On Pairwise Semi-p-separation Axioms in Bitopological Spaces

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Abstract

In this paper, we define a new type of pairwise separation axioms called pairwise semi-p-separation axioms in bitopological spaces, also we study some properties of these spaces and relationships of each one with the ordinary separation axioms in the bitopological spaces.
Keywords: Bitopological space, pairwise semi-p-T_0- space, pairwise semi-p-T_1- space, pairwise semi-p-T_2- space, pairwise semi-p-regular space, pairwise semi-p-normal space.

1-Introduction

The theory of bitopological spaces started with the paper of Kelly in [1]. A set equipped with two topologies is called a bitopological space. Since then several authors continued investigating such spaces. Furthermore, Kelly extended some of the standard results of separation axioms in a topological space to a bitopological space, such extensions are pairwise regular, pairwise Hausdorff and pairwise normal, concepts of pairwise T_0 and pairwise T_1 were introduced by Murdeshwar and Naimpally in [2].

The purpose of this paper is to introduce and investigate the notion of pairwise semi- p-separation axioms in bitopological spaces and study some properties of these spaces and relationships of each one with the ordinary separation axioms in the bitopological spaces.

2- Preliminaries

In this section, we introduce some definitions and propositions, which is necessary for the paper.

Definition 2.1[3]:

A subset *A* of a topological space (X,τ) is called a *pre-open set* if $A \subseteq \overline{A}^{\circ}$. The complement of pre-open set is called *pre-closed set.*

The family of all pre-open subsets of *X* is denoted by PO(*X*). The family of all pre-closed subsets of *X* is denoted by PC(*X*).

Proposition 2.2 [4]:

Let (X,τ) be a topological space, then:

1-Every open set is a pre-open set.

2-Every closed set is a pre-closed set.

But the converse of (1) and (2) is not true in general.

Proposition 2.3 [4]:

The union of any family of pre-open sets is a pre-open set.

Definition 2.4[3]:

The union of all per-open sets contained in *A* is called the *pre-interior of A*, denoted by pre-int *A*.

The intersection of all pre-closed sets containing *A* is called the *per-closure of A*, and is denoted by pre-cl *A*.

Proposition 2.5 [4]:

Let (X,τ) be a topological space and A, B be any two subsets of X, then:

pre-cl $A \cup \text{pre}-\text{cl}\, B \subseteq \text{pre}-\text{cl}\,(A \cup B)$.

Definition 2.6 [4]:

A subset *A* of a topological space (X,τ) is said to be *semi-p-open set* if and only if there exists a pre-open set in X, say U, such that $U \subseteq A \subseteq pre-cl\ U$.

The family of all semi-p-open sets of X is denoted by S-P(X).

The complement of semi-p-open set is called *semi-p-closed set.*

The family of all semi-p-closed sets of X is denoted by S-P-C(X).

Proposition 2.7 [4]:

1- Every open (closed) set is semi-p-open (closed) set respectively.
2- Every pre-open (pre-closed) set is semi-p-open (semi-p-closed) set respectively.

Also, the converse of (1) and (2) is not true in general.

Proposition 2.8:

The union of any family of semi-p-open sets is semi-p-open set.

Proof:

Let $\{A_\alpha\}, \alpha \in \Lambda$ be any family of semi-p-open sets in X, we must prove $\bigcup_{\alpha\in\Lambda} A_\alpha$ is a semi-p-open set, since A_α is semi-p-open set, for all $\alpha \in \Lambda$, which implies there exists a pre-open set U_α such that $U_\alpha \subseteq A_\alpha \subseteq pre-clU_\alpha$.

Thus $\bigcup_{\alpha\in\Lambda} U_\alpha \subseteq \bigcup_{\alpha\in\Lambda} A_\alpha \subseteq \bigcup_{\alpha\in\Lambda} pre-clU_\alpha$ and from (Proposition 2.3 and 2.5) we have a pre-open set $\bigcup_{\alpha\in\Lambda} U_\alpha$ such that $\bigcup_{\alpha\in\Lambda} U_\alpha \subseteq \bigcup_{\alpha\in\Lambda} A_\alpha \subseteq pre-cl(\bigcup_{\alpha\in\Lambda} U_\alpha)$. Hence $\bigcup_{\alpha\in\Lambda} A_\alpha$ is a semi- p-open set. ■

Definition 2.9 [4]:

Let (X,τ) be a topological space and let *A* be any subset of *X*, then:

1- The union of all semi-p-open sets contained in *A* is called the *semi-p-interior of* ***A***, denoted by semi-p-int *A*.
2- The intersection of all semi-p-closed sets containing *A* is called the *semi-p-closure of A*, and denoted by semi-p-cl *A*.

