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Comparison of Euler and Euler-Cromer Numerical Methods for Undamped and Damped Spring Oscillation

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ABSTRACT

This study aimed to numerically analyze damped and undamped oscillations of a spring using the Euler and Euler-Cromer methods via Spreadsheet software. The varied parameters in this study were the damping constant, namely 0.1 (damped) and 0.0 (undamped). Various quantities analyzed in this study were position (x), velocity (v), kinetic energy (K), potential energy (U), mechanical energy (E), and phase space as a function of time (t). Iteration was done in t < 60-time steps (seconds). The results of this study indicated that when the spring experiences damping, the numerical results of x, v, K, U, E, and the phase space decrease periodically to zero due to the damping force, both for the Euler and Euler-Cromer methods. Meanwhile, for the undamped spring (zero damping constant), there was a difference in the results for the Euler and Euler-Cromer methods. For the Euler method, the resulting values of x, v, K, U, E, and the phase space increased periodically with time, which was not following the actual situation. According to the simple harmonic oscillation, the Euler-Cromer method values of x, v, K, U, E, and the phase space were stable over time.

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I. Introduction

Science and technology, especially in the field of electronics, have rapidly progressed. This has resulted in the rapid development of computational physics [1], which examines physics problems based on numerical and/or simulation studies. Computational physics can provide accurate calculations of various physical phenomena to serve as a visualization tool [2] or simulation of physical phenomena [3]. Computational physics can also provide solutions that can be compared with analytical results.

One of the uses of computational physics is solving differential equations using computer software, e.g., Spreadsheet. Differential equations are a part of mathematics, especially calculus, which has a very important role in physics. Various laws of physics can be expressed in the form of differential equations, including Newton's laws [4], Maxwell's equations [5], and the first law of thermodynamics [6]. These various differential equations need to be solved to obtain physical information. In the process, only a small number of differential equations have analytical solutions, especially linear differential equations that have been idealized. Most differential equations that describe physical phenomena are non-linear and difficult to solve analytically. This is where one of the strengths of computational physics, i.e., computational physics offers numerical solutions to various differential equations that are difficult to solve, such as solving atoms with more than two electrons using the configuration weight functions method [7] and density functional theory [8].

Vibration is the movement of a system that can be in the form of regular and repeated movements continuously or can also be irregular or random movements. When vibration or oscillation repeats itself (back and forth motion) on the same trajectory, then the motion is called periodic [9]. In this study, the damped and undamped oscillations are numerically studied. A study on damped oscillations has been carried out [10] that analyzed the oscillations of vertical springs in fluids using Tracker software (https://physlets.org/tracker/). In addition, damped harmonic motion using high-speed video has been studied [11]. A computational analysis of damped harmonic motion on a vertical spring using a Spreadsheet is performed [12]. However, the study is still limited to comparing positions (functions of time) for analytical and numerical results, comparison of damping parameters and spring constants.

The numerical method used in this study is the Euler and Euler-Cromer methods. These two methods have been used to discuss the physical properties of damped oscillators [12,13]. However, there has been no further study on the mechanical energy of oscillations and phase space, especially for damped and undamped oscillations. Therefore, this study is aimed to analyze the damped and undamped oscillations of a spring using the Euler and Euler-Cromer methods.

Meanwhile, as mentioned above, the software used is Spreadsheet as used in [12,14]. The main advantages of using the Spreadsheet are ease in data handling and flexible in data presentation. Moreover, the various quantities being studied are position (x), velocity (v), kinetic energy (K), potential energy (U), mechanical energy (E), and phase space as a function of time (t). These quantities are essential in the mechanics of the spring system. As mentioned above, the aforementioned quantities are determined using the Euler and Euler-Cromer methods and finally compared.

II. Theory

A Brief Description of a Spring System

One law that has been successfully revealed by physics is that when energy is transferred or changed through any process, there is no increase (gain) or reduction (loss) of energy in that process. This is known as the law of conservation of energy.

For the spring system, the potential energy is given as:

$$U = \frac{1}{2}kx^2 \tag{1}$$

Where x is the spring's displacement from its normal or unstretched length, and k is the spring's constant. The kinetic energy of the spring can be given as follows:

$$UK = \frac{1}{2}mv^2 \tag{2}$$

Where m is the mass of the object and v is the object's velocity. If there is no friction or other forces acting on this system, then the principle of energy conservation produces the following equation:

$$U_1 + K_1 = U_2 + K_2 \tag{3}$$

The subscripts 1 and 2 refer to the potential and kinetic energies at two points at different times.

