A. in [3] introduced and studied max modules, where an R-module M is said to be max module if $ann_R N$ is a maximal ideal of R, for each nonzero submodule N of M.

1. Recall that an ideal I of R is called semimaximal if I is an intersection of finitely many maximal ideals of R, [4].
2. In this paper, we introduced and studied annsemimaximal and coannsemimaximal modules where an R-module M is called annsemimaximal (resp. coannsemimaximal) if $ann_R N$ (resp. $ann_R \frac{M}{N}$) is a semimaximal ideal of R.

1- Annsemimaximal Modules

In this section, we introduce the concept of annsemimaximal modules. We give some characterizations to this concept and establish some basic properties of this concept.

1.1 Definition:

Let M be an R-module. M is called annsemimaximal module if $ann_R N$ is a semimaximal ideal of R for each non-zero submodule N of M.

1.2 Remarks and Examples:

(1) Z_{p^∞} is not annsemimaximal Z-module.
(2) Z_6 as a Z-module is annsemimaximal module.
(3) Z as a Z-module is not annsemimaximal module.
(4) Q as a Z-module is not annsemimaximal module.
(5) Z_p as a Z-module is annsemimaximal module.
(6) for each $n \in Z_+$, $Z \oplus Z_n$ is not annsemimaximal Z-module.
(7) Every submodule N of an R-module M (where M is annsemimaximal module) is annsemimaximal module.

Proof: Let K be a nonzero submodule of N. Then K be a non-zero submodule of M and so that $ann_R K$ is semimaximal ideal (since M is annsemimaximal module).

(8) Let M be annsemimaximal module and let $N \leq M$. Then $\frac{M}{N}$ is annsemimaximal module.

Proof: Let $\pi: M \longrightarrow M/N$ be the natural epimorphism and M is annsemimaximal module. Then for each non-zero submodule W of M, $ann_R W$ is semimaximal ideal of R. But $ann_R W \subseteq ann_R W/N$. Hence $ann_R W/N$ is semimaximal ideal by [5,prp.(1.2.11)]. Thus $\frac{M}{N}$ is annsemimaximal module.

(9) The homomorphic image of annsemimaximal module is annsemimaximal module.

Proof: Let $f: M \longrightarrow M'$ be an epimorphism such that M is annsemimaximal module. Then by the first fundamental theorem of homomorphisim, $\frac{M}{\ker f} \cong M'$. But $\frac{M}{\ker f}$ is annsemimaximal by (8). Hence M' is annsemimaximal module.

Now, we have the following characterization of annsemimaximal module.

1.3 Proposition:

Let M be an R-module. Then M is annsemimaximal module if and only if $ann_R M$ is a semimaximal ideal of R.

Proof: ($\Rightarrow$) It follows directly by definition (1.1).
($\Leftarrow$) let $(0) \neq N$ be a submodule of M. Then $ann_R N \supseteq ann_R M$. But $ann_R M$ is semimaximal, so by [5,prop. (1.2.11)], $ann_R N$ issemimaximal. Thus M is annsemimaximal module.

1.4 Corollary:

An R-module M is annsemimaximal if and only if $R/ann_R M$ is semisimple ring.

Proof: By proposition (1.3) M is annsemimaximal module
$\Leftrightarrow$ $ann_R M$ is a semimaximal ideal.
$\Leftrightarrow$ $R/ann_R M$ is semisimple ring.

Now, we have the following theorem.

1.5 Theorem:

Let M be an R-module. Then (1) $\Rightarrow$ (2), (2) $\Rightarrow$ (3), (3) $\Rightarrow$ (4), (4) $\Rightarrow$ (1) if M is finitely generated

(1) M is annsemimaximal module.
(2) $[ann_R N \underset{R}{:} A]$ is a semimaximal for each non-zero submodule N of M and for each ideal A of R such that $A \not\subseteq ann_R N$.

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$[\text{annN} \underset{R}{:} r]$ is a semimaximal ideal of R for each non-zero submodule N of M, $r \in R$ such that $(r) \not\subseteq ann_R N$.