Definition 2.10 [4]:

Let (X,τ) be a topological space and let $x \in X$. A subset *N* of *X* is said to be *semi-p-neighborhood of x* if and only if there exists a semi-p-open set *G*, such that $x \in G \subseteq N$. We shall use the symbol nbd. instead of the word neighborhood.

If *N* is semi-p-open subset of *X*, then *N* is a semi-p-open nbd of *x*.

Proposition 2.11:

Let (X,τ) be a topological space, then every semi-p-nbd is a semi-p-open set.

Proof:

Let N be any semi-p-nbds for each of its points, that is means for each $x \in N$, there exists a semi-p- open set G such that $x \in G \subset N$, now we must prove N is a semi-p-open set, since $N = \bigcup_{x\in N}\{x\}$ and since N is a semi-p- nbd for all $x \in N$.

Thus $N = \bigcup_{x\in G}\{G: G \text{ is a semi}-p-\text{open set such that } x \in G \subset N\}$, and from (Proposition 2.8) we have N is a semi-p-open set. ■

Definition 2.12 [1]:

Let X be a non-empty set, let τ_1, τ_2 be any two topologies on X, then (X,τ_1,τ_2) is called a bitopological space.

Note 2.13:

In the space (X,τ_1,τ_2), we shall denote to the set of all semi-p- open sets in τ_1(τ_2) by S-P(X, τ_1)(S-P(X, τ_2)) respectively.

Definition 2.14 [2]:

A bitopological space (X,τ_1,τ_2) is said to be:

1- ***Pairwise*** $\boldsymbol{T_0 - space}$ if for every pair of points x and y in X such that $x \neq y$, there exists a τ_1-open set containing x but not y or y but not x or a τ_2-open set containing y but not x or x but not y.

2- ***Pairwise*** $\boldsymbol{T_1 - space}$ if for every pair of points x and y in X such that $x \neq y$, there exists a τ_1-open set U and a τ_2-open set V such that $x \in U, y \notin U \text{ and } y \in V,\ x \notin V.$

Definition 2.15[1]:

A bitopological space (X,τ_1,τ_2) is said to be:

1- ***Pairwise*** $\boldsymbol{T_2 - space}$ if every two distinct points in X can be separated by disjoint τ_1-open set and τ_2-open sets.

2- ***Pairwise regular space***, if for each point $x \in X$ and each τ_i-closed set F not containing x, there exists a τ_i-open set U and τ_j-open set V such that $x \in U, F \subset V \text{ and } U \cap V = \emptyset$, where $i \neq j \text{ and } i,j = 1,2.$

3- ***Pairwise normal space***, if for each τ_i-closed set A and τ_j-closed set B such that $A \cap B = \emptyset$, there exist sets U and V such that U is τ_j-open, V is τ_i-open, $A \subset U, B \subset V, \text{and } U \cap V = \emptyset, i,j = 1,2, i \neq j.$

3-Pairwise semi-p-separation axioms

We begin with the definition of pairwise semi-p-T_0- spaces.

Definition 3.1:

A space (X,τ_1,τ_2) is called *pairwise semi-p-$\boldsymbol{T_0}$- space* if for any pair of distinct points x and y in X, there exists a τ_1-semi-p-open set or τ_2-semi-p-open set which contains one of them but not the other.

Proposition 3.2:

If a space (X,τ_1,τ_2) is pairwise T_3- space, then (X,τ_1,τ_2) is pairwise semi-p-T_3- space.

Proof:

For any $x,y \in X$ such that $x \neq y$, we must prove there exists a semi-p-open in τ_1 or τ_2 which contains one of them but not the other.

