A Complete (k,r)-Cap in PG(3,p) Over Galois Field GF(4)

F. F. Kareem
Department of Mathematics, Ibn-Al-Haitham College of Education , University of Baghdad

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Abstract

The aim of this paper is to construct the (k,r)-caps in the projective 3-space PG(3,p) over Galois field GF(4). We found that the maximum complete (k,2)-cap which is called an ovaloid , exists in PG(3,4) when k = 13. Moreover the maximum (k,3)-caps, (k,4)-caps and (k,5)-caps.

Key words: Projective Space Maximum Complete (k,r) –cap Galois Field

Introduction

Many of the researchers worked on the construction and classification of the (k,n)-arcs in the projective planes PG(2,P),2≤ P ≥17. Now,I study of a finite proective spaces PG(3,P) over Galois field GF(P),It is the largest of the projective plane over Galois field,Hirschfeld, [1] give the basic definition and theorems of projective geometrics over finite fields, and Al-Mukhtar, A.SH. in [2] give the complete Arcs and surfaces in three dimensional projective space over Galois field and give (k,r)-caps in PG(3,q) over Galois fields GF(q), q = 2,3, and 5. In this work we construct the (k,r)-caps in PG(3,4). This paper is divided into six sections, section one is the preliminaries of projective 3-space which contains some definitions and theorems for that concept and section two consists of the additions and multiplications operations of GF(4). In section three to section six the construction of maximum complete (k,r)-caps for r = 2,3,4,5.This work I have done manually without using the computer program.

1- Preliminaries

1.1 Definition: "Projective 3-Space", [3]

A projective 3-space PG(3,k) over a field k is a 3-dimensional projective space which consists of points, lines and planes with the incidence relation between them. The projective 3-space satisfies the following axioms:

A) Any two distinct points are contained in a unique line.
B) Any three distinct non-collinear points, also any line and point not on the line are contained in a unique plane.
C) Any two distinct coplanar lines intersect in a unique point.
D) Any line not on a given plane intersects the plane in a unique point.
E) Any two distinct planes intersect in a unique line.

A projective space PG(3,p) over Galois field GF(p) , $p = q^m$ for some prime number q and some integer m, is a 3-dimensional projective space. Any point in PG(3,p) has the form of a quadrable (x_1,x_2,x_3,x_4), where x_1,x_2,x_3,x_4 are elements in GF(p) with the exception of the quadrable consisting of four zero elements.

Two aquadrables (x_1,x_2,x_3,x_4) and (y_1,y_2,y_3,y_4) represent the same point if there exists λ in GF(p)\{0} such that $(x_1,x_2,x_3,x_4) = \lambda (y_1,y_2,y_3,y_4)$.

Similarly, any plane in PG(3,p) has the form of a quadrable $[x_1,x_2,x_3,x_4]$, where x1,x2,x3,x4 are elements in GF(p) with the exception of the quadrable consisting of four zero elements.

Two quadrables $[x_1,x_2,x_3,x_4]$ and $[y_1,y_2,y_3,y_4]$ represent the same plane if there exists λ in $GF(p)\backslash\{0\}$ such that $[x_1,x_2,x_3,x_4] = \lambda\,[y_1,y_2,y_3,y_4]$.

Also a point $p(x_1,x_2,x_3,x_4)$ is in cident with the plane π [a1,a2,a3,a4] iff $a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 = 0$.

1.2 Definition: "Plan π", [1]

A plan π in PG(3,p) is the set of all points $p(x_1,x_2,x_3,x_4)$ satisfying a linear equation $u_1x_1 + u_2x_2 + u_3x_3 + u_4x_4 = 0$. This plane is denoted by $\pi\,[u_1,u_2,u_3,u_4]$.

1.3 Theorem: [2]

Four distinct points $A(x_1,x_2,x_3,x_4)$, $B(y_1,y_2,y_3,y_4)$, $C(z_1,z_2,z_3,z_4)$ and $D(w_1,w_2,w_3,w_4)$ are coplanar iff $\Delta = \begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \\ w_1 & w_2 & w_3 & w_4 \end{vmatrix} = 0$.

1.4 Corollary: [2]

If four distinct points $A(x_1,x_2,x_3,x_4)$, $B(y_1,y_2,y_3,y_4)$, $C(z_1,z_2,z_3,z_4)$ and $D(w_1,w_2,w_3,w_4)$ are collinear, then $\Delta = 0$.

1.5 Theorem: [2]

The points of PG(3,p) have unique forms which are (1,0,0,0), (x,1,0,0), (x,y,1,0), (x,y,z,1) for all x, y, z in GF(p).

1.6 Theorem: [2]

The planes of PG(3,p) have unique forms which are [1,0,0,0], [x,1,0,0], [x,y,1,0], [x,y,z,1] for all x, y, z in GF(p).

1.7 Theorem: [2]

A projective 3-space PG(3,p) satisfies the following:

A) Every line contains exactly $p + 1$ points and every point is on exactly $p + 1$ lines.

B) Every plane contains exactly $p^2 + p + 1$ points (lines) and every point is on exactly $p^2 + p + 1$ planes.

C) There exists $p^3 + p^2 + p + 1$ of points and there exists $p^3 + p^2 + p + 1$ of planes.

D) Any two planes intersect in exactly $p + 1$ points and any line is on exactly $p + 1$ planes, and any two points are on exactly $p + 1$ planes.