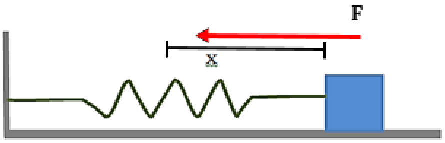

Figure 1. Oscillation of a spring.

When an object vibrates or oscillates in the same trajectory, and each oscillation takes the same time, then the motion is periodic. Suppose a spring is attached horizontally, as shown in Figure 1, with an object of mass m sliding frictionless on a horizontal surface. The spring has an initial length where the spring does not exert a force on the object. The position of the object at this point is called the equilibrium position. If the object is pushed to the left, then the spring is compressed.

On the other hand, if the spring is pulled to the right, then the spring is stretched. The spring exerts a force on the object in the opposite direction by pushing or pulling it to its equilibrium position. Therefore, this force is called the restoring force, F (red arrow in Figure 1). An oscillatory system with a restoring force directly proportional to the negative displacement can be given mathematically as:

$$F = -kx \tag{4}$$

This oscillatory system is said to perform simple harmonic motion (SHM).

SHM can also be modified by generating other forces that affect the system's motion, namely damping forces. For example, a study in [15] shows the phenomenon of damped oscillations caused by a wooden rod given a magnet swinging near an aluminum rod. The damping is a function of the distance between the magnet and the aluminium that can produce underdamped, overdamped, and critically damped oscillations.

This study investigates the phenomenon of damped and undamped oscillations of a horizontal spring (see Figure 1). To stretch or compress a spring, work must be done. Thus, the potential energy is stored in a stretched or compressed spring. If the damping force is directly proportional to the velocity, i.e., $F' = -bv$, then using Newton's second law and equation (4), the differential equation for the spring system can be written as follows:

$$\ddot{x} = -\omega_0^2 x - \frac{b}{m}\dot{x} \tag{5}$$

and

$$\omega_0 = \sqrt{\frac{k}{m}}. \tag{6}$$

The damping parameter may be stated as $\beta = b/(2m)$. Therefore, the damped oscillatory system has a frequency of

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$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} = \sqrt{\omega_0^2 - \beta^2} \tag{7}$$

for the underdamped condition where $\beta < \omega_0$. In this study, equation (5) is the differential equation used to model the spring system using the numerical methods of Euler and Euler-Cromer.

Euler and Euler – Cromer Methods

Euler's method is utilized for solving differential equations by utilizing the description of the Taylor series. The initial step of deriving this method starts from the explicit relationship,

$$\frac{dy}{dx} = f(x, y) \tag{8}$$

Equation (8) can be approximated using the finite difference form, i.e.:

$$\frac{dy}{dx} = \frac{\Delta y}{\Delta x} = \frac{y_{i+1} - y_i}{x_{i+1} - x_i} = f(x, y) \tag{9}$$

By denoting $\Phi = f(x,y)$, equation (9) can be written as:

$$y_{i+1} = y_i + \Phi_i \Delta x \tag{10}$$

The value of Φ is an estimate of the gradient for the extrapolation from y_i to y_{i+1} with a distance of $\Delta x = x_{i+1} - x_i$ with $i = 1, 2, 3, \ldots, n$. Equation (10) can calculate the y step by step with linear extrapolation on Δx grid. However, the Euler method has a disadvantage, which is its error is relatively high, i.e., more than 60%, so that the numerical results may not be correct. Hence, the Euler method can be modified to the Euler-Cromer method via substituting Φ_i to Φ_{i+1} in equation (10), that is:

$$y_{i+1} = y_i + \Phi_{i+1} \Delta x \tag{11}$$

The Model

The physical model of the spring is based on the differential equation (5). Substituting y and x for v and t, respectively, in equation (10), we obtain

$$v_{i+i} = v_i + \Phi_i (t_{i+1} - t_i) \tag{12}$$

Comparing equation (5) and equation (12), an Euler numerical equation is obtained as

$$v_{i+i} = v_i + \left(-\frac{k}{m} x_i - \frac{b}{m} v_i\right)(t_{i+1} - t_i). \tag{13}$$

Another equation is obtained by substituting y and x in equation (10) with x and t, respectively.

$$x_{i+1} = x_i + v_i (t_{i+1} - t_i). \tag{14}$$

Equations (13) and (14) are a pair of numerical equations based on the Euler method used in this study. A modification of the Euler method results in the Euler-Cromer method by modifying equation (14) according to equation (11), namely:

$$x_{i+1} = x_i + v_{i+1} (t_{i+1} - t_i). \tag{15}$$

Equations (14) and (15) are a pair of equations for the Euler-Cromer method.