(3) $Ann_R(m)$ is a semimaximal ideal of R for each $m \neq 0$, $m \in M$.

Proof: (1) $\Rightarrow$ (2), suppose that M is annsemimaximal module. Let N be a non-zero submodule of M. Then $ann_R N$ is semimaximal ideal of R. Assume that A is an ideal of R such that $A \not\subseteq ann_R N$. It is clear that $ann_R N \subseteq [ann_R N \underset{R}{:} A]$. So, according to [5,coro.(1.2.12)], $[ann_R N \underset{R}{:} A]$ is semimaximal ideal of R.

(2) $\Rightarrow$ (3), take A = (r) the ideal of R generated by r, the result follows by (2).

(3) $\Rightarrow$ (4), let $0 \neq m \in M$. Because $1 \notin ann_R(m)$. $[ann_R(m):R]$ is semimaximal ideal of R by (3). But $[ann_R(m):R] = ann_R(m)$. So $ann_R(m)$ is semimaximal ideal of R.

(4) $\Rightarrow$ (1), since M is finitely generated, $M = \sum_{i=1}^{n} Rx_i$, $x_i \in M$, $ann_R M = \bigcap_{i=1}^{n} ann x_i$. But $ann(x_i)$ for all $i=1,\ldots,n$ is semimaximal ideal. So, by [5,coro.(1.2.15)], $ann_R M$ is semimaximal. Thus M is ansemimaximal by prop. (1.3).

Recall that an R-module M is called semisimple if every submodule of M is a direct summand of M. And a ring R is said to be semisimple ring if and only if R is a semisimple R-module, [6].

1.6 Proposition:

Every semisimple R-module M is annsemimaximal.

Proof: By [6,prop.(1.1.46)], we get $R/ann_R M$ is a semisimple ring. Therefore $ann_R M$ is a semimaximal ideal by [4,prop.(1.2.5)]. Thus M is annsemimaximal by prop. (1.3).

The following corollary is an application of proposition (1.6).

1.7 Corollary:

Let R be a semisimple ring. Then every R-module M is annsemimaximal.

Proof: It is known that if R is semisimple ring, then M is semisimple module [5,prop.(1.1.44)]. Hence M is annsemimaximal module by previous proposition. Next, we have the following proposition.

1.8 Proposition:

If M is an Artinian and annsemimaximal R-module, then M is semisimple.

Proof: We have M is annseimmaximal, then $ann_R M$ is semimaximal. Thus $J(M) = 0$ by [5,coro,(1.3.6)]. But M is an artinian and $J(M)=0$, then M is semisimple, [5].

1.9 Example:

Z_{24} as a Z-module is not annsemimaximal module and Z_{24} is not semisimple.

The following result is consequence of proposition (1.8).

1.10 Corollary:

Let M is an Artinian R-module.Then M is semisimple module if and only if M is annsemimaximal.

1.11 Proposition:

If M is annsemimaximal R-module, then every cyclic submodule of M is semisimple.

Proof: If M is annsemimaximal R-module, then $ann_R(x)$ is semimaximal ideal by Th.((1.5),(4)), so by [5,prop.(2.3.15)], we get the result Now, we induced the following corollary.

1.12 Corollary:

If M is finitely generated and annsemimaximal R-module, then M is semisimple R-module.

Proof: Let $M=Rx_1 + Rx_2 + \ldots + Rx_n$ for some $x_1, x_2, \ldots, x_n$. But Rx_i is semisimple by previous proposition. Therefore $M=\sum_{i=1}^{n} Rx_i$ is semisimple By combining corollary (1.12), proposition (1.6), we get the following:

1.13 Corollary:

Let M be a finitely generated R-module. Then M is annsemimaximal module if and only if M is semisimple.

1.14 Corollary:

R is a semisimple ring if and only if R is annsemimaximal ring.

Now, we turn our attention to direct sum of annsemimaximal modules.