1.8 Theorem: [2]

There exists $(p^2+1)(p^2+p+1)$ of lines in PG(3,P).

1.9 Definition: [2]

A (k,ℓ)-set in PG(3,p) is a set of k spaces π_ℓ. A k-set is a (k,0)-set that is a set of k-points.

1.10 Definition: "(k,r)-cap", [1]

A (k,r)-cap is a set of k points in PG(n,p) with $n \geq 3$, such that at most r points on any line. Thus (k,2)-cap is a set of k points in PG(3,p), such that no three of them are collinear.

1.11 Definition: "Complete (k,r)-cap", [2]

A (k,r)-cap is a complete if it is not contained in a (k+1,r)-cap.

1.12 Definition: [2]

Let C_i be the number of points of index i in PG(3,p) which are not on a (k,r)-cap then the constants C_i of (k,r)-cap satisfy the following:

i) $\sum_{\alpha}^{\beta} C_i = p^3 + p^2 + p + 1 - k$

ii) $\sum_{\alpha}^{\beta} iC_i = \dfrac{k(k-1)\ldots(k-n+1)}{n!}(p^2 + p + 1 - n)$

where α is the smallest i for which $C_i \neq 0$, β be the largest i for which $C_i \neq 0$.

1.13 Remark: [3]

The (k,r)-cap is complete iff $C_0 = 0$.

1.14 Definition: [2]

The i-secant of a (k,r)-cap is a line intersects the cap in exactly i points, that is 0-secant is an external line, 1-secant is a unisecant line, 2-secant is a bisecant line and 3-secant is atrisecant line.

1.15 Remark:[3]

A (k,r)-cap is maximum iff every line in PG(3,p) is a 0-secant or r-secant.

1.16 Theorem: [1]

A maximum (k,2)-cap in PG(3,p) is an ovaloid.

2- The Additions and Multiplications Operation of GF(4): [4]

To find the addition and multiplication tables in GF(4), we have the order pairs (x_1,x_2) such that x_1, x_2 in GF(2), as follows:

$0 \equiv (0,0)$, $1 \equiv (1,0)$, $2 \equiv (0,1)$, $3 \equiv (1,1)$

Put these points in one orbit, (1,0) at the first point and by the principle of $(1,0)\,A^i$, $i = 0,1,2,3$ and $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $(1,0)A \equiv (0,1)$ and $(1,0)A^2 \equiv (1,1)$, so $(1,0) = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{vmatrix} (0,1) \\ (1,1) \end{vmatrix}$.

Now, in the left of the following table, m is the operation of multiplication and in the right n is the operation of addition in multiplication side we write the numeration of points as last, and the addition side takes the normal sequence.

m(*)		(+)n = f(m)
1	(1,0)	0
2	(0,1)	1
3	(1,1)	2
mod 3		

In addition table, we have the following relation:

$(x_1,x_2) + (y_1,y_2) = (z_1,z_2)$ where $z_i = (y_i + x_i) \bmod (2)$, for $i = 1, 2$.

In multiplication table, we have the following relation

$$((1,0)\,A^{f(m_1)})\,A^{f(m_2)} \Leftrightarrow m_1 * m_2 = m_3$$
$$= (1,0)\,A^{(f(m_1)+f(m_2))\ (\mathrm{mod}\ 3)}$$
$$= (x_1,x_2)$$

For example: $2*3=1 \Leftrightarrow ((1,0)A^1)A^2 = (1,0)A^3$
$= (1,0)A^0$
$= (1,0)$

where (1,0) equal to 1 in multiplication side.

Now we have addition and multiplication tables:

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

*	1	2	3
1	1	2	3
2	2	3	1
3	3	1	2

3- The (k,2)-caps in PG(3,4):

PG(3,4) contains 85 points and 85 planes such that each point is on 21 planes and every plane contains 21 points and every line contains 5 points and it is the intersection of 5 planes, (table 1 and 2).

In table (1), the set A={1,2,6,22,43} is taken which is the set of unit and reference points 1(1,0,0,0), 2(0,1,0,0), 6(0,0,1,0), 22(0,0,0,1), 43(1,1,1,1), this set is a (5,2)-cap since no three points of A are collinear as in table (2).

A is a (5,2)-cap, which is not complete since there exists some point of index zero for it, which are (12,13,15,16,17,19,20,21,28,29,31,32,33,35,36,37,40,41,46,48,49,50,52,53,55,56, 57,58,60, 1,62,63,65,66,67,68,69,71,72,73,74,76,77,78,79,80,81,82,83,84), then one can add some of them to A in order to obtain a complete (13,2)-cap B; B=A∪{12,15,21,28,31,37,40,46}={1,2,6,12,15,21,22,28,31,37,40,43,46}, B is the maximum (13,2)-cap in PG(3,4), since every line is a 0-secant or 2-secant, B is called an ovaloid.

4- The (k,3)-caps in PG(3,4):

Let B={1,2,6,12,15,21,22,28,31,37,40,43,46} be a (13,2)-cap. The points of index zero are (2,4,5,7,8,9,10,11,12,13,14,16,17,18,19,20,23,24,25,26,27,29,30,32,33,34,35,36,38,39,41, 42,44,45,47,…,85). The distinct (k,3)-cap can be constructed by adding some points of index zero for B, which are 3,7,10,23,26,39,42,54,57,61,64,66,67. Then C=B∪{3,7,10,23,26,39,42, 54,57,61,64,66,67}={1,2,3,6,7,10,12,15,21,22,23,26,28,31,37,39,40,42,43,46,54,57,61,64,66, 67},C is complete (26,3)-cap, since there are no points of index zero, i.e. $C_0=0$. B is a maximum complete (k,3)-cap.