The kinetic energy of the spring system of equation (2) can be modified according to equation (13), i.e. :

$$K_i = \tfrac{1}{2} m v_i^2. \tag{16}$$

Similarly, the potential energy of the spring can be determined from equations (14) and (15) for the Euler and Euler-Cromer methods, respectively, viz.:

$$U_i = \tfrac{1}{2} k x_i^2. \tag{17}$$

Hence, the mechanical energy of the spring system can be obtained by adding equations (16) and (17), that is:

$$E_i = K_i + U_i. \tag{18}$$

Finally, the momentum of the spring system can be calculated using equation (13), i.e.:

$$p_i = m v_i. \tag{19}$$

Equation (19) is used to generate the phase space.

III. Computational Method

This numerical study was conducted using a personal computer (PC) with the hardware of Dell Desktop-FAODG3T and the Inspiron 1.1 3000 Series model. The processor used was Intel(R) CPU N3&10@1.60HZ with 400 GB RAM. Meanwhile, various software used in this numerical study were MS Word and MS Excel. The coding was based on the Spreadsheet using the Euler-Cromer and Euler methods.

The coding steps for the Euler-Cromer method were given as follows: 1) opening the worksheet in Spreadsheet (MS Excel); 2) declaring the known parameters, namely: m, k, b, and t in cells (boxes) B22 to B25 as can be seen in Figure 2; 3) creating a table consisting of 8 columns containing iterations (i), t, x, v, K, U, E, and p, as can be observed in Figure 2. Especially for iteration $i = i + 1$ starts from 1 in cell B32. 4) Declaring the initial conditions, namely $t = 0$, $x = 0$, and $v = 1$, in cells C31, D31, and E31, respectively. The time step $\Delta t = t_{i+1} - t_i$ used is 0.01 with the number of iterations $t < 60$-time steps (seconds). 5) Writing the formula for iteration of v_i based on equation (13) starting from cell E33, namely: = E32 + ((((-B$23/B$22)*D32) - ((B$24/B$22)*E32))*(C33 - C32));

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6) writing the formula for iteration of x_i based on equation (15) starting from cell D33, namely: =D32+(E33*(C33-C32)); 7) writing the formula for the iteration of K_i based on equation (16); 8) writing the formula for the iteration of U_i based on equation (17); 9) writing a formula for the iteration of E_i based on equation (18); 10) writing the formula for the iteration of p_i based on equation (19). Moreover, 11) plotting the graphs of v against t, x against t, K and U against t, K and E against t, and p against x (for the phase space). Finally, 12) for undamped oscillation, steps 1) to 11) were carried out by setting the parameter b = 0.