1.15 Proposition:

Let M be a faithful R-module. Then R is semisimple if and only if M is annsemimaximal.

Proof: (⇒) directly from [5,prop.(1.1.44)] and proposition (1.6).

(⇐) if M is annsemimaxi, then $ann_R M$ is a semimaximal ideal; that is (0) is a semimaximal ideal. Thus $R/(0) \cong R$ is semisimple.

By combining corollary (1.13), proposition (1.15) and corollary (1.14), we get the following:

1.16 Corollary:

Let M be a faithful finitely generated R-module. The following statements are equivalent:

(1) M is annsemimaximal.
(2) M is semisimple.
(3) R is semisimple.
(4) R is annsemimaximal.

Now, we give the following proposition.

1.17 Proposition:

If R is a local ring and M is annsemimaximal R-module, then M is semisimple..

Proof: M is annsemimaximal module. Then $ann_R M$ is semimaximal ideal. Thus the result follows by [5,coro.(1.3.7)].

1.18 Proposition:

Let M_1, M_2 be two R-modules, $M=M_1 \oplus M_2$. Then M is annsemimaximal if and only if M_1, M_2 are annsemimaximal R-module.

Proof: (⇒) let $\rho_1 : M \longrightarrow M_1$, $\rho_2 : M \longrightarrow M_2$ be the natural projections. Thus M_1 and M_2 are annsemimaximal modules by remarks and examples ((1.2),(9)).

(⇐) we have $ann_R M_1$ is semimaximal ideal and $ann_R M_2$ is semimaximal by proposition (1.3). On the other hand $ann_R(M_1 \oplus M_2) = ann_R M_1 \cap ann_R M_2$. But by [5, coro.(1.2.15)],

$ann_R M_1 \cap ann_R M_2$ is semimaximal. Therefore $ann_R(M_1 \oplus M_2)$ is semimaximal. Thus $M_1 \oplus M_2$ is annsemimaximal module, by prop.(1.3).

.. Recall that an R-module M is called semiprime if and only if $ann_R N$ is a semiprime ideal of R, for each non-zero R-submodule N of M, [7,Def.(4.1.1)]. By using this concept, we have the following.

1.19 Proposition:

Every annsemimaximal R-module is semiprime R-module.

Proof: Let M be an annsemimaximal module. Then for each non-zero submodule N of M, $ann_R N$ is semimaximal ideal of R. Thus by [5,prop.(1.2.21)], $ann_R N$ is semiprime and hence M is a semiprime module.

The converse of this proposition is not true in general. For example:Z as a Z-module is semiprime module, but it is not annsemimaximal module by remarks and examples ((1.2),(3)).

For our next corollary the following definitions are needed.

An R-module M is said to be serial (chain) R-module if the R-submodules of M are linearly orderd with respect to inclusion, [6], [7].

An R-module M is said to be a prime module if $ann_R M = ann_R N$ for every non-zero submodule N of M, [8], [9].

As an application of proposition (1.19), we give the following corollary.

1.20 Corollary:

Let M be a serial annsemimaximal module. Then M is prime R-module.

Proof: From proposition (1.19), M is semiprime module and from [7,prop.(4.2.1)], we get the result.

Recall that an R-module M is said to be a max-module if $\sqrt{ann_R N}$ is maximal ideal of R for each non-zero submodule N of M, [3].

In the class of max-module. The two concept of annsemimaximal module and semiprime module are equivalent.

1.21 Proposition:

Let M be a max-module. Then M is annsemimaximal module if and only if M is semiprime module.

Proof: Suppose that M is semiprime R-module. Then for each a non-zero submodule N of M, $ann_R N$ is semiprime ideal of R, that is $ann_R N = \sqrt{ann_R N}$ for each non-zero submodule N of M. But M is max-module which implies that $\sqrt{ann_R N}$ is maximal ideal of R for each non-zero submodule N of M and hence $ann_R N$ is maximal ideal for each non-zero submodule N of M by [5,Rem.(1.2.2),(2)], $ann_R N$ is semimaximal ideal of R and hence M is annsemimaximal module.