5- The (k,4)-caps in PG(3,4):

We can construct complete (k,4)-caps by adding some points of index zero for C which are (4,5,8,9,11,13,14,16,17,18,19,20,24,25,27,29,30,32,33,34,35,36,38,41,44,45,47,48,49,50, 51,52,53,55,56,58,59,60,62,63,65,68,…,85), by adding to C nineteen of these points which are 4,9,11,14,17,24,27,33,38,44,47,48,55,59,60,62,65,68,74. Thus can get a complete (k,4)-cap call D={1,2,3,4,6,7,9,10,11,12,14,15,17,21,22,23,24,26,27,28,31,33,37,38,39,40,42,43, 44,46,47,48,54,55,57,59,60,62,64,65,66,67,68,74}. D is the maximum complete (45,4)-cap.

6- The (k,5)-caps in PG(3,4):

In section five, D is a complete (45,4)-cap. The points of index zero for D are (5,8,13,16, ,18,19,20,25,29,30,32,34,35,36,41,45,49,50,51,52,53,56,58,63,69,70,71,72,73,75,…,85), all of these points can be added to D, then E=D∪{5,8,13,16,18,19,20,25,29,30,32,34,36,41,45, 49,50,51,52,53,56,58,63,69,70,71,72,73,75,…,85} is the whole space PG(3,4). E is the maximum complete (85,5)-cap which can be obtained for any line of PG(3,4) contains five points and hence there are no more than five are collinear.

Conclusions;

From the above results, the distinct complete (K,n)-caps in PG(3,4),2≤ n≤ 5
Is as follows:

(k,2)-cap, where k=13, is a complete maximum cap which is ovaloid.
(k,3)-cap, where k=26, is a complete maximum cap.
(k,4)-cap, where k=45, is a complete maximum cap.
(k,5)-cap, where k=85, is a complete maximum cap, which is the whole space PG(3,4).

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Table (1) Points and Plans of PG (3 , 4)