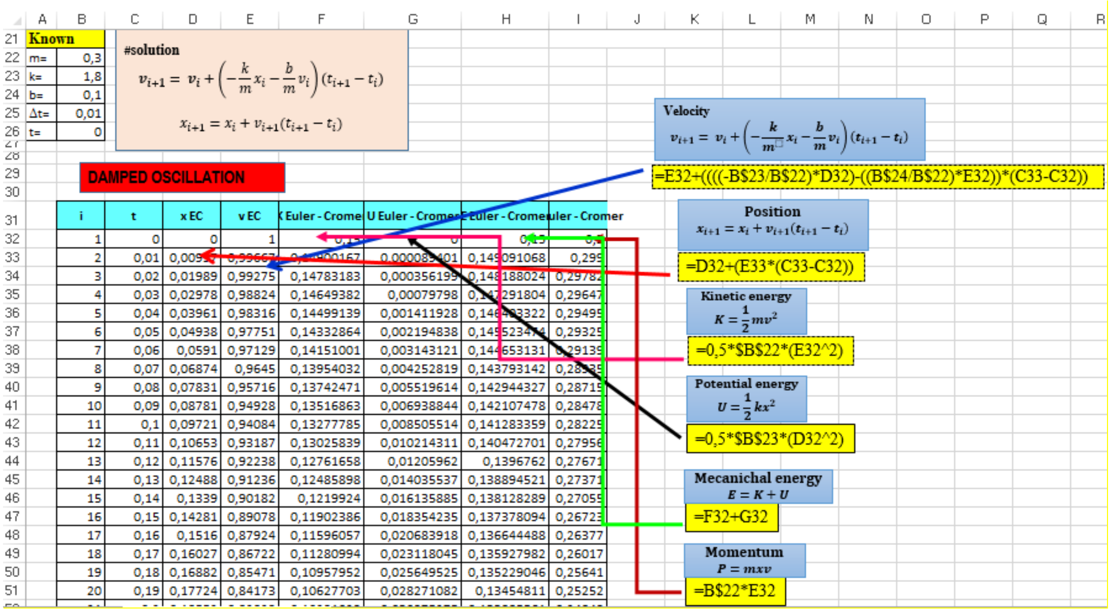

Figure 2. Display parameters and formulas on the Spreadsheet using the Euler-Cromer method.

The coding steps for the Euler method were carried out by repeating steps 1) to 12) in the Euler-Cromer method in the cells that have been determined. For step 5) the formula used was =E27+((((-B$16/B$15)*D27)-((B$17/B$15)*E27))*(C28-C27)). Meanwhile, the formula in step 6) is changed to equation (14), namely: =D27+(E27*(C28-C27)). This can be seen in Figure 3

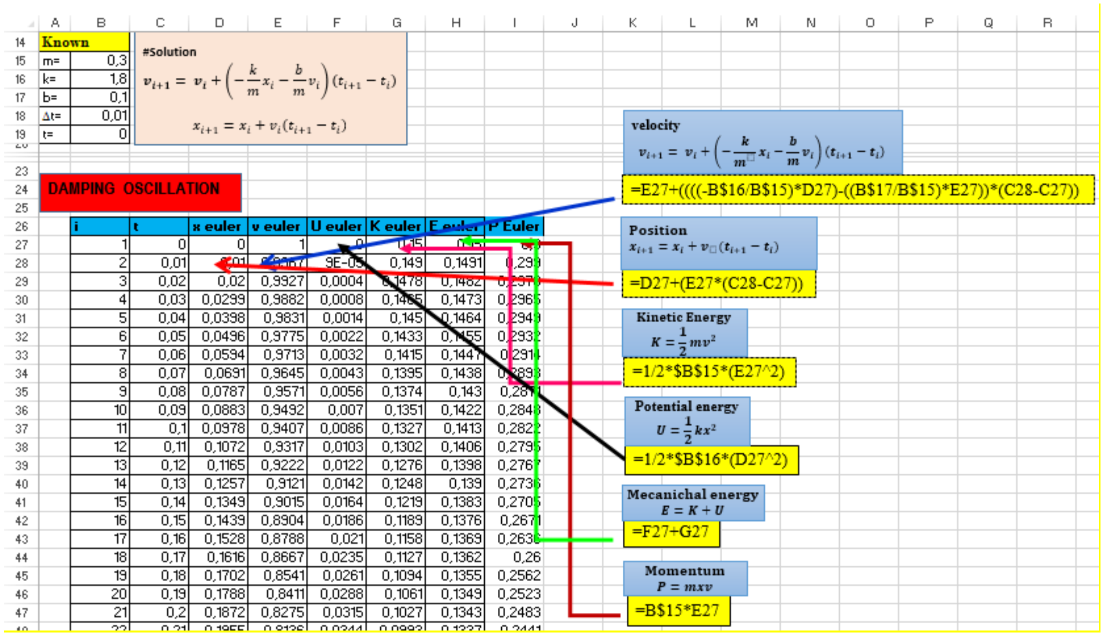

Figure 3. Display parameters and formulas on a Spreadsheet using the Euler method.

IV. Results and Discussion

The numerical analysis of damped and undamped oscillations of the spring in this study uses two numerical methods: the Euler-Cromer and Euler methods. Furthermore, the various numerical results generated from these two methods are discussed. The first result obtained in this study is a graph of position versus time (x vs. t). This can be observed in Figure 4.