Conversely: It follows by proposition (1.19).

Now, the following results are other consequences of proposition (1.21), but first we need to recall some definitions.

An R-module M is called Z-regular module if for all $m \in M$, there exists $f \in Hom_R(M,R) = M^*$ such that $f(m)m = m$, [10].

An R-submodule N of M is called essential in M if for each non-zero R-submodule L of M, $N \cap L \neq 0$, [6].And an R-module M is called uniform if every non-zero R-submodule of M is essential.

An R-submodule N of M is called quasi-invertible if $Hom(\frac{M}{N}, M) = 0$. And an R-module M is called quasi-Dedekind if every non-zero R-submodule of M is quasi-invertible, [11].

Hence, we have the following consequences of (1.21).

1.22 Corollary:
If M is max-module and Z-regular module. Thus M is annsemimaximal module.
Proof: It follows directly from proposition (1.21) and [7,prop.(4.2.2)].

1.23 Corollary:
Let M be a uniform annsemimaximal R-module. Then M is quasi-Dedekind.
Proof: M is annsemimaximal module, then M is semiprime by proposition (1.21) and by [7,prop.(4.2.4)], we get the result.
Now, we can give the following proposition.

1.24 Proposition:
Let M be a uniform max-R-module. Then the following statements are equivalent.
(1) M is annsemimaximal module.
(2) M is seiprime module.
(3) M is quasi-Dedekind.
(4) M is prime.
Proof: (1) $\Rightarrow$ (2) by proposition (1.19).
(2) $\Rightarrow$ (3) by [7,prop.(4.2.4)].
(3) $\Rightarrow$ (4) by [11,prop.(1.7), ch.2].
(4) $\Rightarrow$ (1) It is clear that every prime module is semiprime module and hence by proposition (1.21) we get the result.

Recall that an R-module M is said to be regular module if $R/ann_R(x)$ is regular ring for all $0 \neq x \in M$, [5].
By using this concept, we have the following.

1.25 Remark:
Every annsemimaximal module is regular module.
Proof: Let M be annsemimaximal R-module. Then $ann_R M$ is semimaximal ideal and by [5,prop.(1.3.5)], M is regular module.

1.26 Proposition:
If M is annsemimaximal R-module, then M/N is regular R-module for all submodules N of M.
Proof: Let M is annsemimaximal module. Then $ann_R M$ is semimaximal. But $ann_R M \subseteq [N \underset{R}{:} M]$ for all submodule N of M, so $[N \underset{R}{:} M]$ is semimaximal ideal by [5,prop.(1.2.11)]. Hence M/N is regular R-module by [5,prop.(1.3.8)].
The Jacobson radical of an R-module M denoted by J(M), is defined to be the intersection of all maximal submodules of M, in case M has maximal submodules and J(M)=M in case M has no maximal submodule, [6].

1.27 Remark:
Let M be an annsemimaximal R-module. Then J(M)=0.
Proof: It is abvious according to [5,coro.(1.3.6)].
Recall that an R-module M is called F-regular if every submodule of M is pure [12,ch.2].
By using this concept, we give the following proposition.

1.28 Proposition:
If M is annsemimaximal R-module, then M is F-regular.

Proof: We have M is annsemimaximal, then $ann_R M$ is semimaximal. Thus every cyclic submodule is pure by [5,prop.(1.3.9)]. Hence M is F-regular.

1.29 Proposition:

Let R be a PID, $ann_R M \neq 0$, M is prime R-module. Then M is annsemimaximal module.

Proof: Since M is prime R-module. Then $ann_R M$ is prime ideal which implies that $ann_R M$ is maximal ideal (since R is PID). Thus $ann_R M$ is semimaximal ideal of R. Hence M is annsemimaximal R-module, by proposition (1.3).

The converse of proposition (1.29) is not true, for example:Z_6 as Z-module is annsemimaximal. But M is not prime.