i	P_i	Π_i																				
1	(1,0,0,0)	2	6	10	14	18	22	26	30	34	38	42	46	50	54	58	62	66	70	74	78	82
2	(0,1,0,0)	1	6	7	8	9	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
3	(1,1,0,0)	3	6	11	16	21	22	26	30	34	39	43	47	51	56	60	64	68	73	77	81	85
4	(2,1,0,0)	5	6	13	15	20	22	26	30	34	41	45	49	53	55	59	63	67	72	76	80	84
5	(3,1,0,0)	4	6	12	17	19	22	26	30	34	40	44	48	52	57	61	65	69	71	75	79	83
6	(0,0,1,0)	1	2	3	4	5	22	23	24	25	38	39	40	41	54	55	56	57	70	71	72	73
7	(1,0,1,0)	2	7	11	15	19	22	27	32	37	38	43	48	53	54	59	64	69	70	75	80	85
8	(2,0,1,0)	2	9	13	17	21	22	29	31	36	38	45	47	52	54	61	63	68	70	77	79	84
9	(3,0,1,0)	2	8	12	16	20	22	28	33	35	38	44	49	51	54	60	65	67	70	76	81	83
10	(0,1,1,0)	1	10	11	12	13	22	23	24	25	42	43	44	45	62	63	64	65	82	83	84	85
11	(1,1,1,0)	3	7	10	17	20	22	27	32	37	39	42	49	52	56	61	62	67	73	76	79	82
12	(2,1,1,0)	5	9	10	16	19	22	29	31	36	41	42	48	51	55	60	62	69	72	75	81	82
13	(3,1,1,0)	4	8	10	15	21	22	28	33	35	40	42	47	53	57	59	62	68	71	77	80	82
14	(0,2,1,0)	1	18	19	20	21	22	23	24	25	46	47	48	49	66	67	68	69	74	75	76	77
15	(1,2,1,0)	4	7	13	16	18	22	27	32	37	40	45	46	51	57	60	63	66	71	74	81	84
16	(2,2,1,0)	3	9	12	15	18	22	29	31	36	39	44	46	53	56	59	65	66	73	74	80	83
17	(3,2,1,0)	5	8	11	17	18	22	28	33	35	41	43	46	52	55	61	64	66	72	74	79	85
18	(0,3,1,0)	1	14	15	16	17	22	23	24	25	50	51	52	53	58	59	60	61	78	79	80	81
19	(1,3,1,0)	5	7	12	14	21	22	27	32	37	41	44	47	50	55	58	65	68	72	77	78	83
20	(2,3,1,0)	4	9	11	14	20	22	29	31	36	40	43	49	50	57	58	64	67	71	76	78	85
21	(3,3,1,0)	3	8	13	14	19	22	28	33	35	39	45	48	50	56	58	63	69	73	75	78	84
22	(0,0,0,1)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
23	(1,0,0,1)	2	6	10	14	18	23	27	31	35	39	43	47	51	55	59	63	67	71	75	79	83
24	(2,0,0,1)	2	6	10	14	18	25	29	33	37	41	45	49	53	57	61	65	69	73	77	81	85
25	(3,0,0,1)	2	6	10	14	18	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84
26	(0,0,1,1)	1	2	3	4	5	26	27	28	29	42	43	44	45	58	59	60	61	74	75	76	77
27	(1,0,1,1)	2	7	11	15	19	23	26	33	36	39	42	49	52	55	58	65	68	71	74	81	84
28	(2,0,1,1)	2	9	13	17	21	25	26	32	35	41	42	48	51	57	58	64	67	73	74	80	83
29	(3,0,1,1)	2	8	12	16	20	24	26	31	37	40	42	47	53	56	58	63	69	72	74	79	85
30	(0,0,2,1)	1	2	3	4	5	34	35	36	37	50	51	52	53	66	67	68	69	82	83	84	85
31	(1,0,2,1)	2	8	12	16	20	23	29	32	34	39	45	48	50	55	61	64	66	71	77	80	82
32	(2,0,2,1)	2	7	11	15	19	25	28	31	34	41	44	47	50	57	60	63	66	73	76	79	82
33	(3,0,2,1)	2	9	13	17	21	24	27	33	34	40	43	49	50	56	59	65	66	72	75	81	82
34	(0,0,3,1)	1	2	3	4	5	30	31	32	33	46	47	48	49	62	63	64	65	78	79	80	81
35	(1,0,3,1)	2	9	13	17	21	23	28	30	37	39	44	46	53	55	60	62	69	71	76	78	85
36	(2,0,3,1)	2	8	12	16	20	25	27	30	36	41	43	46	52	57	59	62	68	73	75	78	84
37	(3,0,3,1)	2	7	11	15	19	24	29	30	35	40	45	46	51	56	61	62	67	72	77	78	83
38	(0,1,0,1)	1	6	7	8	9	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53
39	(1,1,0,1)	3	6	11	16	21	23	27	31	35	38	42	46	50	57	61	65	69	72	76	80	84
40	(2,1,0,1)	5	6	13	15	20	25	29	33	37	38	42	46	50	56	60	64	68	71	75	79	83
41	(3,1,0,1)	4	6	12	17	19	24	28	32	36	38	42	46	50	55	59	63	67	73	77	81	85
42	(0,1,1,1)	1	10	11	12	13	26	27	28	29	38	39	40	41	66	67	68	69	78	79	80	81
43	(1,1,1,1)	3	7	10	17	20	23	26	33	36	38	43	48	53	57	60	63	66	72	77	78	83
44	(2,1,1,1)	5	9	10	16	19	25	26	32	35	38	45	47	52	56	59	65	66	71	76	78	85
45	(3,1,1,1)	4	8	10	15	21	24	26	31	37	38	44	49	51	55	61	64	66	73	75	78	84
46	(0,1,2,1)	1	14	15	16	17	34	35	36	37	38	39	40	41	62	63	64	65	74	75	76	77
47	(1,1,2,1)	3	8	13	14	19	23	29	32	34	38	44	49	51	57	59	62	68	72	74	79	85
48	(2,1,2,1)	5	7	12	14	21	25	28	31	34	38	43	48	53	56	61	62	67	71	74	81	84
49	(3,1,2,1)	4	9	11	14	20	24	27	33	34	38	45	47	52	55	60	62	69	73	74	80	83
50	(0,1,3,1)	1	18	19	20	21	30	31	32	33	38	39	40	41	58	59	60	61	82	83	84	85
51	(1,1,3,1)	3	9	12	15	18	23	28	30	37	38	45	47	52	57	58	64	67	72	75	81	82
52	(2,1,3,1)	5	8	11	17	18	25	27	30	36	38	44	49	51	56	58	63	69	71	77	80	82
53	(3,1,3,1)	4	7	13	16	18	24	29	30	35	38	43	48	53	55	58	65	68	73	76	79	82
54	(0,2,0,1)	1	6	7	8	9	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85
55	(1,2,0,1)	4	6	12	17	19	23	27	31	35	41	45	49	53	56	60	64	68	70	74	78	82
56	(2,2,0,1)	3	6	11	16	21	25	29	33	37	40	44	48	52	55	59	63	67	70	74	78	82
57	(3,2,0,1)	5	6	13	15	20	24	28	32	36	39	43	47	51	57	61	65	69	70	74	78	82
58	(0,2,1,1)	1	18	19	20	21	26	27	28	29	50	51	52	53	62	63	64	65	70	71	72	73
59	(1,2,1,1)	4	7	13	16	18	23	26	33	36	41	44	47	50	56	61	62	67	70	75	80	85
60	(2,2,1,1)	3	9	12	15	18	25	26	32	35	40	43	49	50	55	60	62	69	70	77	79	84
61	(3,2,1,1)	5	8	11	17	18	24	26	31	37	39	45	48	50	57	59	62	68	70	76	81	83
62	(0,2,2,1)	1	10	11	12	13	34	35	36	37	46	47	48	49	58	59	60	61	70	71	72	73
63	(1,2,2,1)	4	8	10	15	21	23	29	32	34	41	43	46	52	56	58	63	69	70	76	81	83
64	(2,2,2,1)	3	7	10	17	20	25	28	31	34	40	45	46	51	55	58	65	68	70	75	80	85
65	(3,2,2,1)	5	9	10	16	19	24	27	33	34	39	44	46	53	57	58	64	67	70	77	79	84
66	(0,2,3,1)	1	14	15	16	17	30	31	32	33	42	43	44	45	66	67	68	69	70	71	72	73
67	(1,2,3,1)	4	9	11	14	20	23	28	30	37	41	42	48	51	56	59	65	66	70	77	79	84
68	(2,2,3,1)	3	8	13	14	19	25	27	30	36	40	42	47	53	55	61	64	66	70	76	81	83
69	(3,2,3,1)	5	7	12	14	21	24	29	30	35	39	42	49	52	57	60	63	66	70	75	80	85
70	(0,3,0,1)	1	6	7	8	9	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69
71	(1,3,0,1)	5	6	13	15	20	23	27	31	35	40	44	48	52	54	58	62	66	73	77	81	85