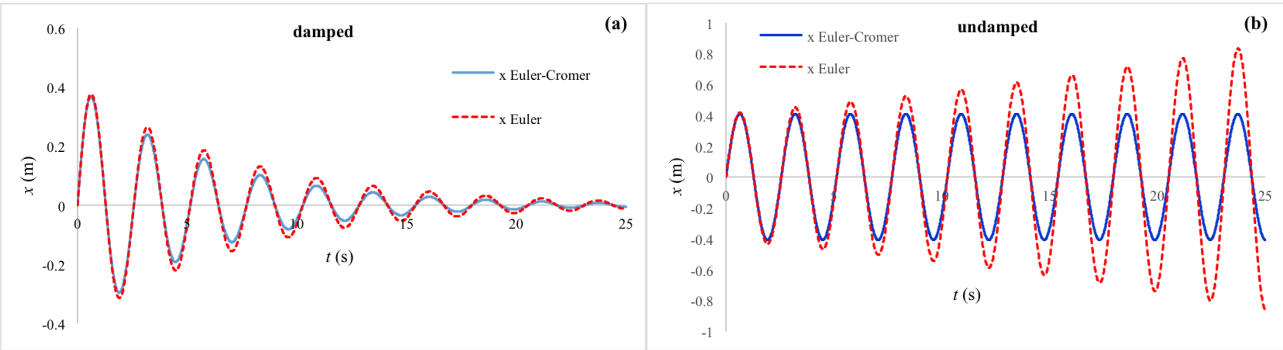

Figure 4. Position the spring concerning time (a) with damping and (b) without damping.

Figure 4(a) shows a comparison of the spring's position as a function of time resulting from the Euler method and Euler-Cromer methods for the damped oscillation. As time advances, the amplitude of the spring gets smaller. This means that the spring's displacement is getting smaller and will become zero at the equilibrium position. Thus, the spring undergoes a damped oscillation to decrease the spring's oscillation. The damping may be caused by, e.g., the friction of the spring at the interface as applied in [16] or viscous fluid in the liquid-spring-magnetorheological-fluid-damper system [17]. Furthermore, the Euler-Cromer method produces a smaller oscillation amplitude than the Euler method. On the other hand, the same oscillation phase is obtained between Euler and Euler-Cromer methods, which means that both methods produce the same damping evolution.

Figure 4(b) shows the graph of the spring's position for the Euler and Euler-Cromer methods in the undamped condition, which is obtained by changing the parameter b to 0. It can be observed that the Euler method gives the position of the spring, which the amplitude gets larger as time progresses. Of course, this does not satisfy the law of conservation of energy. In contrast, the Euler-Cromer method produces stable amplitude, which is according to the SHM system without damping. Thus, the Euler-Cromer method is more stable than the Euler method in the spring vibrations without damping.

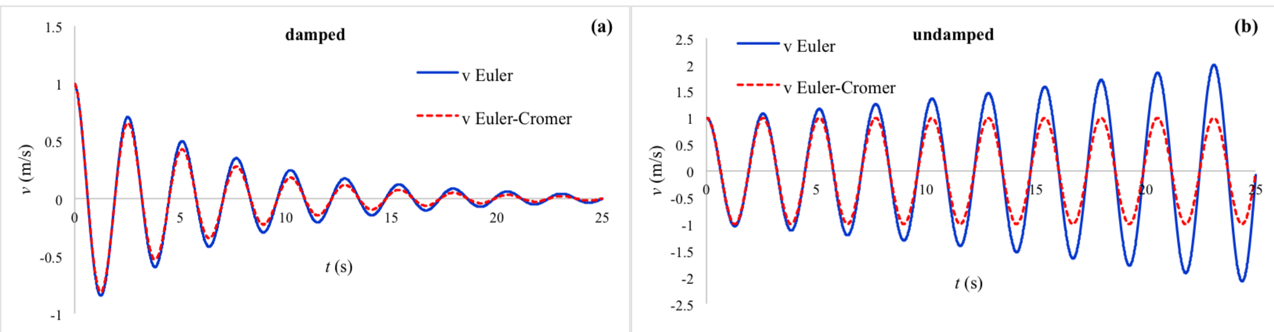

Figure 5. The spring's velocity concerning time (a) with damping and (b) without damping.

Figures 5(a) and 5(b) show v vs. t graphs obtained from two methods, namely: Euler and Euler-Cromer methods, for the damped and undamped oscillations, respectively. In general, the profiles of the velocity of the spring oscillation are not different from the position profile of the spring in Figure 4. It can be observed that the velocity of the oscillation of the spring with damping decreases and eventually becomes zero at the equilibrium position, both for the Euler and Euler-Cromer methods. Meanwhile, the spring's oscillation velocity without damping is stable for the Euler-Cromer method compared to the velocity of the spring's oscillation obtained from the Euler method.