Recall that an R-module M is flat if for each injective homomorphisim f: $N' \longrightarrow N$ from one R-module into another, the homomorphisim $1_M \otimes f: M \underset{R}{\otimes} N' \longrightarrow M \underset{R}{\otimes} N$ is injective, where 1_M is the identity isomorphisim of M, [6].

1.30 Proposition:

If M is flat annsemimaximal R-module, then every homomorphic image of M is flat.

Proof: We have M is annsemimaximal, then $ann_R M$ is semimaximal ideal. Thus by [5,prop.(1.3.10)], we get the result.

Next, we introduce the following definition.

1.31 Definition:

Let N be a proper submodule of an R-module M. N is called quasi-semimaximal if $[N \underset{R}{:} (m)]$ is a semimaximal ideal for each $m \notin N$.

1.32 Remark:

Let M be a finitely generated R-module, N be semimaximal submodule of M. Then $[N \underset{R}{:} M]$ is semimaximal ideal.

1.33 Remark:

Let M be a finitely generated R-module, N be semimaximal submodule of M. Then N is quasi-semimaximal submodule.

Proof: By remark (1.32), $[N \underset{R}{:} M]$ is semimaximal ideal. But for each $m \notin N$, $[N \underset{R}{:} (m)] \supseteq [N \underset{R}{:} M]$. Thus by [5,prop.(1.2.11)], we get $[N \underset{R}{:} (m)]$ is semimaximal ideal of R. We end this section by the following result.

1.34 Proposition:

Let M be a finitely generated R-module. Then M is annsemimaximal module if and only if (0) is quasi-semimaximal submodule of M.

Proof: Suppose that M is annsemimaximal module. Then $ann_R(m)$ is semimaximal ideal for each $m \in M$. Thus $[0 \underset{R}{:} m]$ is semimaximal ideal for each $m \in M$. Hence (0) is semimaximal ideal.

Conversely: if (0) is quasi-semimaximal submodule of M, then $[0 \underset{R}{:} m]$ is semimaximal ideal for each $m \in M$. Therefore M is annsemmaximal by theorem ((1.5),(4)).

2- Coannsemimaximal Modules:

In this section, we introduce the concept of coannsemimaximal module which is strongly from the concept of annsemimaximal module in section one. We give some characterizations about this concept and many results are studied.

We start with the following definition.

2.1 Definition:

An R-module M is called coannsemimaximal module if $\text{ann}_R \frac{M}{N}$ is semimaximal ideal of R for each non-zero proper submodule N of M. Equivalently, M is coannsemimaximal if $\frac{M}{N}$ is annsemimaximal module for each non-zero proper submodule N of M.

2.2 Examples:

(1) Z_{12} is not coannsemimaximal Z-module, since if N = $<\overline{4}>$, then $\text{ann}_Z \frac{Z_{12}}{N} = \text{ann}_Z Z_4 = 4Z$ which is not semimaximal ideal.

(2) Z_{p^2} as a Z-module is coannsemimaximal, where p is a prime number.

Proof: Since $<\overline{p}>$ is only non-zero proper submodule of Z_{p^2}. $Z_{p^2}/<\overline{p}> \simeq Z_p$ and $\text{ann}_Z Z_p = pZ$ which is clear semimaximal ideal.

Next, we have the following proposition.

2.3 Proposition:

Let M be an R-module. Then every annsemimaximal module is coannsemimaximal module.

Proof: Let M be an annsemimaximal module. Then $\frac{M}{N}$ is annsemimaximal module by remarks and examples ((1.2),(9)). Thus $\text{ann}_R \frac{M}{N}$ is semimaximal which implies that M is coannsemimaximal module.

The converse of proposition (2.3) is not true in general. For example: Let Z_9 be a Z-module. Then Z_9 is coannsemimaximal module but not annsemimaximal. And Z_4 as a Z-module is coannsemimaximal module but it is not annsemimaximal module.

The following proposition proves that the converse of (2.3) is true under the condition that M is coprime module, but first we need to recall the definition of coprime module.