72	(2,3,0,1)	4	6	12	17	19	25	29	33	37	39	43	47	51	54	58	62	66	72	76	80	84
73	(3,3,0,1)	3	6	11	16	21	24	28	32	36	41	45	49	53	54	58	62	66	71	75	79	83
74	(0,3,1,1)	1	14	15	16	17	26	28	29	45	46	47	48	49	54	55	56	57	82	83	84	85
75	(1,3,1,1)	5	7	12	14	21	23	26	33	36	40	45	46	51	54	59	64	69	73	76	79	82
76	(2,3,1,1)	4	9	11	14	20	25	26	32	35	39	44	46	53	54	61	63	68	72	75	81	82
77	(3,3,1,1)	3	8	13	14	19	24	26	31	37	41	43	46	52	54	60	65	67	71	77	80	82
78	(0,3,2,1)	1	18	19	20	21	34	35	36	37	42	43	44	45	54	55	56	57	78	79	80	81
79	(1,3,2,1)	5	8	11	17	18	23	29	32	34	40	42	47	53	54	60	65	67	73	75	78	84
80	(2,3,2,1)	4	7	13	16	18	25	28	31	34	39	42	49	52	54	59	64	69	72	77	78	83
81	(3,3,2,1)	3	9	12	15	18	24	27	33	34	41	42	48	51	54	61	63	68	71	76	78	85
82	(0,3,3,1)	1	10	11	12	13	30	31	32	33	50	51	52	53	54	55	56	57	74	75	76	77
83	(1,3,3,1)	5	9	10	16	19	23	28	30	37	40	43	49	50	54	61	63	68	73	74	80	83
84	(2,3,3,1)	4	8	10	15	21	25	27	30	36	39	45	48	50	54	60	65	67	72	74	79	85
85	(3,3,3,1)	3	7	10	17	20	24	29	30	35	41	44	47	50	54	59	64	69	71	74	81	84

Table (2) Plans and lines of PG (3 , 4)