Now, let $x \neq y$ in X, since (X,τ_1,τ_2) is pairwise T_3- space, then there exists open set U in τ_1 or τ_2 such that $x \in U$ and $y \notin U$. But from (Proposition 2.7 part (1)) there exists semi-p-open set U such that $x \in U$ and $y \notin U$. Thus (X,τ_1,τ_2) is pairwise semi-p-T_3- space. ■

Remark 3.3:

The converse of (Proposition 3.2) is not true in general, as the following example shows:

Example 1:

Let X={1, 2, 3}, $\tau_1 = \{\emptyset, X, \{1\}\}$, $\tau_2 = \{\emptyset, X, \{2,3\}\}$, PO(X, τ_1) = S-P(X, τ_1)= { $\{\emptyset, X, \{1\}, \{1,2\}, \{1,3\}\}$, PO(X, τ_2) = S-P(X, τ_2)= { $\{\emptyset, X, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$.

Then, clearly the space (X,τ_1,τ_2) is pairwise semi-p-T_3- space, but not pairwise T_3- space, since 2 $\neq$ 3 in X but there is no open set U$\in \tau_1$ or U $\in \tau_2$ such that 2$\in$ U and $3 \notin$ U.

Theorem 3.4 :

For a space (X,τ_1,τ_2), the following are equivalent :

(1) (X,τ_1,τ_2) is pairwise semi-p-T_3 -space .

(2) For every $x \in X$, $\{x\} = \tau_1 - semi - p - cl\{x\} \cap \tau_2 - semi - p - cl\{x\}$.

(3) For every $x \in X$, the intersection of all $\tau_1 - semi - p - neighbourhoods\ of\ x$ and all $\tau_2 - semi - p - neighbourhoods\ of\ x$ is {x} .

Proof: $(1) \Rightarrow (2)$

Suppose x$\neq$ y in X, there exists a τ_1-semi-p-open set U containing x but not y or a τ_2-semi-p-open set V containing y but not x .That means mean either $x \notin \tau_1 - semi - p - cl\{y\}$ or $y \notin \tau_2 - semi - p - cl\{x\}$.

Hence for a point x, y$\notin \tau_1 - semi - p - cl\{x\} \cap \tau_2 - semi - p - cl\{x\}$ .Thus $\{x\} = \tau_1 - semi - p - cl\{x\} \cap \tau_2 - semi - p - cl$ {x} .

$(2) \Rightarrow (3)$

Suppose there exists $y \neq x$ such that y belongs to the intersection of all $\tau_1 - semi - p - nbds\ of\ x$ and all $\tau_2 - semi - p - nbds\ of\ x$ .Hence (X,τ_1,τ_2) is not pairwise semi-p-T_3 -space, implies τ_1 -semi -pcl {x} $\cap$ $\tau_2 - semi - pcl\{x\} \neq \{x\}$ which is a contradiction, thus the intersection of all$\tau_1 - semi - p - nbds\ of\ x$ and all $\tau_2 - semi - p - nbds\ of\ x\ is\ \{x\}$.

$(3) \Rightarrow (1)$

Let x$\neq$ y in X, since {x} =the intersection of all $\tau_1 - semi - p - nbds\ of\ x$ and $\tau_2 - semi - p - nbds\ of\ x$. Hence, there exists either on $\tau_1 - semi - p - nbds of\ y$ not containing x or a $\tau_2 - semi - p - nbds\ of\ y$ not containing x .Therefore (X,τ_1,τ_2) is pairwise semi-p -T_3 -space.■

IBN AL- HAITHAM J. FOR PURE & APPL. SCI. VOL.24 (3) 2011

Theorem 3.5:

The product of an arbitrary family of pairwise semi -p-T_3 -spaces is pairwise semi- p-T_3 -space.

Proof:

Let $(X,\tau_1,\tau_2) = \prod_{\alpha\in\Lambda}(X_\alpha,\tau_{1_\alpha},\tau_{2_\alpha})$ be the product of an arbitrary family of pairwise semi -p-T_1 -spaces, where τ_1 and τ_2 are the product topologies on X generated by $\tau_{1_\alpha},\tau_{2_\alpha}$ respectively and X $=\prod_{\alpha\in\Lambda}X_\alpha$.