An R-module M is called coprime module if $\text{ann}_R M = \text{ann}_R \frac{M}{N}$ for every proper submodule N of M, [13].

2.4 Proposition:

Let M be a coprime and coannsemimaximal R-module. Then M is annsemimaximal R-module.

Proof: Since M is coprime module. Then $\text{ann}_R M = \text{ann}_R \frac{M}{N}$ for every proper submodule N of M. But $\text{ann}_R \frac{M}{N}$ is semimaximal ideal of R for each non-zero proper submodule N of M (since M is coannsemimaximal). Thus $\text{annn}_R M$ is semimaximal ideal of R and hence M is annsemimaximal module by proposition (1.3).

As an application of (2.4), we have the following.

2.5 Corollary:

Let M be a coannsemimaximal R-module and M is a coprime E-module, where $E=End_R(M)$. Then M is annsemimaximal R-module.

Proof: M is coprime E-module, then M is coprime R-module by [14,coro.(2.2.3)] and from proposition (2.4), we get the result.

. Recall that a non-simple R-module M is called antihopfian if $M \cong M/N$ for all proper submodules N of M, [15].

By using this concept we get the following.

2.6 Proposition:

Let M be an antihopfian, N is semimaximal submodule of M. Then M is coannsemimaximal R-module.

Proof: We have N is semimaximal submodule. Then $\frac{M}{N}$ is semisimple by [5,Def.(2.1.1)]. Thus $\frac{M}{N}$ is annsemimaximal module by prop.(1.5). But $\frac{M}{W} \cong \frac{M}{N}$ for each proper submodule W of M, since M is antihopfian. That means $M \cong \frac{M}{N}$. Thus $\frac{M}{W}$ is annsemimaximal for all proper submodule W of M. Therefore M is coannsemimaximal module.

Now, we prove the following lemma.

2.7 Lemma:

Let M be an R-module. If N is a semimaximal submodule, then $[N \underset{R}{:} M]$ is semimaximal ideal.

Proof: Suppose that N is a semimaximal submodule. Then by [5,def.(2.1.1)], $\frac{M}{N}$ is semisimple R-module and hence by proposition (1.6), $\frac{M}{N}$ is annsemimaximal module. Then by proposition (1.3), $ann_R \frac{M}{N}$ is semimaximal ideal. But $[N \underset{R}{:} M] = ann_R \frac{M}{N}$, thus $[N \underset{R}{:} M]$ is a semimaximal ideal.

The following result follows immediately by lemma (2.7).

2.8 Proposition:

If every submodule N of an R-module M is semimaximal, then M is coannsemimaximal.

..Next, we have the following remark.

2.9 Remark:

The direct sum of coannsemimaximal modules need not be coannsemimaximal. For example:

Let $M=Z_4 \oplus Z_3$ be a Z-module. Z_4 and Z_3 are two coannsemimaximal Z-modules. But $M \cong Z_{12}$ which is not coannsemimaximal.

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المقاسات شبه الاعظمية التالفة وشبه الاعظمية التالفة المضادة

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استلم البحث في :20 أيلول 2010

قبل البحث في : 8 شباط 2011

الخلاصة

بعض الباحثين درسوا المقاسات التي تالف كل مقاس جزئي غير صفري منها هو أولي، ابتدائي أو اعظمي. في هذا البحث قدمنا ودرسنا المقاسات شبه الاعظمية التالفة والمقاسات شبه الاعظمية التالفة المضادة، حيث يدعى المقاس M على الحلقة R شبه اعظمي تالف (على التوالي شبه اعظمي تالف مضاد) اذا كان تالف N على الحلقة R (على التوالي تالف $\frac{M}{N}$ على الحلقة R) هو مثالي شبه اعظمي في R لكل مقاس جزئي غير صفري N في M.

الكلمات المفتاحية :

Annsemimaximal module, semisimple module, semisimple ring, semiprime module, max-module, uniform module, Z-regular module, F-regular module, Artinian module, flat module, coprime module, coannsemimaximal module.