	2	6	10	14	18	22	26	30	34	38	42	46	50	54	58	62	66	70	74	78	82
	6	22	22	22	22	2	2	2	2	6	14	10	18	6	18	14	10	6	10	18	14
1	10	26	42	50	46	38	42	46	50	42	30	34	26	58	30	34	26	74	30	34	26
	14	30	62	58	66	54	58	62	66	46	66	58	62	62	38	38	38	78	50	42	46
	18	34	82	78	74	70	74	78	82	50	70	70	70	66	82	74	78	82	54	54	54
	1	6	7	8	9	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
	6	22	22	22	22	1	7	6	6	9	6	1	8	9	7	1	9	1	7	8	8
2	7	26	27	28	29	23	26	28	29	25	23	26	23	23	25	30	24	35	24	25	24
	8	30	32	33	31	24	33	32	33	32	31	27	32	28	28	31	27	36	29	27	26
	9	34	37	35	36	25	36	36	37	35	35	29	34	37	34	33	34	37	30	30	31
	3	6	11	16	21	22	26	30	34	39	43	47	51	56	60	64	68	73	77	81	85
	6	22	22	22	22	3	11	21	16	6	3	16	21	6	11	3	3	16	11	21	6
3	11	26	43	51	47	39	39	39	39	43	26	26	26	60	34	30	34	30	30	34	73
	16	30	64	60	68	56	68	60	64	47	60	56	64	64	47	47	51	43	51	43	77
	21	34	85	81	77	73	81	85	77	51	77	85	73	68	73	81	85	68	56	56	81
	5	6	13	15	20	22	26	30	34	41	45	49	53	55	59	63	67	72	76	80	84
	6	22	22	22	22	5	13	13	20	6	15	13	20	6	20	15	5	6	5	5	15
4	13	26	45	53	49	41	41	53	45	45	30	34	26	59	30	34	34	76	26	30	26
	15	30	63	59	67	55	67	55	55	49	67	59	63	63	41	41	53	80	45	49	49
	20	34	84	80	76	72	80	76	80	53	72	72	72	67	84	76	84	84	59	63	55
	4	6	12	17	19	22	26	30	34	40	44	48	52	57	61	65	69	71	75	79	83
	6	22	22	22	22	4	4	4	4	6	17	17	12	6	12	17	12	19	6	19	19
5	12	26	44	52	48	40	44	48	52	44	30	26	30	61	34	34	26	26	71	34	30
	17	30	65	61	69	57	61	65	69	48	69	57	57	65	48	40	40	52	79	44	40
	19	34	83	79	75	71	75	79	83	52	71	83	75	69	71	75	79	65	83	57	61
	1	2	3	4	5	22	23	24	25	38	39	40	41	54	55	56	57	70	71	72	73
	2	22	22	22	22	1	2	2	2	1	5	3	4	5	1	5	3	1	3	4	4
6	3	38	39	40	41	23	39	40	41	39	24	25	23	23	54	25	23	71	24	25	24
	4	54	56	57	55	24	55	56	57	40	57	55	56	40	56	38	38	72	41	39	38
	5	70	73	71	72	25	71	72	73	41	70	70	70	73	57	71	72	73	54	54	55
	2	7	11	15	19	22	27	32	37	38	43	48	53	54	59	64	69	70	75	80	85
	7	22	22	22	22	2	2	2	2	7	15	11	19	15	7	15	11	7	11	19	19
7	11	27	43	53	48	38	43	48	53	43	32	37	27	27	54	37	27	75	32	37	32
	15	32	64	59	69	54	59	64	69	48	69	59	64	48	64	38	38	80	53	43	38
	19	37	85	80	75	70	75	80	85	53	70	70	70	85	69	75	48	85	54	54	59
	2	9	13	17	21	22	29	31	36	38	45	47	52	54	61	63	68	70	77	79	84
	9	22	22	22	22	2	2	2	2	21	9	13	21	13	9	17	13	17	9	21	17
8	13	29	45	52	47	38	45	47	52	31	38	36	29	31	54	36	29	31	70	36	29
	17	31	63	61	68	54	61	63	68	61	47	61	63	52	63	38	38	45	79	45	47
	21	36	84	79	77	70	77	79	84	84	52	70	70	77	68	77	79	68	84	54	54
	2	8	12	16	20	22	28	33	35	38	44	49	51	54	60	65	67	70	76	81	83
	8	22	22	22	22	2	2	2	2	8	16	16	12	8	12	16	12	20	8	20	20
9	12	28	44	51	49	38	44	49	51	44	33	28	33	60	35	35	28	28	70	35	33
	16	33	65	60	67	54	60	65	67	49	67	54	54	65	49	38	38	51	81	44	38
	20	35	83	81	76	70	76	81	83	51	70	83	76	67	70	76	81	65	83	54	60
	1	10	11	12	13	22	23	24	25	42	43	44	45	62	63	64	65	82	83	84	85
	10	22	22	22	22	1	10	10	10	1	12	11	12	13	1	13	13	1	11	11	12
10	11	42	43	44	45	23	43	44	45	43	25	25	23	23	62	25	24	83	24	23	24
	12	62	64	65	63	24	63	64	65	44	62	63	64	44	64	42	43	84	45	42	42
	13	82	85	83	84	25	83	84	85	45	84	82	82	85	65	83	82	85	62	65	63
	3	7	10	17	20	22	27	32	37	39	42	49	52	56	61	62	67	73	76	79	82
	7	22	22	22	22	3	3	10	10	10	7	17	20	20	20	7	17	7	17	3	3
11	10	27	42	52	49	39	42	52	49	27	39	27	27	37	32	56	32	76	37	32	37
	17	32	62	61	67	56	61	56	61	67	49	56	62	42	39	61	42	79	39	49	52
	20	37	82	79	76	73	76	76	73	79	52	82	73	79	82	67	73	82	62	62	67
	5	9	10	16	19	22	29	31	36	41	42	48	51	55	60	62	69	72	75	81	82
	9	22	22	22	22	5	10	10	10	16	16	16	9	9	5	5	5	19	9	19	19
12	10	29	42	51	48	41	41	51	48	36	31	29	41	60	29	31	36	29	72	36	31
	16	31	62	60	69	55	69	55	60	62	69	55	42	62	42	48	51	51	81	42	41
	19	36	82	81	75	72	81	75	72	75	72	82	48	69	75	81	82	62	82	55	60
1	4	8	10	15	21	22	28	33	35	40	42	47	53	57	59	62	68	71	77	80	82