Let x and y be two distinct points of X. Hence $x_\lambda \neq y_\lambda$ for some $\lambda\in\Lambda$. But $(X_\lambda,\tau_{1_\lambda},\tau_{2_\lambda})$ is pairwise semi -p-T_1 -space, therefore, there exists either a τ_{1_λ}-semi-p-open set U_λ containing x_λ but not y_λ or a τ_{2_λ}-semi-p-open set V_λ containing y_λ but not x_λ. Define $U=\prod_{\alpha\neq\lambda}X_\alpha\times U_\lambda$ and $V=\prod_{\alpha\neq\lambda}X_\alpha\times V_\lambda$. Then U is a τ_1- semi-p-open set and V is τ_2-semi-p-open set, also, U contains x but not y. Hence $\prod_{\alpha\in\Lambda}(X_\alpha,\tau_{1_\alpha},\tau_{2_\alpha})$ is pairwise semi-p-T_1 -space. ■

Definition 3.6:

A space (X,τ_1,τ_2) is called *pairwise semi-p-T_1- space*, if for any pair of distinct points x and y in X, there exists a τ_1-semi-p-open set U and τ_2-semi-p-open set V such that $x\in U, y\notin U$ and $y\in V, x\notin V$.

Proposition 3.7:

If a space (X,τ_1,τ_2) is pairwise -T_1- space, then (X,τ_1,τ_2) is pairwise semi-p-T_1- space.

Proof:

For any $x\neq y$ in X, since (X,τ_1,τ_2) is pairwise -T_1- space, then there exists τ_1-open set U and τ_2-open set V such that $x\in U, y\notin U$ and $y\in V, x\notin V$. And since every open set is semi-p-open set (by Proposition 2.7 part (1)), which implies U is semi-p-open set in τ_1 containing x but not y and V is semi-p-open set in τ_2 containing y but not x. Hence (X,τ_1,τ_2) is pairwise semi-p-T_1- space. ■

Remark 3.8:

The converse of (Proposition 3.7) is not true in general as the following example shows: Consider Example 1, where:

X={1, 2, 3}, $\tau_1=\{\emptyset,X,\{1\}\}$, $\tau_2=\{\emptyset,X,\{2,3\}\}$,

PO(X, τ_1) = S-P(X, τ_1)= { $\{\emptyset,X,\{1\},\{1,2\},\{1,3\}\}$,

PO(X, τ_2) = S-P(X, τ_2)= { $\{\emptyset,X,\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\}\}$. Then, clearly that the space (X,τ_1,τ_2) is pairwise semi-p-T_1- space, but not pairwise -T_1- space, since $2\neq 3$ in X, but there is noτ_1-open set containing 2 but not containing 3 and there is no τ_2-open set containing 3 but not 2.

Theorem 3.9:

The product of an arbitrary family of pairwise semi -p-T_1 -spaces is pairwise semi- p-T_1 -space.

Proof: Similar to the proof of (Theorem 3.5). ■

Definition 3.10:

A space (X,τ_1,τ_2) is called *pairwise semi-p-T_2- space*, if for any pair of distinct points x and y in X, there exists a τ_1-semi-p-open set U and τ_2-semi-p-open set V such that $x \in U$ $y \in V$ and $U \cap V = \emptyset$.

Proposition 3.11:

If a space (X,τ_1,τ_2) is pairwise -T_2- space, then (X,τ_1,τ_2) is pairwise semi-p-T_2- space.

Proof: similar of the proof of (Proposition 3.7). ■

Remark 3.12:

The converse of (Proposition 3.11) is not true in general; consider example 1:

X={1, 2, 3}, $\tau_1 = \{\emptyset, X, \{1\}\}, \tau_2 = \{\emptyset, X, \{2,3\}\}$,

PO(X, τ_1) = S-P(X, τ_1)= { $\{\emptyset, X, \{1\}, \{1,2\}, \{1,3\}\}$,

PO(X, τ_2) = S-P(X, τ_2)= { $\{\emptyset, X, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$, clearly (X,τ_1,τ_2) is pairwise semi-p-T_2- space, but not pairwise -T_2- space, since $2 \neq 3$ in X, but there is no two disjoint open sets in τ_1 and τ_2, which contain 2 and 3 respectively.

Theorem 3.13:

For a space(X,τ_1,τ_2), the following are equivalent:

1-(X,τ_1,τ_2) is pairwise semi-p-T_2- space.

2- For each $x \in X$and for each $y \in X$ such that $y \neq x$, there exists a τ_1-semi-p-open set U containing x such that $y \notin \tau_2$-semi-pclU.

3- For each $x \in X$, $\{x\} = \cap \{\tau_2$-semi-pclU: $x \in U$ and U is τ_1-semi-p-open set}.