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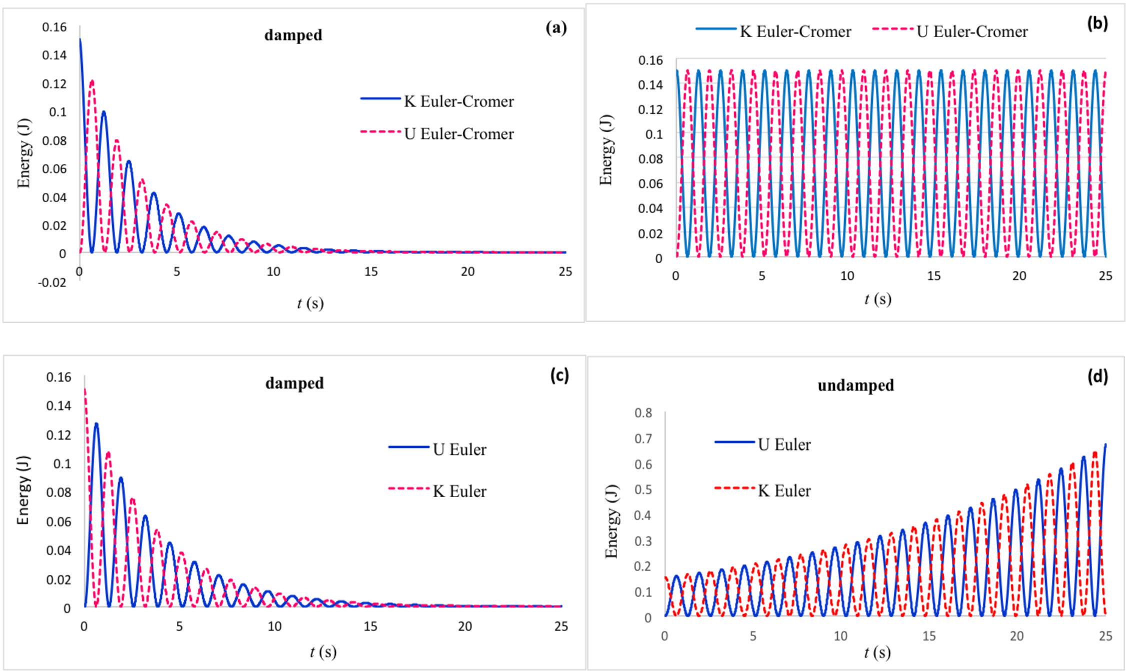

Figure 6. The kinetic and potential energies for damped spring oscillation using (a) Euler-Cromer and (c) Euler methods, and undamped using (b) Euler-Cromer and (d) Euler methods.

It can be shown from Figures 6(a) and (c) that for the damped springs, *K* and *U* profiles decrease with time for both Euler-Cromer and Euler methods. In this case, the law of energy conservation applies: when *K* is maximum, then *U* is minimum, and vice versa. Furthermore, the *K* and *U* profiles obtained through the Euler-Cromer method undergo damping faster than the Euler method. Furthermore, Figures 6(b) and (d) show the *K* and *U* profiles for undamped spring oscillation obtained from the Euler-Cromer and Euler methods. Via the Euler-Cromer method, *K* and *U* profiles are stable over time. Meanwhile, in Figure 6(d) *K* and *U* profiles are getting larger with time.