3	8	22	22	22	22	4	10	10	10	15	15	15	8	21	21	21	8	8	4	4	4
	10	28	42	53	47	40	40	53	47	35	33	28	40	35	33	28	57	77	28	33	35
	15	33	62	59	68	57	68	57	59	62	68	57	42	42	40	53	59	80	42	47	53
	21	35	82	80	77	71	80	77	71	77	71	82	47	80	82	71	62	82	59	62	68
14	1	18	19	20	21	22	23	24	25	46	47	48	49	66	67	68	69	74	75	76	77
	18	22	22	22	22	1	18	18	18	1	20	21	21	20	1	19	21	1	20	19	19
	19	46	48	49	47	23	47	48	49	47	24	25	24	23	66	23	23	75	25	25	24
	20	66	69	67	68	24	67	68	69	48	69	67	66	48	68	49	46	76	46	47	46
	21	74	75	76	77	25	75	76	77	49	74	74	75	77	69	74	76	77	68	66	67
15	4	7	13	16	18	22	27	32	37	40	45	46	51	57	60	63	66	71	74	81	84
	7	22	22	22	22	4	18	18	18	7	16	16	13	7	13	16	13	7	4	4	4
	13	27	45	51	46	40	51	40	45	45	32	27	32	60	37	37	27	74	27	32	37
	16	32	63	60	66	57	63	60	57	46	66	57	57	63	46	40	40	81	45	46	51
	18	37	84	81	74	71	71	84	81	51	71	84	74	66	71	74	81	84	60	63	66
16	3	9	12	15	18	22	29	31	36	39	44	46	53	56	59	65	66	73	74	80	83
	9	22	22	22	22	3	18	18	18	15	15	15	9	12	12	9	12	9	3	3	3
	12	29	44	53	46	39	53	39	44	36	31	29	39	31	36	56	29	74	29	31	36
	15	31	65	59	66	56	65	59	56	65	66	56	44	53	46	59	39	80	44	46	53
	18	36	83	80	74	73	73	83	80	74	73	83	46	74	73	66	80	83	59	65	66
17	5	8	11	17	18	22	28	33	35	41	43	46	52	55	61	64	66	72	74	79	85
	8	22	22	22	22	5	11	11	11	18	18	8	18	8	5	17	5	17	8	5	17
	11	28	43	52	46	41	41	52	46	33	35	41	28	61	28	35	35	33	72	33	28
	17	33	64	61	66	55	66	55	61	61	55	43	64	64	43	41	52	43	79	46	46
	18	35	85	79	74	72	79	74	72	85	79	52	72	66	74	74	85	66	85	64	55
18	1	14	15	16	17	22	23	24	25	50	51	52	53	58	59	60	61	78	79	80	81
	14	22	22	22	22	1	14	14	14	1	17	15	16	1	16	17	15	1	15	16	17
	15	50	53	51	52	23	51	52	53	51	25	23	24	59	25	23	24	79	25	23	24
	16	58	59	60	61	24	59	60	61	52	58	58	58	60	52	53	51	80	50	50	50
	17	78	80	81	79	25	79	80	81	53	80	81	79	61	78	78	78	81	60	61	59
19	5	7	12	14	21	22	27	32	37	41	44	47	50	55	58	65	68	72	77	78	83
	7	22	22	22	22	5	14	14	14	21	21	5	7	12	7	21	5	12	5	12	7
	12	27	44	50	47	41	47	44	41	32	37	32	41	32	55	27	37	37	27	27	72
	14	32	65	58	68	55	55	68	65	58	55	65	44	50	65	50	50	47	44	41	77
	21	37	83	78	77	72	83	72	77	83	78	78	47	77	68	72	83	58	58	68	78
20	4	9	11	14	20	22	29	31	36	40	43	49	50	57	58	64	67	71	76	78	85
	9	22	22	22	22	4	14	14	14	20	20	11	9	9	4	4	4	20	11	11	9
	11	29	43	50	49	40	49	43	40	31	36	36	40	58	29	31	36	29	31	29	71
	14	31	64	58	67	57	57	67	64	58	57	58	43	64	43	49	50	50	50	40	76
	20	36	85	78	76	71	85	71	76	85	78	71	49	67	76	78	85	64	57	67	78
21	3	8	13	14	19	22	28	33	35	39	45	48	50	56	58	63	69	73	75	78	84
	8	22	22	22	22	3	3	3	13	14	14	8	13	14	19	8	13	19	18	19	3
	13	28	45	50	48	39	45	48	48	28	33	39	33	28	33	56	28	28	73	35	35
	14	33	63	58	69	56	58	63	58	35	69	45	56	48	39	58	39	50	78	45	50
	19	35	84	78	75	73	75	78	73	63	73	50	75	84	84	69	78	63	84	56	69
22	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	2	6	6	6	6	1	2	2	2	1	5	3	4	1	4	5	3	1	3	4	5
	3	10	11	12	13	7	11	12	13	11	8	9	7	15	8	9	7	19	8	9	7
	4	14	16	17	15	8	15	16	17	12	17	15	16	16	10	10	10	20	13	11	12
	5	18	21	19	20	9	19	20	21	13	18	18	18	17	21	19	20	21	14	14	14
23	2	6	10	14	18	23	27	31	35	39	43	47	51	55	59	63	67	71	75	79	83
	6	23	23	23	23	2	10	10	10	14	14	6	18	18	18	6	2	6	2	2	14
	10	27	43	51	47	39	39	51	47	35	31	39	27	35	31	55	35	75	27	31	27
	14	31	63	59	67	55	67	55	59	63	67	43	63	43	39	59	51	79	43	47	55
	18	35	83	79	75	71	79	75	71	75	71	51	71	79	83	67	83	83	59	63	47
24	2	6	10	14	18	25	29	33	37	41	45	49	53	57	61	65	69	73	77	81	85
	6	25	25	25	25	2	10	10	10	18	18	6	18	14	2	6	2	14	14	2	6
	10	29	45	53	49	41	41	53	49	33	37	41	29	29	29	57	37	33	37	33	73
	14	33	65	61	69	57	69	57	61	61	57	45	65	49	45	61	53	45	41	49	77
	18	37	85	81	77	73	81	77	73	85	81	53	73	85	77	69	85	69	65	65	81
25	2	6	10	14	18	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84
	6	24	24	24	24	2	14	14	18	18	2	6	10	6	10	18	10	6	14	2	2
	10	28	44	52	48	40	48	44	44	32	28	40	32	60	36	28	28	76	36	32	36
	14	32	64	60	68	56	56	68	56	60	60	44	56	64	48	52	40	80	40	48	52
	18	36	84	80	76	72	84	72	80	84	76	52	76	68	72	72	80	84	64	64	68
26	1	2	3	4	5	26	27	28	29	42	43	44	45	58	59	60	61	74	75	76	77
	2	26	26	26	26	1	2	2	2	1	5	3	4	1	4	5	3	1	3	4	5
	3	42	43	44	45	27	43	44	45	43	28	29	27	59	28	29	27	75	28	29	27
	4	58	60	61	59	28	59	60	61	44	61	59	60	60	42	42	42	76	45	43	44
	5	74	77	75	76	29	75	76	77	45	74	74	74	61	77	75	76	77	58	58	58
27	2	7	11	15	19	23	26	33	36	39	42	49	52	55	58	65	68	71	74	81	84
	7	23	23	23	23	2	15	15	15	11	19	11	7	11	19	7	2	19	2	2	7
	11	26	42	52	49	39	49	42	39	26	36	36	39	33	33	55	36	26	26	33	71
	15	33	65	58	68	55	55	68	65	68	55	58	42	52	39	58	52	52	42	49	74
	19	36	84	81	74	71	84	71	74	81	81	71	49	74	84	68	84	65	58	65	81
28	2	9	13	17	21	25	26	32	35	41	42	48	51	57	58	64	67	73	74	80	83
	9	25	25	25	25	2	2	2	13	9	21	17	2	13	21	21	9	17	17	13	9
	13	26	42	51	48	41	42	48	48	42	35	26	35	32	32	26	57	32	35	26	73