4- The diagonal $\Delta = \{(x,x); x \in X\}$ is a semi-p-closed subset of $(X \times X, \tau_{X\times X})$.

Proof: $(1) \Rightarrow (2)$

Let $x \in X$, be given and consider $y \in X$ such that $y \neq x$, since (X,τ_1,τ_2) is pairwise semi-p-T_2- space, there exists τ_1-semi-p-open set U and τ_2-semi-p-open set V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$. Hence $y \notin \tau_2$-semi-pclU, since we have a semi-p-open set V such that $y \in V$, but $U \cap V = \emptyset$.

$(2) \Rightarrow (3)$

Suppose that there exists $x \neq y$ in X, such that $y \in \cap \{\tau_2$-semi-pclU; $x \in U$ and U is τ_1-semi-p-open set}; implies $y \in \tau_2$-semi-pclU; $x \in U$ for all τ_1-semi-p-open set U, which is a contradiction, thus for each $x \in X$, $\{x\} = \cap \{\tau_2$-semi-pclU: $x \in U$ and U is τ_1-semi-p-open set}.

$(3) \Rightarrow (4)$

To prove $\Delta = \{(x,x); x \in X\}$ is a semi-p-closed subset of $(X \times X, \tau_{X\times X})$, that is mean we must prove $X \times X \backslash \Delta$ is semi-p-open subset of $(X \times X, \tau_{X\times X})$.

Let $(x,y) \in X \times X \backslash \Delta$, which implies that $x \neq y$. In view of (3), there exists a τ_1-semi-p-open set U containing x and $y \notin \tau_2$-semi-pclU.

We know that $U \cap (X \setminus \tau_2\text{-semi-pcl}U) = \emptyset$. Also, we have $y \in (X \setminus \tau_2\text{-semi-pcl}U)$. So $(x, y) \in U \times (X \setminus \tau_2\text{-semi-pcl}\,U) \subseteq X \times X \setminus \Delta$. But $U \cap (X \setminus \tau_2\text{-semi-pcl}U)$ is a $\tau_{X \times X}$- semi-p-open set, so $X \times X \setminus \Delta$ is a $\tau_{X \times X}$-semi-p-nbd of each of its points. Thus Δ is $\tau_{X \times X}$-semi-p-closed set.

$(4) \Rightarrow (1)$

Let $x \neq y$ in X, hence $(x, y) \in X \times X \setminus \Delta$. Since Δ is $\tau_{X \times X}$-semi-p-closed set, $X \times X \setminus \Delta$ is a semi-p-nbd of each of it is points. Therefore, there exists a $\tau_{X \times X}$-semi-p-open set $U \times V$ containing (x, y) and contained in $X \times X \setminus \Delta$. then U is τ_1-semi-p-open set and V is τ_2-semi-p-open set, also $x \in U$ and $y \in V$, since $U \times V \subseteq X \times X \setminus \Delta$, $U \cap V = \emptyset$. Thus (X, τ_1, τ_2) is pairwise semi-p-T_2- space. ■

Definition 3.14:

A space (X, τ_1, τ_2) is said to be *pairwise semi-p-regular- space*, if for each τ_i-closed set F and for each point $x \notin F$, there exist τ_i- semi-p-open set U and τ_j- semi-p-open set V such that $x \in U, F \subseteq V$ and $U \cap V = \emptyset$, where i, j=1, 2 , $i \neq j$.

Proposition 3.15:

Every pairwise regular space (X, τ_1, τ_2) is pairwise semi-p-regular- space.

Proof:

Let F be any τ_i-closed set and let $x \in X$, such that $x \notin F$, since (X, τ_1, τ_2) is pairwise regular space, there exist τ_i- open set U and τ_j- open set V such that $x \in U, F \subseteq V$ and $U \cap V = \emptyset$.