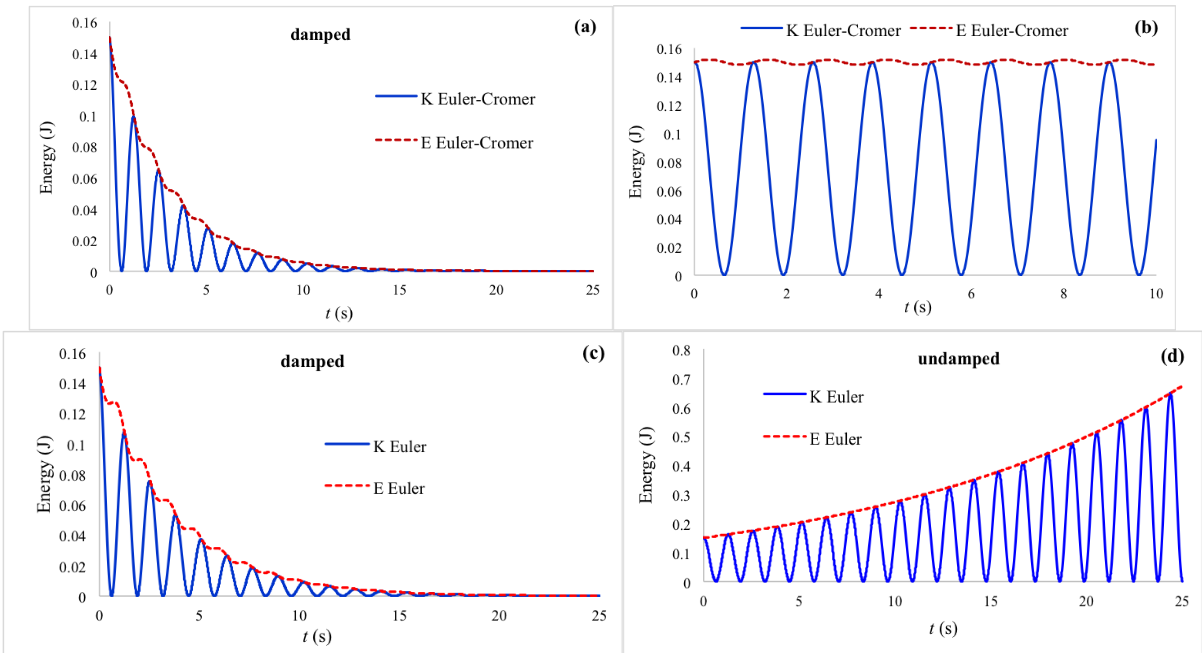

Figure 7. Kinetic and mechanical energies for damped spring oscillation using (a) Euler-Cromer and (c) Euler methods and undamped using (b) Euler-Cromer and (d) Euler methods.

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Based on Figures 7(a) and (c), a relationship is found between the velocity and the total loss of mechanical energy in damped spring oscillation using the Euler-Cromer and Euler methods, respectively. With the increase of time, the value of the velocity of the oscillation gets smaller. This causes the mechanical energy produced to dissipate and eventually go to zero. This phenomenon of mechanical energy is a physical phenomenon and not due to computational calculations. The area with the highest change in the mechanical energy occurs when the slope is greatest on the graph. Figures 7(b) illustrate the relationship between velocity and the mechanical energy of the undamped spring via the Euler-Cromer method. It can be observed that the oscillation velocity of the spring appears to be stable, while the mechanical energy of the system is also stable and shows the ripple effect. This is different from Figure 7(d) obtained using Euler's method. In this case, it is obtained that the spring's velocity increases periodically with time, while the mechanical energy also increases but appears smoother without any ripples.

The phase space of the spring oscillation can be observed in Figure 8. The phase space is the *p* vs. *x* graph. Figures 8(a) and (c) show the damped spring oscillation phase space generated by the Euler-Cromer and Euler methods, respectively. It can be observed that the spiral profiles move from the outside to the inside and are getting closer to the zero point value (origin). This means that the momentum and position of the damped spring oscillation decrease towards zero. The greater the damping parameter, the less number of spirals that occur because the faster the spring is damped. In addition, it can be observed that the damped spring oscillation phase spaces from the Euler-Cromer [Fig. 8(a)] and Euler [Fig. 8(b)] methods produce similar profiles.

However, the phase spaces for the spring's oscillation without damping show different profiles for the Euler-Cromer and Euler methods. This can be observed in Figures 8(b) and 8(d). Figure 8(b) is the phase space of the spring oscillation without damping from the Euler-Cromer method. It can be observed that this phase space is circular. This is because the phase space is stable, so there is no spiral, according to SHM. On the other hand, Figure 8(d) shows the phase space of a spring oscillation without damping based on Euler's method resulting in a spiral that is directed outward and gets larger with time. This is unrealistic because the spring dynamics by itself (without force) can't produce larger values of momentum and position.