	17	32	64	58	67	57	58	64	58	48	57	57	67	51	41	51	58	42	41	41	74
	21	35	83	80	74	73	74	80	73	51	80	83	83	74	83	73	64	67	64	67	80
29	2	8	12	16	20	24	26	31	37	40	42	47	53	56	58	63	69	72	74	79	85
	8	24	24	24	24	2	20	2	2	8	20	16	12	8	20	16	16	12	2	12	8
	12	26	42	53	47	40	53	47	53	42	37	26	31	58	31	37	31	37	26	26	72
	16	31	63	58	69	56	63	63	69	47	56	56	56	63	40	40	42	47	42	40	74
	20	37	85	79	74	72	72	79	85	53	79	85	74	69	85	74	72	58	58	69	79
30	1	2	3	4	5	34	35	36	37	50	51	52	53	66	67	68	69	82	83	84	85
	2	34	34	34	34	1	2	2	2	3	1	5	3	1	3	5	5	4	1	4	4
	3	50	51	52	53	35	51	52	53	35	50	35	36	67	37	37	36	35	82	37	36
	4	66	68	69	67	36	67	68	69	69	52	66	66	68	52	50	51	53	84	51	50
	5	82	85	83	84	37	83	84	85	84	53	85	83	69	82	83	82	68	85	66	67
31	2	8	12	16	20	23	29	32	34	39	45	48	50	55	61	64	66	71	77	80	82
	8	23	23	23	23	2	2	2	2	8	16	16	12	8	12	20	12	8	16	20	20
	12	29	45	50	48	39	45	48	50	45	32	29	32	61	34	29	29	77	34	34	32
	16	32	64	61	66	55	61	64	66	48	66	55	55	64	48	50	39	80	39	45	39
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	12	29	42	52	49	39	42	49	52	42	30	29	30	60	35	35	29	29	70	35	30
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	8	62	64	65	63	56	63	64	65	60	62	63	62	64	58	61	58	59	58	67	60
	9	66	69	67	68	57	67	68	69	61	68	66	67	65	69	66	68	65	64	69	62
71	5	6	13	15	20	23	27	31	35	40	44	48	52	54	58	62	66	73	77	81	85
	6	23	23	23	23	5	5	5	5	6	15	15	13	6	13	15	13	20	6	20	20
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73	3	6	11	16	21	24	28	32	36	41	45	49	53	54	58	62	66	71	75	79	83
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79	5	8	11	17	18	23	29	32	34	40	42	47	53	54	60	65	67	73	75	78	84
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	18	34	85	78	76	71	76	78	85	51	71	85	76	68	71	76	78	63	85	54	61
82	1	10	11	12	13	30	31	32	33	50	51	52	53	54	55	56	57	74	75	76	77
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83	5	9	10	16	19	23	28	30	37	40	43	49	50	54	61	63	68	73	74	80	83
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	19	37	83	80	74	73	74	80	83	50	73	83	74	68	73	74	80	63	83	54	61
84	4	8	10	15	21	25	27	30	36	39	45	48	50	54	60	65	67	72	74	79	85
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	20	35	84	81	74	71	74	81	84	50	71	84	74	69	71	74	81	64	84	54	59

الغطاء (k,r) الكامل في الفضاء الاسقاطي ثلاثي الابعاد على حقل كالوا GF(4)

فاطمة فيصل كريم

قسم الرياضيات ،كلية التربية – ابن الهيثم ،جامعة بغداد

استلم البحث في: 27، حزيران، 2010

قبل البحث في: 12 ،كانون الاول،2010

الخلاصة

الهدف من هذا البحث هو بناء الغطاء (k,r) في الفضاء الاسقاطي ذي ثلاثة ابعاد PG(3,p) حول حقل كالوا GF(4). وقد وجدنا ان اعظم غطاء كامل (k,2) الذي يدعى اهليلجي، موجود في PG(3,4) عندما k=13. فضلا عن ذلك وجدنا اعظم غطاء لـ (k,3) و (k,4) و (k,5).

الكلمات المفتاحية: الفضاء الاسقاطي أعظم غطاء كامل حقل كالوا.