And from (Proposition 2.5 part (1)), we have τ_i- semi-p-open set U and τ_j- semi-p-open set V such that $x \in U, F \subseteq V$ and $U \cap V = \emptyset$. Hence (X, τ_1, τ_2) is pairwise semi-p-regular- space. ■

Remark 3.16:

The converse of (Proposition 3.15) is not true in general, as the following example shows:

Let X={1, 2, 3}, $\tau_1 = \{\emptyset, X, \{1,2\}\}, \tau_2 = \{\emptyset, X, \{1,3\}\}$, then

S-P(X, τ_1)= { $\{\emptyset, X, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{2,3\}\}$,

S-P(X, τ_2)= { $\{\emptyset, X, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{3,3\}\}$. Then X is pairwise semi-p-regular- space, but not pairwise regular space since {3} is closed set in τ_1 and $1 \notin \{3\}$, but for any τ_1-open set containing 1 and for any τ_2-open set containing {3}, its intersection is not empty.

Theorem 3.17:

A space (X, τ_1, τ_2) is pairwise semi-p-regular- space if and only if for each point x in X and every τ_i- closed set F not containing x there is a τ_i- semi-p-open set U such that $x \in U$ and $(\tau_j\ \ \text{semi}\ \ \text{pcl}U) \cap F = \emptyset$.

Proof:

Suppose (X, τ_1, τ_2) is pairwise semi-p-regular- space, let $x \in X$ and F is any τ_i- closed set such that $x \notin F$, implies $X \setminus F$ is τ_i-open set containing x and since (X, τ_1, τ_2) is pairwise semi-p-regular- space, there is a τ_i- semi-p-open set U such that $x \in U \subset \tau_j - semi - pclU \subset X \setminus F$. Hence $(\tau_j - semi - pclU) \cap F = \emptyset$.

Conversely, let F be any τ_i- closed set and $x \notin F$, then there exists a τ_i- semi-p-open set U such that $x \in U$ and $(\tau_j - semi - pclU) \cap F = \emptyset$.

Let V=$X \setminus (\tau_j - semi - pclU)$, then V is τ_j-semi-p-open set such that $F \subseteq V, x \in U$ and $U \cap V = \emptyset$, thus (X, τ_1, τ_2) is pairwise semi-p-regular- space. ■

Definition 3.18:

A space (X, τ_1, τ_2) is said to be *pairwise semi-p-normal- space*, if for each τ_i-closed set A and τ_j- closed set B disjoint from A, there exist τ_j- semi-p-open set U and τ_i- semi-p-open set V such that $A \subseteq U, B \subseteq V$ and $U \cap V = \emptyset$, where i, j=1, 2 , $i \neq j$.

Proposition 3.19:

Every pairwise normal space (X, τ_1, τ_2) is pairwise semi-p-normal- space.

Proof:

Let A, B be two closed disjoint sets in $\tau_i, \tau_j; i, j = 1, 2$ (respectively), since X is pairwise normal space, there exist τ_j- open set U and τ_i- open set V such that $A \subseteq U, B \subseteq V$ and $U \cap V = \emptyset$, but from (Proposition 2.4 part (1)) U, V semi-p-open sets which contains A and B respectively. Thus (X, τ_1, τ_2) is pairwise semi-p-normal- space. ■

Remark 3.20:

The converse of Proposition 3.19 is not true in general, as the following example shows:
Consider example 2, where:

X={1, 2, 3}, $\tau_1 = \{\emptyset, X, \{1,2\}\}, \tau_2 = \{\emptyset, X, \{1,3\}\}$,

S-P(X, τ_1)= { $\{\emptyset, X, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{2,3\}\}$,

S-P(X, τ_2)= { $\{\emptyset, X, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$.

Then (X, τ_1, τ_2) is pairwise semi-p-normal- space, but not pairwise normal space, since {3} and {2} are closed disjoint sets in τ_1 and τ_2 respectively but for any open set in τ_2 which containing {3} and any open set in τ_1 which containing {2}, its intersection is not empty.

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حول بديهيات الفصل شبه – P – على الفضاءات التبولوجية الثنائية

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الخلاصة

في هذا البحث قمنا بتعريف نوع جديد من بديهيات الفصل على الفضاءات التبولوجية الثنائية التي اسميناها بديهيات الفصل شبه – P ، كذلك درسنا بعض خواص هذه الفضاءات وعلاقات كل نوع مع بديهيات الفصل الاعتيادية في الفضاءات التبولوجية الثنائية.

الكلمات المفتاحية: الفضاء التبولوجي الثنائي، الفضاء شبه –p – T_0 ، الفضاء شبه –p – T_1 ، الفضاء شبه –p – T_2 ، الفضاء شبه –p – القياسي، الفضاء شبه –p – الاعتيادي.