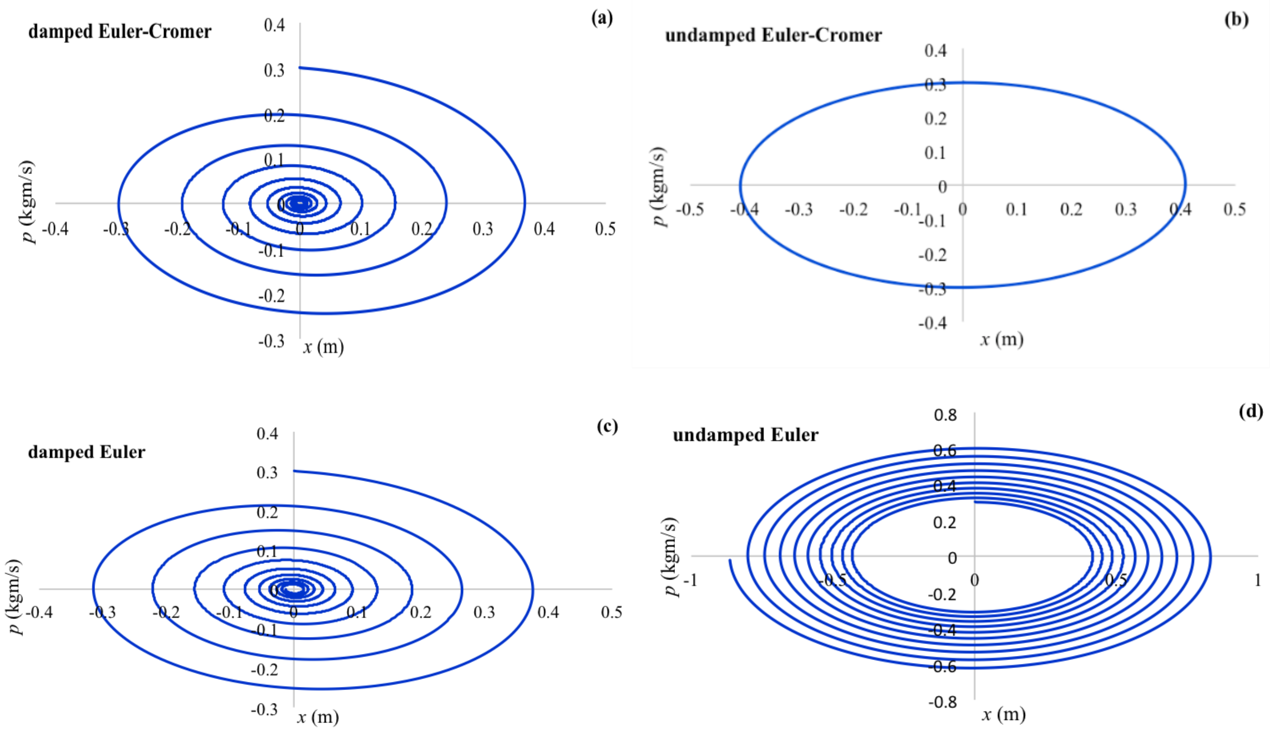

Figure 8. The phase spaces for damped spring oscillations using (a) Euler-Cromer and (c) Euler methods, and undamped using (b) Euler-Cromer and (d) Euler methods.

The numerical results above show that the Euler-Cromer and Euler methods work equally well in the case of damped spring oscillations. This is due to the damping factor *b*, whose value is not zero so that the Hamiltonians of the spring system via Euler-Cromer and Euler methods obey the principle of conservation of energy.

However, the performance of these two computational methods is different when faced with the

Hamiltonian case, which is independent of time, namely, in this case, the vibration without damping ($b = 0$). The two methods have different performances due to their numerical structure. Especially Euler's method cannot maintain the principle of conservation of energy. In a spring without damping, the structure of Euler's numerical equation causes the oscillation to get larger. This results in all the quantities becoming increasingly large and unrealistic. On the other hand, changing the parameter v_i in equation (14) to v_{i+1} in equation (15) causes the oscillation of the harmonic motion to become stable, and the principle of conservation of energy is fulfilled. Thus, the Euler-Cromer method produces quantities that are more stable and idealistic.

V. Conclusion

A numerical study of the spring oscillation with damping and without damping has been carried out. The numerical methods used in this study are Euler and Euler-Cromer. The quantities of x, v, K, U, E, and phase space produce the same profiles for the Euler and Euler-Cromer methods for the spring oscillation with damping. Moreover, the numerical results of *x*, *v*, *K*, *U*, *E*, and the phase space decrease periodically to zero due to the damping force. Meanwhile, when the spring does not experience damping, there is a difference in numerical results from the Euler and Euler-Cromer methods. For the Euler method, the resulting profiles of *x*, *v*, *K*, *U*, *E*, and phase space increase periodically with time, not matching the ideal condition. Then, on the profiles of x, v, K, U, E, and phase space from the Euler-Cromer method, stable profiles are obtained with time, according to the ideal condition of SHM. This means that the Euler method only works well for the spring oscillation with damping, whereas the Euler-Cromer method is good for spring oscillation with and without damping. Hence, we recommend using this study's appropriate computational method for the spring oscillation. Moreover, this study can be used as a part of physics learning for senior high school and undergraduate students.

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